COMPETING TECHNOLOGIES AND LOCK-IN

BY RANDOM EVENTS

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ABSTRACT

In this paper we study the choice of competing technologies with increasing returns occasioned by learning-by-doing and learning-by-using phenomena. We study under what circumstances one technology can achieve a monopoly and eventually take the whole market by formulating a firm's optimal decision problem, when both uncertainty and increasing returns to adoption are present. In contrast with Arthur (1989) we allow agents to learn: the formulation of the decision problem takes into account the revision of the probabilities on the future states of the adoption process, according to a version of the two-armed bandit problem.

1. INTRODUCTION

This paper explores the dynamics of allocation under increasing returns occasioned by learning-by-doing and learning-by-using phenomena. We study the choice between technologies competing for adoption.

This issue has been tackled by Brian Arthur and Paul David in their writings on cumulative causation occurring in path-dependent processes. They argue that "history matters" when increasing returns to adoption are introduced. If one technology gets ahead by good fortune, it gains an advantage, with the result that the adoption market may "tip" in its favour and may end up dominated by it (Arthur, 1989). Given other circumstances, a different technology might have been favoured early on, and it might have come to dominate the market. Thus in competition between technologies with increasing returns ordinarily there are multiple equilibria. As to which actual outcome is selected from these multiple candidate outcomes, it is argued that the prevailing outcome turns out to depend on the path which has been initially chosen. In particular, the resulting outcome may be inefficient; that is, the market may be locked-in to the "wrong" technology.

The possibility of ending up with a "wrong" choice has also been stressed by the literature of statistical decision theory referring to two-armed bandit problems. In the basic model of a two-armed bandit problem (Bellman, 1956), a decision maker who follows optimal strategies will after an initial period of sampling settle on one arm and play it in preference to the other. However, the arm chosen will not necessarily be the correct one. Notice, incidentally, that the literature on two-armed bandit problems appears quite fragmentary and has rarely been applied by economists.

The purpose of this paper is to provide a link between the two views. Such a link follows from the following remark. In Arthur's paper (1989) there is a weakness: agents behave myopically. That is, they do not modify their expectations as the fortunes of alternatives change during the adoption process itself. In our paper, on the contrary, we try to bring together the two features of increasing returns to adoption and non-myopic behaviour.

In Arthur's model randomness is introduced by lack of knowledge of the arrival sequence of the adopters who have natural preferences on different technologies. His model is developed within an equilibrium analysis.

In our paper, on the contrary, randomness enters in a homogeneous adopter-type model because technological improvements occur in part by unpredictable breakthroughs. We study under what circumstances one technology can achieve a monopoly and eventually take the whole market, by formulating a firm's optimal decision problem. The formulation of the decision problem takes into account the revision of the probabilities on the future states of the adoption process. This yields a version of the two-armed bandit problem.

We show that by allowing agents to learn in a Bayesian way, lock-in to inferior technologies is still possible (Section 3). An early run of bad luck with a potentially superior technology may cause the decision-maker, perfectly rational, to abandon it. Put it another way, escape from inferior technological paths is not guaranteed. Moreover, the stronger the learning-by-doing, learning-by-using effects, the more likely is that a "wrong" technology is locked-in (Section 4). That is to say, increasing returns have the role of strengthening the probability of getting to an inefficient result.

The plan of the paper is as follows. In Section 2 the model and the main assumptions are introduced. Our central result is proved and discussed in Sections 3 and 4. Finally, in Section 5 some conclusions and final remarks are contained.

2. THE MODEL

We consider two unsponsored technologies, A and B, which are available for performing the same task and are competing passively for a market[1]. Randomness enters in this homogeneous adopter-type model because technological improvements occur in part by unpredictable breakthroughs. Moreover, as adoptions of A (or B) increase, learning-by-using and learning-by-doing effects[2] take place and improved versions of A (or B) become available, with correspondingly higher payoffs or returns to the adopter. We shall suppose that the one-period returns to the two technologies A and B are described by the following Table, where n_A and n_B denote the number of adoptions of A and B respectively.

