ECONOMIC AND POLITICAL EQUILIBRIA IN OPTIMAL TAXATION
Is incentive compatibility enough?

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Abstract
This paper is an attempt to describe the fiscal structure of the State as an equilibrium of a non-cooperative game where decentralised agents play both political and economic strategies. We propose a theory of fiscal equilibria (describing features of direct and indirect taxation as well as of the expenditure side of the public intervention) arising from the voluntary interaction among decentralised and fully informed agents. Among the main results we show that: (i) the greater is the size of the public sector, the more income taxation prevails on expenditure taxation; (ii) the better is the monitoring technology on tax evasion the larger the proportion of income over expenditure taxation in the fiscal revenue; (iii) the larger is the group of agents who can work in the shadow economy without paying income taxation, the larger the public budget; (iv) agents in group with larger labour income tend to attract more public subsidies unless they constitute a relatively small minority of the population; (v) self-seeking political activity has an ambiguous effect on different welfare indicators.

* Thanks are due to Vincenzo Denicolo', Flavio Delbono and Meg Meyer. The usual caveats apply.
1 Introduction

"Many authors... focus on the insignificant effects of a tax (on prices or incomes) and forget about the political and economic equilibria, by far more important which allow the government to impose them".

V. Pareto - Trattato di Sociologia Generale (1916 p.525)

This paper is an attempt to describe the fiscal structure of the State as an equilibrium of a non-cooperative game where decentralized agents play both political and economic strategies. This is because, notwithstanding the authoritative quotation above, most theoretical studies on taxation following the Pareto contribution failed to integrate the analysis of those political and economic equilibria into a homogeneous setting. In particular we think that such a sharp distinction between political and economic stages in the analysis of public interventions constitutes a serious shortcoming in the recent literature on optimal taxation. Looking at the economic and political decisions as two sides of the same game, when a fiscal mechanism is designed in order to maximize a social welfare function (SWF, henceforth), such SWF is changed by the implementation of the mechanism. Consequently the latter - though incentive compatible and individually rational - may well not represent an equilibrium of the game including political strategies and therefore may not be feasible. The analysis of fiscal mechanisms should therefore take into account that self-seeking decentralized agents can manipulate not just the rules in a given mechanism, but the rules (SWFs) from which mechanisms are derived.

We propose a theory of fiscal equilibria (describing features of direct and indirect taxation as well as of the expenditure side of the public intervention) arising from the voluntary interaction among decentralized and fully informed agents. No fiscal agency is described because we try to found as many

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1 Of course, this is more so as far as the Anglo-saxon tradition in welfare economics is concerned. See Fasiani (1932) and Buchanan (1960).

2 The most important reference here is Mirlees (1976), but see also Stern (1987) for an introduction to the field and Slemrod (1990) for a critical survey on its recent developments.
social institutions as possible (taxes in this context) on pure non-cooperative behaviour on part of self-seeking agents.

More precisely we shall study a situation where a given population in a community is grouped into two types according to their characteristics in the labour market. Given his type any agent chooses a level of political activity knowing that the latter affects his welfare in two ways. First, the amount of fiscal revenue distributed to each citizen is proportional to his investment in political activity. Second, the weights in the SWF - which works as a fiscal constitution - are given by the aggregate political activity of each group of agents of the same type.

Accordingly, citizens in choosing their level of political activity look forward to the solution of a game of social interaction where - once the weights in the SWF are determined as above - the latter is maximized in order to determine the labour and expenditure tax rates compatible with the fiscal constitution. Furthermore, such tax rates - as functions of the levels of political activity in the two groups - are used by each player to determine the post-tax wage and price levels so that consumption and labour supply can take place.

Such a framework can therefore be seen as one where political and economic strategies are two aspects of the same simultaneous maximization problem. From a logical viewpoint, one can see three stages. First, the labour supply and consumption decisions are made as a function of the fiscal structure. Second, taxes on labour and consumption are determined by using a fiscal constitution (i.e., by maximizing a SWF). Such tax rates are not single values but functions of the aggregate level of political activity in the two groups. Third, individual (and aggregate) levels of political activity are chosen by citizens of the two types who are fully aware of the implications of their choice on both the redistribution of the fiscal revenue and on their income and consumption via their influence on the tax rates.

The "economic stage" of our game presents a simple instance of general equilibrium with one commodity and two sectors which can be seen as the regular (where income taxes are implemented) and the shadow sectors of our economy. Agents adopt Nash strategies in the mix between hours worked in
either sectors. In the "political stage" citizens play Nash strategies in the level of political activity which can be seen either as lobbying or as support to political parties.

