REGULATING INNOVATIVE ACTIVITY: THE ROLE OF A PUBLIC FIRM

Flavio Delbono * and Vincenzo Denicolò **

* University of Verona, Italy
** University of Parma, Italy

June 1991

To be presented at the Round Table "Innovation Policy and EEC Growth", 18th EARIE Conference, Ferrara (Italy), September 2, 1991.

Abstract. Without spillovers and under the "winner-take-all" hypothesis, there is overinvestment in R & D in a non cooperative equilibrium. This is due to the so-called "common pool problem", i.e., duplication of efforts. We show that a public firm can represent a useful instrument in the hands of a policymaker to mitigate such a problem. More precisely, it is proved that, in a mixed duopoly: (i) each firm invest less than in a private duopoly, (ii) although the expected time of innovation is postponed, social welfare is higher than in a private duopoly.
1. Introduction

In the so-called "strategic approach" to market structure and innovation, a standard result is that, under some hypotheses we shall discuss later, the equilibrium of an industry where a fixed number of firms behave non cooperatively is characterised by overinvestment in R & D. This is because each firm does not recognize the parallel nature of its effort and those of the other firms, and this produces a "common-pool problem".

Then, there is room for policy. In this paper we wish to analyse the role of a public firm in reducing the gap between the optimal level of R & D and the level emerging in a Nash equilibrium.

In designing the optimal form of intervention, one could argue that imposing taxes or awarding appropriate prizes (incentives) to the innovating firms can allow for the implementation of a first-best arrangement. By way of contrast, when the policy instrument is a public firm competing on equal basis with one or more private firms the outcome is very often bound to be sub-optimal. Nevertheless, we believe that the analysis of the regulatory role of a public firm is worth pursuing for at least two reasons: first, the first best policy might be difficult to implement for lack of the necessary information, whereas the public firm may have access to better information about the industry than an external regulator; second, public firms already operate in many oligopolistic industries (at least in Europe) and therefore it is interesting to study the rules that should inform their behaviour.

Furthermore, at the theoretical level, the conduct of public firms as instruments of policy in mixed oligopolies (i.e., oligopolistic industries where firms with different objectives coexist)
has typically been analysed assuming that firms compete only in prices or in quantities \(^1\). It then seems desirable to explore the role of public firms in oligopolistic markets also when forms of non-price competition (quality, R & D, capacity) are crucial to understand the performance of the industry.

The paper is organised as follows. Section 2 sets forth the background of our subsequent analysis. In Section 3 we present the model. Section 4 analyses the Nash equilibrium of the R & D game and contains our results, and Section 5 concludes the paper.

2. Background

Much of the recent theoretical literature on the relationships between market structure and innovation adopt a game-theoretic formulation \(^2\). Starting with the seminal contributions of Loury (1979), Dasgupta and Stiglitz (1980) and Lee and Wilde (1980), some old questions - that can be dated back to Schumpeter's (1943) famous contribution - have been revisited with the lenses of modern microeconomics. Among such questions, one that attracted a great deal of attention is the issue of the social optimality of innovative activity undertaken by profit seeking investors.

In fact, this question was not tackled explicitly by Schumpeter, who instead compared the expected paces of R & D under perfect competition and under monopoly. In a more formal, albeit somehow oversimplified way, the same comparison was carried out by Arrow (1962) in

\(^1\) Exceptions are Cremer, Marchand and Thisse (1991) and Delbono, Denicolò and Scarpa (1991); both papers analyse the choice of the quality level by a public firm in a mixed oligopoly.

\(^2\) See Baldwin and Scott (1987) and Reinganum (1989) for fairly comprehensive surveys. For a survey of other approaches see Dosi (1988).
a justly famous article. A welfare criterion, however, is at least implicit in both of these classical contributions. Thus, Schumpeter and Arrow’s analyses could in principle be extended to the social optimum. This was done explicitly by Dasgupta and Stiglitz (1980), who compared the equilibrium R & D expenditure level in a perfectly competitive, monopolistic and socially managed industry.

