

ON THE OPTIMAL RATE OF OBSOLESCENCE

by

Elettra Agliardi*

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Abstract. In this paper the decision of adoption of new technologies is studied through a multistage model. It incorporates multiple sequential technological innovations and the possibility of implementing technologies that have already been discovered. It is shown how such decision is affected by the relevant parameters, that is, the cost of adoption, the probability of discovery, the discount factor. Moreover, it is studied how the decision about adoption is affected by the presence of a rival.

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* University of Bologna, Department of Economics, and Churchill College, University of Cambridge (U.K).

1. INTRODUCTION

This paper deals with the problem of adoption of new technologies and explores how rivalry affects the decision about adoption, and therefore the evolution of an industry subject to technical change.

There are two aspects of technical change. Technical change may be "evolutionary": after you have chosen one technology, there are still improvements on it. Technical change may also have a "revolutionary" nature: new technologies may turn out to be more profitable because of new discoveries. In this context, firms have to decide whether to go on with the old method or to switch to the new one, i.e. whether to adopt immediately or to delay adoption. As Schumpeter states: "A new type of machine is in general but a link in a chain of improvements and may presently become obsolete". In a case like this, it would be convenient to wait, in order to see "how the chain behaves".

The reason is in the following circumstance. Although potential benefits from innovations largely take the form of what are called "first-mover advantages"¹, there may also be substantial first-mover disadvantages as well as late-mover advantages. If there are significant spillovers of knowledge between firms, then a late-mover could gain the same knowledge at a lower cost while, at the same time, avoiding the major mistakes that the first-mover made en route. In some circumstances the disadvantage of an early start may be such that old capital embodying old methods hinders the adoption of new methods and is worse than no capital at all. The question arising in this context is whether first-mover advantages may survive the offsetting disadvantages so that they provide an incentive for the performance of basic research.

Most models of innovations abstract enormously from the richness of reality. Models of the supply of innovations generally represent the process as a racing game (see Lee and Wilde (1980), Loury (1979), Reinganum (1981a), (1985)). In this game

the potential suppliers invest in research whose outcome – either the time until discovery or the size of the discovery – is random. The winner reaps all the benefits of the research investment. Models of the demand for innovation generally represent the problem as a one-time decision to adopt the new technology (see Jensen (1982), Reinganum (1981b), Kamien and Schwartz (1972)). The timing of the adoption of the innovation may depend on strategic considerations or on market structure.

In a different way with respect to Reinganum (1981b) and Kamien and Schwartz (1972), we consider a multistage model. It incorporates multiple sequential technological innovations and the possibility of implementing technologies that have already been discovered. Although many of the conclusions in this paper are economically intuitive, they have not, to our knowledge, been systematically derived from a multistage framework. Our analysis shows that there is a threshold such that the firm will immediately adopt the best technology if the advantage in terms of cost with respect to the old technology exceeds this threshold; otherwise it temporarily avoids the switching costs and postpones adoption of the new technology. Furthermore, this threshold increases and postponement becomes more attractive if the cost of adoption increases.

This result may be linked to Ames and Rosenberg (1963). They say that if the cost of moving from a lower to a higher technology is an increasing function of the level of technology already reached², then, at each stage, the firm must contemplate a larger "leap" than it made last time. Before it can afford another step, at some point, it must wait for so long a time that technological progress would actually come to a stop. Put it another way, if the cost of moving from a lower to a higher technology is increasing, then the rate of development may slow down as the economy develops.

Our analysis also shows that postponement is more attractive if there is an increase in the pace of technological progress: if the probability of making a discovery increases, then the firm will try to avoid locking itself in. We study how the decision

problem of a firm is affected by the presence of a rival. We show that the presence of a rival retards the adoption of the current best technology. The last circumstance is in keeping with the so-called Schumpeterian hypothesis, according to which the incentive to innovate is larger in a monopolistic market than in a competitive one. Our result can shed light on the question about the role of entry in preventing or not the market to lock-in to an inefficient technology.

Obviously, other salient factors should be introduced in the analysis to flesh out the desired realism. A few of them are discussed in the final remarks.