In order to check how different economic settings affect the fiscal equilibrium, we focus on three settings of the labour market which characterize the differences between groups of citizens in the community. The first is a setting where agents differ exogenously for their productivity so that they can work either in the more efficient regular economy or in the less efficient shadow economy. We define this setting the less-developed community (LC). This is because here there are inherent and significant differences among agents - which have not been reduced by schooling or social programs -. Moreover here there is a completely untaxed sector which is a realistic feature only in communities where costs of implementing public interventions are relatively large.

In the second setting, agents differ exogenously for their utility from leisure, so that there is one type supplying a given amount of work in the regular economy and another one who, on the top of that amount, works a variable number of hours in the shadow economy. We define such a setting the developed community (DC). The reasons for such a definition should be clearer thinking of the above definition for the LC. Here differences among agents do not rest on their productivity but only on the utility arising from leisure. Moreover, the bulk of the economy is regular while its shadow part arises to provide further incentives to supply labour above the standard.

In the last setting, agents of one type again supply a given amount of work in the regular economy, whereas the other type while working the same number of hours can decide on how to allocate them between the regular and the shadow economy depending on the level of labour taxation and on the efficiency of the fiscal monitoring system. We define this setting the free-riding community (FC). Such a definition is due to the fact that here one group in the community can decide how much to free ride in
terms of its contribution to the overall fiscal revenue, being limited in doing so only by the existence of decreasing returns in the shadow economy and of a monitoring mechanism\(^3\).

Among the main results - obtained by the working of the three settings - we show that:

(i) the greater is the size of the public sector in terms of the possibility of administering large fiscal revenues the more income taxation prevails on expenditure taxation;
(ii) the better is the monitoring technology on tax evasion the larger the proportion of income over expenditure taxation in the fiscal revenue;
(iii) the larger is the group of agents who can work in the shadow economy without paying income taxation, the larger the public budget;
(iv) agents in group with larger labour income tend to attract more public subsidies unless they constitute a relatively small minority of the population;
(v) self-seeking political activity has an ambiguous effect on different welfare indicators.

2 Related literature

2.1 Political and economic equilibria

Models which tackle normative issues usually regard as exogenous that stage of the social interaction where a political equilibrium is reached. Accordingly, the interaction between political and economic stages is often studied through comparative statics with respect to the weights in the SWF. Clearly, this interaction does not rest on a fully-fledged theory of agents’ behaviour in both stages. On the other hand such a restriction allows one to investigate for instance how informational asymmetries limit the design of mechanisms or the effects of such limitation on the equity-efficiency trade-off

\(^3\) We regard this setting as an acceptable description of a State where workers are divided between autonomous and corporated sectors.
through incentive constraints.

However, disregarding the issue of how a SWF comes about - or founding it on normative axioms - introduces a serious ambiguity on the implementability of social contracts, on which SWF are implicitly assumed to rest. More generally, it allows one to introduce surreptitiously a cooperative stage in which SWF are accepted and where agreements are binding, and a non-cooperative one where decentralized agents try to manipulate the SWF agreed on in the previous stage. A defense for such an argument has been that the SWF is not a normative concept, but a shorthand description for the existing non-cooperative political equilibrium⁴. While such a position is thoroughly coherent, we maintain that its implications have not been fully drawn in the welfare economics tradition.

If one sticks to non-cooperative behaviour through the stages of the process of public decision making, both economic and political equilibria should be explicitly determined in a game where decentralized agents act simultaneously in both stages. This is not the same as studying how different weights in a SWF change the outcome of public interventions. In that case the focus is on the dependence of economic allocations on political weights. In our view, such a dependence cannot be fully founded on non-cooperative individual behaviour, since both stages are solved simultaneously by self-seeking agents. Therefore, while this practice can be fruitful in many applications, there is a need to look at the political and economic spheres not not merely as interacting, but as essentially one domain of agents’ maximizing activity.

Our position is also different from the one taken by most models in the public choice tradition. A major feature of such models is their focusing on the dependence of the political activity on the agents’ interests prevailing in the economic stage⁵. In this line of research one can trace the literature on pressure groups, on bureaucracy behaviour and on voting. We deal with each of them very briefly in order to outline their main differences with respect to our approach.

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⁴ For such a “positive” interpretation of the SWF see the literature cited in Stern (1987).
⁵ See Mueller (1989) for a survey of the most relevant literature.
In bureaucratic models of public decision making, no foundations, apart from those arising from informational asymmetries, are given for the existence of a bureaucracy furthering its own aims. Such models do not explain why the political equilibrium whose properties are investigated is, in fact, an equilibrium, when self-seeking behaviour by decentralized agents is assumed.

