Quality Choice in a Vertically Differentiated Mixed Duopoly+

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Abstract

We model a duopoly with a private and a public firm under the hypothesis of vertical product differentiation. Firms choose their quality levels first and then prices. We ask which firm will choose to serve the higher (lower) segment of the market. When firms act simultaneously in each stage, there are two subgame perfect Nash equilibria entailing opposite rankings between the quality levels. If the State-owned firm has a move advantage, then there is a unique Stackelberg equilibrium in which the public firm serves the upper segment of the market.

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1. Introduction

In most Western economies, there are oligopolistic industries where State-owned enterprises compete with private companies (see Parris _et al._, 1987, for an excellent account). The banking sector, the insurance industry, the oil industry are significant examples. These industries are called _mixed oligopolies_ in that the objective functions of the operating firms differ: the publicly owned firm aims at maximizing social targets, whereas private firms maximize profits.

The primary goal of this paper is to analyze a mixed duopoly in which the private firm and the public one compete both in quality levels and prices. Specifically, following Shaked and Sutton (1982) - where a private oligopoly is considered - we model a duopolistic industry in which technology and consumers' preferences allow for _vertical_ differentiation. In such a setting, it is of some interest to ask which firm will choose to serve the higher (lower) segment of the market: should a public firm serve the low segment of the market, allowing the private one to "skim the cream"? Casual empiricism suggests that public enterprises usually supply relatively low quality levels (see Kamerman and Kahn, 1989, for some examples), but the rationale for such behaviour is far from obvious.

The small but growing literature on mixed oligopolies typically analyzes an industry where one public firm maximizes social welfare competing with a small number of profit maximizing companies (see De Fraja and Delbono, 1990, for a survey). It is usually assumed that firms compete in output levels. This choice is quite natural when the product is assumed to be homogeneous, as it allows one to avoid the unpalatable conclusion which would emerge under price competition (the equilibrium being the same as under perfect competition). Of course, allowing for product differentiation would whitewash the interest for price competition.

Our results seem to suggest that the case for a public firm to serve the lower segment of the market is quite weak. When firms act simultaneously there are two subgame perfect Nash equilibria [1], entailing opposite rankings between the quality levels: the public firm might supply either the lowest or the highest quality level. However, if the State-owned firm has a move advantage, then there is a unique Stackelberg equilibrium in which the public firm serves the upper segment of the market. If the public firm is to be used as a regulatory instrument, it seems natural to look at the latter setting as the most plausible one.

The paper is organized as follows. After a brief presentation of the model (Section 2), we examine the choices made by a private or a public monopolist when they can produce two qualities (Section 3). We then consider a private duopoly, under the assumption that each firm produces and sells one quality level only (Section 4). Next we turn to the analysis of a mixed duopoly. In Section 5 we show that in a mixed duopoly, when firms compete in prices, the public firm may find it optimal to deviate from marginal cost pricing. In Section 6 we consider the subgame perfect Nash equilibria of the game, where the welfare maximizing firm and the profit maximizing one act non-cooperatively and choose quality first, and then prices. We then analyze the case in which the public firm is the Stackelberg leader in the quality choice (Section 7). Section 8 summarizes and comments the results while Section 9 contains some concluding remarks.

2. The Model

We consider a vertically differentiated market in which two goods of quality θ_i ($i = 1, 2$) are supplied. Production costs are [2]:

(1) $C(\theta,x) = \theta^2 x$

the same for both goods, where x denotes quantity.

In modelling the demand side of the industry, we follow Shaked and Sutton (1982). There is a continuum of consumers whose total number is normalized to one. The good is indivisible and each consumer buys at most one unit of it. Preferences are represented by the following utility function:

(2) $U = m\theta_i - p_i$

where p_i is the price of good i. Consumers are indexed by m, which is uniformly distributed over the interval [0,1]; the parameter m can be interpreted as the marginal willingness to pay for quality.

In general, consumers can be divided into three groups: those who do not buy anything (getting $U = 0$), those who buy the <u>low</u> quality good (θ_L), and those who buy the <u>high</u> quality good (θ_H). Let us denote by k and h (with $h \geq k$) the marginal consumers, i.e., the two critical thresholds in the interval [0,1] which separate the three groups (see figure 1 below).

(figure 1 about here)

Clearly,

$$(3) \quad k = \frac{p_L}{\theta_L}$$

$$(4) \quad h = \frac{p_H - p_L}{\theta_H - \theta_L}$$

It follows that the demand functions for the high quality good and the low quality good are given, respectively, by:

$$(5) \quad x_H = (1 - h) = \frac{(\theta_H - \theta_L) - (p_H - p_L)}{\theta_H - \theta_L}$$

and

$$(6) \quad x_L = (h - k) = \frac{\theta_L p_H - \theta_H p_L}{\theta_L(\theta_H - \theta_L)}$$

Of course, we will have to require these expressions to be positive in equilibrium. Notice that both demand functions are linear in prices, given quality levels.

3. Monopoly

For future reference, let us first determine the quality levels that would be chosen by a social planner that can produce two distinct goods. The social planner maximizes social welfare, defined as the sum of total profits and consumers' surplus:

$$(7) \quad W = \int_k^h [m\theta_L - \theta^2{}_L]\, dm + \int_h^1 [m\theta_H - \theta^2{}_H]\, dm$$

$$= x_L \left[\frac{(h+k)\theta_L}{2} - \theta^2{}_L \right] + x_H \left[\frac{(1+h)\theta_H}{2} - \theta^2{}_H \right]$$

Clearly, the social planner will price at marginal cost. As regards quality choice, maximizing (7), it is easy to show that the quality levels are:

(8) $\theta_H = 0.400$ and $\theta_L = 0.200$

and the corresponding prices are:

(9) $p_H = 0.160$ and $p_L = 0.040$

It is worth noting that the quality levels given by (8) solve the maximization problem of a profit-maximizing firm as well. The objective function of such a firm is:

(10) $\pi = [p_H - \theta^2{}_H]x_H + [p_L - \theta^2{}_L]x_L$

A profit maximizing firm will price above marginal cost; more precisely, equilibrium prices in this case are:

(11) $p_H = 0.227$ and $p_L = 0.075$

However, as already mentioned, a profit-maximizing firm will choose the same quality levels as the social planner. This feature of the model relies upon the linearity in prices of demand curves (5) and (6), which is due to

Shaked and Sutton's assumptions on preferences (cf. Spence, 1975, fn. 7). This will be relevant in the evaluation of some of our results (see Section 8).

