DISCONTINUOUS ADOPTION PATHS WITH DYNAMIC SCALE ECONOMIES

by

Agliardi Elettra

Department of Applied Economics, University of Cambridge,

and Churchill College, Cambridge (U.K.)

September 1991

ABSTRACT

This paper examines the equilibrium adoption path for two innovations when there are "network externalities". We show that the existence of significant increasing returns in system-scale can give the result that one variant will drive out the other one and so emerges as the unique standard for the industry. Moreover, if "network externalities" are sufficiently strong, then the equilibrium adoption path is discontinuous, that is, it includes a catastrophe point.

1. INTRODUCTION

The possibility that agents may react discontinuously to continuous changes in their environments does not seem to have been sufficiently investigated in the literature. Intuition suggests that continuously changing causes should produce continuous effects.

The occurrence of discontinuities in smoothly evolving systems has come to be recognized as a not unlikely event with catastrophe theory. One of the purposes of this paper is to show that "catastrophes" can arise in a simple model of the adoption of innovations with increasing returns to adoption.

The focus of the paper is on the equilibrium adoption path for two innovations. In the presence of more than one innovation the decision of adopters is not only when to adopt but also which innovation to adopt. We consider this problem when there are "network externalities", that is, when one agent's benefit from adoption increases with the number of other adopters. Typical examples of "network externalities" include communication networks (e.g. fax machines) and technology standards.

These examples describe the two forms of externalities which arise typically. The first form of externality impacts immediately upon the technical performance of a particular system, and feeds back to affect the cost and profits of other component suppliers through the influence of the system's performance characteristics upon users' demand for it in comparison with alternative technologies. The second form operates directly upon the demand side of the market, where the benefits derived by users increase with the number of others whose decision to use compatible products enlarges the coverage of the network. For both sets of circumstances, there have been several models in recent years (Dybvig and Spatt,1983; Farrell and Saloner,1985; Katz and Shapiro, 1986a,1986b). However, these are static models, in which the core problem is seen to be the fact that markets are likely to work poorly as mechanisms for quickly achieving the degree of compatibility or standardization required to maximize the benefits obtainable with an already existing network technology. A common theme of these approaches is that "network externalities" likely lead to multiple equilibria, some of which Pareto dominate others.

In this paper we put forward a dynamic formulation: things take on a very different appearance when one turns to consider the dynamics of market rivalries among alternative variants of a network technology. We assume that there is a continuum of heterogeneous potential adopters and that benefits from adoption depend positively on the measure of adoption. First, we show that the existence of significant increasing returns in system-scale can give the result that one variant will drive out the other one and so emerge as the de facto standard for the industry, a result which is in keeping with Arthur (1989). Second, but as the main innovation of this paper, we provide a qualitative characterization of the equilibrium adoption path. We show that if increasing returns to adoption, in the form of network externalities, are sufficiently strong, then the equilibrium adoption path is discontinuous (i.e., it includes a catastrophe point). This is in sharp contrast with the case of diffusion without increasing returns to adoption. In the latter, indeed, if the basic functions of the model are smooth, also the equilibrium adoption path is smooth and therefore continuous.

This paper is organized as follows. Section 2 presents the model and proves the existence of an equilibrium adoption path for two innovations. Section 3 discusses the dynamics of the sequential process of innovation choice and Section 4 shows under which circumstances the equilibrium adoption path is discontinuous. Section 5 contains some final remarks.

2. THE MODEL

We consider two innovations, α and β, which have to be adopted by a continuum of potential adopters. At a particular point in time t, each agent has to decide whether to adopt or not, and which innovation to adopt. The choice between α and β is irreversible. We assume that such decision is influenced by consideration of the magnitude of the net externalities, which are positively correlated with the proportion of adoptions: there are benefits from compatibility. This can occur either in the case of direct physical or communications "network externalities", that is, the case where one consumer's value for a good increases when another consumer has a compatible good (as in the case of telephones or personal computer software); or in the case of market-mediated externalities, that is, a complementary good (servicing, software, spare parts, etc.) becomes cheaper and more readily available the greater the extent of the compatible market.

Firms are heterogeneous with respect to the effectiveness of the innovation on their profits. Let v_α be a parameter measuring the inherent net benefit that a firm would derive from selecting α rather than β, and v_β be the analogous parameter for β. Define $v = v_\alpha - v_\beta$. The higher and positive is v, the more preferred α is; the lower and negative is v, the more preferred β is. We assume that each agent is characterized by a parameter v, and that v is distributed according to the function F(v), which is smooth, has a finite and convex support (v^l, v^h), where $F(v^l) = 0$, $F(v^h) = 1$, and is increasing in v.