As a matter of fact, however, most R & D investment appears to be made by firms which operate in oligopolistic industries. In this context, the issue at stake can be re-phrased as follows: Is the Nash equilibrium R & D investment of a given number (n) of uncoordinated private firms greater or smaller than the optimum level?

Here the key word is optimum, as it can be given different meanings. In what follows, we interpret it as the amount that would be decided by a social planner controlling n units (laboratories), who takes into account the overall benefits (for consumers as well as for producers) accruing from the R & D activity. From this viewpoint, both Arrow (1962) and Dasgupta and Stiglitz (1980) have proved that the social value of an innovation - and then the incentive to do R & D - is greater than the private one both under monopoly and perfect competition. This conclusion implies a tendency by private firms to underinvest in R & D, and can be extended to the case of oligopoly.

However, in oligopolistic industries the difference between the private and social benefit from the innovation is by no means the only reason why the socially optimal level of R & D may differ from the one resulting from a Nash equilibrium. Indeed, suppose that the R & D competition is a race where the first firm to innovate is awarded a prize, whereas the latecomers obtain no benefit (i.e., the winner takes all), and there are no spillovers in the innovative acti-

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3 For an analysis of R & D equilibria under Bertrand and Cournot competition in the product market see Delbono and Denicolò (1990).
vity. Then, non cooperative behaviour implies that firms do not operate at the jointly optimal scale as a result of their strategic interaction. The reason is that all firm jockey for being first in the race.

The distortion due to the strategic interaction between competing firms has been called "duplication of effort" by Loury (1979) and Lee and Wilde (1980). In order to isolate this effect, they assume that the social benefit from the innovation is equal to the private one. Then, if the \( n \) private firms behaved cooperatively, the private oligopoly equilibrium level of R & D would coincide with the social optimum. But in a Nash equilibrium firms do not recognize the parallel nature of their efforts and thus overinvest in R & D.

To assume that the private and the social benefit from the innovation coincide, however, may be inappropriate in several contexts. For instance, in the case of cost reducing innovations, when firms are quantity-setting Cournot players in the product market, the social and private benefits from the innovation differ. Indeed, the private benefit, which is given by the increase in future profits of a single firm, is generally speaking lower than the social benefit, which also includes the change in other firms’ profits and in consumers’ surplus \(^4\). It might therefore be thought that retaining Lee and Wilde’s assumption significantly limits the interest of our analysis. However, we believe that, as a working hypothesis, it can be defendend on twofold grounds.

First of all, there are contexts in which this assumption seems appropriate. Consider, for instance, the case of a completely new product, when the firm which innovates first can produce the new product under constant returns to scale, whereas the losers of the R & D race cannot enter the market. If there is perfect and infinitely lived patent protection, and no further innovation is anticipated, the market will be monopolised forever by the firm which innovates first.

\(^4\)This has been formally shown in Delbono and Denicolò (1991), under linear demand function and constant marginal costs.
Assuming that perfect price discrimination is available to the monopolist, the flow of profits accruing to the private firm in case it wins the race will be equal to the post-innovation level of social welfare (i.e., the sum of producers' and consumers' surpluses).

Secondly, even when the assumption that the private prize equals the social benefit from the innovation is debatable, it allows us to disentangle the distortions caused by strategic interaction and those arising from differences in benefits accruing to different players. Focussing on the former simplifies the analysis and produces neat results.

In this paper we therefore retain Lee and Wilde's assumption on the equality of the private and social benefit from the innovation, we set \( n = 2 \), and we focus on the effects of one firm being a social welfare maximiser.

3. The model

We consider an industry where two firms, P and S, operate. P is a profit maximising firm and S is a State-owned firm which maximises social welfare.

We assume that only one innovation is in prospect. The private firm gets a positive profit if it innovates first, whereas it gets nothing if the public firm wins the R & D race. The innovation entails also a social benefit, which we assume to be the same as the prize obtained by the private firm if it wins. However, the public firm is indifferent as to who wins the R & D race; to firm S, the only thing that matters is the expected date of innovation.
The timing of the innovation is uncertain. Following the prevailing literature, we assume that each firm’s probability of innovating is an increasing exponential function of its R & D expenditure \( x_i \) (see Reinganum 1989 for details). As in Lee and Wilde (1980), \( x_i \) is assumed to be a flow cost that firm \( i \) pays until one player succeeds. R & D investment affects the expected date of innovation, but does not affect its nature.

