Upward Sloping Reaction Functions under Quantity Competition in Mixed Oligopolies

Flavio Delbono *

Carlo Scarpa **

* University of Verona, Italy

** University of Bologna, Italy

August 1991

Abstract

We show that in a mixed duopoly under quantity competition, the public firm's reaction function may be upward sloping. This is the case under concavity of the demand function when the public firm attaches private profits a smaller weight than its own profits.

1. Introduction

It is well known that in oligopoly, under quantity competition, firms' reaction functions are downward sloping. This is true for profit-maximizing firms as long as each firm's marginal revenue decreases when a rival increases its output (see Shapiro (1989), for an excellent account). Notice that such a condition (which is known as Hahn's condition) is implied by the strict concavity of the demand function. Moreover, firm i's optimal output is a decreasing function of its rivals' output also when firm i maximizes social welfare, i.e., the sum of consumer surplus and producer surplus (see, for instance, some of the papers, surveyed in De Fraja and Delbono, 1989).

In this note we show that, under concavity of the demand function, firm i's reaction function may be upward sloping in the space of output levels whenever firm i maximizes social welfare but gives less weight to its rivals' profits than to its own profit. Firm i may be understood as a public enterprise which competes with private (possibly foreign) companies, or as a mixed firm in which different shareholders end up maximizing a weighted average of consumer and producer surpluses.

The model and the main result are presented in section 2. An example is provided in section 3.

2. The model

Consider a duopolistic market [(1)] in which firm S (State-owned) and firm P (Private) compete in output levels. The product is homogeneous, and we consider the Nash equilibrium of a one-shot non cooperative game with complete information.

The inverse market demand function is $p(Q)$, where Q is total output ($Q = q_S + q_P$). This function is twice continuosly differentiable, with $dp/dQ < 0$; we also assume that $d^2p/dQ^2 < 0$, i.e., that the demand function is concave.

The cost function of firm i is $c(q_i)$, $i = P,S$. While we assume that both firms share the same cost function, this is inessential to our result. We assume that the marginal cost is increasing.

Firm P maximizes its own profits, i.e., $\pi_P = p(Q)q_P - c(q_P)$.

Firm S maximizes V as given by:

$$(1) \quad V = \int_{z=0}^{Q} p(z)dz - p(Q)Q + \pi_S + \tau\pi_P$$

where π_S is the profit of firm S, and τ ($0 \leq \tau \leq 1$) is the weight of P's profits in the maximand of firm S.

When $\tau = 1$, (1) is total surplus in the industry, which is usually assumed to be the objective function of a public enterprise. In this paper we consider also the case in which $\tau < 1$, which can be interpreted in at least two ways. On the one hand, it may be thought that a share of π_P accrues to foreign shareholders and then it does not enter into the domestic welfare function. On the other hand, political reasons may induce the public authority to attach private profits a smaller weight.

Let us consider the maximization problem of firm S. It chooses q_S in order to maximize V, given q_P. The first order condition is then:

$$(2)\quad \delta V/\delta q_S \equiv p - Q\,\delta p/\delta q_S - p + \delta R_S/\delta q_S - \delta c/\delta q_S + \tau\,\delta R_P/\delta q_S = 0$$

where R_i is firm i's revenue. (2) can be rewritten as:

$$(2')\quad p - \delta c/\delta q_S - (1 - \tau)\, p\, \delta p/\delta q_S = 0$$

Notice that $\tau = 1$ implies the usual rule of marginal cost pricing. Whenever $\tau < 1$, consumer surplus is given a larger weight than private profits, and thus firm S prices below its marginal cost.

Clearly, equation (2') defines the reaction function of firm S. Let F_S be the left-hand side of (2'); then, the slope of the reaction function of firm S is given by:

$$\frac{dq_S}{dq_P} = - \frac{\delta F_S/\delta q_P}{\delta F_S/\delta q_S}$$

Assuming that the second order condition is met, this expression has the same sign as $\delta F_S/\delta q_P$, which, exploiting the fact that $\delta p/\delta q_P = \delta p/\delta q_S = dp/dQ$, can be written as:

$$(3)\quad \delta F_S/\delta q_P = \tau\, dp/dQ - (1 - \tau)\, q_P\, d^2p/dQ^2$$

The first term on the right-hand side is negative for any $\tau > 0$, whereas the concavity of the demand function implies that the second term is positive for any $\tau < 1$. Clearly, given the demand function, $\delta F_S/\delta q_P$ is more likely to be positive the closer τ is to zero. Thus, we have proved the following:

Proposition. Under concavity of the demand function, when τ is sufficiently small, the public firm's reaction function is upward sloping.

Remark 1. When $\tau = 0$, $\delta F_S/\delta q_P$ is certainly positive for any Q, as long as $q_P^{(2)} > 0$. By continuity, for any concave demand function there exist some values of τ which make the public firm's reaction function upward sloping.

Remark 2. When $\tau = 1$, $\delta F_S/\delta q_P$ is always negative.

Remark 3. For any $\tau > 0$, the concavity of the demand function is necessary, but not sufficient to yield the result summarized in the Proposition.

Remark 4. The reaction function need not be monotonic. Indeed, as the values of dp/dQ and d^2p/dQ^2 depend on Q, the sign of $\delta F_S/\delta q_P$ may vary with Q. As a consequence, the reaction functions might intersect several times, generating multiple Nash equilibria.

It is worth noting that when the equilibrium occurs in the increasing region of the public firm's reaction function, some of the standard stability conditions may not be met.

3. An example

We provide now a simple example to illustrate the findings of section 2. Let $p = 10 - Q^2$ be the inverse demand function, and $c(q_i) = q_i^2$ $(i=S,P)$ be the cost function. The objective functions of firms P and S are those specified in the previous section. Hence, the private firm's reaction function is implicitly defined by

$$- 3q_P^2 - 2q_P(1 + 2q_S) + 10 - q_S^2 = 0$$

whereas the public firm's reaction function is defined by

$$- q_S^2 - 2q_S(1 + 2\tau q_P) + q_P^2(1 - 2\tau) + 10 = 0$$

The latter reaction function is upward sloping if and only if:

$$(4) \qquad \tau < \frac{q_P}{2(q_P + q_S)}$$

Not surprisingly condition (4) defines an upper bound to τ. For τ small enough, (4) is met. As long as $\tau < 0.5$, the public firm's reaction function is upward sloping for some values of q_S and q_P. For instance, with τ = 0.05, the unique Nash equilibrium is $q_S = 2.310$ and $q_P = 0.377$. By simple substitution of these values into (4), it is easy to see that the equilibrium occurs where the reaction function of the public firm is upward sloping. Of course, for other values of τ the equilibrium may occur in the decreasing region of the public firm's reaction function.

Footnotes

(1) A generalization to the case of oligopoly would be straightforward.

(2) Clearly, when $\tau = 0$ and the demand function is linear, $\delta F_S/\delta q_P = 0$, so that the optimal value of q_S does not depend on q_P. This case is considered in Delbono and Scarpa (1991).

References

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