In models of voting in representative democracies, the presence of politicians maximizing their own payoffs - often in contrast to voters' interests - fails analogously to explain how this is possible without introducing crucial assumptions on the information sets of the voters. On the other hand, in median voter models in direct democracies - when standard conditions are met - agents cannot be seen as acting strategically in the political stage to manipulate the outcome of the collective decision process. This is because it is assumed that the maximization problem in the economic stage is not modified by the individual's choices in the political stage. In other words, it is excluded that an agent can purposefully modify, through his economic strategies, his position in the relevant distribution in order to change the median voter. A majority voting scheme is assumed not to be manipulable because a separation is imposed between the maximization problem in the two stages. Our thesis is that, although majority voting schemes are not manipulable taking the relevant distribution as given, they can be manipulated when agents can modify such distribution acting strategically at the economic level.

Analogously, in models of pressure groups, coalitions are usually given and agents do not change

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7 This has been noticed by Mueller (1989) for instance with respect to the works of Becker (1983), (1985).
8 We are here fully endorsing what Wittman (1989) writes on this issue: "Behind every model of government failure there is an assumption of extreme voter stupidity, serious lack of competition, or excessively high negotiation/transfer costs. Economists are very suspicious of similar assumptions regarding economic markets. This skepticism should be carried over to model of government behaviour".
9 Fiorentini (1990a) investigates a model where a community is made of two coalitions of employees in a firm and autonomous workers and where collective decisions are made by majority voting. In such a case the firm can overemploy with respect to the optimal number of workers in absence of the political stage, in order to reap fiscal advantages. This is because the employees benefit from such advantages through an increased productivity, and therefore vote for this fiscal equilibrium.
their behaviour in the economic stage in order to augment their effectiveness in the political arena. On the contrary, while individuals are seen as acting strategically in the political arena, the advantages that can be gathered in the latter do not depend on their actions in the economic stage.

Summing up, one can say that both welfare economics models and public choice models focus on strategic behaviour by decentralized agents in collective decision making. The former analyze such behaviour in the stage where agents modify their economic actions to manipulate collective decision mechanisms derived from exogenous political equilibria. The latter study strategic behaviour in the political arena, where strategies are chosen to take advantage of the collective mechanisms in the economic stage whose equilibria are exogenously given.

Needless to say, we regard both such approaches as very fruitful, but we also think that there is a need to analyze both stages of public decision making in models where non-cooperative agents act strategically in a coherent framework. This is not just for methodological reasons, but because when the notion of mechanism manipulation is accepted, it is difficult to limit its domain to the strategic use of private information. When decentralized agents act strategically, they try also to change the political equilibrium from which the mechanism is derived in the first place. Hence, in order to understand the features of public intervention one needs to study mechanisms which not only are incentive compatible (that is strategy-proof with respect to informational manipulation), but also equilibria in the game including the political stage (that is strategy-proof with respect to political manipulations). Only when mechanisms possess both features, they can be regarded as implementable in the game where economic and political strategies are intertwined.

2.2 Game-theoretic constitutional economics

Integrated models of strategic actions in public decision making are in order also to compare different institutional settings (i.e., general rules of political and/or economic interaction) within the
same model. As we said above, the literature on public decision making has been extensively investigated under a plethora of different institutional settings in isolation. To our knowledge, however, very little if anything has been done in order to compare rigorously the performances of different public decision making frameworks according to some normative criteria. Such task is not the same as comparing the normative characteristics of different public interventions under the same institutional setting, but is nearer in spirit to what has been recently defined as constitutional economics\textsuperscript{10}.

Our approach suits the comparison between general rules of public decision making because we focus only on mechanisms which are equilibria with respect to both economic and political manipulations. This means that the rules produced by such mechanisms are equilibria in a stronger sense than usually understood in public economics. But adopting such notion of equilibrium, one is allowed to compare more general rules knowing that "lower order" rules which are derived by the working of the "constitutional" settings are indeed comparable. However, we are aware that any game theoretic model needs to take some rules as completely exogenous in order to be self-consistent. In this respect we do not aim at modelling a regressio ad infinitum of rules of higher order, but instead at shifting the emphasis on a more integrated modelling of political and economic strategic behaviours.