The two firms have access to the same R & D technology; the hazard function \( \mu h(x_i) \), \( (i = P, S) \), is assumed to satisfy the following conditions:

\[
\begin{align*}
(A.1) & \quad h'(x_i) > 0 \\
(A.2) & \quad h''(x_i) < 0 \\
(A.3) & \quad h(0) = 0 \\
(A.4) & \quad \lim_{x_i \to \infty} h(x_i) = \infty \\
(A.5) & \quad \lim_{x_i \to \infty} h'(x_i) = 0 \\
(A.6) & \quad \lim_{x_i \to 0} h'(x_i) = \infty
\end{align*}
\]

Conditions (A.2), (A.5) and (A.6) guarantee that the firms’ maximisation problems will always yield an interior solution. Condition (A.3), together with (A.1) and (A.2), ensures that the "average productivity" of R & D expenditure is greater than its "marginal productivity", i.e. \( h(x_i) > x_i h'(x_i) \). (A.3) will be useful, together with (A.4), to provide a sharper characterisation of the reaction curves of the two firms. The positive parameter \( \mu \) measures the productivity of R & D expenditure.
The payoff function of firm P is the present value of expected profits, net of its R & D costs: i.e.,

\[ V_P = \int_0^\infty e^{-\mu h(x_p) + \mu h(x_s) + r} \left[ \frac{\mu h(x_p)}{r} W - x_p \right] dt = \frac{w}{r} \frac{\mu h(x_p) - x_p}{\mu h(x_p) + \mu h(x_s) + r} \]

where \( W \) is the flow of profits accruing to the private firm if it innovates first, which is assumed to be given and independent of the level of R & D investment, and \( r \) is the discount rate.

Firm S, on the other hand, maximises the present value of the expected benefit from the innovation, net of all R & D costs: i.e.,

\[ V_S = \int_0^\infty e^{-\mu h(x_p) + \mu h(x_s) + r} \left[ \frac{\mu h(x_p)}{r} W + \mu h(x_s) \frac{W}{r} - x_p - x_s \right] dt = \frac{w}{r} \frac{[\mu h(x_p) + \mu h(x_s)] - x_p - x_s}{\mu h(x_p) + \mu h(x_s) + r} \]

Notice that the payoffs of the public firm differs from that of the private firm in two respects. First, the public firm is not concerned about who wins the R & D race: the only thing that matters to S is the date of innovation. Second, firm S takes into account the R & D costs of both firms, whereas firm P obviously cares only about its own costs.
4. The Nash equilibrium of the R & D game

The two firms non cooperatively and simultaneously choose the R & D expenditure in order to maximise their payoffs. Thus, firm P maximises (1) and firm S maximises (2). From the first order conditions for a maximum we can obtain the reaction curves in the R & D space $(x_p, x_s)$. The reaction curve of the private firm is implicitly defined by:

\[(3) \quad \theta h'(x_p)[1 + h(x_s)\theta]W - 1 - \theta h(x_s) - \theta h(x_p) + \theta x_p h'(x_p) = 0\]

or, rearranging terms

\[(3') \quad [\theta h'(x_p)W - 1][1 + h(x_s)\theta] = \theta[h(x_p) - x_p h'(x_p)]\]

(where $\theta = \frac{\mu}{r}$) whereas the reaction curve of the public firm is implicitly defined by:

\[(4) \quad \theta h'(x_s)(W + x_p) - 1 - \theta h(x_p) - \theta h(x_s) + \theta x_s h'(x_s) = 0\]

Notice that $\mu$ and $r$ do not enter separately into (3) and (4), but only through the ratio $\theta$.