4. Private Duopoly

We now consider a private duopoly. Each firm maximizes:

$$(12)\quad \pi_i = [p_i - \theta^2{}_i]x_i \qquad i = 1,2$$

By symmetry, there will be two equilibria, depending on which firm chooses the higher quality. Let us denote by H the high quality firm, and by L the low quality one. Using (5) and (6), we obtain the following expressions:

$$(13)\quad \pi_H = \left(1 - \frac{p_H - p_L}{\theta_H - \theta_L}\right)(p_H - \theta^2{}_H)$$

and

$$(14)\quad \pi_L = \frac{p_H\theta_L - p_L\theta_H}{\theta_H(\theta_H - \theta_L)}\,(p_L - \theta^2{}_L)$$

Following Shaked and Sutton (1982), we model the duopolistic interaction as a noncooperative two-stage game. In the first stage, firms set quality. In the second stage, they compete in prices in the product market. We consider the subgame perfect Nash equilibria in which firms act simultaneously in both stages.

Solving the model backwards, we obtain first the reaction functions in prices, which are:

$$(15) \quad p_H = \frac{1}{2}\,(p_L + \theta^2_H + \theta_H - \theta_L)$$

and

$$(16) \quad p_L = \frac{\theta_L}{2\theta_H}\,(p_H + \theta_H\theta_L)$$

Given these expressions, we can solve for the Nash equilibrium prices of the second stage of the game as functions of the quality levels:

$$(17) \quad p_H = \frac{\theta_H}{4\theta_H - \theta_L}\,(\theta^2_L + 2\theta^2_H + 2\theta_H - 2\theta_L)$$

$$(18) \quad p_L = \frac{\theta_L}{4\theta_H - \theta_L}\,(\theta^2_H + 2\theta_H\theta_L + \theta_H - \theta_L)$$

Substituting (17) and (18) into the objective functions (13) and (14), and taking the first order conditions (FOCs), we get the reaction functions of the high quality and low quality firm, respectively, at the first stage of the game:

$$(19) \quad 48\theta^4_H - 4\theta^3_H - 4\theta^2_H(3\theta^2_L - 12\theta_L - 4) + 3\theta_H\theta_L(\theta_L - 2)(3\theta_L + 2) + \\ + 2\theta^2_L(\theta_L - 2)^2 = 0$$

and

$$(20) \quad 8\theta^4_H - 4\theta^3_H(9\theta_L - 4) + \theta^2_H(51\theta^2_L - 48\theta_L + 8) + \\ + 2\theta_H\theta_L(13\theta^2_L - 18\theta_L + 6) + \theta^2_L(\theta_L - 1)(3\theta_L - 1) = 0$$

These expressions can be solved by numerical computation, yielding the equilibrium quality levels:

(21) $\theta_H = 0.469$ and $\theta_L = 0.305$

Equilibrium prices are $p_L = 0.130$ and $p_H = 0.257$. Firm H's profit is 0.008 while firm L has a profit equal to 0.013. The welfare level is 0.0756. Notice that the distortion usually associated with oligopolistic competition here is expressed both in a positive price-cost margin and in quality levels, with a clear tendency to excessive quality upgrading.

5. Optimal pricing by a public firm in a mixed duopoly

Let us now turn to the analysis of a mixed duopoly, in which a profit maximizing firm (P) competes against a State-owned firm (S) which maximizes social welfare, i.e., the sum of consumers' and producers' surpluses.

The first thing to notice is that in a mixed duopoly the public firm does not necessarily find it optimal to price at marginal cost. To illustrate this point in a more general setting, let us write the usual social welfare function as:

(22) $W = GS(x_S, x_P) - C_S(x_S) - C_P(x_P)$

where GS denotes total gross surplus and C_i is the total cost of firm i ($i = P, S$). In a Bertrand equilibrium, the public firm will maximize (22) with respect to price, taking p_P as given. Therefore, the FOC is:

$$(23) \quad \frac{\delta GS}{\delta x_S}\frac{\delta x_S}{\delta p_S} + \frac{\delta GS}{\delta x_P}\frac{\delta x_P}{\delta p_S} - \frac{\delta C_S}{\delta x_S}\frac{\delta x_S}{\delta p_S} - \frac{\delta C_P}{\delta x_P}\frac{\delta x_P}{\delta p_S} = 0$$

Denoting with C'_i i's marginal cost, since $\delta GS/\delta x_i = p_i$ equation (23) reduces to:

$$(24) \quad (p_S - C'_S)\frac{\delta x_S}{\delta p_S} + (p_P - C'_P)\frac{\delta x_P}{\delta p_S} = 0$$

It follows that, unless the two goods are completely independent, the public firm prices at marginal cost if and only if the private firm does the same. In the case we are considering, the derivatives of x_S and x_P with respect to p_S will depend on the relative position of the two goods in the quality space, but are in any case different from zero. More precisely, since the output of the private firm is a decreasing function of the price set by the public firm, the public firm will price above marginal cost, as long as the private one makes positive profits. We shall see in the next section how this condition applies to our model.