TABLE 1

Returns to adoptions

A:	$H_A(n_A)$	with probability Π_A
	$L_A(n_A)$	with probability $1 - \Pi_A$
B:	$H_B(n_B)$	with probability Π_B
	$L_B(n_B)$	with probability $1 - \Pi_B$

[1]"Unsponsored" technologies is a term coined by Arthur (1989). Following this author, we will say that technologies are "sponsored" if they compete strategically, that is, if they are products that can be priced and manipulated. Technologies are "unsponsored" if they compete passively, and adoptions of one technology displace or preclude adoptions of its rivals.

[2]The two phenomena are different qualitatively. Learning-by-doing is a phenomenon due to which cost of production decreases with accumulated knowledge, which is usually measured by the volume of production. Given a certain volume of production, cost of production also decreases the more a technology is used, and the more is learned about it; that is, the technology improves because of learning-by-using.

and the following assumptions hold:

Assumption 1. $H_i(.)$ and $L_i(.)$ are continuous functions and are non-decreasing in n_i, $i = A, B$.

Assumption 2. For any $n_A = n_B = \bar{n}$ we have $H_A(\bar{n}) > H_B(\bar{n}) > L_B(\bar{n}) > L_A(\bar{n})$

Assumption 3. Returns to adoption increase but are bounded:

$$\lim_{n_i \to \infty} H_i(n_i) = \bar{H}_i < \infty$$
$$\lim_{n_i \to \infty} L_i(n_i) = \bar{L}_i < \infty$$
$$i = A, B.$$

Assumption 1 is a central assumption. It means that there exist experience advantages. Assumption 2 implies that technology B is less dispersed than technology A[3]. Assumption 3 states that experience effects vanish, at least in infinite time.

With this formulation two aspects of technological improvement can be captured: the "endogenous" aspect due to learning-by-doing and learning-by-using effects, such that as adoptions increase , improved versions of the technologies become available (Table 1 and Assumption 1); and the "exogenous" and uncertain aspect of technological change, because of which a "high" payoff or a "low" payoff can result (Table 1 and Assumption 2).

The decision-maker does not know the parameters Π_A and Π_B with certainty. He decides which technology to adopt at each stage after consulting his prior beliefs about Π_A

[3]For simplicity, we do not consider the case where H_A and H_B (and L_A and L_B) cross each other: the analysis, however, could be generalized to this case as well.

and Π_B, and examining the record of "high" payoffs (subscript H) and "low" payoffs (subscript L) on the technologies so far.

2.1. PRELIMINARIES

Let us denote by n_A^H, n_B^H (and n_A^L, n_B^L) the number of times that a "high" payoff (or a "low" payoff) is recorded for technology A and B respectively. Obviously, $n_A^H + n_A^L = n_A$ and $n_B^H + n_B^L = n_B$. We define the following statistics:

$$r_i = \frac{1}{n_i + 1}, \qquad m_i = \frac{n_i^H}{n_i + 1}, \qquad i = A, B$$

Simple rules for updating m_i and r_i can be derived.

If technology i is chosen, then m_i becomes $(m_i + r_i)/(1 + r_i)$ if a "high" payoff is obtained, while it becomes $m_i/(1 + r_i)$ if a "low" payoff is obtained. The information from the sample is contained in $(m,r) = (m_A, m_B, r_A, r_B)$ belonging to a fourfold copy of the closed interval [0, 1].

Given our assumption that the decision-maker does not know the parameters Π_i with certainty, we suppose that he possesses prior beliefs about the parameters, summarised by a prior density function $g(\Pi_A, \Pi_B)$ such that[4] $g(\Pi_A, \Pi_B) > 0$ for all $(\Pi_A, \Pi_B) \in [0,1]^2$.

[4]In order to avoid those situations where one cannot lock-in to the wrong technology, since it looks so bad at the start that it will never be tried, we assume that the supports of the initial priors are not disjoint, so that both technologies will be used with positive probability.