2. THE BASIC MODEL

We consider a monopolist in a homogeneous-product market. The model is a multistage one with a sequence of process innovations. For simplicity, we characterize innovations with the one-dimensional variable c , as specifying the constant marginal cost of production, $c \in \mathbb{R}_+$. Time is divided into an infinite sequence of discrete periods, $t = 0, 1, 2, \dots$. We assume that the discovery process of technological improvements is exogenous and stochastic. In each period t , we denote by p the probability that a discovery takes place, and by $(1 - p)$ the probability that no discovery takes place. Hence, the stochastic process is independent and identically distributed through time. We denote by $\Pi(c)$ the one-period profit³ associated with the cost of production c .

We consider the following decision problem for the monopolist. In each period, the firm has to decide about adoption of new technologies. In any period t , the firm knows whether a discovery took place or not, by the time it makes a decision for t . If the firm adopts, it incurs a fixed cost of adoption K , $K > 0$. Once a discovery takes place and the firm adopts, the cost of production decreases by α , $0 \leq \alpha \leq 1$, that is, it becomes $c(1 - \alpha)$. We assume that α is certain and constant through time⁴. If the firm does not adopt, the cost of production remains unchanged. However, the discovery is not lost and can be adopted subsequently. Indeed we assume that an innovation embodies all the previous discoveries that have not been adopted yet.

The firm maximizes the expected stream of discounted profits. Let δ , $0 < \delta < 1$, be the discount factor. We can write the problem formally as:

$$(1) \quad \text{Max } E \left[\sum_{t=0}^{\infty} \delta^t (\Pi(c_t) - Kx_t) \right]$$

$$\text{subject to } x_{1t}, x_{2t} \in \{0, 1\}$$

and the sequential constraints for all t

$$\left. \begin{aligned} x_t &= x_{1t} \\ c_t &= x_{1t} d_{t-1} c_{t-1} (1-\alpha) + (1-x_{1t}) c_{t-1} \\ d_t &= x_{1t} + (1-x_{1t}) d_{t-1} (1-\alpha) \end{aligned} \right\} \text{ with probability } p$$

$$\left. \begin{aligned} x_t &= x_{2t} \\ c_t &= x_{2t} d_{t-1} c_{t-1} + (1-x_{2t}) c_{t-1} \\ d_t &= x_{2t} + (1-x_{2t}) d_{t-1} \end{aligned} \right\} \text{ with probability } 1-p$$

where d_t is a measure of accumulated innovation at time t , $0 \leq d_t \leq 1$, for any t . Let us explain the constraints above. With probability p a discovery takes place. For any t , if the firm adopts ($x_{1t} = 1$), then the cost decreases by α , and since an innovation embodies all previous improvements not already adopted, we get c_t by multiplying $(1-\alpha)c_{t-1}$ by d_{t-1} . The value of d_t equals 1. If the firm does not adopt ($x_{1t} = 0$), then the current cost remains unchanged, but next period value for d is multiplied by $(1-\alpha)$. With probability $1-p$ a discovery does not take place. If the firm decides to adopt the previous innovation ($x_{2t} = 1$), then the cost decreases by d and next period value for d equals 1. If the firm postpones adoption ($x_{2t} = 0$), then the current cost remains unchanged and next period value for d remains unchanged as well.

We make the following assumptions on the one-period profit function $\Pi(c)$:

- A.1. For all $c \geq 0$:
- (i) $\Pi(\cdot)$ is continuously differentiable on \mathbb{R}_+
 - (ii) $\Pi(c)$ is nonnegative
 - (iii) $\Pi(\cdot)$ is non-increasing and convex

A.2. $\Pi(0) < \infty$

Assumption A.1. does not require special comments and is satisfied, for example, in the case of a linear demand function. Assumption A.2. implies that $\Pi(c)$ is bounded for any $c \geq 0$.

The value of the optimal program (1) depends, for given α , K , δ and p , on the value of initial (c,d) . Define the value of the program for initial (c,d) as:

$$V(c,d) = \max E \left[\sum_{t=0}^{\infty} \delta^t (\Pi(c_t) - Kx_t) \right]$$

etc, as in (1). Associated with the function $V(c,d)$, there is the functional equation:

$$\begin{aligned} (2) \quad V(c,d) = & \\ p \max_{x_1 \in \{0,1\}} & [\Pi(x_1cd(1-\alpha) + (1-x_1)c) - Kx_1 + \delta V(x_1cd(1-\alpha) + (1-x_1)c, x_1 + (1-x_1)d(1-\alpha))] + \\ & + (1-p) \max_{x_2 \in \{0,1\}} [\Pi(x_2cd + (1-x_2)c) - Kx_2 + \delta V(x_2cd + (1-x_2)c, x_2 + (1-x_2)d)] \end{aligned}$$

which can be written as:

$$\begin{aligned} (3) \quad V(c,d) = & p \max [\Pi(cd(1-\alpha) - K + \delta V(cd(1-\alpha), 1); \Pi(c) + \delta V(c, d(1-\alpha))] + \\ & + (1-p) \max [\Pi(cd) - K + \delta V(cd, 1); \Pi(c) + \delta V(c, d)] \end{aligned}$$