However, this simple result is quite general and deserves some further comments. In the first place, it is interesting to contrast our finding to the existing literature on mixed oligopolies (see De Fraja and Delbono, 1990). Under quantity competition and without budget constraint, it is quite well known that in a Nash equilibrium it is optimal for the public firm to price at marginal cost (Sheshinsky, 1986). On the other hand, in a Stackelberg equilibrium where the public firm acts as leader, pricing at marginal cost is not optimal (De Fraja and Delbono, 1989). The reason is that in a Cournot-Nash equilibrium $\delta x_P/\delta x_S = 0$, whereas in a Stackelberg equilibrium the public firm is able to "move" some of its production to the private one (so that $\delta x_P/\delta x_S \neq 0$).

The case of price competition, as noted in the Introduction, has been paid little attention because the literature has concentrated especially on the case of homogeneous products. In this situation an oligopoly equilibrium, when it exists, always entails marginal cost pricing. Our result can be interpreted by saying that in general with price competition $\delta x_P/\delta x_S \neq 0$ and thus the public firm prices above marginal cost. This holds as long as products are differentiated, so that in equilibrium $\delta x_P/\delta p_S$ (and thus $\delta x_P/\delta x_S$) actually exist.

A tentative conclusion is therefore that marginal cost pricing applies only when the public firm perceives that the rival's output will not react to a change in the public firm's strategic variable (i.e., when the "conjectural variation" coefficient is equal to zero). This holds, for instance, when the public firm acts in isolation (e.g. it is a monopolist). In games with simultaneous moves (without imposing a budget constraint on the public firm) there are two crucial factors. The first one is the degree of product differentiation: when products are homogeneous, in a mixed oligopoly equilibrium the public firm shall price at marginal cost. Without homogeneity, a second important factor is the strategic variable considered: marginal cost pricing prevails in a Cournot-Nash equilibrium, while a deviation from it is optimal with price competition.

6. The mixed duopoly: subgame perfect Nash equilibria

We are now ready to study the choice of price and quality levels in a mixed duopoly. Since we consider a two-stage game with complete information, we adopt the notion of subgame perfect equilibrium. Proceeding backwards, we consider first the Nash equilibrium in prices of the second stage of the game, and then proceed to the determination of quality levels. From our previous remarks it follows that modelling market competition as a Bertrand game might be crucial to some of our results.

A firm's decision on its quality level can be decomposed into two "stages". In the first place, a firm has to decide which market segment to serve; then, given its relative position in the quality space, it has to decide the actual quality level. Therefore, also because a firm's demand function is different depending on its relative position in the quality space, we shall distinguish two cases, depending on whether θ_P is higher or lower than θ_S. In each case, we are able to identify what would be the best response of each firm given its market segment. However, in order to define the actual reaction function, we will have to check for what range of the parameters that position is actually optimal.

Case I: $\theta_P > \theta_S$

The objective function for firm P is given by (13), setting H = P and L = S. Maximizing (13) with respect to p_P yields the FOC in prices, which is (15). On the other hand, the public firm chooses p_S to maximize social welfare. This yields:

$$(25)\quad p_S = \theta^2{}_S + \frac{\theta_S}{\theta_P}(p_P - \theta^2{}_P)$$

Clearly, in line with the remarks contained in the previous section, the price of the public firm is greater than its marginal cost, as long as $p_P > \theta^2{}_P$. Notice that

$$\frac{\theta_S}{\theta_P} = - \frac{\delta x_P/\delta p_S}{\delta x_S/\delta p_S}$$

so that (25) can be directly derived from (24). Comparing (25) to (16), one can check that the public firm's pricing policy is not necessarily more aggressive than the one pursued by a private firm. When $\theta_P > \theta_S$, a public firm would charge a lower price than a private one when $p_P = 0$, but its best response increases more steeply when p_P increases. Nash equilibrium prices are:

$$(26) \quad p_P = \frac{\theta_P}{2\theta_P-\theta_S}(\theta^2_P + \theta^2_S - \theta_P\theta_S + \theta_P - \theta_S)$$

and

$$(27) \quad p_S = \frac{\theta_S}{2\theta_P-\theta_S}(2\theta_P\theta_S - \theta^2_P + \theta_P - \theta_S)$$

Given (26) and (27), output levels become:

$$(28) \quad x_P = \frac{\theta_P}{2\theta_P-\theta_S}(1 - \theta_P - \theta_S)$$

and

$$(29) \quad x_S = \theta_P$$

For the private output level to be positive we have to assume:

$$(30) \quad \theta_P + \theta_S < 1$$

Thus, in the first stage of the game the private firm chooses θ_P to maximize the following expression:

$$(31) \quad \pi_P = \frac{\theta^2_P}{(2\theta_P-\theta_S)^2}(\theta_P - \theta^2_P - \theta_S + \theta^2_S)(1 - \theta_P - \theta_S)$$

The FOC gives:

$$(32) \quad 6\theta_P^4 + (\theta_S+8)\theta_P^3 - 2(3\theta^2_S-4\theta_S-1)\theta^2_P + 3\theta_P\theta_S(\theta^2_S-1) + 2\theta^2_S(\theta_S-1)^2 = 0$$

Given (30), equation (32) admits only one acceptable value of θ_P for any admissible value of θ_S. Moreover, numerical computation in the relevant range shows that θ_P is an increasing function of θ_S if and only if $\theta_S > 0.09$. As mentioned at the beginning of this section, however, this cannot yet be considered a reaction function, as we have to check whether to set $\theta_P > \theta_S$ is at all optimal for firm P. As we shall see analyzing the case $\theta_P < \theta_S$, (32) represents the best response for firm P only when an additional condition is met.