Let $\Pi_\alpha(v_\alpha, x_\alpha, t)$ and $\Pi_\beta(v_\beta, x_\beta, t)$ be the net profit flow obtained upon adoption at time t of innovation α and β, where x_α and x_β denote the measure of adoptions of α and β, $0 \leq x_\alpha + x_\beta \leq 1$. Let us make the following assumption:

Assumption.

(1) $\Pi_\alpha(v_\alpha, x_\alpha, t) = v_\alpha + f(x_\alpha, t)$

(2) $\Pi_\beta(v_\beta, x_\beta, t) = v_\beta + g(x_\beta, t)$

f(.) and g(.) are smooth and $\partial\Pi_i/\partial v_i > 0$, $\partial\Pi_i/\partial x_i > 0$, $\partial\Pi_i/\partial t > 0$, $i = \alpha, \beta$.

Expressions (1) and (2) capture the fact that the net benefits from adopting an innovation are both "inherent" benefits and "system-use" benefits. That is, the effectiveness of an innovation is measured by the parameter v_i and by the benefits deriving from the network externalities. The additive formulation in expressions (1) and (2) is a simplifying one , although not crucial, and is in keeping with David (1987). The sign of the derivatives have a straightforward interpretation: the net profit obtained upon adoption increases the larger the inherent net benefit, and the larger the measure of adoptions of that innovation. The assumption $\partial\Pi_i/\partial t > 0$ implies that there exists learning: over time, knowledge about innovation i increases and the benefits upon adoption increase as well. This is a crucial assumption in this model. It is a reasonable assumption if we consider the case of "unsponsored" innovations, like in Arthur (1989), that is, innovations which do not compete strategically, so that they cannot be priced and manipulated.

Let $P = \Pi_\alpha - \Pi_\beta$, which under our specification of Π_α and Π_β becomes:

(3) $$P(v, x_\alpha, x_\beta, t) = v + f(x_\alpha, t) - g(x_\beta, t)$$

If $P > 0$ at t, then innovation α will be chosen, while if $P < 0$ then innovation β will be chosen. $P = 0$ is the case of indifference between α and β. Since by assumption benefits at time t depend only on the measure of adoption at time t, we can begin by looking at the static problem of finding the equilibrium values of x_a, x_β for each value of t.

For a given time t, let $v^* = v^*(x_\alpha, x_\beta, t)$ be a threshold value such that $P(v^*, x_\alpha, x_\beta, t) = v^* + f(x_\alpha, t) - g(x_\beta, t) = 0$. In equilibrium, all types v for whom $P > 0$ will adopt innovation α. We have the following:

Definition 1

For a given time t an equilibrium is defined by the pair (x_α, x_β) such that:

$$x_\alpha = 1 - F(v^*(x_\alpha, x_\beta, t)) \tag{4}$$

$$x_\beta = F(v^*(x_\alpha, x_\beta, t)) \tag{5}$$

Indeed, for those types with $v > v^*$, we have $P > 0$ and therefore adoption of α is preferred. The measure of firms whose best response is to adopt innovation α at or before time t is $\int_{v^*}^{v^h} dF(v) = (1 - F(v^*(x_\alpha, x_\beta, t))$, which in equilibrium must be equal to the measure of firms who actually adopted at or before time t, as in (4). Analogously, for those types with $v < v^*$, adoption of β is preferred. The measure of these adopters is $\int_{v^l}^{v^*} dF(v) = F(v^*(x_\alpha, x_\beta, t))$, which in equilibrium must be equal to the measure of agents who actually adopted at or before t, as in (5). We denote by $X(t)$ the set of equilibrium measures of adoption for α and β for each time t. An equilibrium adoption path for the two innovations is a vector $x(t) = (x_\alpha(t), x_\beta(t)) \in X(t)$ for all t. The following Proposition holds:

PROPOSITION 1. There exists an equilibrium adoption path for the two innovations.

