FIXED AGENDA SOCIAL CHOICE THEORY: CORRESPONDENCE AND IMPOSSIBILITY THEOREMS FOR SOCIAL CHOICE CORRESPONDENCES AND SOCIAL DECISION FUNCTIONS

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1. INTRODUCTION

Arrow's Impossibility Theorem is concerned with the problem of finding a collective choice rule which selects one or more alternatives from every non empty subset of the universal set of alternatives. However, in actual situations, it is only necessary to choose from one particular subset of alternatives - the set of all alternatives which are feasible under the given circumstances. On the basis of this observation, it might be argued that in fact Arrow's Impossibility Theorem does not preclude the existence of a satisfactory collective choice rule.

Indeed, Arrow's theorem rests heavily on *inter-agenda* conditions, i.e. conditions which relate the choice from certain subsets of alternatives to the choice from other subsets. In the relation-theoretic version of the theorem (cf. Sen, 1986), a social preference relation is defined, which may be thought of as a complete specification of social choice from every pair of alternatives. Then, by means of appropriate collective rationality conditions, one may infer from information on social preferences over the pairs $\{x,y\}$ and $\{y,z\}$ what the social preference over $\{x,z\}$ will be. The use of inter-agenda conditions is even more eplicit in the choice-theoretic version of Arrow's theorem. A crucial hypothesis of the theorem is the Axiom of Revealed Preferences, which states that, given two sets of alternatives A and B, if B is a subset of A, and if some elements of the choice set from A belong to B, then the choice set from B must be made of those elements of B which were chosen from A. This is clearly an inter-agenda condition. More generally, all the choice consistency conditions are of the inter-agenda type, and are automatically satisfied by any collective choice rule which works on a fixed agenda.

The condition of Independence of Irrelevant Alternatives also looses much of its strength in a fixed agenda framework, i.e. when it is only necessary to choose from one particular set of alternatives. As a matter of fatc, if we identify the feasible set with the universal set of alternatives, the condition of Independence of Irrelevant Alternatives is also automatically satisfied.

This opens the way to the definition of efficient, anonymous and neutral collective choice rules on a fixed agenda. The Borda counting rule is an easy example, but many other, more sophisticated methods can be devised.

Incidentally, some version of this argument must lie behind the rapidly growing literature on the implementation of collective choice rules. In the light of Arrow's Impossibility Theorem, before embarking on the analysis of the implementation problem, it would seem necessary to ask whether a satisfactory rule can indeed be devised. One answer, of course, is that Arrow's theorem requires unrestricted domain, and this assumption is often relaxed when dealing with implementability problems. However, no lower importance has the argument that the literature on implementation is concerned with fixed agenda collective choice rules which are immune from Arrow's theorem and related multi-agenda impossibility results.

However, new problems may arise if we impose *intra-agenda* conditions which are in the same spirit as the inter-agenda conditions of the Arrowian tradition. The main purpose of this paper is to show that there is a close correspondence between the fixed agenda and the multiagenda approaches: if a fixed agenda collective choice rule satisfies certain intra-agenda conditions, then there exists a Social Decision Function that satisfies appropriate inter-agenda conditions, and viceversa. As a consequence, one can prove a fixed agenda counterpart of the Arrow Impossibility Theorem (Arrow, 1963). Analogously, Gibbard's oligarchy theorem for quasi-transitive Social Decision Functions [1] can be translated into a fixed agenda framework.

After some preliminaries in section 2, we introduce in section 3 two fixed agenda counterparts of the condition of Independence of Irrelevant Alternatives, i.e., Independence and Weak Independence. In section 4 we

[1] Though Gibbard is universally credited for this result, the first published proofs are due to Guha (1972) and Mas Collel and Sonnenschein (1972).

show that, provided weak conditions of Pareto efficiency hold, the existence of a Social Choice Correspondence satisfying Independence is equivalent to the existence of a Social Welfare Function satisfying Independence of Irrelevant Alternatives, and the existence of a Social Choice Correspondence satisfying Weak Independence is equivalent to the existence of a Quasi-Transitive Social Decision Function satisfying Independence of Irrelevant Alternatives. Exploiting this equivalence, we prove in section 5 the fixed agenda counterparts of Arrow's Impossibility Theorem and of Gibbard's oligarchy result.