At each stage the most reliable estimate for Π_A, Π_B is given by the posterior mean of the decision-maker's beliefs about the values Π_A, Π_B, given the sample information (m, r) and the prior density $g(\Pi_A, \Pi_B)$. In particular, if experience gives (m, r), the prior beliefs will be updated from $g(\Pi_A, \Pi_B)$ to $j(\Pi_A, \Pi_B, m, r)$ according to Bayes's rule. Therefore the posterior mean of the decision-maker's belief about the value Π_i, given (m,r) and $g(\Pi_A, \Pi_B)$ is:

(1) $$\lambda_i(m, r) = \int_0^1 \int_0^1 \Pi_i \, j(\Pi_A, \Pi_B, m, r) d\Pi_A d\Pi_B, \qquad i = A, B.$$

Notice that λ_i is the mean of a posterior distribution based on $n_i = (1-r_i)/r_i$ observations. The function $\lambda_i(m, r)$ is defined and continuous for all (m, r) such that $r_i > 0$, $i = A, B$. It can be extended by continuity to $[0, 1]^4$ since $\lim_{r_i \to 0} \lambda_i(m,r) = m_i$. This follows from the fact that as $r_i \to 0$ the posterior distribution approaches a normal distribution with mean equal to the sample mean.

2.2. THE CHOICE OF TECHNOLOGIES

The decision-maker will choose the technology that maximizes the expected discounted value of his profits over an infinite horizon. Let δ, $0 < \delta < 1$, be the discount factor. In order to compute the expected discounted value of the profits we will state the problem of the decision-maker as a dynamic programming problem.

(2) $$\max_{y_t} E \left\{ \Sigma_{t=o}^{\infty} \delta^t [y_t \, F_A(\frac{1-r_A(t)}{r_A(t)}) + (1-y_t) F_B(\frac{1-r_B(t)}{r_B(t)})] \right\}$$

subject to $y_t \in \{0, 1\}$

and the sequential constraints that:

$y_t = 1$

$r_A(t) = r_A(t-1) / (1 + r_A(t-1))$

$r_B(t) = r_B(t-1)$

$m_B(t) = m_B(t-1)$

$$m_A(t) = \begin{cases} (m_A(t-1) + r_A(t-1))/(1 + r_A(t-1)), & \text{if "high"} \\ m_A(t-1)/(1 + r_A(t-1)), & \text{if "low"} \end{cases}$$

$$F_A\left(\frac{1-r_A(t)}{r_A(t)}\right) = \begin{cases} H_A\left(\frac{1-r_A(t)}{r_A(t)}\right), & \text{if "high"} \\ L_A\left(\frac{1-r_A(t)}{r_A(t)}\right), & \text{if "low"} \end{cases}$$

$y_t = 0$

$r_B(t) = r_B(t-1)/(1 + r_B(t-1))$

$r_A(t) = r_A(t-1)$

$$m_B(t) = \begin{cases} (m_B(t-1) + r_B(t-1))/(1 + r_B(t-1), & \text{if "high"} \\ m_B(t-1)/(1 + m_B(t-1)), & \text{if "low"} \end{cases}$$

$m_A(t) = m_A(t-1)$

$$F_B\left(\frac{1-r_B(t)}{r_B(t)}\right) = \begin{cases} H_B\left(\frac{1-r_B(t)}{r_B(t)}\right) & \text{if "high"} \\ L_B\left(\frac{1-r_B(t)}{r_B(t)}\right) & \text{if "low"} \end{cases}$$

where $[1 - r_i(t)]/r_i(t)$ is the number of adoptions of i at time t. Obviously,

$[1 - r_i(0)]/r_i(0) = 0$, $i = A, B$.