Quite obviously, something analogous holds for the public firm as well. In the first stage of the game its objective function is:

$$(33) \quad W = \frac{\theta^2_P}{(2\theta_P-\theta_S)^2}\{3\theta^2_P(1 - \theta^2_P) + 5\theta_P\theta^2_S(\theta_S - \theta_P) + 2\theta_S[(\theta_S - \theta_P)^2 + \\ - \theta_P + 2\theta^2_P - \theta^3_S]\}$$

Maximizing (33) with respect to θ_S we obtain the FOC of the public firm:

$$(34) \quad 4\theta^4_S - (21\theta_P + 2)\theta^3_S + (30\theta^2_P + 12\theta_P)\theta^2_S + \\ - (20\theta^3_P + 10\theta^2_P + 2\theta_P)\theta_S + 6\theta^4_P + 2\theta^2_P = 0$$

which defines implicitly the optimal value of θ_S given θ_P and $\theta_S > \theta_P$. Given (30), equation (34) admits only one economically acceptable value of

θ_S for any admissible value of θ_P. Moreover, numerical computations show that this solution is an increasing function of θ_P. Again, however, this does not necessarily represent a part of the reaction function of firm S. To discuss this aspect, we must now turn to the analysis of the case in which $\theta_P < \theta_S$.

Case II: $\theta_P < \theta_S$

The private firm's objective function is now (14), setting H = S and L = P. Maximizing (14) with respect to p_P yields (16). The public firm still maximizes (7), and the FOC in the second stage of the game is:

$$(35) \quad p_S = \theta^2{}_S + (p_P - \theta^2{}_P)$$

Again, we can see that the public firm prices above marginal cost. Analogously to what we observed earlier on, equation (35) can be derived from (24) as:

$$- \frac{\delta x_P/\delta p_S}{\delta x_S/\delta p_S} = 1$$

A comparison between (35) and (15) indicates that for $p_P = 0$ the pricing policy of the public firm is more aggressive than the private firm's. However, p_S increases with p_P more rapidly than a private firm's price would, exactly as in the case $\theta_P > \theta_S$.

Using (16) and (35), it turns out that equilibrium prices are:

$$(36) \quad p_P = \frac{\theta_P}{2\theta_S - \theta_P} (\theta^2{}_S - \theta^2{}_P + \theta_S\theta_P)$$

and

$$(37)\quad p_S = \frac{\theta_S}{2\theta_S-\theta_P}(2\theta^2{}_S - \theta^2{}_P)$$

Equilibrium output levels are:

$$(38)\quad x_S = 1 - \theta_P - \theta_S$$

$$(39)\quad x_P = \frac{\theta^2{}_S}{2\theta_S-\theta_P}$$

Clearly, x_P is always positive, while (30) still ensures that the output of the high quality firm (here firm S) is positive.

In the first stage of the game the private firm's profit is:

$$(40)\quad \pi_P = \frac{\theta_P\theta^3{}_S(\theta_S - \theta_P)}{(2\theta_S - \theta_P)^2}$$

Maximization of (40) with respect to θ_P yields:

$$(41)\quad \theta_P = 2\theta_S/3$$

As regards the public firm, in the first stage of the game, its objective function becomes:

$$(42)\quad W = \frac{\theta_S}{2(2\theta_S-\theta_P)^2}(4\theta^4{}_S - 8\theta^3{}_S - \theta_S{}^3\theta_P - 5\theta^2{}_S\theta^2{}_P + 8\theta^2{}_S\theta_P + 4\theta^2{}_S + - 4\theta_S\theta_P - 2\theta_S\theta^2{}_P + 4\theta_S\theta^3{}_P + \theta^2{}_P - \theta^4{}_P)$$

Maximizing (42) with respect to θ_S we obtain the FOC:

$$(43)\quad 24\theta_S^5 - 32\theta_S^4 - 24\theta_S^4\theta_P - 6\theta_S^3\theta_P^2 + 48\theta_S^3\theta_P + 8\theta_S^3 + 15\theta_S^2\theta_P^3 + \\ - 24\theta_S^2\theta_P^2 - 12\theta_S^2\theta_P + 6\theta_S\theta_P^2 + 4\theta_S\theta_P^3 - 6\theta_S\theta_P^4 - \theta_P^3 + \theta_P^5 = 0$$

which implicitly defines the optimal value of θ_S given θ_P and $\theta_S > \theta_P$. Again, given (30), equation (43) admits only one economically acceptable value of θ_S for any admissible value of θ_P. Moreover, numerical computations indicate that this expression is always increasing in θ_P within the relevant range.

Reaction functions and equilibria

So far we have analyzed the two cases separately, determining each time the FOCs for the two firms in the first stage of the game; we can now determine the actual reaction functions. To this end, we shall check for firm i for what ranges of θ_j it is optimal to set a quality level greater or smaller than θ_j.

Starting from the private firm, using expressions (31), (32), (40) and (41) we can now determine the reaction function for θ_P.

Fact 1. The reaction function of the private firm in the first stage of the game is implicitly defined by (32) when $\theta_S \leq 0.323$ and is given by (41) when $\theta_S \geq 0.323$.

This result can be proved by numerical computation, calculating the profit levels corresponding to different points along (32) and (41) as θ_S varies. Thus, when θ_S is "low enough", the reaction function of the private firm is given by (32), as firm P would prefer to serve the higher segment of the market. When $\theta_S > 0.323$ the private firm will maximize profits by producing the lower quality, and its best response is described by (41). The reaction curve of the private firm (R_P) is drawn in figure 2 below.

(figure 2 about here)

Analogously, we can now turn to the public firm. Using expressions (33), (34), (42) and (43) we can determine its reaction function in the first stage of the game.

Fact 2. The reaction function of the public firm in the first stage of the game is implicitly defined by (34) when $\theta_P \geq 0.327$ and by (43) when $\theta_P \leq 0.327$.