2. NOTATION AND DEFINITIONS

Let $X = \{x,y,z,...\}$ be the universal set of alternatives, with $\# X > 2$, and let $N = \{1,2,...,n\}$ be the finite set of individuals. Each individual i is endowed with a weak ordering R_i on X (i.e., a complete, reflexive and transitive binary relation on X). Strict preference P_i and indifference I_i are defined in the usual way. Given a subset L of N, xR_Ly means that xR_iy for all $i \in L$. Analogously, xP_Ly means xP_iy for all $i \in L$.

Let W be the set of all weak orderings on X. Elements of the n-fold Cartesian product W^n are called preferences profiles and denoted by $p = \{R_1, R_2,..., R_n\}$. Given a subset S of X, $p{:}S$ is the restriction of p to S.

DEFINITION 1. A Social Choice Correspondence (SCC) on X is a correspondence C which to any profile $p \in W^n$ on X assigns a non empty subset of X, $C(p,X)$.

$C(p,X)$ is the set of the choosable alternatives from X at profile p. Notice that the condition of unrestricted domain is implicit in Definition 1. Notice also that we have identified the universal set of alternatives with the set of feasible alternatives among which a choice has to be made.

Let R denote social preference, with P and I denoting strict social preference and social indifference, respectively. Recall that R is quasi-transitive if P is transitive. Let Q denote the set of all quasi-transitive, complete and reflexive binary relations on X.

DEFINITION 2. A Social Welfare Function (SWF) on X is a function which to any profile $p \in W^n$ on X assigns a weak ordering on X, $R(p) \in W$.

DEFINITION 3. A Quasi-transitive Social Decision Function (Q-SDF) on X is a function which to any profile $p \in W^n$ on X assigns a quasi-transitive social preference relation on X, $R(p) \in Q$.

Notice again that the condition of unrestricted domain is implicit in Definitions 2 and 3. If p, p', p'' are distinct profiles, we denote by $R(p)$, $R(p')$, $R(p'')$ the corresponding social preference relations.

Next, we introduce two simple conditions which formalize the notion of weak Pareto efficiency. The first one is the well known Weak Pareto Principle; the second one is a condition called Weak Pareto Optimality.

DEFINITION 4. A SWF (Q-SDF) satisfies the Weak Pareto Principle if, for all $x,y \in X$, xP_Ny implies $xP(p)y$.

DEFINITION 5. A SCC satisfies Weak Pareto Optimality if, for all $x,y \in X$, xP_Ny implies $y \notin C(p,X)$.

3. INDEPENDENCE OF IRRELEVANT ALTERNATIVES AND ITS FIXED-AGENDA RELATIVES

The condition of Independence of Irrelevant Alternatives is based upon the notion that the relative desirability of two alternatives is completely determined by information pertaining directly to that pair of alternatives. In the relation-theoretic framework, this notion is formalized as follows.

DEFINITION 6. A SWF (Q-SDF) satisfies the condition of Independence of Irrelevant Alternatives if, for all $x,y \in X$ and all $p,p' \in W^n$, if $p:\{x,y\} = p':\{x,y\}$, then $xR(p)y$ if and only if $xR(p')y$.

Actually, Arrow's own formulation of the condition of Independence of Irrelevant Alternatives was slightly different (cf. Arrow, 1963). He first defines social choice in terms of maximisation of social preferences; thus, for any non empty (finite) subset S of X, the choice set is

$$C(p,S) = \{x;\ x \in S \text{ and } xR(p)y \text{ for all } y \in S\}.$$

Then, it clearly follows from Definition 6 that, if a SWF or a Q-SDF satisfies (relational) Independence of Irrelevant Alternatives, $p:S = p':S$ implies $C(p,S) = C(p',S)$, for all $\varnothing \neq S \subseteq X$ [2]. The last implication is taken by Arrow as his definition of Independence of Irrelevant Alternatives. This is indeed the definition commonly used in the choice-theretic framework.