Proof. Define $Y(x_\alpha,x_\beta,t) = 1 - F(v^*(x_\alpha,x_\beta,t))$. For each t and for each x_β, $Y(x_\alpha,x_\beta,t)$ is continuous on x_α. (Apply the implicit function theorem to $P(v^*, x_\alpha,x_\beta,t) = 0$). Moreover, $Y(x_\alpha,x_\beta,t)$ is defined for all $x_\alpha \in [0,1]$, for any t and x_β. Therefore, there exists at least one fixed point, for any t and x_β, such that $x_\alpha = Y(x_\alpha,x_\beta,t)$. Since $F(v^*(x_\alpha,x_\beta,t))$ is continuous on x_β, for each t and for each x_α, and is defined for all $x_\beta \in [0,1]$, there exists at least one fixed point, for any t and x_α, such that $x_\beta = F(v^*(x_\alpha,x_\beta,t))$.

□

Remark 1. Consider the following system:

(6) $$\varphi_1(x_\alpha,x_\beta,t) = x_\alpha - 1 + F(v^*(x_\alpha,x_\beta,t)) = 0$$

(7) $$\varphi_2(x_\alpha,x_\beta,t) = x_\beta - F(v^*(x_\alpha,x_\beta,t)) = 0$$

Assume that the Jacobian matrix J is not singular, that is:

$$J = \begin{matrix} 1 + \frac{\partial F}{\partial v}\frac{\partial v^*(x_\alpha,x_\beta,t)}{\partial x_\alpha} & \frac{\partial F}{\partial v}\frac{\partial v^*(x_\alpha,x_\beta,t)}{\partial x_\beta} \\ -\frac{\partial F}{\partial v}\frac{\partial v^*(x_\alpha,x_\beta,t)}{\partial x_\alpha} & 1 - \frac{\partial F}{\partial v}\frac{\partial v^*(x_\alpha,x_\beta,t)}{\partial x_\beta} \end{matrix}$$

$$\det J = 1 - \frac{\partial F}{\partial v}\frac{\partial v^*(x_\alpha, x_\beta, t)}{\partial x_\beta} + \frac{\partial F}{\partial v}\frac{\partial v^*(x_\alpha, x_\beta, t)}{\partial x_\alpha} \neq 0.$$

By the implicit function theorem we get:

$$\partial x_\alpha / \partial t = -[\frac{\partial F}{\partial v}\frac{\partial v^*(x_\alpha, x_\beta, t)}{\partial t}]/[1 - \frac{\partial F}{\partial v}\frac{\partial v^*}{\partial x_\beta} + \frac{\partial F}{\partial v}\frac{\partial v^*}{\partial x_\alpha}]$$

$$\partial x_\beta / \partial t = -[\frac{\partial F}{\partial v}\frac{\partial v^*(x_\alpha, x_\beta, t)}{\partial t}]/[\frac{\partial F}{\partial v}\frac{\partial v^*}{\partial x_\beta} - 1 - \frac{\partial F}{\partial v}\frac{\partial v^*}{\partial x_\alpha}]$$

which implies that $\partial x_\alpha(t)/\partial t \gtrless 0 \qquad \partial x_\beta(t)/\partial t \lessgtr 0$.

Remark 2. *$Y(x_\alpha, x_\beta, t)$ is increasing in x_α, t, and decreasing in x_β.* Indeed,

$$\partial Y/\partial x_\alpha = -\frac{\partial F}{\partial v}\frac{\partial v^*(x_\alpha, x_\beta, t)}{\partial x_\alpha} > 0, \text{ because } \partial v^*/\partial x_\alpha = -\frac{\partial P/\partial x_\alpha}{\partial P/\partial v} < 0$$

$$\partial Y/\partial t = -\frac{\partial F}{\partial v}\frac{\partial v^*(x_\alpha, x_\beta, t)}{\partial t} > 0, \text{ because } \partial v^*/\partial t = -\frac{\partial P/\partial t}{\partial P/\partial v} < 0$$

$$\partial Y/\partial x_\beta = -\frac{\partial F}{\partial v}\frac{\partial v^*(x_\alpha, x_\beta, t)}{\partial x_\beta} < 0, \text{ because } \partial v^*/\partial x_\beta = -\frac{\partial P/\partial x_\beta}{\partial P/\partial v} > 0.$$

Analogously, $F(x_\alpha, x_\beta, t)$ is non-decreasing in x_β and non-increasing in x_α, t. Indeed,

$$\partial F/\partial x_\alpha = \frac{\partial F}{\partial v}\frac{\partial v^*(x_\alpha, x_\beta, t)}{\partial x_\alpha} < 0,$$

$$\partial F / \partial t = \frac{\partial F}{\partial v} \frac{\partial v^*(x_\alpha, x_\beta, t)}{\partial t} < 0,$$

$$\partial F / \partial x_\beta = \frac{\partial F}{\partial v} \frac{\partial v^*(x_\alpha, x_\beta, t)}{\partial x_\beta} > 0.$$