2.3 The political activity

Of course, the optimally chosen levels of political activity depend on the constitutional rules imposed on the working of both the economic and the political stages. In this respect, we define constitutional settings by means of SWFs which are not chosen by any external agency, but are rules of aggregation of individual preferences known \textit{ex-ante} by each player. The latter uses the knowledge of

\textsuperscript{10} See Buchanan and Brennan (1985) and Brennan, Bohanon and Carter (1984) for an introduction to this concept. The latter authors propose also a simple and interesting model where "quasi constitutional" policies are discussed. In their approach, however, neither political nor economic institutions are founded on agents' non-cooperative behaviour.
such rules acting strategically at the political level as to modify the "weights" in the SWF. In this respect our model tackles the issue of the manipulability of constitutional rules by part of self-seeking agents and not that of rules derived by the adoption of a given constitution as in the traditional literature of optimal taxation.

A few caveats are in order. Our theory of political behaviour can be seen as methodologically opposite to Brennan and Buchanan's Leviathan, that is to principal-agent schemes applied to political interaction\textsuperscript{11}. This is because, in our opinion, if one pushes the explanation of citizens' behaviour based on informational asymmetries and transaction costs far enough, one can justify any public intervention varying the objective function of the agent (Leviathan). Though we recognize that in complex political settings there is scope for incentive problems on the part of politicians and bureaucrats\textsuperscript{12}, there is a major obstacle to a complete acceptance of such a thesis. This is because in doing so one ends up adopting extremely different viewpoints in the analysis of individual behaviour depending on whether the economic or the political stage is at stake.

As Wittman (1989) notices, such a dichotomy is not tenable since "many of the arguments claiming that economic markets are efficient apply equally well to democratic political markets, and, conversely, that economic models of political market failure are no more valid than the analogous arguments for economic market failure". In our perspective this is naturally so, for economic and

\textsuperscript{11} See Buchanan and Brennan (1977) for a discussion of different constitutional frameworks where the agent (the Leviathan) holds a monopolistic power over the principals (the citizens).

\textsuperscript{12} When this perspective is taken seriously, one is immediately confronted with an host of models where agents in the economic sphere are described as forward looking (possibly on an infinite horizon), completely informed, rational maximizers, and the institutions originated by their interactions are invariably Pareto (constrained) efficient. On the other hand, in the political sphere, presumably the same agents are described as irrational, poorly informed (and incapable of getting information) and their institution as inefficient due to monopoly, rent-seeking and lack of proper incentives.
political strategies are two sides of the same game. From this need of basing firmly social rules on individual rationality, we are naturally led to rule out any room for third agents such as politicians and bureaucrats.

3 The model

We consider a community formed by a finite number (N) of agents exogenously distributed into two groups. Depending on which of the aforesaid settings is at stake, agents in group A can work either exclusively or partially in the shadow sector of the economy, whereas agents in group B always work in the regular sector. In both sectors the same homogeneous commodity is produced.

To solve their maximization problem agents in sector A proceed as follows. First, they choose simultaneously an individual labour supply ($x_i$) in the shadow economy and a consumption level and Nash equilibria are established in both markets. Agents in sector B only choose the consumption level since they supply a given amount of work in the regular sector ($x$). Second, the fiscal constitution determines the tax rates. Third, agents in both sectors simultaneously solve for the optimal level of political activity ($s_i, i = A, B$) in order to take advantage of both the distribution of the fiscal revenue and of the effect of taxation on the economic sphere. Clearly, the equilibrium levels of political activity depend on the number of agents operating in the two sectors ($n_A, n_B$). Finally, a condition to close the model is that the commodity market is in equilibrium, so that the price level is determined only by the parameters of the model (i.e., $n_A, n_B$ and $x$).

To summarize, there are six unknowns in the model - the labour supply in the shadow sector, the tax rates on both expenditure and the labour supply, the level of political activity for both groups of agents, and the price level - that are simultaneously determined by using the first order conditions (FOCs) in the SWF and in the political stage and the equilibrium conditions in the labour and commodity markets. We now turn to the formal description of the model.
3.1 In the regular sector a homogeneous commodity is produced using the following linear technology:

\[ Q_r = L_r \]

where \( L_r \) indicates the aggregate labour supply in the regular sector. Accordingly, the after-tax nominal equilibrium wage in such sector is \( w_r = p(1 - t_r) \) that is equal to the value of the marginal productivity of labour. \( p \) indicates the price level in the commodity market and \( t_r \) is the marginal tax rate on labour. On the other hand, in the shadow sector the technology, characterized by decreasing returns to scale is as follows:

\[ Q_s = \log(L_s) \]

so that the nominal wage in equilibrium level is \( w_s = p/L_s \). The demand for the final commodity is given by the after-tax overall income aggregated across agents in both groups. By overall income we mean the labour income plus the redistributive quota of fiscal revenue net of the costs to undertake political activity. Therefore, the equilibrium in the commodity market implies:

\[ p(L_r + \log(L_s)) = (1 - t_r)(w_rL_r + w_sL_s) + T - S \]  \hspace{1cm} (1)

where:

\[ T = t_r(w_rL_r + s_rL_r) + t_s p L_r \]

and

\[ S = s_s p r + s_m p \]

where \( t_r \) is the marginal tax rate on final consumption and \( s_r \) is the level of political activity produced by agent i in group j. The payoff function for the representative agent working only in the regular sector is:

\[ U_r = (1 - t_r)w_r x + \frac{s_r}{S} T - s_s \]