Again, this result can be proved by numerical computation, calculating the welfare levels corresponding to different points along (34) and (43) as θ_P varies.

Thus, when θ_P is "low enough", the public firm would choose to serve the higher segment of the market. When $\theta_P > 0.327$ the public firm will maximize welfare by producing the lower quality, and its best response is described by (34). The reaction curve of the public firm (R_S) is drawn in figure 3 below.

(figure 3 about here)

We are now ready to prove the existence of equilibria of our game.

Proposition 1. There exist two subgame perfect Nash equilibria in pure strategies, denoted by (E.1) and (E.2).
In (E.1), $\theta_P = 0.380 > \theta_S = 0.259$, while $p_P = 0.177$ and $p_S = 0.089$.
In (E.2), $\theta_P = 0.260 < \theta_S = 0.390$, while $p_P = 0.093$ and $p_S = 0.177$.

Proof. The existence of Nash equilibria is a delicate issue, since both reaction functions are discontinuous. (E.1) is obtained by solving first equations (32) and (34). Because of the constraint (30) the solution is unique. From Fact 2 we know that when $\theta_P = 0.380$, W is maximized setting $\theta_S < \theta_P$, so that (34) is the relevant part of the public firm's reaction function. On the other hand (see Fact 1), given $\theta_S = 0.259$ profits are maximized setting $\theta_P > \theta_S$, so that (32) is really the relevant part of the private firm's reaction function. Prices are then obtained using (26) and (27).

(E.2) is obtained solving first equations (41) and (43). Under the constraint (30) the solution is still unique. Again, given $\theta_P = 0.260$, W is maximized when $\theta_S > \theta_P$. Moreover, given $\theta_S = 0.390$, profits are maximized setting $\theta_P < \theta_S$. Equilibrium prices are then obtained using (36) and (37). In both cases it is possible to show that second order conditions are at least locally met.∎

Figure 4 illustrates the content of the Proposition.

(figure 4 about here)

Comparing the two subgame perfect Nash equilibria, it is possible to observe that the lowest θ_L is supplied in (E.1), when the public firm serves the lower segment of the market; analogously, the highest θ_H is supplied in (E.2), when the public firm serves the upper segment of the market. Furthermore, available quality levels are maximum in (E.2), and minimum in (E.1).

Both mixed duopoly equilibria yield less extreme quality levels than monopoly situations. As regards a comparison to the private duopoly equilibrium, one can observe that quality levels are closer between them when a public firm operates in the duopoly. This means that setting-up a mixed duopoly decreases product diversity. As regard quantities, the size of the market (i.e., the fraction of consumers served) is higher in (E.1), when the public firm serves the lower segment of the market [(3)].

Since there are two subgame perfect equilibria, the problem arises as to which one would be selected. It could be argued that, if one equilibrium Pareto dominates the other one, the former would be more likely to be chosen. To analyze this issue, let us now calculate which one of the two equilibria is preferred by the public firm and by the private firm, respectively. Easy calculations show that in (E.1), (i.e., when $\theta_P > \theta_S$), $\pi_P = 0.00907$ and $W = 0.07755$, whereas in (E.2) (with $\theta_P < \theta_S$), $\pi_P = 0.00741$ and $W = 0.07792$. It follows that each firm would prefer to serve the higher segment of the market, and the two equilibria cannot be Pareto ranked.

7. The mixed duopoly: (subgame perfect) Stackelberg equilibrium

We now suppose that the public firm can choose its quality level before the private firm chooses its ones, that is, we consider the Stackelberg equilibrium of the (first stage of the) game. As for the second stage (i.e, the competition in prices, given quality levels), we continue to assume simultaneous moves. This scenario may be of interest in cases where the public firm can credibly commit itself into choosing a pre-specified quality level, as for instance when its market segment is determined as the result of a binding political process.

The second stage of the game is the same as in the previous section, and therefore equilibrium prices are given by (26) and (27) when $\theta_P > \theta_S$, and by (36) and (37) when $\theta_P < \theta_S$. New problems arise in the choice of the quality levels, however. The behaviour of the private firm is still described by the reaction function characterized in Fact 1. Taking into account the private firm's reaction, the public firm chooses θ_S to maximize social welfare. Thus, we have the following:

Proposition 2. The Stackelberg equilibrium in pure strategies when the public firm acts as leader is: $\theta_P = 0.249$, $\theta_S = 0.374$, $p_P = 0.085$ and $p_S = 0.163$.

Proof. In view of the discontinuity of the private firm's reaction curve, we proceed by distinguishing the two cases $\theta_P > \theta_S$ and $\theta_P < \theta_S$, calculating the two candidate equilibria, and then comparing them in order to see which one yields the maximum payoff for firm S.

Let us consider first the case $\theta_P < \theta_S$. Substituting (41) into (42) and simplifying, we get:

$$(44)\quad W = \frac{1}{144}\left(85\theta^3_S - 144\theta^2_S + 72\theta_S\right)$$

The FOC for the choice of θ_S is therefore

$$(45)\quad 85\theta^2_S - 96\theta_S + 24 = 0$$

The maximum is attained at

(46) $\theta_S = 0.374$

and from (41) it follows that $\theta_P = 0.249$. The welfare level is $W = 0.07801$.

The case $\theta_P > \theta_S$ is more difficult to deal with analytically, since the FOC of the private firm is given by (32). We therefore proceed by numerical computation within the admissible range defined by (30). For any given θ_S we calculate, using the private firm's reaction function, the corresponding θ_P, and then substitute into the public firm's objective function. It turns out that social welfare is maximum for $\theta_S = 0.269$, which yields $\theta_P = 0.386$ and $W = 0.07759$.