3. DYNAMICS OF THE SEQUENTIAL PROCESS OF INNOVATION CHOICE

In this Section we discuss the dynamics of the sequential process of innovation choice. From the viewpoint of the formal theory of stochastic processes the model is equivalent to a generalized "Polya urn" scheme. Indeed, the density function associated with the distribution F(v) measures the "frequency of arrival" of firm types with parameter v. Given that $Y(x_\alpha, x_\beta, t)$ is increasing in x_α, for given x_β, t, it implies that the "frequency of arrival" of firm types with $v > v^*$ increases with the measure of adoptions of α. As a consequence, the probability that one particular innovation is chosen is an increasing function of the measure of adoptions of that innovation.

A basic tool for investigating limit properties in path-dependent dynamic processes of this sort is in Arthur, Ermoliev and Kaniovski (1985). Their theorem says, in essence,that if the process is extended indefinitely, the respective shares of the adopters population x_α and x_β must converge with probability one to a fixed point. In particular, it can converge only to points of stable equilibrium and not to unstable fixed points. Actually, we can show the following result:

PROPOSITION 2. Both innovations cannot coexist indefinitely in the market, and one of the two will take all but a finite set of firms with probability one.

Proof. Let the vector $x(n) = (x_\alpha(n),x_\beta(n))$ denote the measure of adoptions of innovations α and β. Denote by $\{p(x(n),n) = (p_\alpha(x(n),n),p_\beta(x(n),n)\}$, where $p_\alpha(x(n),n) = (1 - F(x_\alpha(n),x_\beta(n),n))$, $p_\beta(x(n),n) = F(x_\alpha(n),x_\beta(n),n)$, a sequence of continuous functions mapping the measures of adoptions into the probabilities at time n. Thus, starting at time 0 with an initial vector of number of firms (which for convenience we set at zero), the industry forms by the addition of one firm at a time, choosing one and only one of the two innovations; and at time n, a firm chooses innovation i with probability $p_i(x(n),n)$, $i = \alpha,\beta$. The vector of measures of adoptions evolves as:

$$x(n+1) = x(n) + \frac{1}{n+1}(b(x(n),n) - x(n)), \qquad x(0) = 0 \tag{8}$$

where $b(x(n),n)$ is a unit vector with 1 in the i–th place with probability $p_i(x(n),n)$ and zero elsewhere. We can write (8) in the form:

$$x(n+1) = x(n) + \frac{1}{n+1}(p(x(n),n) - x(n)) + \frac{1}{n+1}\mu(x(n),n) \tag{9}$$

where μ is defined as the random vector:

$$\mu(x(n),n) = b(x(n),n) - p(x(n),n) \tag{10}$$

Equation (9) is the basic description of the dynamics of the measures of adoptions. It consists of a determinate "driving" part (the first two terms on the right of (9)) and a perturbational part (the μ–term). Notice in (10) that the conditional expectation of $\mu(n)$ with respect to $x(n)$ is zero, so that we can show that the expected motion of $x(n + 1)$ is given by the "driving" part of (9) as :

$$E(x(n+1)\,|\,x(n)) - x(n) = \frac{1}{n+1}(p(x(n),n) - x(n)) \tag{11}$$

If the probability $p_i(x(n),n)$ is larger than the current measure of adoption of i, then this measure of adoption increases, at least on an expected basis. Conversely, if it is less, then the measure of adoption decreases. Equation (9) is analogous to the basic dynamic equation in Arthur, Ermoliev and Kaniovski (1985). The proof of Proposition 2 is now an application of Theorems 4.1, 5.1 and 5.2 in Arthur, Ermoliev and Kaniovski (1985). Here we merely sketch the argument.

Consider the set $B = \{x \in X\text{: two elements of x are maximal}\}$, and B_ε its ε-neighborhood. Partition X/B_ε into two separate sets C^i designated by the nearest extreme i, $i = \alpha,\beta$. Now consider a given point z in C^i. There exists a finite time t(z) such that $P(v, z_i,z_j,t(z)) > 0$. Further, it is easy to show that t(z) has an upper bound t^i in C^i. Therefore, if the process x(t) is in C^i at a time $t > t^i$, then innovation i will have maximal return for all firm types. It will then be chosen with probability 1; the process will never exit C^i and it will converge to 100% share for i. Alternatively, the process might not enter C^i; it could stay within B_ε forever. By Lemma 4 of Ljung (1978) which shows that for points lying in an unstable region (the neighborhood of a separatrix) the process must exit the region in a finite time, we get that the process cannot stay in B_ε forever and therefore x(t) converges to an extreme, which is an attracting fixed point. □

4. THE CHARACTERISTICS OF THE EQUILIBRIUM ADOPTION PATH

In this Section we give a characterization of the equilibrium adoption path. In particular, we want to examine whether the presence of "network externalities", and therefore of benefits from compatibility, yields an equilibrium adoption path whose characteristics markedly differ from the case of diffusion with no "network externalities".