Clearly, the choice-theoretic version of the condition of Independence of Irrelevant Alternatives is restrictive only in a multi-agenda framework. For consider a fixed agenda SCC, where social choice is defined only on the universal set of alternatives X. Then, $p:X = p':X$ means $p = p'$, so that $C(p,X) = C(p',X)$ follows by the very definition of a SCC. Thus Independence of Irrelevant Alternatives (in Arrow's sense) is automatically satisfied.

One must therefore look for other ways of translating the notion of independence into the fixed agenda framework. One way to proceed is to

[2] For a formal proof, cf. e. g. Suzumura (1983, p. 68).

define the choice set $C(p,X)$ as did Arrow (that is, $C(p,X) = \{x;\ x \in X$ and $xR(p)y$ for all $y \in X\}$), and then to consider the implications of the condition of (relational) Independence of Irrelevant Alternatives (Definition 6).

Suppose that $R(p)$ is transitive. Suppose also that, at profile p, x is chosen and y is rejected. By the transitivity of R, this means that $xP(p)y$. Now consider a profile, p', whose restriction to $\{x,y\}$ is the same as that of p. By the condition of Independence of Irrelevant Alternatives, it must then be $xP(p')y$. It follows that y is dominated at profile p' and therefore cannot be chosen. We take this implication of the condition of Independence of Irrelevant Alternatives as a primitive axiom.

DEFINITION 7. A SCC satisfies the condition of Independence if, for all $x,y \in X$ and for all $p,p' \in W^n$, the following implication holds: if $x \in C(p,X)$, $y \notin C(p,X)$ and $p{:}\{x,y\} = p'{:}\{x,y\}$, then $y \notin C(p',X)$.

A weaker conclusion is obtained if one postulates that $R(p)$ is quasi-transitive. Under quasi-transitivity, from the fact that x is chosen and y is rejected at profile p we cannot conclude that $xP(p)y$: it may happen that y is dominated by x only indirectly, e.g. xIz, zPy and xIy. However, if x were the *unique* alternative chosen at profile p, then we would be sure that $xP(p)y$ for any other alternative y, and we could proceed as we did above. This motivates the following weakening of the condition of Independence.

DEFINITION 8. A SCC satisfies the condition of Weak Independence if, for all $x,y \in X$ and for all $p,p' \in W^n$, the following implication holds: if $\{x\} = C(p,X)$ and $p{:}\{x,y\} = p'{:}\{x,y\}$, then $y \notin C(p',X)$.

In words, Independence requires that, whenever x is chosen and y is rejected at profile p, y cannot be chosen at profile p' if p and p', restricted to the pair $\{x,y\}$, coincide. Weak Independence weakens this condition by requiring that y be not choosable only if x were the unique alternative chosen at p. These conditions capture and formalize, in a fixed agenda framework, the vague notion that the "relative desirability" of x and y should depend only on the way individuals order x *vis-a-vis* y. Their motivation is therefore the same as that of Arrow's condition of Independence of Irrelevant Alternatives (which however, as we have seen above, is automatically satisfied in a fixed agenda framework).

4. CORRESPONDENCE THEOREMS

In this section we shall show that, if there exists a SWF that satisfies the Weak Pareto Principle and Independence of Irrelevant Alternatives, then there exists a SCC that satisfies Weak Pareto Optimality and Independence, and viceversa. Analogously, if there exists a Q-SDF that satisfies the Weak Pareto Principle and Independence of Irrelevant Alternatives, then there exists a SCC that satisfies Weak Pareto Optimality and Weak Independence, and viceversa. These results imply that there is a close correspondence between the fixed agenda and the multi agenda frameworks. For sake of simplicity, the results are stated and proved for the case of a finite universal set of alternatives.