It can be easily shown that, given $x_s$, there is a unique $x_p$ which solves equation (3); analogously, given $x_p$, equation (4) has exactly one real solution. Hence, the reaction functions are well defined.

\[5\text{The second order conditions are always satisfied by (A.2).}\]
The following Lemmas help to describe the shape of the reaction curves. The first lemma is well known (cf. Beath et al., 1989) but we provide a simple proof for the sake of completeness.

**Lemma 1.** The reaction function of the private firm is upward sloping.

Proof. Implicitly differentiating (3), and exploiting the second order condition, we get:

\[
\text{sign} \left( \frac{dx_p}{dx_s} \right) = \text{sign} \left[ \theta h'(x_p)W - 1 \right]
\]

(5)

Notice now that the FOC can be written as:

\[
\theta h'(x_p)W - 1 = \frac{\theta h(x_p) - \theta x_p h'(x_p)}{1 + \theta h(x_s)} > 0
\]

(6)

by the concavity of \( h \) and (A.3); hence it follows that \( x_p \) is an increasing function of \( x_s \). Q.E.D.

**Lemma 2.** The reaction function of the public firm is downward sloping if \( x_s > x_p \), and is upward sloping if \( x_p > x_s \).

Proof. Implicitly differentiating (4), and exploiting the second order condition, we get:

\[
\text{sign} \left( \frac{dx_s}{dx_p} \right) = \text{sign} \left[ h'(x_s) - h'(x_p) \right]
\]

(7)
whence the Lemma follows by the concavity of $h$. Q.E.D.

The intuition behind Lemma 2 can be explained as follows. When total R & D investment is low, an increase in $x_p$ induces firm S to reduce its investment because the marginal social benefit from investment goes down. If total investment is high, however, firm S responds to an increase in $x_p$ by enhancing its effort because this reduces the waiting time to the innovation and then reduces also total R & D expenditure. Clearly, this crucially depends on R & D costs being a flow which is sustained until one player succeeds.

Let us now denote $x_i^0$ the optimal R & D investment of firm $i$, given that the other firm does not invest, and $x_i^{\infty}$ the limit of the optimal R & D investment of firm $i$, given that the other firm’s investment tends to infinity.

**Lemma 3.** $x_p^0 = x_s^0$.

Proof. Setting $x_s = 0$ into (3) and $x_p = 0$ into (4) we get:

\[ \theta h'(x_i) W - 1 - \theta h(x_i) + \theta x_i h'(x_i) = 0 \]

that is, the FOCs of the two firms now coincide. The Lemma follows by simply observing that $x_i^0$ is the solution of (8). Q.E.D.
Let us now define $\bar{x}_p$ by:

\[
\frac{h(\bar{x}_p)}{\bar{x}_p} = \frac{1}{\Theta W}
\]

Since (A.1), (A.2) and (A.3) imply, as we have already noted, that the "average productivity" of R & D expenditure is greater than its "marginal productivity", it clearly follows that $\bar{x}_p > x_p^\ast$.

**Lemma 5.** The reaction function of firm S if it acts as a social welfare maximiser lies below the one arising if it behaved as a profit maximiser if $\bar{x}_p > x_p$, and lies above it if $x_p > \bar{x}_p$.

Proof. If firm S maximised its own profit, behaving like a private firm, its reaction function would be:

\[
\theta h'(x) [1 + h(x)\Theta]W - 1 - \Theta h(x_p) - \Theta x_\delta h'(x_\delta) = 0
\]

Comparing (12) and (4) it follows clearly that the graph of (4) lies below that of (12) if $\Theta h(x_p)W - x_p > 0$, that is, if $\bar{x}_p > x_p$. Q.E.D.

Since $\bar{x}_p > x_p^\ast$, it follows that the relevant part of the public firm's actual reaction curve lies below that of a profit maximising firm.
Lemma 4. \( x_p^- \times x_s^- = \infty \).