The following Proposition holds:

Proposition 1. For given c , the value of problem (1), $V(c,d)$, is a continuous, non-increasing, convex function of d that satisfies equation (3).

Proof. The proof is by induction on the sequence of finite horizon problems with the same constraints and objective function. Consider the sequence of functions:

$$V^0(c,d) = 0$$

$$V^1(c,d) = p \max[\Pi(cd(1-\alpha)) - K, \Pi(c)] + (1-p) \max[\Pi(cd) - K, \Pi(c)]$$

.....

$$V^T(c,d) = p \max[\Pi(cd(1-\alpha)) - K + \delta V^{T-1}(cd(1-\alpha), 1), \Pi(c) + \delta V^{T-1}(c, d(1-\alpha))] + \\ + (1-p) \max[\Pi(cd) - K + \delta V^{T-1}(cd, 1), \Pi(c) + \delta V^{T-1}(c, d)]$$

Let c be assigned. By assumption A.1, $V^1(c,d)$ is a non-increasing, convex function of d . Assume that $V^{T-1}(c,d)$ satisfies all the claims. Then $V^T(c,d)$ is non-increasing as a function of d , because Π and V^{T-1} are. Moreover $V^T(c,d)$ is convex, because the maximum of convex functions is convex and the sum of convex functions is convex.

In what follows we show that $V^T(c,d)$ is monotonic in T and uniformly bounded above for every T and for given (c,d) . Therefore $V^T(c,d)$ is convergent to the limit $V(c,d)$ and the properties of $V^T(c,d)$ are inherited by $V(c,d)$.

Let us show that $V^T(c,d)$ is monotonic in T . Obviously $V^1(c,d) \geq V^0(c,d)$. Assume that $V^{T-1}(c,d) \geq V^{T-2}(c,d)$ for every (c,d) . Then

$$V^T(c,d) \geq p \max[\Pi(cd(1-\alpha)) - K + \delta V^{T-2}(cd(1-\alpha), 1), \Pi(c) + \delta V^{T-2}(c, d(1-\alpha))] + \\ + (1-p) \max[\Pi(cd) - K + \delta V^{T-2}(cd, 1), \Pi(c) + \delta V^{T-2}(c, d)]$$

$$= V^{T-1}_{(c,d)}.$$

Finally we show the property of uniform boundedness of $V^T_{(c,d)}$. By A.2.(i) and for $0 < \delta < 1$, $\Pi(0)/(1-\delta)$ is finite. As Π is non-increasing, it holds:

$$V^1_{(c,d)} \leq \Pi(0) \leq \Pi(0)/(1-\delta).$$

If we assume that $V^{T-1}_{(c,d)} \leq \Pi(0)/(1-\delta)$ for any (c,d) , then it follows:

$$\begin{aligned} V^T_{(c,d)} &\leq p \max\left[\Pi(cd(1-\alpha)) - K + \delta \frac{\Pi(0)}{1-\delta}, \Pi(c) + \delta \frac{\Pi(0)}{1-\delta}\right] \\ &\quad + (1-p) \max\left[\Pi(cd) - K + \delta \frac{\Pi(0)}{1-\delta}, \Pi(c) + \delta \frac{\Pi(0)}{1-\delta}\right] \\ &= V^1_{(c,d)} + \delta \frac{\Pi(0)}{1-\delta} \leq \Pi(0) + \delta \frac{\Pi(0)}{1-\delta} = \frac{\Pi(0)}{1-\delta}. \end{aligned}$$