Theorem 1. *Let X be finite. A SCC defined on X satisfying Weak Pareto Optimality and Independence exists iff there exists a SWF defined on X satisfying the Weak Pareto Principle and Independence of Irrelevant Alternatives.*

Proof. i) Sufficiency

Suppose there exists a SWF satisfying the Weak Pareto Principle and Independence of Irrelevant Alternative. For all $p \in W^n$, let us define $C(p,X) = \{x;\ x \in X \text{ and } xR(p)y \text{ for all } y \in X\}$. Since X is finite and $R(p)$ is transitive for all p, $C(p,X)$ is always non empty and thus we have defined a SCC on X. We must now show that this SCC satisfies Weak Pareto Optimality and Independence. Weak Pareto Optimality follows immediately from the Weak

Pareto Principle. We now prove that Independence holds. Consider two profiles, p and p', and two alternatives, x and y, such that $x \in C(p,X)$, $y \notin C(p,X)$ and $p{:}\{x,y\} = p'{:}\{x,y\}$. We must prove that $y \notin C(p',X)$. We first show that $xP(p)y$. Indeed, since $x \in C(p,X)$, it must be $xR(p)z$ for all $z \in X$; hence, if $yR(p)x$, then by the transitivity of R it would follow $yR(p)z$ for all $z \in X$, and therefore it would be $y \in C(p,X)$, violating our hypothesis. Now, from $xP(p)y$ and the condition of Independence of Irrelevant Alternatives if follows $xP(p')y$; hence $y \notin C(p',X)$.

ii) Necessity

Suppose there exists a SCC satisfying Weak Pareto Optimality and Independence. We then define a SWF in the following way: for all $p \in W^n$ and for all $x,y \in X$, $xR(p)y$ iff $x \in C(p^*_{\{x,y\}},X)$ and $xP(p)y$ iff $\{x\} = C(p^*_{\{x,y\}},X)$, where $p^*_{\{x,y\}}$ is a profile obtained from p by shifting all alternatives different from x and y to the bottom of the preference ordering of each individual while leaving unaltered the relative ordering of x and y. More formally and generally, given a subset A of X and a profile p, we define p^*_A throught the following conditions:

a) $p{:}A = p^*_A{:}A$,

b) for all $i \in N$, $x \in A$ and $y \in X\text{-}A$ implies xP_iy,

c) for all $i \in N$ and $y,x \in X\text{-}A$ implies xI_iy.

By Weak Pareto Optimality it follows that $R(p)$ is complete and reflexive. To prove transitivity, suppose that $xR(p)y$ and $yR(p)z$. This means that $x \in C(p^*_{\{x,y\}},X)$ and $y \in C(p^*_{\{y,z\}},X)$. We now show that it must be $x \in C(p^*_{\{x,y,z\}},X)$. If not, by Weak Pareto Optimality only three cases can arise:

(i) $C(p^*_{\{x,y,z\}},X) = \{y,z\}$. Since $x \in C(p^*_{\{x,y\}},X)$, this violates Independence.

(ii) $C(p^*_{\{x,y,z\}},X) = \{y\}$. Again, this violates Independence since $x \in C(p^*_{\{x,y\}},X)$.

(iii) $C(p^*_{\{x,y,z\}},X) = \{z\}$. Since $y \in C(p^*_{\{y,z\}},X)$, this also violates Independence.

We have therefore $x \in C(p^*_{\{x,y,z\}},X)$. This implies that $x \in C(p^*_{\{x,z\}},X)$, for otherwise Independence would be violated. It follows $xR(p)z$, hence R is transitive.

It remains to be proven that the SWF we have defined satisfies the Weak Pareto Principle and Independence of Irrelevant Alternatives. The Weak Pareto Principle follows immediately from Weak Pareto Optimality. Independence of Irrelevant Alternatives follows from the observation that, if $p{:}\{x,y\} = p'{:}\{x,y\}$, then $p^{*}_{\{x,y\}} = p'^{*}_{\{x,y\}}$.

Q.E.D.