Analogously, the payoff function for the representative agent who may work in either sector is:
\[ U_A = (1 - s_A)(w_0 x + w_e x_e) + \frac{s_0 T - s_A}{S} \, . \]

In both the above expressions the linear cost of production for the political activity is normalized to one. In these payoff functions the first term can be seen as the economic component, while the second and third terms define the political component. Notice, however, that while the second term define explicitly the rule through which the fiscal revenue is distributed back to each agent, in the economic term these political elements are present as far as equilibrium tax rates are concerned. The latter, in fact, are determined by maximizing the following SWF:

\[ W = U_A^\omega x, \quad U_B^\omega x \]

which will be used in what follows in its logarithmic form.

As the tax rates are determined as functions of the equilibrium levels of political activity in both groups, agents of either type maximize their payoffs wrt such activities in order to define \( x^*_A \) and \( x^*_B \). The latter are then substituted back in the expression for the tax rates, for the wages in the shadow economy and hence for the equilibrium in the commodity market. Some attention should be given to the role played in this framework by the SWF which is not derived by any external agency, but instead is an external rule used by each player in order to define his equilibrium strategies in both the political and economic stages.

We shall analyze the model under the LC setting in next section and under the DC and FC settings in sections 5 and 6 respectively.

4 The less developed community

Recall that in this setting agents are exogenously divided into two groups which can work exclusively in either the regular or the shadow sector. We start considering the maximization problem
faced by the representative agent in the regular sector. His labour supply is given and equal to \( x \) so that his payoff function is:

\[
U_a = p(1-t_f)(1-t_a)x + \frac{s_a}{S} T - s_a
\]  

(3a)

In the shadow sector, due to decreasing returns to scale, the equilibrium wage is \( w_s = p/(n_a x_s) \) so that the representative agents’s payoff function is:

\[
U_a = \frac{p(1-t_f)}{n_a} + \frac{s_a}{S} T - s_a
\]  

(3b)

where \( T \) in both expression results:

\[
T = t_i p[1 + (1-t_s) n_b] + t_s p n_b x.
\]  

(4)

Given our definition of the SWF, from the FOC obtained wrt \( t_i \) and \( t_s \) it is easy to derive the following:

\[
(1-t_s) = \frac{s_a}{s_a n_b x}
\]  

(5a)

\[
t_s = 1.
\]  

(5b)

Notice that, due to the logarithmic form adopted for the SWF, the SOCs are easily met. Plugging (4 a and b) into (3 a and b) and deriving the latter wrt \( s_a \) and \( s_s \), one gets the reaction curves in the political activity strategy space as:

\[
s_a^* = s_a^* \frac{n_b(1 + Q_s - Q_s)}{n_a(1 + Q_s - (1 + Q_s - Q_s))}.
\]

(6)

Finally, plugging the equilibrium levels of the individual political activity and tax rates into the equivalent of (1) in this setting the equilibrium price level can be obtained as:

\[
p^* = \frac{s^*}{1 + Q_s - Q_s^*}
\]  

(7)

whence a condition of non-negativity for the price level is:
\[ Q_i' > Q_k' - 1. \] (8)

Notice that (8) is also a necessary condition for (6) to be positive. The equilibrium level of labour supply in the shadow sector is accordingly obtained plugging the equilibrium price level in the definition of \( x_k \).

From the above expressions straightforward differentiation allows one to derive:

Remark 1. \( \frac{\partial s^*_h}{\partial q} < 0 \quad \frac{\partial s^*_h}{\partial x^*_h} > 0 \quad \frac{\partial x^*_h}{\partial q} > 0 \).

Remark 2. \( \frac{\partial x^*_h}{\partial q} = 0 \quad \forall j = (A, B) \).

Remark 3. If \( n_A = n_B \), then \( s^*_h > s^*_A \).