A comparison between the levels of social welfare in correspondence of the two candidate equilibria proves the Proposition.∎

Notice that in the Stackelberg equilibrium the total fraction of consumers served (i.e., $x_S + x_P$) is 0.659 and then is higher than in both subgame perfect Nash equilibria of the mixed duopoly.

8. Discussion of the results

Our main findings are summarized in the Table below.

(Table 1 about here)

First of all, notice that the prices set by a private monopolist exceed marginal costs, while quality levels are the same that would have been chosen by a public monopolist. As pointed out by Spence (1975), this occurs because demand functions are linear in prices. Generally speaking, the quality levels chosen by a private monopolist would differ from the socially optimal ones, depending on the shape of the demand curve. Therefore, focusing on a linear demand structure, we can identify the additional sources of distortion arising from the strategic interaction of competing oligopolists.

Secondly, in a private duopoly competition pushes firms to choose higher quality levels but at the same time reduces price-cost margins with respect to a private monopoly. The resulting improvement in the quality/price ratio increases the number of consumers served in equilibrium, and significantly enhances social welfare.

Thirdly, in a mixed duopoly absolute quality levels and prices are lower than in a private duopoly, despite that the public firm prices above marginal cost. It turns out that the number of consumers served in equilibrium increases (even if the public firm does not consider distributional issues), and social welfare is even higher than in a private duopoly.

Fourthly, two aspects of the problem of quality choice by a public firm in a mixed duopoly must be distinguished: the public firm is interested in the <u>absolute</u> quality levels and in its <u>relative</u> position in the quality space. In our model, the effect of the nationalization of a duopolistic firm would be a reduction in quality levels, but this does not mean that the public firm should choose to serve the lower segment of the market. On the contrary, comparing the two subgame perfect Nash equilibria of the mixed duopoly, it turns out that social welfare is higher when the public firm chooses the higher quality level. This finding is strengthened by the analysis of the Stackelberg equilibrium, in which the public firm, acting as a leader in quality choice, will find it optimal to serve the higher segment of the market.

Finally, it is worth noting that in all mixed duopoly equilibria quality levels are "too close" relatively to the social optimum, and are closer than the equilibrium quality levels of the private duopoly. This may be interpreted as the result of the attempt by a public firm to make price competition more effective by reducing product differentiation.

9. Concluding remarks

In this paper we have tried to analyze the issue of the quality choice of a public firm in a mixed duopoly. The policy conclusions that can be drawn from the model, although obviously limited by the partial equilibrium framework we have adopted, require some further qualifications. In particular, we shall concentrate on three aspects of our setting, indicating possibile extensions of our model.

First of all, we have assumed that the marginal willingness to pay is uniformly distributed across consumers. In this, we have closely followed existing literature, but the issue addressed in this paper might suggest a more general specification of consumers' distribution. Our conclusions may change if we relax this assumption, considering, as may be plausible in many industries, that consumers are more concentrated in the low segment of the market.

Moreover, it may be worth considering the case of quantity competition in the final stage of the game. In the light of the remarks developed in Section 5, marginal cost pricing would be the outcome of the public firm's maximization problem, and this might change some of the previous conclusions [4].

Finally, it may be interesting to analyze the case in which the public firm is concerned about distribution, in that it cares about the total number of consumers that have access to the market. When this is the case, the public firm might decide to serve the lower segment of the market, and this might actually justify the fact that in practice many public firms, as already noted, prefer to supply the lower quality.

Footnotes

(1) We will use the following terminology. The term subgame perfect Nash equilibrium is used to characterize a situation in which both players move simultaneously in both stages of the game. The (subgame perfect) Stackelberg equilibrium we consider is one where a player (namely, the public firm) has a move advantage in the first stage of the game.

(2) We neglect fixed costs in the following analysis, as they have no effect on first order conditions, and would only entail straightforward limitations of the parameters. Moreover, there would be no gain in generality in introducing a parameter α such that production costs would be $C = \alpha\theta^2 x$. Indeed, as shown by Spence (1975) and Shaked and Sutton (1982) costs usually have little relevance to these results. However, given the structure of preferences (2), here we need assume that marginal cost is convex in quality to ensure the existence of an equilibrium in the mixed duopoly.

(3) In (E.1) $x_S + x_P = 0.656$, whereas in (E.2) $x_S + x_P = 0.642$.

(4) For an analysis of quantity competition in a vertically differentiated oligopoly and a comparison to the case of price competition, see Bonanno (1986).

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Table 1

\	θ_H	θ_L	p_H	p_L	x_H	x_L	$x_H + x_L$	W	π_H	π_L
SM	.400	.200	.160	.040	.400	.400	.800	.0800	.000	.000
PM	.400	.200	.280	.120	.200	.200	.400	.0600	.024	.016
PD	.469	.305	.257	.130	.226	.348	.574	.0756	.008	.013
E.1 (H=P)	.380	.259	.177	.089	.276	.380	.656	.0775	.009	.008
E.2 (H=S)	.390	.260	.177	.093	.350	.292	.642	.0779	.009	.007
SE (H=S)	.374	.249	.163	.085	.376	.283	.659	.0780	.009	.007