In view of the fact that the market ends up monopolized by a single innovation (Proposition 2), let us concentrate on the equilibrium adoption path for one innovation, say α. An analogous argument can be made for innovation β. The following Proposition holds:

PROPOSITION 3. (i) If $\partial Y/\partial x_\alpha < 1$ for all t, x_β at all equilibria, then there is a unique equilibrium adoption path, which is continuous;

(ii) If $\partial Y/\partial x_\alpha > 1$ at some fixed point, for all t, x_β,then there is a continuum of discontinuous equilibrium adoption paths.

PROOF. Part (i). If $\partial Y/\partial x_\alpha$ is of one sign at all equilibria, then there can be only one equilibrium, for each t, x_β. By varying t, we get a continuous equilibrium adoption path, which is unique.

Part (ii). If $\partial Y/\partial x_\alpha > 1$ at some fixed point, for all t, then we have multiple static equilibria. It implies that $x_\alpha(t)$ is a correspondence, not a function, and therefore there is a continuum of discontinuous equilibrium adoption paths.

Figure 1 illustrates three typical cases. In particular, from the implicit function theorem applied to $\varphi_1(x_\alpha, x_\beta, t) = x_\alpha - 1 + F(v^*(x_\alpha, x_\beta t)) = 0$ we obtain that $\partial x_\alpha(t)/\partial t \gtrless 0$ according to $1 + \partial F/\partial x_\alpha \gtrless 0$, that is, $\partial Y/\partial x_\alpha \lesseqgtr 1$, which is in keeping with Figure 1.