Remark 4. \( \frac{\partial s^*_h}{\partial q} > 0 \quad \frac{\partial s^*_h}{\partial q} < 0 \quad \frac{\partial x^*_h}{\partial q} > 0 \).

Remark 5. \( U_A > (n) U_B \) if \( n_B \) is sufficiently large.

Remark 6. \( \frac{\partial x^*_h}{\partial q} < 0 \).

Comments on these remarks will be postponed to section 7.

5 The developed community

In this setting agents A can work for a variable number of hours in the shadow sector on the top of the exogenously given labour supply in the regular sector. In the shadow sector the wage is again given as \( w_r = p/(n_A x_A) \) but now the equilibrium in the labour market (that is the mobility of agents of group A across sectors) requires that:

\[ w_r = p(1 - t) = p/(n_A x_A) = w_r. \]

Accordingly, the payoff function of the representative agent in groups A and B are respectively:
\[ U_a = p(1-t_a)(1-t_o) \left( x + \frac{1}{1-t_o} \right) + \frac{S_a}{S}T - s_a \]  \hspace{1cm} (9a)

\[ U_b = p(1-t_o)^2(1-t_a)x + \frac{S_b}{S}T - s_b \]  \hspace{1cm} (9b)

where \( T \) in both expression results:

\[ T = t_o p(1-t_o) \left[ N x + \frac{1}{1-t_o} \right] + t_o p N x \]  \hspace{1cm} (10)

As in section 4, from the FOC of (2) wrt \( t_a \) and \( t_o \) one gets:

\[ (1-t_o) = \frac{s_b}{n_o p_x (s_a - s_b)} \]  \hspace{1cm} (11a)

\[ t_o = 0 \]  \hspace{1cm} (11b)

Again the SOC{s for (11a) and (11b) are met due to the analytical form of the adopted SWF. Following the same procedure as above, the reaction curves in the political activity strategy space can be derived as:

\[ s_a = s_a \frac{n_b (n_a + p Q_o)}{n_a (1 + p Q_o)} \]  \hspace{1cm} (12)

Finally, plugging the equilibrium price level can be obtained as:

\[ p^* = \frac{S^*}{(1-t_o)(Q_o^* + N x) + (Q_a^* - Q_o^*)} \]  \hspace{1cm} (13)

From the expressions above one can easily derive the following:

**Remark 1D.** \( \frac{\partial x}{\partial x_a} < 0 \quad \frac{\partial x}{\partial x_b} > 0 \quad \frac{\partial x}{\partial x_o} > 0 \).  

**Remark 2D.** \( \frac{\partial x}{\partial x_j} = 0 \quad \forall j = (A, B) \).

**Remark 3D.** If \( n_a = n_b \), then \( s_a > s_b \).
Remark 4D. \( \frac{\partial u^*}{\partial x} > 0 \quad \frac{\partial u^*}{\partial z} < 0 \).

Remark 5D. \( U_A > (\leq) U_B \) if \( n_A \) (\( n_B \)) is sufficiently large.

Remark 6D. \( \frac{\partial u}{\partial z} > 0 \quad \frac{\partial u}{\partial x} < 0 \quad \frac{\partial u}{\partial x} > 0 \).

As before, comments on these remarks will be postponed to section 7.

6 The free-riding community

For the sake of analytical tractability, in dealing with the free-riding community, we shall focus only on the effect of the political activity on the definition of the tax structure in terms of labour and expenditure tax rates leaving aside the problem of the distribution of the fiscal revenue. This is not a loss of generality as great as it may appear if one assumes that a public good is provided which enters separately and equally the payoff of every citizen. In this setting agents A supply the same amount of work as agents in group B, but the former have the opportunity to allocate their work in either sectors. Such a decision depends on the usual variables and on the level of a a parameter \( \beta \) indicating the costs involved in tax evasion, that is in being caught working in the shadow sector. Such costs increase with the fiscal pressure on labour.

As in section 5 there is an equality constraint for net wages across sectors. The payoffs functions for the representative agents in sectors a and B are:

\[
U_a = p(1 - t_a)(1 - t_b)[x - x_a + \alpha(t_a)x_a] - s_a = p(1 - t_a)(1 - t_b)[x + \beta(t_a)x_a] - s_a \quad (14a)
\]

\[
U_b = p(1 - t_b)(1 - t_a)x - s_a \quad (14b)
\]

where \( \beta(t_a) = \alpha(t_a) - 1 < 0 \), with \( d\beta/dt_a > 0 \), is the aforesaid parameter measuring the efficiency of the fiscal organization to tackle income tax evasion. Proceeding as above for the FOCs of the SWF, we get:

\[
(1 - t_a) = \frac{s_{sa}x_n(s_nx_n + S)}{xS\beta} \quad (15a)
\]
\[ t^*_p = \frac{\beta' x_a n_a + (1 - \beta)S}{s_a n_a p (s_a n_a + S)}. \]  \hspace{1cm} (15b)

