PRICE UNCERTAINTY AND INTERNATIONAL SPECIALIZATION

by

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Introduction

Trade and specialization among similar countries has been so far explained by resorting to market structure imperfections coupled to product differentiation and economies of scale\(^1\), to barriers to trade\(^2\), to cultural differences\(^3\). The purpose of this paper is to model trade and specialization within a framework of decisions under uncertainty. Firms face two kinds of uncertainty, one concerns prices and the other one concerns exchange rates. The main thread of this paper is based on the assumption that firms and trading companies adopt, in their market choices, criteria which are similar to those of financial agents.

In section 1, decisions criteria of firms under uncertainty will be shortly reviewed.

In section 2 a portfolio approach will be utilized to assess the role of price and exchange rate uncertainty in the distribution of the exports of a firm.

In section 3 imports will be introduced and the effect of different degree of risk aversion on international specialization will be analysed.

In section 4 international specialization and Intraindustry Trade will be analysed.

Eventually some conclusions will be drawn and some hints will be proposed for further empirical tests.

1. Decisions of firms under uncertainty

It can be assumed that each firm operates in a quasi-perfectly competitive market. Each firm can sell any quantity at a market price
which is given to the firm. Yet the market price is not known to the firm. Moreover firms make a profit out of a mark-up over costs, unlike perfect competitors.

The output price of each firm is a random variable whose expected value is

\[ E(p^o) = \int p f(p)dp \]  \hspace{1cm} (\S\S\S)

where \( f(p) \) is the probability density function of the random variable \( p \) (price of the output). The variance is

\[ V(p^o) = \int (p - E(p^o))^2 f(p)dp \]

Given this kind of uncertainty Sandmo (1971) assumes that each entrepreneur maximizes the expected utility of profits of the firm, using the hypothesis that firms are risk averse. By a generalization of a theorem of McCall (1967) he proves that the level of output is lower under price uncertainty than under conditions of certainty. The optimal output level will be determined by the condition

\[ g'(x) \leq E(p^o) \]  \hspace{1cm} (3)

where \( g'(x) \) is the marginal cost of producing quantity \( x \). This has to be weakly lower than the expected value of the market price. The gap between \( g'(x) \) and \( E(p^o) \) is determined by the degree of risk aversion. Neutrality implies equality. Ishii (1977) showed that, as risk increases, output is being further reduced.

Firms can reduce, in some circumstances, total risk they face by diversifying either products or markets. This second alternative is the one pursued here.

In a mean-variance context every firm will export as to reduce price variance, like a portfolio holder.

\[ \]  \hspace{1cm} \S\S\S

(\S\S) \S is for random variables
2. The international market for goods as an asset market.

We are mainly interested in total production of each firm and its distribution over separated markets. To this purpose we can follow two alternative routes. We might assume that firms have constant average costs on which they impose a mark-up which is a function of expected market price. Total profits will be proportional to prices over separated markets, weighted by the quantities sold on each market. This is a typical maximization process of a portfolio investor, with the only difference concerning markets instead of financial assets.

Alternatively we could think of a firm as undertaking a two-stage maximization process (Green, 1964) whereby the firm decides, first how to distribute the output between the home market and the rest of the world, second how to distribute output over foreign markets, following a process of optimal geographic diversification. We shall follow the first alternative.

We can then write a profit function for each firm-industry, without considering costs because of the assumption made above about their constancy. We have two industries, A and B, and three countries (1=home, foreign country 2 and 3). Let us write home profit functions:

\[
\pi_a = \pi_a (x_{ap} + x_{ae} + x_{ae})
\]

\[
\pi_b = \pi_b (x_{bp} + x_{be} + x_{be})
\]

where

\[x_{1a} = \text{quantity of good } a \text{ sold at home}\]
\[x_{1b} = \text{ } \quad b \quad \text{ } \]
\[x_{2a} = \text{ } \quad a \text{ sold in country } 2\]
\[x_{2b} = \text{ } \quad b \quad \text{ } \]
\[ x_{3a} = \text{quantity of good } a \text{ sold in country } 3 \]
\[ x_{3b} = b \]
\[ e_{2a} = \text{home currency price of exports to country } 2 \text{ for good } a \]
\[ e_{2b} = b \]
\[ e_{3a} = 3 - a \]
\[ e_{3b} = 3 - b \]
\[ p_a = \text{home price of good } a \]
\[ p_b = b \]