□

3. PROPERTIES OF THE OPTIMAL POLICY

The function $V(c, d)$ gives the value of the program that follows the optimal policies $x_1(c, d)$, $x_2(c, d)$. Let us concentrate our attention to the case of a discovery, that is, $x = x_1(c, d)$. By simple inspection on expression (3) we notice that the case of no discovery, that is, $x = x_2(c, d)$, has a similar behaviour. If the following inequality holds:

$$(4) \quad \Pi(cd(1-\alpha)) - K + \delta V(cd(1-\alpha), 1) > \Pi(c) + \delta V(c, d(1-\alpha))$$

then the optimal policy is to adopt immediately, i.e. $x_1(c, d) = 1$. Let $\Delta = (1-\alpha)d$, $0 \leq \Delta \leq 1$. Expression (4) can be written in the following way:

$$(5) \quad \Pi(c\Delta) - \Pi(c) - K > \delta(V(c, \Delta) - V(c\Delta, 1)).$$

The following properties hold:

Proposition 2. For given c and for suitable K , there exists Δ^* such that the firm adopts the best available technology iff $\Delta \leq \Delta^*$; otherwise it defers adoption.

Proof. Let c be assigned. Let $F(\Delta) = \Pi(c\Delta) - \Pi(c) - K - \delta(V(c, \Delta) - V(c\Delta, 1))$. Obviously, $F(1) = -K < 0$. Consider:

$$(6) \quad F(0) = \Pi(0) - \Pi(c) - \delta(V(c, 0) - V(0, 1)) - K$$

For K sufficiently small, expression (6) is positive, because $\Pi(0) > \Pi(c)$ for any $c > 0$ and $V(c, 0) < V(0, 1)$, as can be proved by induction. Since $\Pi(c)$, $V(c, \Delta)$ and $V(c\Delta, 1)$ are

continuous functions, also $F(\Delta)$ is a continuous function. Therefore there exists Δ^* , $0 < \Delta^* < 1$, such that $F(\Delta^*) = 0$. (See figure 1).

□

Remark. If K is sufficiently large, such that expression (6) is negative, then $F(\Delta) < 0$ for $0 \leq \Delta \leq 1$, and therefore the firm will not adopt. In what follows we consider K as required by Proposition 2.

Proposition 2 states that the firm will adopt the new technology if the size of innovation is large enough; otherwise it will postpone adoption. We examine how the critical value Δ^* depends on the relevant parameters K , p and δ .

Proposition 3. The critical value Δ^* is non-increasing in K . That is, the firm will postpone adoption, if the cost of adoption increases.

Proof. In this proof the following simple Lemma will be used.

Lemma 1. Let $a \leq b$. Then, for any x :

$$\max(a, x) - \max(b, x) = \begin{cases} x-b & \text{if } a \leq x \leq b \\ 0 & \text{if } a \leq b \leq x \\ a-b & \text{if } x \leq a \leq b \end{cases}$$

Consider expression (7):

$$(7) \quad \Pi(c\Delta) - \Pi(c) - K = \delta(V(c, \Delta|K) - V(c, 1|K))$$

which is satisfied for $\Delta = \Delta^*(K)$. The function $\Pi(c\Delta) - \Pi(c) - K$ is decreasing in K . In order to investigate how the right-hand side of expression (7) behaves as K changes, let us

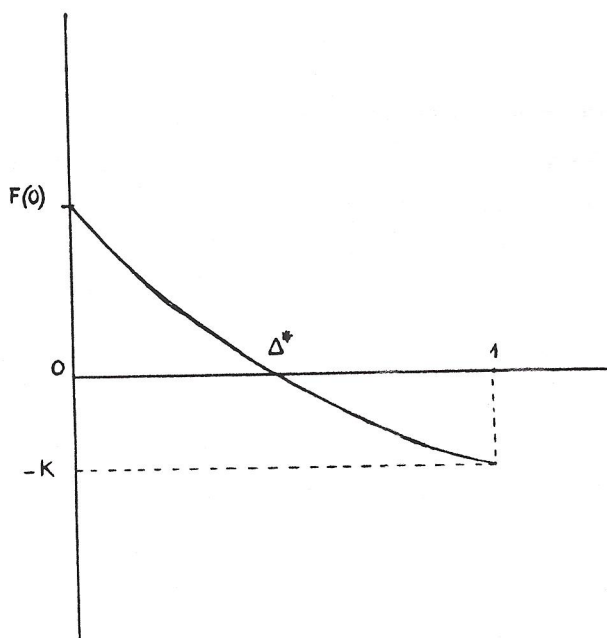


Figure 1

prove the following inequality:

$$(8) \quad V(c, \Delta | K) - V(c, \Delta, 1 | K) > V(c, \Delta | K + \epsilon) - V(c, \Delta, 1 | K + \epsilon)$$

which holds for any K and for small ϵ , $\epsilon > 0$.