Let
$$h_A(m, r) = \left(\frac{m_A + r_A}{1 + r_A}, m_B, \frac{r_A}{1 + r_A}, r_B\right);$$

$$l_A(m, r) = \left(\frac{m_A}{1 + r_A}, m_B, \frac{r_A}{1 + r_A}, r_B\right);$$

$$h_B(m, r) = \left(m_A, \frac{m_B + r_B}{1 + r_B}, r_A, \frac{r_B}{1 + r_B}\right);$$

$$l_B(m, r) = \left(m_A, \frac{m_B}{1 + r_B}, r_A, \frac{r_B}{1 + r_B}\right).$$

Associated with (2) there is the functional equation:

(3)
$$W(m, r) =$$

$$\max \{\lambda_A(m,r)[H_A(\frac{1-r_A}{r_A}) + \delta W(h_A(m,r)] + (1 - \lambda_A(m,r))[L_A(\frac{1-r_A}{r_A}) + \delta W(l_A(m,r))];$$

$$\lambda_B(m,r)[H_B(\frac{1-r_B}{r_B}) + \delta W(h_B(m,r))] + (1 - \lambda_B(m,r))[L_B(\frac{1-r_B}{r_B}) + \delta W(l_B(m,r))]\}$$

which may be written as:

(4)
$$W(m, r) = \max_{i = A,B} \{V_i(m, r)\}$$

where V_i is defined by:

(5) $$V_i(m, r) = [\lambda_i(m,r)H_i(\frac{1-r_i}{r_i}) + (1-\lambda_i(m,r))L_i(\frac{1-r_i}{r_i})] +$$

$$+ \delta\{\lambda_i(m,r)W(h_i(m,r)) + (1-\lambda_i(m,r))W(l_i(m,r))\}$$

Lemma. The functions W(m,r) and V_i(m,r) are continuous.

Proof. The proof is by an inductive argument. Let:

$W^o(m, r) = 0$

$W^1(m, r) = \max_{i=A,B} \{ V_i^1(m,r)\},$

where $V_i^1(m,r) = \lambda_i(m,r)H_i(\frac{1-r_i}{r_i}) + (1-\lambda_i(m,r))L_i(\frac{1-r_i}{r_i})$

....

$W^T(m, r) = \max_{i=A,B} \{ V_i^T(m,r)\},$

where $V_i^T(m,r) = \lambda_i(m,r)H_i(\frac{1-r_i}{r_i}) + (1-\lambda_i(m,r))L_i(\frac{1-r_i}{r_i}) +$
$\delta\{\lambda_i(m,r)W^{T-1}(h_i(m,r)) + (1-\lambda_i(m,r))W^{T-1}(l_i(m,r))\}$

The Lemma can be proved through the following steps.

STEP 1. The functions W^T(m,r) and V_i^T(m,r) are continuous.
Indeed, the continuity of λ_i(m,r) and $W^o(m,r) = 0$ establish the property for t = 1. If we suppose that V_i^{T-1}(m,r) and W^{T-1}(m,r) are continuous, then also V_i^T(m,r) and W^{T-1}(m,r) are, because, by the induction hypothesis, they are sums of continuous functions.

STEP 2. The functions W^T(m,r) and V_i^T(m,r) are monotonic in T.
Indeed, obviously $W^1(m,r) \geq W^o(m,r)$. If we assume that $W^{T-1}(m,r) \geq W^{T-2}(m,r)$, for every (m,r), then we get:

$$W^T(m,r) \geq \max\{\lambda_A(m,r)H_A(\frac{1-r_A}{r_A}) + (1-\lambda_A(m,r))L_A(\frac{1-r_A}{r_A}) + \delta[\lambda_A(m,r)W^{T-2}(h_A(m,r)) + (1-\lambda_A(m,r))W^{T-2}(l_A(m,r)]; \lambda_B(m,r)H_B(\frac{1-r_B}{r_B}) + (1-\lambda_B(m,r))L_B(\frac{1-r_B}{r_B}) + \delta[\lambda_B(m,r)W^{T-2}(h_B(m,r)) + (1-\lambda_B(m,r))W^{T-2}(l_B(m,r))]\} = W^{T-1}(m,r).$$