FIGURE 1

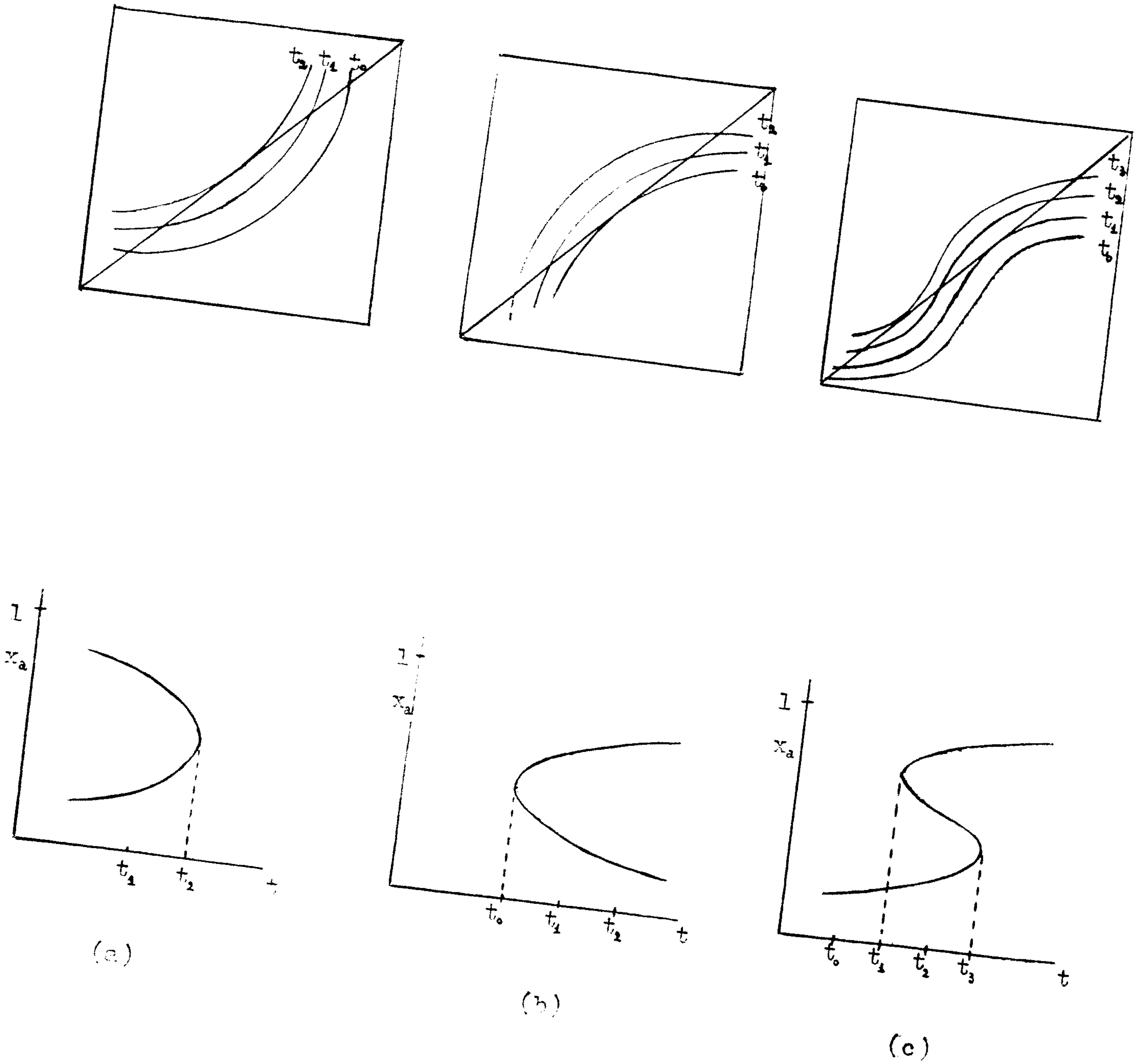

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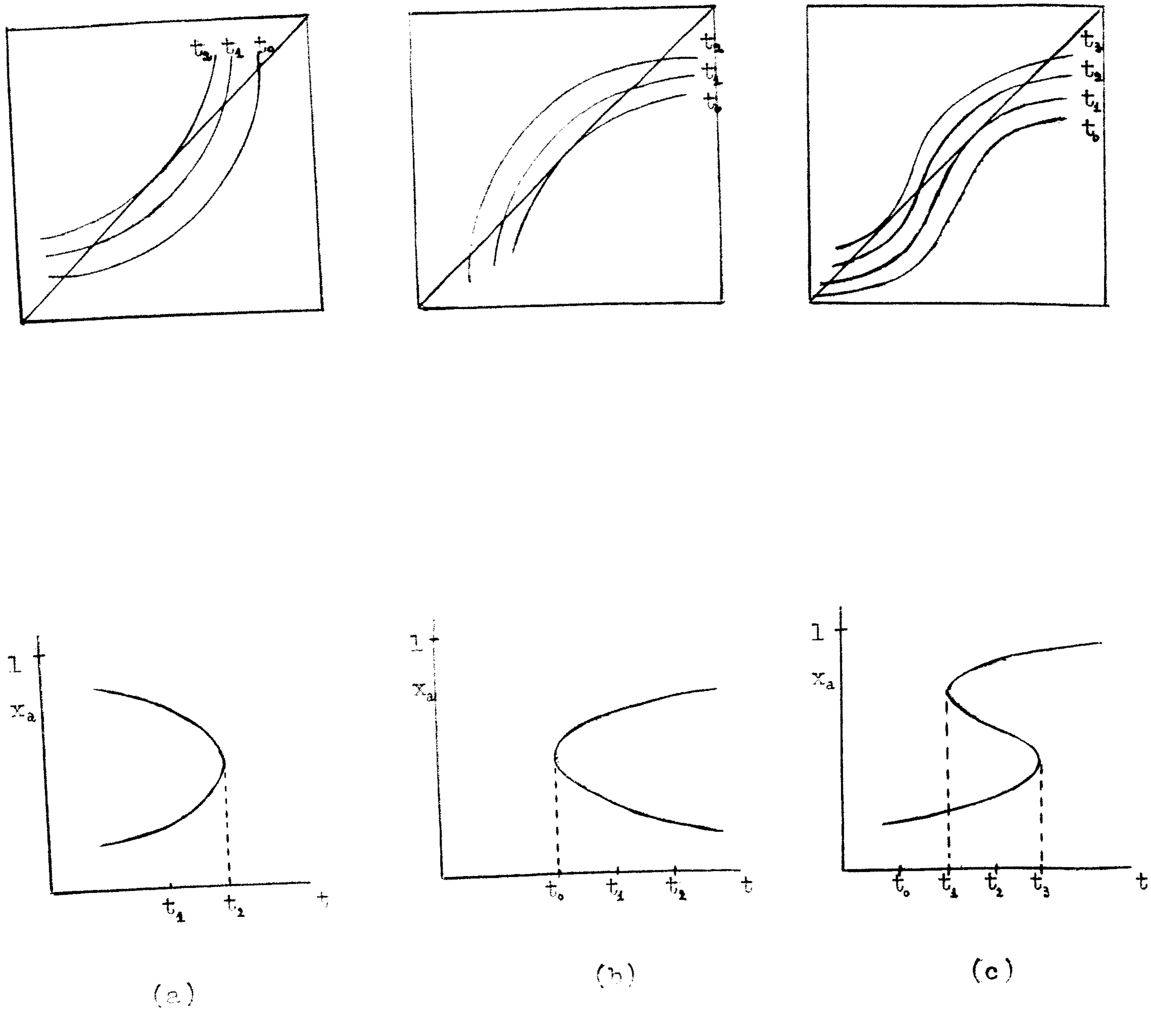
FIGURE 1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Remark 3. The equilibrium adoption path is discontinuous in situations of strong network externalities, or relative homogeneity among adopters.

