A mixed firm objective function: some instances of bizarre behaviour

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ABSTRACT

We analyse the objective function of a **mixed firm**, i.e. a firm whose controllers have different objectives (profit and value added per worker). It seems that the optimal output behaves in a bizzarre manner near critical values of a weight parameter. Output jumps up or down whenever we approach the critical value respectively from above or from below.

1. INTRODUCTION

It is well known that, in the short run, labour - managed (LM hereafter) monopolies are flawn by endemic underproduction whenever fixed costs are not too high (see for instance Ireland - Law, 1982; Bonin - Putterman, 1987). Moreover, the LM equilibrium output shows the traditional perverse effect with respect to market size (or price) change, i.e., output shrinks when demand increases.

If one aims at giving some weight to consumers' interest, it should appear desirable to alter the LM firm behaviour in a way resulting in output expansion and therefore lower prices. Such a change of behaviour could be reached by introducing an appropriate (i.e., output enhancing) argument in the original objective function (a similar approach is adopted in Delbono - Rossini, 1990).

As both a profit seeking monopolist and a welfare maximising monopolist would produce an equilibrium level of output greater than the LM monopoly, a horizontal merger between the latter and one of the former monopolists, leading to the creation of a so called **mixed firm**, will clearly give rise to output enhancing solutions.

The purpose of this note is twofold. On one side we wish to show that a tiny change in the original LM firm's maximand, as those envisaged above, can give rise to a huge change in output. On the other hand, we also show that the new equilibrium resulting from such changes may display regular (i.e., non perverse) responses to demand increases. Hence the allocative losses due to the LM short run behaviour may be neutralized, in the mixed firm, by giving some weight to consumers' interests.

Some implications of these findings are not only interesting on their own, yet turn out to be of some relevance for policy issues.

2. THE SETTING

Let us consider an industry monopolized by a two plant LM firm whose technology is:

$$q_i = \sqrt{l_i} \quad i = 1,2 \tag{1}$$

where q_i is the output of plant i, l_i is labour in plant i.

Costs are then represented by the following convex cost function

(we have normalized money wage setting $w = 1$):

$$c(q_i) = F + q_i^2 \tag{2}$$

where F are fixed costs.

The inverse demand function is

$$p = a - Q \tag{3}$$

where $Q = \Sigma_i q_i$ while a is a market size parameter.

The objective function of the LM firm is

$$V_c = \frac{pQ - 2F}{L} \tag{4}$$

where $L = l_1 + l_2$

Optimal output of the LM firm is:

$$Q^*_c = \frac{4F}{a} \quad \text{if} \quad a^2 \geq 12F \tag{5}$$

$$= 0 \quad \text{otherwise}$$

The solution satisfies the condition of increasing returns (see Cremer - Cremer, 1990).

It is worth noting that in the same industry a two plant profit maximizer monopolist would maximize:

$$V_p = pQ - c(Q) \tag{6}$$

Assuming the same costs of the previously considered LM two plant monopolist, the profit seeking monopolist will produce

$$Q^*_p = a/3 \quad \text{if} \quad a^2 \geq 12F \tag{7}$$
$$= 0 \quad \text{otherwise.}$$

A monopolist, maximizing social welfare under a break - even constraint, should maximize

$$V_s = S_c + \pi \tag{8}$$

where S_c is the consumers' surplus and π is profit. Then it will produce:

$$Q^*_s = a/2 \quad \text{if} \quad a^2 \geq 16F \tag{9}$$
$$= 0 \quad \text{otherwise}$$

Let us now consider the effects of changing (4) by giving some weight to either (6) or (8).

The arrangements may be thought of as the result of some kind of horizontal merger between firms identical in all respects but the maximand. A rather straightforward way of figuring out the merger objective function is to devise a weighted average of partners objective functions (for another solution see Prasnikar - Svejnar - Mihaljek - Prasnikar, 1990; Rossini - Scarpa, 1991).