First let us show that inequality (8) holds with V^1 replacing V . Indeed

$$V^1(c, \Delta | K) - V^1(c, \Delta, 1 | K) = p B_1(K) + (1-p) B_2(K),$$

where $B_1(K) = \max[\Pi(c\Delta(1-\alpha)) - K, \Pi(c)] - \max[\Pi(c\Delta(1-\alpha)) - K, \Pi(c\Delta)]$ and $B_2(K) = \max[\Pi(c\Delta) - K, \Pi(c)] - \max[\Pi(c\Delta) - K, \Pi(c\Delta)]$. Analogously

$$V^1(c, \Delta | K + \epsilon) - V^1(c, \Delta, 1 | K + \epsilon) = p B_1(K + \epsilon) + (1-p) B_2(K + \epsilon).$$

By applying Lemma 1 with $a = \Pi(c)$, $b = \Pi(c\Delta)$, $x = \Pi(c\Delta(1-\alpha)) - K$, we get $B_1(K + \epsilon) \leq B_1(K)$, for small $\epsilon > 0$. Moreover it is clear that $B_2(K + \epsilon) \leq B_2(K)$. Thus inequality (8) holds for V^1 . By the same argument we can prove by induction that (8) holds with V^T replacing V for any T , and hence (8) follows. Since the change in the right-hand side of (7) due to a change in K is smaller than the change in the left-hand side, it follows that $\Delta^*(K + \epsilon) \leq \Delta^*(K)$ for any $\epsilon > 0$. \square

A corollary of Proposition 3 is the following. If the cost of adoption is an increasing function of the level of technology already achieved, then the firm will postpone adoption. At a certain point, when the cost of adoption becomes sufficiently large, the firm will not adopt any longer. A consequence is that the rate of development will slow down as knowledge increases.

We consider now the implications of a change in p . The following Proposition

holds:

Proposition 4. The critical value Δ^* is non increasing in p . That is, an increase in the pace of technological progress renders postponement more attractive.

Proof. Consider expression (9):

$$(9) \quad \Pi(c\Delta) - \Pi(c) - K = \delta(V(c, \Delta|p) - V(c\Delta, 1|p))$$

which is satisfied for $\Delta = \Delta^*(p)$. The left-hand side of expression (9) does not change with p . Moreover, the following inequality holds:

$$(10) \quad V(c, \Delta|p) - V(c\Delta, 1|p) \leq V(c, \Delta|p + \epsilon) - V(c\Delta, 1|p + \epsilon)$$

for any p and for small ϵ , $\epsilon > 0$. Indeed:

$$V^1(c, \Delta|p) - V^1(c\Delta, 1|p) - (V^1(c, \Delta|p + \epsilon) - V^1(c\Delta, 1|p + \epsilon)) = -\epsilon[\max(\Pi(c\Delta(1-\alpha)) - K, \Pi(c)) - \max(\Pi(c\Delta) - K, \Pi(c)) + \Pi(c\Delta) - \max(\Pi(c\Delta(1-\alpha)) - K, \Pi(c\Delta))] \leq 0$$

It can be proved by induction that inequality (10) holds for V^T replacing V for any T , and hence (10) follows. Therefore $\Delta^*(p + \epsilon) \leq \Delta^*(p)$ for any $\epsilon > 0$.

□

Finally, let us consider the implications of a change in δ . Following the same procedure of the previous proof we can obtain this Proposition:

Proposition 5. The critical value Δ^* is non increasing in δ . That is, an increase in the

discount factor renders postponement more attractive.

4. THE ROLE OF RIVALRY

In this section we study how the decision problem of a firm is affected by the presence of a rival. It is often possible for a later entrant to start with an equipment base which begins at a cost level that may be lower than that of an earlier entrant⁵.

We study whether the presence of a rival which is subject to technological improvement may cause the established firm to accelerate the process of adoption or to delay it. In this industry, indeed, a critical decision is to determine when it becomes worthwhile to commit to an investment that will replace the existing technology and when the best strategy is to continue to operate the present technology. The decision must take into account whether or not the rival has adopted its innovation.