STEP 3. The functions $W^T(m,r)$ and $V_i^T(m,r)$ are bounded above and converge. Indeed, consider $\max\{\bar{H}_A, \bar{L}_A, \bar{H}_B, \bar{L}_B\} = \bar{H}_A$. For $0 < \delta < 1$, it follows that $\bar{H}_A/(1-\delta)$ is a finite number. Obviously, $W^o(m,r) = 0 < \bar{H}_A/(1-\delta)$. Moreover $W^1(m,r) \leq \bar{H}_A < \bar{H}_A/(1-\delta)$. If we assume that $W^{T-1}(m,r) \leq \bar{H}_A/(1-\delta)$, then it follows:

$$W^T(m,r) \leq \max\{\lambda_A(m,r)H_A(\frac{1-r_A}{r_A}) + (1-\lambda_A(m,r))L_A(\frac{1-r_A}{r_A}) + \delta\bar{H}_A/(1-\delta); \lambda_B(m,r)H_B(\frac{1-r_B}{r_B}) + (1-\lambda_B(m,r))L_B(\frac{1-r_B}{r_B}) + \delta\bar{H}_A/(1-\delta)\} = W^1(m,r) + \delta\bar{H}_A/(1-\delta) \leq \bar{H}_A + \delta\bar{H}_A/(1-\delta) = \bar{H}_A/(1-\delta).$$

Since $W^T(m,r)$ is monotonic in T and bounded above, then the sequences $\{W^T(m,r)\}$ and $\{V_i^T(m,r)\}$ converge.

STEP 4. The functions $W^T(m,r)$ and $V_i^T(m,r)$ converge uniformly to $W(m,r)$ and $V_i(m,r)$. Indeed, the following inequality clearly holds:
$W(m,r) \leq W^T(m,r) + \delta^T \bar{H}_A/(1-\delta)$ and therefore $|W(m,r) - W^T(m,r)| \leq \delta^T \bar{H}_A/(1-\delta)$. Since this inequality is independent of (m,r), then uniform convergence is established. An immediate consequence is that $W(m,r)$ and $V_i(m,r)$ are continuous. □

Let $Z_B = \{(m, r) \in (0, 1)^4; V_B(m, r) > V_A(m, r)\}$. Since $V_A(m, r)$ and $V_B(m, r)$ are continuous, then Z_B is an open set.

3. WHEN DOES TECHNOLOGY B ACHIEVE A MONOPOLY ?

The following result establishes up to which point the choice of technology A is not convenient.

PROPOSITION I. For every Π_B^1 such that $\Pi_B > \Pi_B^1 > 0$ there exist $\varepsilon > 0$, $\bar{\delta} > 0$ such that $V_B(m, r) > V_A(m, r)$ whenever $m_A + r_A < \varepsilon$, $m_B \geq \Pi_B - \Pi_B^1 > 0$, and $0 < \delta < \bar{\delta}$.

Proof. The proof consists of two steps.

STEP 1. Consider $K = \{(m,r) \in (0,1)^4;\ m_A = r_A = 0,\ m_B \geq \Pi_B - \Pi_B^1 > 0\}$, which is a compact set. Consider $Z_B = \{(m,r) \in (0,1)^4;\ V_B(m,r) > V_A(m,r)\}$.
We want to show the conditions under which it happens that $K \subset Z_B$. Consider:

$$V_B(0, m_B, 0, r_B) = \lambda_B(0, m_B, 0, r_B)H_B(\frac{1-r_B}{r_B}) + (1 - \lambda_B(0, m_B, 0, r_B))L_B(\frac{1-r_B}{r_B}) +$$
$$+ \delta\{\lambda_B(0, m_B, 0, r_B)W(h(m_B)) + (1 - \lambda_B(0, m_B, 0, r_B))W(l_B(m_B))\} > L_B(\frac{1-r_B}{r_B})$$

The last inequality holds under Assumption 2.