Then the new maximand when the ingredients are (4) and (6) becomes:

$$V_{cp} = \alpha V_p + (1 - \alpha) V_c \tag{10}$$

whereas it becomes

$$V_{cs} = \beta V_s + (1 - \beta) V_c \tag{11}$$

when the merger concerns (4) and (8).

Clearly, both α and β belong to the interval (0, 1).

Let us now perform some comparative statics on the first order condition (FOC) emerging from (10); the same qualitative conclusions will apply to the FOC resulting from (11).

The maximization of (10) with respect to Q yields:

$$\alpha(a-3Q)+\frac{2(1-\alpha)(4F-aQ)}{Q^3}=0 \tag{12}$$

Let Q* be the solution(s) of (12). Then we have:

$$\frac{dQ^*}{d\alpha}=-\frac{dh(Q)/d\alpha}{dh(Q)/dQ}=^s dh(Q)/d\alpha>0 \tag{13}$$

where $=^s$ reads: "has the same sign of"; *h(Q)* is the left hand side of (12) and we are taking advantage of the second order condition (SOC).

Moreover,

$$\frac{dQ^*}{dF}=^s dh(Q)/dF>0 \tag{14}$$

and

$$\frac{dQ^*}{da}=^s dh(Q)/da> \quad \text{or} \quad <0 \quad \text{as} \quad \alpha> \quad \text{or} \quad <\frac{2}{Q^2+2} \tag{15}$$

From the comparative statics above we can state the following two claims:

CLAIM 1. In the feasible region of the parameters (a, F) increasing the weight of profit seeking α boosts output.

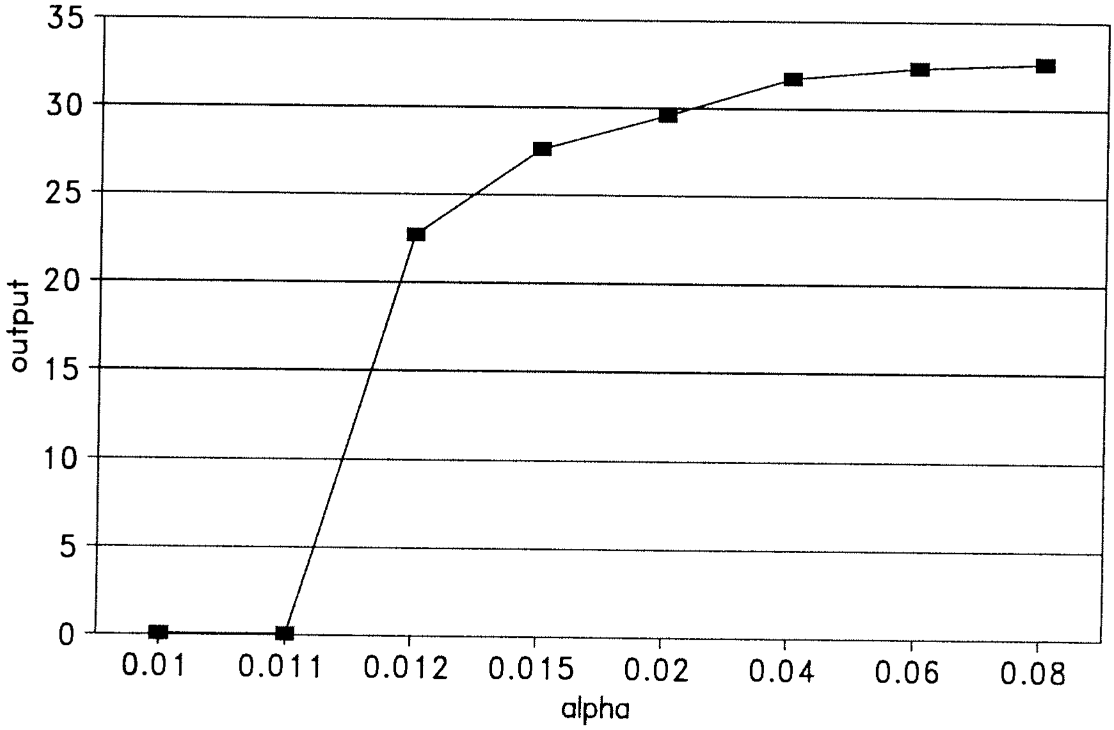
This is rather obvious as the solution given by (7) is always greater than or equal to the solution given by (5).