Such a model should be formulated as a dynamic game, to take into account the interaction between the competing firms. Here, however, we put forward the following formulation as a first step to solve such a problem. We consider the decision problem for the established firm, given the choice of the rival. Let firm 1 be the established firm and firm 2 the rival. We make the simplifying assumption that firm 1's expectation about the probability of adoption of firm 2 is known and constant through time. Let q be the rival's probability of adoption that firm 1 expects, and $1-q$ the probability of no adoption. We denote by $\Pi(c^1, c^2)$ the one-period profit⁶ for firm 1 if firm 1's cost of production is c^1 and firm 2's cost is c^2 , with $c^1, c^2 \geq 0$. The following assumptions on $\Pi(c^1, c^2)$ hold:

- A.3.
- (i) $\Pi(c^1, c^2)$ is continuously differentiable
 - (ii) $\Pi(c^1, c^2)$ is non-negative
 - (iii) $\Pi(c^1, c^2)$ is non-increasing in c^1 and non-decreasing in c^2

(iv) given c^2 , $\Pi(c^1, c^2)$ is convex in c^1

(v) $\partial^2 \Pi(c^1, c^2) / \partial c^1 \partial c^2 < 0$, for all c^1, c^2

A 4 given c^2 , $\Pi(0, c^2) < \infty$

Similar comments to those for A.1 and A.2. can be applied to A.3. and A.4. Assumption A.3 (v) is satisfied, for example, in the case of a Cournot duopoly, with linear demand function. The analysis which follows constitutes an extension of what we showed in the previous sections. Now the problem can be written formally as:

$$(11) \quad \text{Max } E \left[\sum_{t=0}^{\infty} \delta^t (\Pi(c_t^1, c_t^2) - Kx_t) \right]$$

subject to $x_{1t}, x_{2t} \in \{0, 1\}$

and the sequential constraints for all t

$$\left. \begin{aligned} x_t &= x_{1t} \\ c_t^1 &= x_{1t} d_{t-1}^1 c_{t-1}^1 (1-\alpha) + (1-x_{1t}) c_{t-1}^1 \\ d_t^1 &= x_{1t} + (1-x_{1t}) d_{t-1}^1 (1-\alpha) \end{aligned} \right\} \quad \text{with probability } p$$

$$\left. \begin{aligned} x_t &= x_{2t} \\ c_t^1 &= x_{2t} d_{t-1}^1 c_{t-1}^1 + (1-x_{2t}) c_{t-1}^1 \\ d_t^1 &= x_{2t} + (1-x_{2t}) d_{t-1}^1 \end{aligned} \right\} \quad \text{with probability } 1-p$$

$$\left. \begin{aligned} c_t^2 &= d_{t-1}^2 c_{t-1}^2 (1-\alpha) \\ d_t^2 &= 1 \end{aligned} \right\} \quad \text{with probability } pq$$

$$\left. \begin{aligned} c_t^2 &= c_{t-1}^2 \\ d_t^2 &= d_{t-1}^2(1-\alpha) \end{aligned} \right\} \text{ with probability } p(1-q)$$

$$\left. \begin{aligned} c_t^2 &= d_{t-1}^2 c_{t-1}^2 \\ d_t^2 &= 1 \end{aligned} \right\} \text{ with probability } (1-p)q$$

$$\left. \begin{aligned} c_t^2 &= c_{t-1}^2 \\ d_t^2 &= d_{t-1}^2 \end{aligned} \right\} \text{ with probability } (1-p)(1-q)$$

where d^1 and d^2 are a measure of accumulated innovation for firm 1 and firm 2 respectively. The value of the optimal program (11) depends, for given α , K , p and q , on the initial value of (c^1, d^1, c^2, d^2) . From (11) we can obtain the following functional equation:

$$\begin{aligned} (12) \quad & V(c^1, d^1; c^2, d^2) = \\ & p \max \{ q[\Pi(d^1 c^1(1-\alpha), d^2 c^2(1-\alpha)) - K + \delta V(d^1 c^1(1-\alpha), 1; d^2 c^2(1-\alpha), 1)] + \\ & + (1-q)[\Pi(d^1 c^1(1-\alpha), c^2) - K + \delta V(d^1 c^1(1-\alpha), 1; c^2, d^2(1-\alpha))] , \\ & q[\Pi(c^1, d^2 c^2(1-\alpha)) + \delta V(c^1, d^1(1-\alpha); d^2 c^2(1-\alpha), 1)] + \\ & + (1-q)[\Pi(c^1, c^2) + \delta V(c^1, d^1(1-\alpha); c^2, d^2(1-\alpha))] \} + \\ & + (1-p) \max \{ q[\Pi(d^1 c^1, d^2 c^2) - K + \delta V(d^1 c^1, 1; d^2 c^2, 1)] + \\ & + (1-q)[\Pi(d^1 c^1, c^2) - K + \delta V(d^1 c^1, 1; c^2, d^2)] , \end{aligned}$$