Suppose now that $V_B(0, m_B, 0, r_B) \leq V_A(0, m_B, 0, r_B)$. In this case $W(0, m_B, 0, r_B) = V_A(0, m_B, 0, r_B) = \bar{L}_A + \delta W(0, m_B, 0, r_B)$ because $\lambda_A(0, m_B, 0, r_B) = m_A = 0$ if r_A 0; that is, $V_A(0, m_B, 0, r_B) = \bar{L}_A/(1 - \delta)$, which is a contradiction if $L_B(\frac{1-r_B}{r_B}) > \bar{L}_A/(1 - \delta)$, that is if $\delta < \bar{\delta} = 1 - \bar{L}_A/L_B(\frac{1-r_B}{r_B})$. Therefore,[5]

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[5]Notice that $\bar{\delta} > 0$ if r_B is sufficiently small, say, if $r_B < \bar{r}_B$. By assumptions 2 and 3,

if $\delta < \bar{\delta}$, then $V_B(0, m_B, 0, r_B) > V_A(0, m_B, 0, r_B)$.

STEP 2. Since Z_B is open, then for each $(0, m_B, 0, r_B) \in K$ there is a suitable ball (in the max norm) centered at $(0, m_B, 0, r_B)$ which is contained in Z_B. Since K is compact, we can find a finite collection of such balls covering K, i.e. $K \subset \bigcup_{j \in J} B_j$, where J is a finite set. Let ε be the minimum radius of the B_j's with $j \in J$. Then consider:

$$K^{\varepsilon} \{ (m, r) \in (0, 1)^4; m_A + r_A < \varepsilon, m_B \geq \Pi_B - \Pi_B' > 0 \}$$

Notice that every $(m, r) \in K^{\varepsilon}$ belongs to some B_j for a suitable $j \in J$. Hence $\max\{|m_A|, |m_B - m_B^j|, |r_A|, |r_B - r_B^j|\} \leq \varepsilon_j$, for any $(m, r) \in K^{\varepsilon}$, where ε_j is the radius of B_J. Thus we get $K^{\varepsilon} \subset Z_B$. □

In Proposition 1 the expressions:

$$m_A + r_A < \varepsilon, \text{ and } m_B \geq \Pi_B - \Pi_B' > 0 \tag{5}$$

give an exact specification of what is meant by "sufficiently bad" experience on technology A and "sufficiently good" experience on technology B.

Then Proposition 1 can be interpreted as follows. If the decision-maker observes that after a certain number of sequences on A and B, experience on technology A is sufficiently poor, while it is sufficiently good on B, then, for appropriate values of the discount factor δ, it is more profitable to choose B rather than A. To complete the proof,

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there exists $\bar{r}_B$, $0 < \bar{r}_B < 1$. Obviously, we will take r_B as small as necessary for δ to be positive.

we need to show that it is possible that experience on technology B is not so erratic that the decision-maker will never choose technology A again. Actually we can prove the following:

PROPOSITION 2. A decision-maker who follows an optimal strategy will with positive probability choose technology B infinitely often and A only a finite number of times.

Proof. A proof can be given which follows the argument in Rothschild (1974a), p. 197–8. □

4. LOCK-IN TO THE "WRONG" TECHNOLOGY

The results we have obtained in the previous Section make no use of the relation between the true probabilities Π_A and Π_B. In particular, nothing in Proposition 1 and Proposition 2 guarantees that technology B is the more efficient one – that is, the technology the decision-maker would have chosen had he known the exact sequence of "high" and "low" payoffs. As a consequence, the market may be locked-in to the "wrong" technology.

In our model we can identify two distinct forces driving the system to lock in to one technology. The first is the reduction of uncertainty. When the adoption process begins, the merits of neither technology are well-known. An important aspect of early use of the technologies is simply the reduction of this uncertainty. This reduction will, by itself, cause lock-in to occur, and is the sole cause of it in the simple two-armed bandit.

The possibly inferior technology result can be a result of the initial priors, or from bad luck with the first implementations of the superior technology. The inferior technology can then be used. If it does not have very bad luck it will continue to be used , and because the other is not being used, it has no way to demonstrate its superiority and therefore the market is locked in to the former.