The equilibrium levels of political activity are obtained maximizing (14a and b) wrt \( s_a \) and \( s_b \):
\[ s^*_a = s^*_b = \frac{n_a \sqrt{\beta' - \sqrt{\beta' + \beta - 1}}}{\sqrt{\beta' + \beta - 1}}. \]  \hspace{1cm} (16)

The equilibrium price level becomes:
\[ p^* = \frac{S^*}{n_a x_a - Q^*_s + (1 - t_e) + N x[(1 - t_e)(1 - t_f) - 1]}. \]  \hspace{1cm} (17)

It will be useful to define in this setting the dead-weight loss (D) arising from the existence of the shadow sector:
\[ D = p[n_a x_a - \log(n_a x_a)] = p \left[ \frac{x' S'}{s_a s_a n_a (s_a n_a + S)} - \log \left( \frac{x' S'}{s_a s_a n_a (s_a n_a + S)} \right) \right]. \]

From the above expressions we get:

**Remark 1F:** \( \frac{\Delta t}{\Delta s_a} < 0 \); \( \frac{\Delta t}{\Delta n_a} < 0 \); \( \frac{\Delta t}{\Delta \beta} > 0 \).

**Remark 2F:** \( \frac{\Delta t}{\Delta s_a} > 0 \); \( \frac{\Delta t}{\Delta n_a} > 0 \); \( \frac{\Delta t}{\Delta \beta} > 0 \); \( \frac{\Delta t}{\Delta \beta} < 0 \).

**Remark 3F:** With \( n_a = n_b \), \( s^*_a > (<) s^*_b \) for small (large) values of \( \beta \).

**Remark 4F:** \( \frac{\Delta t}{\Delta s_a} > 0 \); \( \frac{\Delta t}{\Delta n_a} > 0 \).

**Remark 5F:** \( \frac{\Delta t}{\Delta s_a n_a} < 0 \); \( \frac{\Delta t}{\Delta n_a} < 0 \).

We shall now comment on the remarks obtained so far.
7 Fiscal structures compared

Remarks 1L, 1D and 1F tell us that the equilibrium tax rate on labour:

(i) provides in all three cases values in the interval $[0,1]$. Notice, however, that under DC $t^*_e$ could be outside such open interval if $s_a > s_d$. But this is excluded by remark 3D;

(ii) is a rather complex function of the equilibrium production of political activity in the society. More specifically, under DC and LC, it depends negatively on the political activity produced by the group working exclusively in the regular sector and positively on that produced by the group working also in the shadow sector. Under FC the political activity produced by both groups affect negatively the tax rate on labour. This is because under FC agents in group A would prefer to work in the regular sector with small tax rates than to work in the shadow sector with rapidly decreasing returns to scale;

(iii) depends positively on the exogenously given supply of labour in the regular economy. This seems to represent a feasibility constraint on the level of labour taxation given by the size of the fiscal revenue;

(iv) under FC depends positively on the rate of increase in the cost of being caught working in the shadow sector. This can be seen as a sort of "deterrence" effect provided by a more efficient (more penalizing) technology of fiscal monitoring. On its turn improvements in such technologies allow for larger levels of taxation on labour.

Summing up, labour taxation seems to represent the more active battleground for the production of political activity. By this we mean that the structure of the economic stage does not determine strictly the equilibrium value of the tax rate. (Contrast this with what happens for the expenditure taxation).

Remarks 2L, 2D and 2F show that the equilibrium tax rate on expenditure:

(i) is a corner solution under DC and LC, while is an internal solution under FC. Notice that under LC $t^*_e = 1$ while under DC $t^*_e = 0$. Such corner values are determined in our model by the structure of the economic stage which gives greater net income (including political payoffs) to group B in LC and group A in DC. However, these features can also be seen as an alternative explanation (to those based on
organizational arguments) for the predominance of expenditure over income taxation in developing countries and vice versa for more developed communities. In our model these configurations of the fiscal structure arise out of the competition in political as well as economic strategies which influence the working of the fiscal constitution;

(ii) under FC depends positively on the effort in political activity from both groups. Recalling the opposite effect of the latter on equilibrium labour taxation this indicates a pressure towards direct taxation which can be explained thinking of how the dead-weight loss increases with \( x \) and therefore with labour taxation (see remark 5F). Of course, such aggregate pressure is counterweighed by the individual incentive to work in the shadow sector for agents in group \( A \);

(iii) again under FC, depends positively on the costs to be caught working in the shadow sector which seem to allow for greater fiscal pressure. Notice, however, that the role played by \( \beta'(t_s) \) is reversed w.r.t. the point (iv) above.