Indeed, if we compute

(12) $$\partial Y/\partial x_\alpha =$$

$$= -\frac{\partial F}{\partial v}\frac{\partial v^*(x_\alpha,x_\beta,t)}{\partial x_\alpha} = \frac{\partial F}{\partial v}\frac{\partial P(v^*(x_\alpha,x_\beta t),x_\alpha,x_\beta,t)}{\partial x_\alpha}\frac{1}{\frac{\partial P(v^*(x_\alpha,x_\beta,t)),x_\alpha,x_\beta t}{\partial v}}$$

we get that $\partial Y/\partial x_\alpha > 1$ implies, other things equal, high density of agents of a given type $(\partial F/\partial v)$ or strong network externalities $(\partial P/\partial x_\alpha)$.

Remark 4. We can characterize the equilibrium adoption path with the tools of catastrophe theory. To formulate the equilibrium analysis above into the standard framework of catastrophe theory[1] we need to show that the equilibrium defined by (4) and (5) minimizes a potential function. That is, the dynamical system:

$$dx_\alpha/dt = \varphi_1(x_\alpha,x_\beta,t) = x_\alpha - 1 + F(v^*(x_\alpha,x_\beta,t))$$

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[1]Let us give here the definition of a catastrophe point:

Definition.

Consider a phenomenon occurring in the state space S and governed by parameters belonging to the control space Ω. Assume that there exists a potential function V defined on $S\times\Omega$ such that the phenomenon under study minimizes V restricted to $S\times\{\omega\}$, for a given control $\omega \in \Omega$. Let E be the subset of $S\times\Omega$ consisting of pairs (s,ω) such that s is a minimum of V restricted to $S\times\{\omega\}$. Let $\tilde{\Pi}$ denote the restriction of the natural projection $\Pi\colon S\times\Omega \to \Omega$. The singularities of $\tilde{\Pi}$ are called catastrophes and Thom's theorem asserts that, if the control space Ω is a Euclidean space of dimension less than or equal to five, then there is a finite number of elementary catastrophes, i.e. of generic singularities of $\tilde{\Pi}$, up to a diffeomorphism of the state space S.

$$dx_\beta/dt = \varphi_2(x_\alpha,x_\beta,t) = x_\beta - F(v^*(x_\alpha,x_\beta,t))$$

must be a gradient system. That is, there must be some functions K_1, K_2, such that $\varphi_1(x_\alpha,x_\beta,t) = -\partial K_1(x_\alpha,x_\beta,t)/\partial x_\alpha$ and $\varphi_2(x_\alpha,x_\beta,t) = -\partial K_2(x_\alpha,x_\beta,t)/\partial x_\beta$. Then the equilibria of the system:

(13) $$dx_\alpha/dt = \varphi_1(x_\alpha,x_\beta,t) = -\partial K_1(x_\alpha,x_\beta,t)/\partial x_\alpha$$

(14) $$dx_\beta/dt = \varphi_2(x_\alpha,x_\beta,t) = -\partial K_2(x_\alpha,x_\beta,t)/\partial x_\beta$$

are precisely the singularities of K_1 and K_2; that is, x_α, x_β are equilibria iff $\partial K_1(x_\alpha,x_\beta,t)/\partial x_\alpha = 0$, $\partial K_2(x_\alpha,x_\beta,t)/\partial x_\beta = 0$. Thus, the study of how the nature of the system (13) (and also (14)) changes as t changes can be reduced to the study of the singularities of $K_1(x_a,x_\beta,t)$ (and of $K_2(x_\alpha,x_\beta,t)$).