$$q[\Pi(c^1, d^2 c^2) + \delta V(c^1, d^1, d^2 c^2, 1)] + (1-q)[\Pi(c^1, c^2) + \delta V(c^1, d^1, c^2, d^2)]\}.$$

As in the previous section we concentrate our attention to the case of a discovery. Let $\Delta_1 = (1-\alpha)d_1$ and $\Delta_2 = (1-\alpha)d_2$, with $0 \leq \Delta_1 \leq 1$, $0 \leq \Delta_2 \leq 1$. If the following inequality holds

$$(13) \quad q[\Pi(c^1 \Delta^1, c^2 \Delta^2) - \Pi(c^1, c^2 \Delta^2)] + (1-q)[\Pi(c^1 \Delta, c^2) - \Pi(c^1, c^2)] - K >$$

$$\delta\{q[V(c^1, \Delta^1; c^2 \Delta^2, 1) - V(c^1 \Delta^1, 1; c^2 \Delta^2, 1)] + (1-q)[V(c^1, \Delta^1; c^2, \Delta^2) - V(c^1 \Delta^1, 1; c^2, \Delta^2)]\}$$

then the optimal policy for firm 1 is to adopt immediately.

With the same argument as in Proposition 2 it is easy to prove the following:

Proposition 6 For given c^1, c^2, Δ^2 and for suitable K , there exists Δ^{1*} such that firm 1 adopts the best available technology iff $\Delta^1 \leq \Delta^{1*}$; otherwise it defers adoption.

If $q = 0$, for all t and for any p , then $c_t^2 = c_{t-1}^2$, that is, c^2 does not change through time. Since firm 2 affects firm 1's behaviour through c^2 only, the presence of a rival does not play any role on firm 1's choice if $q = 0$. In order to study how the presence of a rival affects the decision about adoption of the established firm, let us compare the threshold value Δ^{1*} when $q = 0$ and when $q > 0$. The following Proposition holds:

Proposition 7 For given c^1, c^2, Δ^2 , the critical value Δ^{1*} is decreasing in q . In particular, $\Delta^{1*}(q = 0) > \Delta^{1*}(q > 0)$. That is, the presence of a rival renders postponement more attractive.

Proof. It follows from a comparison between expressions (14) and (15), for given c^1, c^2, Δ^2 :

$$(14) \quad \Pi(c^1 \Delta^1, c^2) - \Pi(c^1, c^2) - K = \delta[V(c^1, \Delta^1; c^2, \Delta^2) - V(c^1 \Delta^1, 1; c^2, \Delta^2)]$$

which is satisfied for $\Delta^1 = \Delta^{1*}(q = 0)$, and

$$(15) \quad q[\Pi(c^1 \Delta^1, c^2 \Delta^2) - \Pi(c^1, c^2 \Delta^2)] + (1-q)[\Pi(c^1 \Delta^1, c^2) - \Pi(c^1, c^2)] - K =$$

$$\delta[q(V(c^1, \Delta^1; c^2 \Delta^2, 1) - V(c^1 \Delta^1, 1; c^2 \Delta^2, 1)) + (1-q)[V(c^1, \Delta^1; c^2, \Delta^2) - V(c^1 \Delta^1, 1; c^2, \Delta^2)]]$$

which is satisfied for $\Delta^1 = \Delta^{1*}(q > 0)$.