Proof. Consider the private firm first. As \( x_s \) goes to \( \infty \), the second term inside square brackets in (3') must go to zero for the FOC to hold. This implies that \( x_p \) converges to a finite value \( x_p^+ \) defined by:

\[
(9) \quad h'(x_p^+) = \frac{1}{\theta W}
\]

Consider now the public firm. Notice that (4) can be rewritten as:

\[
(4') \quad \theta x_p \left[ h'(x_s) - \frac{h(x_p)}{x_p} \right] + \theta h'(x_s)W - 1 - \theta h(x_s) + \theta x_s h'(x_s) = 0
\]

Since (A.3) and (A.4) imply that

\[
(10) \quad \lim_{x_p \to \infty} \frac{h(x_p)}{x_p} = 0
\]

it follows that as \( x_p \) goes to infinity \( h'(x_s) \) must go to zero for the FOC to continue to hold. But this implies that \( x_s \) goes to \( \infty \) by (A.5). This completes the proof of the Lemma. Q.E.D.
The content of Lemmas 1 to 5 is summarized in figure 1.

Before proceeding to characterize the Nash equilibrium of the mixed duopoly, it may be useful to locate in the \((x_p, x_s)\) space the social optimum and the private duopoly equilibrium. This can be done easily, since by the symmetry of the two firms and the concavity of the hazard function it follows that both points must lie on the 45° line. Thus, we have

**Remark 1.** The point where the public firm's reaction curve intersects the 45° line (point S in figure 2) corresponds to the social optimum, i.e. to the levels of R & D investment that would be chosen by a social planner with full control over both firms.

Analogously,

**Remark 2.** The point where the private firm's reaction curve intersects the 45° line (point P in figure 2) corresponds to the private duopoly equilibrium, i.e. to the levels of R & D investment that would result if both firms, behaving non cooperatively, maximised their profits.
Analytically, the level of R & D investment in the social optimum and in the private duopoly equilibrium are given, respectively, by the roots of (13) and (14) below:

\[(13) \quad \theta h'(x) (W + x) - 1 - 2\theta h(x) + \theta x h'(x) = 0\]

\[(14) \quad \theta h'(x) [1 + h(x)\theta]W - 1 - 2\theta h(x) + \theta x h'(x) = 0\]

We can now show that in the mixed duopoly a Nash equilibrium exists.

**Proposition 1.** There exists a Nash equilibrium \((x_p^*, x_3^*)\) of the R & D game described in section 2.

Proof. Follows trivially from lemmas 3 and 4 and continuity of the reaction curves. Q.E.D.

Having established existence, we now want to analyse which firm makes the largest R & D effort. An inspection of figure 1 reveals that the public firm is bound to invest less than the private firm in equilibrium. We now give a formal proof of this result.

**Proposition 2.** In a Nash equilibrium, the public firm invests in R & D less than the private one.

Proof. Subtracting (4) to (3) we get:

\[(15) \quad (\theta W + x_p) [h'(x_p) - h'(x_p)] + \theta [\theta h(x_3)h'(x_p)W - x_3 h'(x_3)] = 0\]
Now suppose, contrary to Proposition 2, that \( x_5^* \geq x_r^* \). Then the first term inside square brackets in (15) would clearly be non-negative. As for the second term inside square brackets, we have:

\[
(16) \quad \theta h(x_5)h'(x_r)W - x_5 h'(x_5) \geq h'(x_5) [\theta h(x_5)W - x_5]
\]

But \([\theta h(x_5)W - x_5] \) must be positive in equilibrium. To see why, suppose that \([\theta h(x_5)W - x_5] \leq 0\). Obviously, since we are in equilibrium and the private firm is maximising its profits, it must be \([\theta h(x_r)W - x_r] > 0\). But then the public firm could obtain a higher payoff setting \( x_5 = 0 \) as

\[
(17) \quad \frac{[\theta h(x_r)W - x_r] + [\theta h(x_5)W - x_5]}{\theta h(x_r) + \theta h(x_5) + r} \leq \frac{[\theta h(x_r)W - x_r]}{\theta h(x_r) + r}.
\]

Therefore it must be \([\theta h(x_5)W - x_5] > 0\), and this implies that the second term inside square brackets in (15) must also be positive. But then (15) cannot hold. This contradiction shows that \( x_5^* < x_r^* \). Q.E.D.