CLAIM 2. Profit seeking will not turn the positive (perverse) response of Q* to fixed costs into a negative one.

This depends on (5) being an increasing function of F and on (7) being independent of F.

Since (12) is not amenable to any analytical treatment, we resorted to numerical analysis. We have scanned the solutions for many intervals of parameters. We present in figure 1 a diagram of Q as a function of α around its critical value (as from (15)) for a set of parameters.

FIGURE 1 (F/a=1/100)
Merger output around critical alpha



From numerical analysis we can derive another claim:

CLAIM 3. In some regions of (a, F) there is a critical value of α (derived in (15)) such that in a left neighbourhood of Q^* tendes to $4F/a=Q^*_c$, and in a right neighbourhood of it Q^* tends to $a/3 = Q^*_p$.

An illustration of this claim is visualized in figure 1.

Moreover it is worth noting that, in the neighbourhood of critical α it happens that $2/(Q^{*2}+2) < \alpha$ so that, by (15), $dQ^*/da > 0$ and the perverse effect disappears.

The reason why figure 1 displays the jump after α critical has something to do with the solutions of (12). A fourth degree polynomial has four roots. From our scanning it appears that when we are far from the critical value of α there is only one acceptable solution since the others are either negative or complex. However, in the neighbourhood of critical α there are usually 3 positive roots. We have been able to drop two roots out of 3 simply by inspecting the second order condition (SOC). Figure 1 has been drawn after having done this selection of roots. The only admissble root is the one responsible for the sudden jump.

3. CONCLUSIONS

We tried to analyse the behaviour of the maximand of a mixed firm in a monopoly market. The mixed firm is the result of joining LM with either profit seeking or welfare maximizing.

For several pairs of parameters (a, F) there is a critical value of α representing the weight of the two firms in the objective of the mixed firm[1]. In a neighbourhood of this critical value the output of the mixed firm switches suddenly from the LM level to the profit seeking level.

Moreover in the right neighbourhood of critical α we do not observe the so-called perverse effect (i.e., output responds positively to increasing market size).

REFERENCES

BONIN J. - L. PUTTERMAN (1987), *Economics of Cooperation and the Labour -Managed Economy*, Harwood Academic Publishers, London.

CREMER H. - J. CREMER (1990), Employee Control and Oligopoly in a Free Market Economy, *Paper presented at the ACES annual conference: Washington D.C., 28-30 December 1990.*

DELBONO F. - G. ROSSINI (1990), Competition Policy versus Horizontal Merger with Public, Entrepreneurial and Cooperative Firms, *Working Paper n. 28, Istituto di Scienze Economiche, University of Verona.*

IRELAND N. - P. LAW (1982), *The Economics of Labour Managed Enterprises*, Croom Helm, London.

1 We have to warn the reader that α has no measure meaning since the objective function of the mixed firm is a linear combination of two heterogeneous objectives. Therefore, the critical value of α is important for the bizzarre behaviour around it, yet not for its level.

PRASNIKAR J. - J. SVEJNAR - D. MIHALJEK - V. PRASNIKAR (1990), A Test of Enterprise Behaviour under Labour Management, *Paper presented at the annual ACES conference, Washington D.C. 28-30 December 1990.*

ROSSINI G. - C. SCARPA (1991), The Transformation of a Labour Managed Monopoly: Competition Policy versus Mixed Firms, *Working Paper n. 102, Dipartimento di Scienze Economiche, University of Bologna.*