Theorem 2. *Let X be finite. A SCC defined on X satisfying Weak Pareto Optimality and Weak Independence exists iff there exists a Q-SDF defined on X satisfying the Weak Pareto Principle and Independence of Irrelevant Alternatives.*

Proof. We proceed as in the proof of theorem 1.

i) Sufficiency

Suppose there exists a Q-SDF satisfying the Weak Pareto Principle and Independence of Irrelevant Alternative. For all $p \in W^n$, let us define $C(p,X) = \{x;\, x \in X \text{ and } xR(p)y \text{ for all } y \in X\}$. Since X is finite and $R(p)$ is quasi-transitive for all p, $C(p,X)$ is always non empty and thus we have defined a SCC on X. We must now show that this SCC satisfies Weak Pareto Optimality and Weak Independence. Weak Pareto Optimality follows immediately from the Weak Pareto Principle. We now prove that Weak Independence holds. Consider two profiles, p and p', and two alternatives, x and y, such that $\{x\} = C(p,X)$ and $p{:}\{x,y\} = p'{:}\{x,y\}$. We must prove that $y \notin C(p',X)$. We first show that $xP(p)y$. Indeed, since $y \notin C(p,X)$, it must be $zP(p)y$ for some $z \in X$. If $z = x$ our claim is proved; if $z \neq x$, since z is not chosen, there must be a w such that $wP(p)z$; if $w = x$, by the quasi-transitivity of R it follows $xP(p)y$; otherwise, the argument proceeds one step further. Since X is finite, by the quasi-transitivity of R it must eventually follow $xP(p)y$. Now, from $xP(p)y$ and the condition of Independence of Irrelevant alternatives if follows $xP(p')y$; hence $y \notin C(p',X)$.

ii) Necessity

Suppose there exists a SCC satisfying Weak Pareto Optimality and Weak Independence. On the basis of this SCC, we then define a Social Decision Function as in the proof of theorem 1. To prove quasi-transitivity, suppose that $xP(p)y$ and $yP(p)z$. This means that $\{x\} = C(p^*_{\{x,y\}},X)$ and $\{y\} = C(p^*_{\{y,z\}},X)$. By Weak Independence, we have $y \notin C(p^*_{\{x,y,z\}},X)$ and $z \notin C(p^*_{\{x,y,z\}},X)$. By Weak Pareto Optimality, it follows $\{x\} = C(p^*_{\{x,y,z\}},X)$. Hence, by Weak Independence we get $\{x\} = C(p^*_{\{x,z\}},X)$. This implies $xP(p)z$.

The Weak Pareto Principle and Independence of Irrelevant Alternatives follow as in the proof of theorem 1.

Q.E.D.

5. FIXED AGENDA IMPOSSIBILITY THEOREMS

The correspondence theorems proved in section 4 allow us to obtain fixed agenda impossibility theorems from Arrow's and Gibbard's results. The strategy of proof is to show that, if a SCC satisfying Weak Pareot Optimality and Independence (resp., Weak Independence) is not dictatorial (resp., not oligarchic), then the corresponding SWF (resp., Q-SDF) is not dictatorial (resp., oligarchic), but this is not possible by the Arrow (resp., Gibbard) theorem.

We first define the notions of dictatorial and oligarchic SDFs and SCCs.

DEFINITION 9. A SDF is dictatorial if there exists a $d \in N$ such that, for all $x,y \in X$, xP_dy implies xPy.

DEFINITION 10. A SDF is oligarchic if there exists a $L \subseteq N$ such that, for all $x,y \in X$, xP_Ly implies xPy and, for all $j \in L$, for all $x,y \in X$, xP_jy implies xRy.

DEFINITION 11. A SCC is dictatorial if there exists a $d \in N$ such that, for all $x,y \in X$, xP_dy implies $y \notin C(p,X)$.

DEFINITION 12. A SCC is oligarchic if there exists a $L \subseteq N$ such that, for all $x,y \in X$, xP_Ly implies $y \notin C(p,X)$ and, for all $j \in L$, for all $x,y \in X$, xP_jy implies $\{y\} \neq C(p,X)$.

We are now ready to prove the following theorems, which represent the fixed agenda versions of Arrow's Impossibility Therems and of Gibbard's oligarchy result.

Theorem 3. Let X *be finite*, with $\#X > 2$. If a SCC defined on X satisfies Weak Pareto Optimality and Independence, then it is dictatorial.