Proposition 2 shows that the public firm invests less than a private firm which competes against a public one. However, in order to determine whether the presence of a public firm is effective in reducing the "duplication of effort" pointed out by Loury (1979) and Lee and Wilde

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\( ^{6} \)The strict inequality follows by (A.6), which implies that the marginal productivity of R & D investment is very high for a low level of R & D expenditure.
(1980), one has to compare the mixed duopoly equilibrium with the private duopoly equilibrium, i.e. the equilibrium that would arise if both firms maximised profits. This is what we do now.

Proposition 3 shows that the mixed duopoly equilibrium does indeed entail a reduction of investment compared to the private duopoly, and Proposition 4 proves that this implies a welfare gain. However, in Proposition 5 we show that the mixed duopoly equilibrium is still characterised by overinvestment in R & D, even if to a lower extent than a private duopoly.

**Proposition 3. In a mixed duopoly, each firm invests in R & D less than in a private duopoly.**

Proof. From Lemmas 1 and 5 it follows directly that the private firm invests in R & D less than it would have invested if it competed against a private opponent; Proposition 2 implies that the reduction in investment for the public firm is even stronger. Q.E.D.

Since the private duopoly is characterised by overinvestment in R & D, one may conjecture that a reduction of R & D investment is associated to a welfare improvement. This is what Proposition 4 shows.

**Proposition 4. Social welfare is higher in a mixed duopoly than in a private duopoly.**
Proof. We prove this Proposition showing that it is possible to move from point P (the private duopoly equilibrium) to point M (the mixed duopoly equilibrium) through a chain of welfare improving steps (see figure 2).

Consider first the move from point P to point Q, which has the same abscissa as P but lies on the public firm's reaction curve. Given the private firm's level of investment, the public firm could have chosen point P; if it chooses point Q, it must be because social welfare is higher in Q than in P.

Next consider the move from point Q to point Z along the 45° line QZ. Point Z has the same abscissa as M, and lies to north-west of point Q (this follows from Proposition 3). Total investment in R & D is unaffected by this move; however, in so far as point Z lies below the 45° line OSP, total investment is spread more evenly among the two firms. Since investment in R & D is subject to decreasing returns, the expected date of innovation would be closer and thus social welfare is enhanced by this move.

Finally, consider the move from point Z to point M. This move is welfare improving by the same argument used to establish that point Q is socially preferred to point P.

This completes the proof of the Proposition. Q.E.D.

**Proposition 5. The mixed duopoly equilibrium is still characterised by overinvestment in R & D with respect to the social optimum.**

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7 If point Z were to lie over the 45° line OSP, the same argument could be applied but it would be necessary to consider a greater number of steps.
Proof. Follows easily from Proposition 2, Lemma 2 and Remark 1. Q.E.D.

5. Concluding remarks

In this paper we have investigated the role of a public enterprise as an instrument to be used by a regulator to shrink the gap between the market level of R & D and the socially optimal level.

Our conclusions have been derived within a setting that seems worth extending in at least two directions. First of all, by relaxing the assumption that the private benefit from the innovation coincides with the social benefit. This is clearly a strong hypothesis, which could be removed by explicitly distinguishing between gains to the private innovator (extra profits) and benefits to consumers. This would require to model the competition between different firms in the product market which is going to be affected by the innovation.

Second, we have considered a fixed number of firms. A natural extension would be to examine the non cooperative equilibrium with unrestricted entry and see whether the presence of a public firm may reduce the excess of entry that characterises the Nash equilibrium (see Loury (1979) and Lee and Wilde (1980)).

Moreover, it would be interesting to provide a firmer justification for the use of a public firm as the policy instrument. As we noted in the introduction, other instruments (like taxes or incentive schemes: see Wright (1983)) are likely to allow the policymaker to force the first best, but may require information that an external regulator may find difficult to collect. Following Shapiro and Willig (1990), one could analyse the choice of the best policy instrument under different informational structures. The point is that using a public firm as a policy instrument may enrich the information set of the policymaker, and this may more than offset the relative
inferiority of such an instrument.
REFERENCES


Figure 1