In our model, however, there is a second mechanism driving lock-in, that is, the presence of increasing returns. This is the driving force behind Arthur's (1989) results. In our model the presence of increasing returns has an effect on the probability of locking in to an inferior technology. In particular, the lock–in result occurs more quickly, the stronger the increasing returns to adoption are. The following result holds:

PROPOSITION 3. The stronger the learning-by-doing, learning-by-using effects, the more likely is that the "wrong" technology is locked-in.

Proof. Following Gittins (1979) we can construct a Dynamic Allocation Index (known as Gittins index) for each technology i, i = A, B. The index is defined as follows. Consider a modification of the above bandit process, which allows the additional option of choosing a known technology, for which the probability of getting a payoff of value 1 is p, and $(1-p)$ is the probability of getting 0 (standard bandit process). Let $M = \Sigma_{t\,0}^{\infty} \delta^t p = p/(1-\delta)$. Now offer the decision-maker the choice between choosing the known technology forever, or choosing technology i, i= A, B, at least once, possibly more times, with the option of switching at some future time to the known one, which must then be chosen forever. The value of p for which the decision-maker is indifferent between these two options is the value of the Gittins index for technology i. The Gittins index policy consists in choosing each period the technology which has the highest index value.

Let $\varphi_i = \max [\, M,\ L_i\varphi_i \,]$, where:

$$L_i\varphi_i(m,r) = \lambda_i(m,r)H_i(\tfrac{1-r_i}{r_i}) + (1-\lambda_i(m,r))L_i(\tfrac{1-r_i}{r_i}) +$$

$$+\ \delta\,\{\lambda_i(m,r)\varphi_i(h_i(m,r)) + (1-\lambda_i(m,r))\varphi_i(l_i(m,r))\}$$

that is, φ_i is the maximal expected reward for the modified process we have described above. The Gittins index is defined by $M = \varphi_i(m,r)$, that is, the options between continuation with technology i or retirement with reward M must be indifferent. This implies $M = L_i\varphi_i(m,r)$. Because of the presence of increasing returns to adoption we get $L_i\varphi_i \geq M$, and therefore continuation is always optimal. It follows that $M = \Sigma_{t\,0}^{\infty}\, \delta^t[\lambda_i(m(t),r(t))H_i(\frac{1-r_i(t)}{r_i(t)}) + (1-\lambda_i(m(t),r(t)))L_i(\frac{1-r_i(t)}{r_i(t)})]$ and therefore we get the following Dynamic Allocation Index for technology i:

(6) $$p = (1-\delta)\Sigma_{t\ 0}^{\infty}\ \delta^t[\lambda_i(m(t),r(t))H_i(\frac{1-r_i(t)}{r_i(t)}) + (1-\lambda_i(m(t),r(t)))L_i(\frac{1-r_i(t)}{r_i(t)})]$$

From (6) it follows that p increases with the degree of increasing returns to adoption because the expression within square brackets increases. As a result, suppose that the inferior technology was used until it had a large lead in learning. The stronger are the increasing returns, the larger p, the easier it is to gain this lead. Even if the estimates of the better technology are accurate, an erroneously high estimate of the inferior technology will allow it to be used, and increasing returns will allow it to continue being used. That is to say, strong increasing returns have the role of strengthening the probability of getting to an inferior technology. □

5. CONCLUSION

This paper has studied the choice of competing technologies with increasing returns to adoption. It has been shown that increasing returns can cause the economy gradually to lock itself into an outcome not necessarily superior to alternatives, even though the decision-maker is allowed to learn in a Bayesian way.

This finding is important for policy implications too. Where a central authority with full information on future returns to alternative adoption paths knows which technology has superior long–run potential, it can attempt to make the market adopt this technology. When there are increasing returns to adoption, in Paul David's phrase (1987), there are only "narrow windows" in time, during which effective public policy interventions can be made at moderate resourse costs. But if it is not clear in advance which technologies have most potential promise, then the central authority intervention is even more problematic. As it is shown in this paper, an early run of bad luck with a potentially superior technology may cause the central authority, perfectly rational, to abandon it. Even with central control, escape from "wrong" technologies is not guaranteed.

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