Proof. Consider a SCC which satisfies Weak Pareto Optimality and Independence, and define a SWF as in the proof of theorem 1. Since, by theorem 1, the SWF satisfies the Weak Pareto Principle and Independence of Irrelevant Alternatives, it must be dictatorial by Arrow's Impossibility Theorem. Let d be the dictator. Then, for all $x,y \in X$, xP_dy implies xPy. But this means that $\{x\} = C(p^*_{\{x,y\}},X)$, where p in any profile with xP_dy. By Independence (actually, Weak Independence would suffice), it follows that y cannot be chosen whenever xP_dy, that is, d is a dictator for the original SCC.

Q.E.D.

Theorem 4. Let X be finite, with $\#X > 2$. If a SCC defined on X satisfies Weak Pareto Optimality and Weak Independence, then it is oligarchic.

Proof. Consider a SCC which satisfies Weak Pareto Optimality and Weak Independence, and define a Q-SDF as in the proof of theorem 2. Since, by theorem 2, the Q-SDF satisfies the Weak Pareto Principle and Independence of Irrelevant Alternatives, it must be oligarchic by Gibbard's theorem. Let L be the oligarchy. Then, for all $x,y \in X$, xP_Ly implies xPy

and xP_jy for at least one $j \in L$ implies xRy. Consider first the case xP_Ly. Since then xPy, it follows that $\{x\} = C(p^*_{\{x,y\}},X)$, where p in any profile with xP_Ly. By Weak Independence, it follows that y cannot be chosen whenever xP_Ly, that is, L is a decisive coalition for the original SCC. Now suppose that xP_jy for at least one $j \in L$. Since then xRy, it follows that $x \in C(p^*_{\{x,y\}},X)$, where p in any profile with xP_jy for at least one $j \in L$. Now consider a profile p' with the same restriction to the pair $\{x,y\}$ as the profile p. If $\{y\} = C(p',X)$, Weak Independence would be violated. It follows that any individual in L has a veto power, which implies that L is an oligarchy.

Q.E.D.

A direct proof of theorems 3 and 4 may be found in Denicolò (1985, 1987)[3]. Together with the correspondence theorems 1 and 2, these results can be used to provide a new proof of Arrow's and Gibbard's theorems. Since the proofs are symmetric to those of theorems 3 and 4, they are left to the reader.

6. CONCLUDING REMARKS

In this paper we have shown that there is a close correspondence between multi-agenda and fixed agenda collective choice rules. We have argued that inter-agenda conditions, like Independence of Irrelevant Alternatives or any type of collective rationality condition, are vacuously satisfied in a fixed agenda framework. However, we have defined fixed agenda conditions, like Independence and Weak Independence, which turn out to have the same effects as combinations of Independence of Irrelevant Alternatives and various degrees of collective rationality.

[3] Actually, the theorems proved in Denicolò (1985, 1987) are sligthly more general, as they do not require the assumption that X be finite.

In view of the correspondence theorems of section 4, it can be mantained that the impossibility theorems proved in section 5 have the same strengths (and the same weakenesses) as Arrow's and Gibbard's theorems. In other words, if, for instance, one regards Independence of Irrelevant Alternatives and transitivity of social preference as compelling conditions, then he/she should consider Independence compelling as well.

It may be conjectured that the equivalence pointed out in this paper carries over to other conditions of collective rationality, so that, for instance, some further weakening of the Weak Independence condition would correspond to Independence of Irrelevant Alternatives plus acyclicity. If this conjecture were correct, then it should be possible to prove fixed agenda counterparts of the results for SDFs obtained, *inter alia*, by Blau and Deb (1977) and by Blair and Pollak (1982).

More generally, one could ask whether, in some sense, *all* the results obtained in the multi-agenda framework can be translated into the fixed agenda framework. In a different context, Rubinstein (1983), using mathematical logic's tools, has asked this question with reference to the relationships between the multi-profile and the single-profile approaches. It would be interesting to analyse the relationships between the multi-agenda and the fixed agenda approaches in a similar way.

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