Since $\varphi_1(x_\alpha,x_\beta,t)$ and $\varphi_2(x_\alpha,x_\beta,t)$ are continuous in x_α and x_β respectively, it follows that $dx_\alpha/dt = \varphi_1(x_\alpha,x_\beta,t)$, $dx_\beta/dt = \varphi_2(x_\alpha,x_\beta,t)$ is a gradient system.

Points in the graph of the equilibrium correspondence where the tangent is vertical are called singular points, or catastrophe points. A necessary condition for the equilibrium adoption path to be discontinuous is that there exists a singular point. This occurs when the following condition is satisfied:

Definition 2.

A singular point is characterized by the condition:

(15) $$\frac{\partial F(v^*(x_\alpha,x_\beta,t))}{\partial v} \frac{\partial P(v^*(x_\alpha,x_\beta,t),x_\alpha,t)/\partial x_\alpha}{\partial P(v^*(x_\alpha,x_\beta,t),x_\alpha,t)/\partial v} = 1$$

Condition (15) easily follows from the fact that $Y(x_\alpha, x_\beta, t)$ must be tangent to x_a in order to have a singular point (see Figure 1). Condition (15) certainly occurs if we have $\partial Y/\partial x_\alpha > 1$ at some fixed point , as required in Proposition 3. It follows that the condition $\partial Y/\partial x_\alpha > 1$, that is, strong network externalities or relative homogeneity among potential adopters, is a sufficient condition for the existence of a singular (or catastrophe) point.

Remark 5. From part (ii) in Proposition 3 we have that $x_\alpha(t)$ can be a correspondence, and hence the possibility of having a continuum of equilibrium adoption paths. The question arises: is there a natural way of selecting among these equilibrium adoption paths? That is, what is the economic meaning of possible jumps from one branch to the other of the graphs of the equilibrium correspondence?

If one supposes that agents make their own decision at time t, based on the extent of adoption at time $t - \varepsilon$, ε small, that is, each agent assumes that the extent of adoption at time t is close to what it was at time $t - \varepsilon$, then there exists a unique equilibrium adoption path, given by the lower envelope of the increasing branches of the graph of the equilibrium correspondence. Notice from Figure 1 that only in case (c) the equilibrium adoption path is discontinuous. Under the circumstances of case (c) we have a fold catastrophe.

Remark 6. One may wonder how realistic the idea of a discontinuous equilibrium adoption path is. There is a vast body of empirical research giving evidence of an S-shaped pattern in the diffusion of innovations. One could argue that a discontinuous equilibrium adoption path can be approximated with a steep S-shaped path (see Figure 1(c)).

5. FINAL REMARKS

The adoption of innovations is a far more complex issue than has been modelled in this paper. For example, our assumption of a continuum of potential adopters abstracts from the possibility of considering the case where agents behave strategically: this would produce a stochastic dynamic oligopoly game. Moreover, we assume adopters have complete information with regard to the benefit of adoption, that is to say, there is no uncertainty. Especially, we assume myopic decisions so that new firms decide to choose the innovation of present maximum return. However, in the presence of "network externalities", since benefits are conferred if future adopters go along with one's choice, expectations about the future states of the adoption process become a very important element, which should instead be taken into account.

Katz and Shapiro (1986a) consider a static version of the problem of competing "networks" of different standards in which "network externalities" accrue to increased network size. It pays firms to provide large networks if potential adopters expect these networks to be large and thereby commit their choice to them. If, prior to adoption, sufficient numbers of agents believe that network α will have a large share of adopters, it will; but if sufficient believe β will have a large share, it will. Katz and Shapiro show that there could be multiple "fulfilled-expectation equilibria", that is, multiple sets of eventual network adoption shares that fulfill prior expectations. In their model, however, expectations are given and fixed before the adoption process takes place.

More realistically, if firms are affected by future innovation choice decisions, one should assume they would form beliefs about these decisions. Firms might then have conditional probabilities of future states and they might base their choices on these beliefs. In choosing they would create an actual stochastic process which would be in fulfilled expectations equilibrium if it were identical to the believed stochastic process. In this model, if one standard gets ahead by "chance" adoptions, its increased probability of doing well in the adoption market will further enhance the expectations of its success, which implies that our result of Proposition 2 now would occur more easily.

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