Remarks 3L, 3D and 3F tell us that the equilibrium level of political activity:

(i) is greater for agents in group \( B \) (A) than for agents in group \( A \) (B) under LC (DC) assuming \( \gamma_a = \gamma_b \).

These remarks describe the strategy space in political activity showing that there is in general an asymmetric equilibrium. In such equilibrium agents in the group with larger labour income invest more heavily in political activity so that they reinforce this inequality through the distribution of the fiscal revenue. Of course, this effect can be reversed if the group with lower labour income is larger in number. In such a case the economic inequality can be overcome through fiscal redistribution;

(ii) is greater for agents in group \( A \) (B) for small (large) values of the costs involved in tax evasion. This indicates that, if the costs of tax evasion are sufficiently small, agents in group \( A \) have greater incentives to invest in political activity.
From remarks 4L, 4D and 4F one gets that the equilibrium price level:

(i) is positively related to the overall equilibrium level of political activity. Needless to say that this implies a negative effect of such activity on real incomes. In this perspective there are some analogies with the results of overall wastefulness of rent-seeking political activity common in many public choice models. In the spirit of our model this remark has not a normative flavour because there is no room for an external framer whose actions are indipendent from the self-seeking strategies of decentralized agents. Normative comments could be made reasoning about the different forms of a fiscal constitution to be adopted;

(ii) is negatively (positively) related with the share of the regular (shadow) sector on the overall economy. This feature is merely due to the existence of decreasing returns to scale in the shadow sector.

Remarks 5L, 5D and 5F show that:

(i) under LC and DC the equilibrium payoff is greater for agents in the group with larger labour income unless they represent a relatively small minority of the community. In our view this formalizes one of the central points of Montemartini (1900) conception of the State as a political enterprise. As he puts it: "the political enterprise may be defined as a productive organization designed to obtain the participation of an entire community in the purchase of given goods or services. ...(T)he political entrepreneur is that economic unit, that group, or that economic class which, at its own risk and peril, coordinates factors of production in a manner such as to compel an entire community to participate in the production of particular goods or in the performances of particular services. The object of the activity of the political entrepreneur is not the satisfaction of one or another need, but rather the creation of a coercive force for the distribution of the cost of certain production processes over a community."

13 But see comments on remark 5 below.

14 In this respect Montemartini could be seen as a very influential forerunner of an approach to public intervention purely based on the interaction of decentralized self-seeking agents. For more about this see Fiorentini (1990b).
(ii) under FC the dead-weight losses arising from working in the less efficient shadow sector are negatively related to the political activity produced by both groups. This can be explained looking back at remark 1 which told us that political activity in each group affects negatively the equilibrium labour tax rate and therefore reduces incentives to work in the shadow economy. Here we have that political activity under FC may have a positive impact on the payoff levels of both groups which counterweights its negative implications in terms of the price level.

Remarks 6L and 6D tell us:

(i) the dimension of the public sector (here equivalent to the size of the fiscal revenue) increases with the share of the regular sector on the whole economy in both LC and DC;

(ii) under LC the levels of political activity do not affect the size of the public sector. This is because the interaction in the political arena affects only the income and not the expenditure taxation on which most of the public budget rests. This is not true under DC where the public sector size increases (decreases) with the political activity produced by agents in group A (B). In other words agents which can work in the shadow sector act in the political arena as to increase the fiscal pressure knowing that they can partially free-ride.

8 Conclusions

In this paper we aimed at providing a description of fiscal equilibria between direct and indirect taxation in a game where decentralized agents play simultaneously economic and political strategies. This because we felt the need to investigate whether fiscal mechanisms are incentive compatible or not with respect to manipulations in the political equilibria on which SWF are usually assumed to rest.

Such a concern has led us to build a framework where the SWF is endogenous and more precisely is determined by the non-cooperative interaction of agents producing costly political activity. In this sense the tax rates which arise are equilibrium tax rates in a more robust sense than is usually understood.
in public economic models as they are strategy-proof on part of agents acting in a setting including political interaction.

We regard the results obtained as very preliminary both because more general economic settings can be sketched out and because of the lack of a comparative analysis of different fiscal constitutions. We think that some normative flavour - in a constitutional political economy prospective - could be added to this model if the latter direction of research is pursued.
References


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