PD: Private Duopoly

SE: Stackelberg Equilibrium

SM: Public Monopoly

PM: Private Monopoly

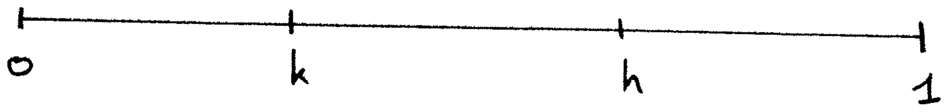

Figure 1

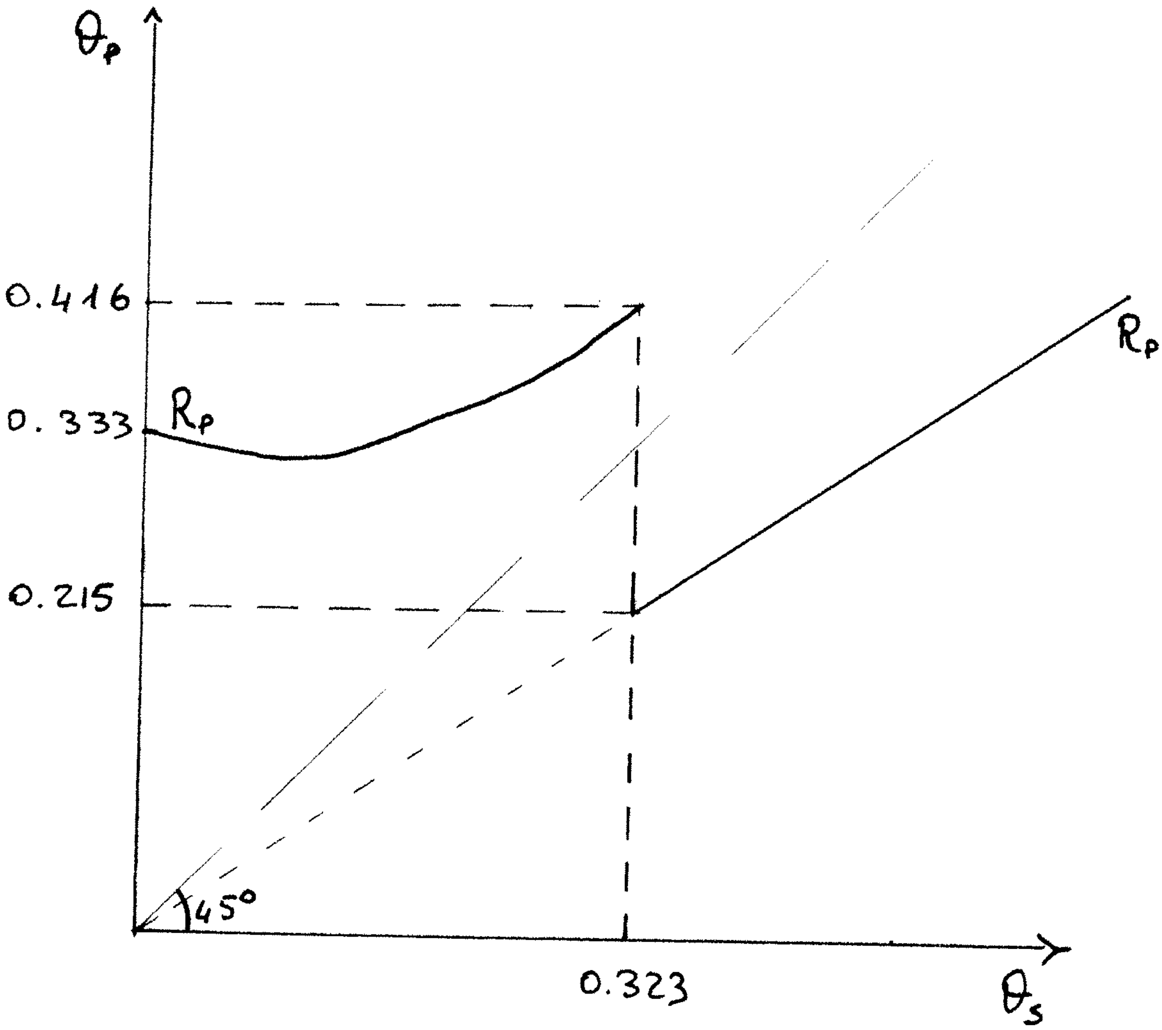

Figure 2

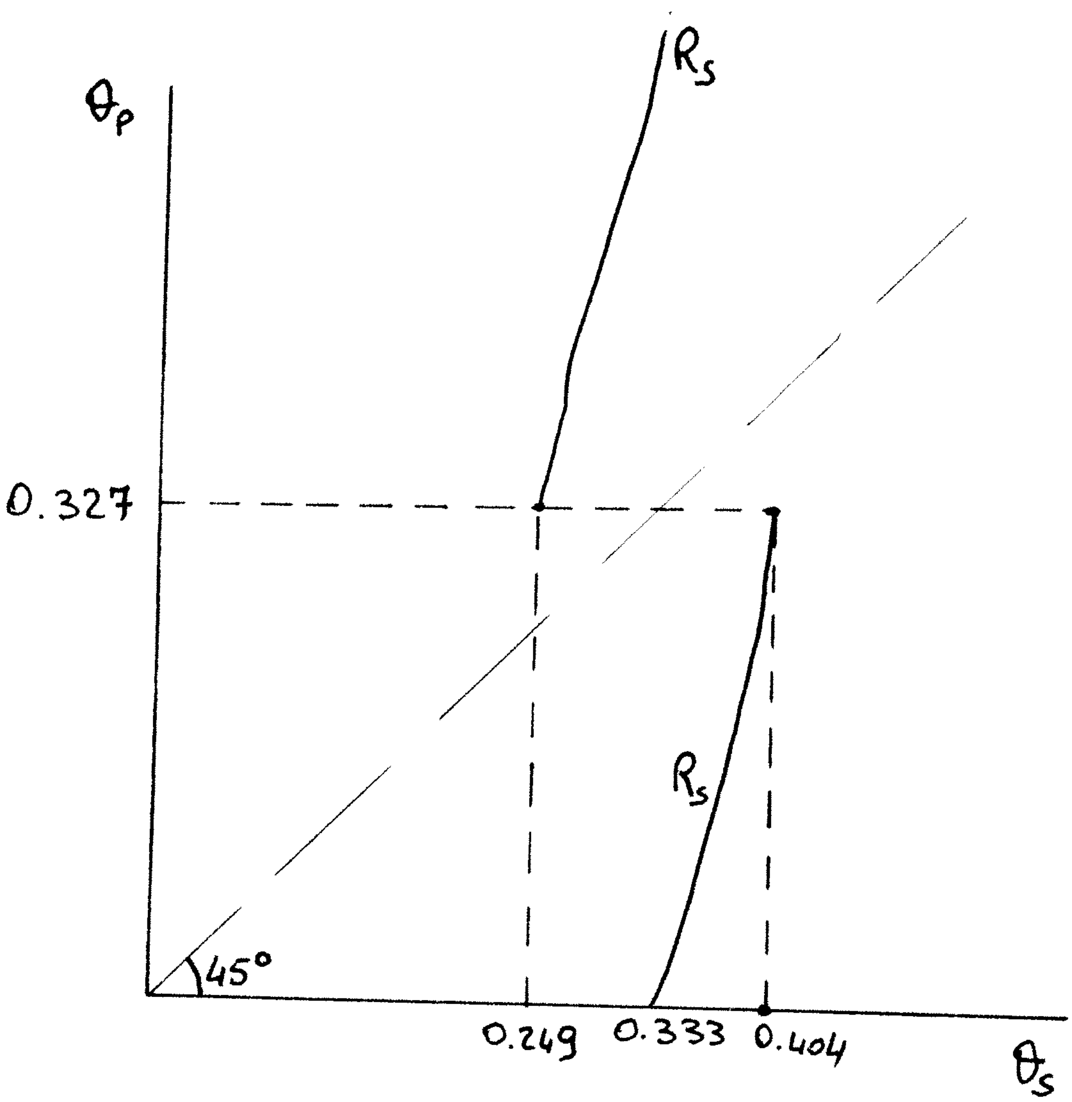

Figure 3

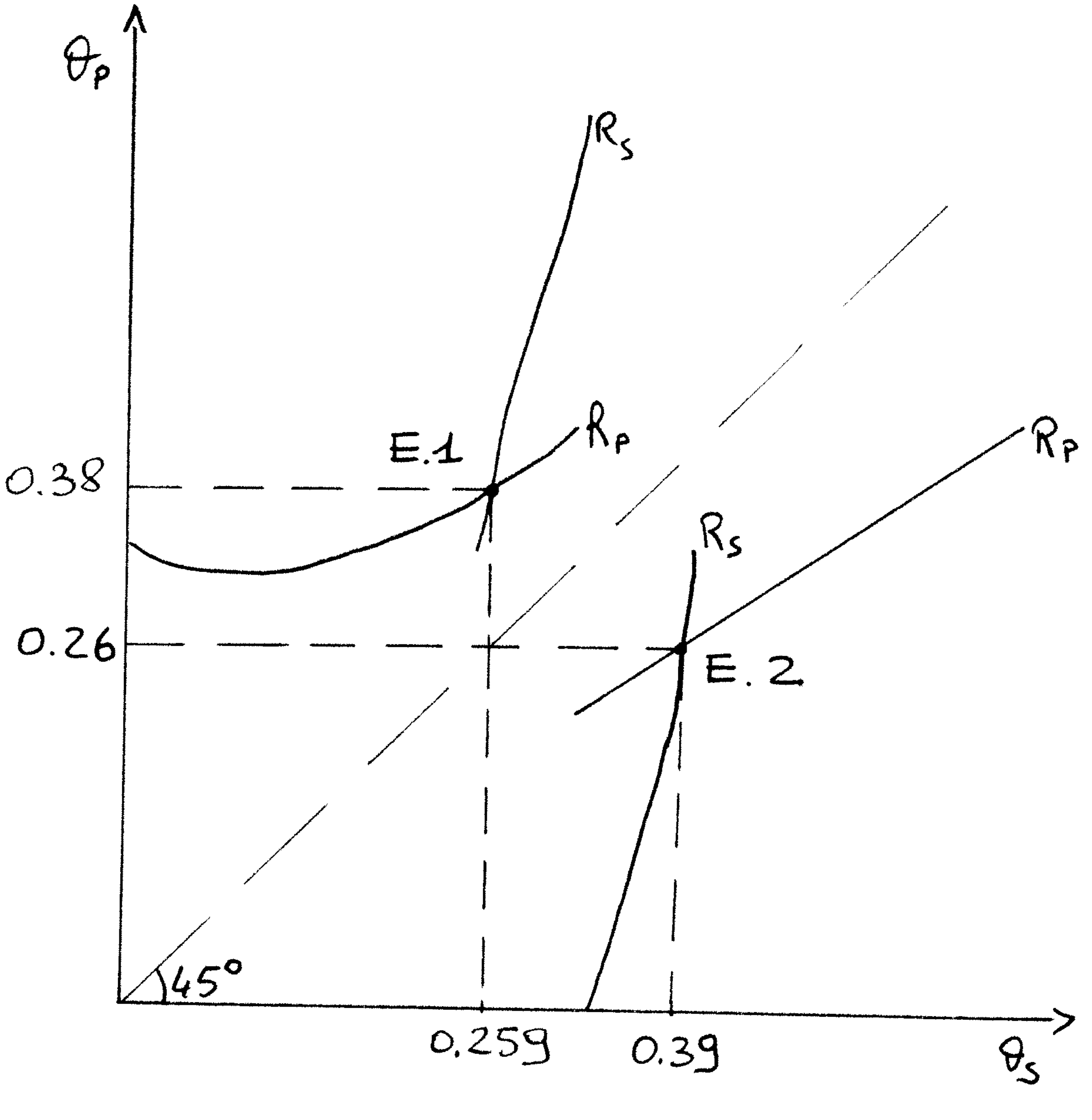

Figure 4