By A.3. we get that the derivative of the left-hand side of expression (15) with respect to q is negative, and in particular:

$$\Pi(c^1 \Delta^1, c^2) - \Pi(c^1, c^2) > q[\Pi(c^1 \Delta^1, c^2 \Delta^2) - \Pi(c^1, c^2 \Delta^2)] + (1-q)[\Pi(c^1 \Delta^1, c^2) - \Pi(c^1, c^2)]$$

Moreover we can show by induction that the right-hand side of expression (15) is non decreasing in q , and in particular that:

$$V(c^1, \Delta^1, c^2, \Delta^2) - V(c^1 \Delta^1, 1, c^2 \Delta^2) \leq q[V(c^1, \Delta^1, c^2 \Delta^2, 1) - V(c^1 \Delta^1, 1, c^2 \Delta^2, 1)]$$

$$+ (1-q)[V(c^1, \Delta^1, c^2, \Delta^2) - V(c^1 \Delta^1, 1, c^2, \Delta^2)]$$

Therefore, Δ^{1*} is decreasing in q and in particular $\Delta^{1*}(q = 0) > \Delta^{1*}(q > 0)$.

□

This result is in keeping with the literature supporting the so-called Schumpeterian

hypothesis, according to which the incentive to innovate is larger in a monopolistic market. With an opposite conclusion with respect to Arrow (1962), this literature argues that it is not always true that competitiveness in industries leads to a higher level of technological advancement than when industries are less competitive. Our result is that competition inhibits technological advancement. When interpreted in terms of potential rivalry, such result can be of some help in the discussion about the role of entry in preventing or not the market to lock-in to an inefficient technology. Potential rivalry would retard the adoption of the best available technology.

5. FINAL REMARKS

This paper examines the decision of adoption of new technologies in a multistage framework and explores how such decision is affected by the relevant parameters, that is, the cost of adoption, the probability of discovery and the probability of adoption of the rival. Even if the framework is a simplified one, our result that rivalry inhibits technological advancement can be of some help in the discussion about the role of entry and of policy interventions in such industries. Our result implies that the common attitude in policy interventions, and especially in anti-trust policies, may be questionable in the presence of technological progress.

A few extensions for possible further research can be identified. In this paper innovations are fully characterized by the one-dimensional abstraction of cost reduction. We consider process innovations. More generally, one can model innovations by considering an index number that identifies the product produced or the production technique employed. That is, such index may be interpreted as an index of product quality, or as the inverse of the minimum average cost associated with the technique in operation.

The timing of decisions is largely influenced by expectations about the time path of future technological changes. An interesting extension of this paper is to model expectations as well. Rosenberg (1976) illustrates that the main determinant of adoption or not is the expectation concerning the future course of technological innovations. One of his conclusions seems to be inescapable in our framework: "A firm may be unwilling to introduce the new technology if it seems highly probable that further technological improvements will shortly be forthcoming". To flesh out the desired realism one should introduce other salient factors such as uncertainty

about the profitability of new discoveries; temporizing measures such as minor adjustments, alterations and additions to existing equipment; compatibility of the various innovations.

NOTES

¹They include a variety of learning experiences. Firms that move down such learning curves first – whether these curves pertain to cost reductions or performance improvements – may be able subsequently to exploit the advantages conferred as a barrier to entry of new firms. To the extent that the findings of basic research can be translated into patentable assets, first-movers may be able to consolidate their market position through patent protection. Furthermore, buyers switching costs may be significant and may constitute a significant form of protection against competitors for firms that are first to enter the new product line.

²This depends on the degree of sunkness, i.e. it is true in the industries where adopting the newest technology requires a huge financial commitment in physical and intangible assets, and where "retrofitting" is not very important.

³Obviously, $\Pi(c) = \max_{q \geq 0} (qf(q) - cq)$, where q is the output level and $f(q)$ is the inverse demand function. In this paper we specify assumptions on $\Pi(c)$, which is our primitive function, instead of specifying assumptions on $f(q)$.

⁴The analysis can be extended easily to the case where α is uncertain, with known distribution function.

⁵This seems to have been the distinctive characteristic of the American chemical processing scene (Rosenberg, 1990). The presence of specialized engineering firms (SEFs) to which most of the design and engineer functions were subcontracted, played a critical role with respect to competition among chemical manufacturers. Latecomers to a particular chemical technology could benefit from their relations with the SEFs, which were able to provide them with the process know-how that they had accumulated through their previous relations with earlier entrants. The availability of such technologies from the SEFs also encouraged many new entrants into the industry from related sectors. A result was intensified competition, including periods of overbuilding and excess capacity.

⁶We omit the superscript 1 to $\Pi(c^1, c^2)$, indicating firm 1's profit, because it does not cause ambiguity. Indeed, we do not have to specify firm 2's profit in this formulation firm 2 affects firm 1's profit through c^2 .

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