The expected value \( E \) and the variance \( V \) of profits for home industry \( a \) will be:

\[ E(\pi^o) = \sum_{a=1}^{a} \left( x_{1a} E(p_a) + x_{2a} E(e_{2a}) + x_{3a} E(e_{3a}) \right) \]

\[ V(\pi^o) = \sum_{a=1}^{a} \left( x_{1a}^2 V(p_a) + x_{2a}^2 V(e_{2a}) + x_{3a}^2 V(e_{3a}) + 2 \text{cov}(p_a, e_{2a}) x_{1a} x_{2a} + 2 \text{cov}(e_{2a}, e_{3a}) x_{2a} x_{3a} \right) \]

Each firm possesses a quadratic utility function of the kind

\[ U = E(\pi^o) - \alpha V(\pi^o) \]

This is equivalent to the maximization of a quadratic expected utility function of Von Neumann-Morgenstern type in Sandmo(1971) model, where the variance does not appear.

First order condition requires

\[ \frac{dU}{dx_{1a}} = \frac{dU}{dx_{2a}} = \frac{dU}{dx_{3a}} = 0 \]

hence
\[ \frac{dU}{dx_{1a}} = U_1 \frac{dE_{(\pi^0)_a}}{dx_{1a}} - \alpha_1 U_2 \frac{dV_{(\pi^0)_a}}{dx_{1a}} = 0 \]

where

\[ U_1 = \frac{dU}{dE_{(\pi^0)_a}} \]

\[ U_2 = \frac{dU}{dV_{(\pi^0)_a}} \]

\[ \frac{dU}{dx_{2a}} = U_1 \frac{dE_{(\pi^0)_a}}{dx_{2a}} - \alpha_2 U_2 \frac{dV_{(\pi^0)_a}}{dx_{2a}} = 0 \]

\[ \frac{dU}{dx_{3a}} = U_1 \frac{dE_{(\pi^0)_a}}{dx_{3a}} - \alpha_3 U_2 \frac{dV_{(\pi^0)_a}}{dx_{3a}} = 0 \]

From equation (10) we can obtain

\[ E(p^0_a) = AP \left( 2x_{1a} V(p^0_a) + 2x_{2a} \text{cov}(p^0_a e^0_{2a}) + 2x_{3a} \text{cov}(p^0_a e^0_{3a}) \right) \]

where

\[ AP = -\left( \frac{U_2}{U_1} \right) \pi_a \]
which is the Arrow-Pratt index of relative risk aversion.

We can then derive:

\[(15) \quad x_{1a} = \frac{1}{2} \frac{E(p^o_a)}{V(p^o_a)} \frac{2(x_{2a} \text{cov}(p^o_e^e)) + x_{3a} \text{cov}(p^o_e^e)}{V(p^o_a)} \]

\[(16) \quad x_{2a} = \frac{1}{2} \frac{E(e^o_{2a})}{V(e^o_{2a})} \frac{2(x_{1a} \text{cov}(e^o_{1a}p^o_a) + x_{3a} \text{cov}(e^o_{3a}e^o))}{V(e^o_{2a})} \]

\[(17) \quad x_{3a} = \frac{1}{2} \frac{E(e^o_{3a})}{V(e^o_{3a})} \frac{2(x_{1a} \text{cov}(e^o_{1a}p^o_a) + x_{2a} \text{cov}(e^o_{2a}e^o))}{V(e^o_{3a})} \]

With the same procedure we can derive \(x_{1b}, x_{2b}, x_{3b}\).

3. Modelling imports

As far as imports are concerned, we can assume that their distribution and magnitude is determined by a maximization process similar to the one used for exports.

In other words we can assume that imports are a proportion of domestic demand. This means that:

\[(18) \quad M_{a2} + M_{a3} + M_{b2} + M_{b3} = d (x_{1a} + x_{1b}) \]

where \(d > 0\)
What matters are the proportions of imports of different countries, of different industries. Equation (18) is not a balanced trade equilibrium, yet only a behavioural equation. Hence equation (18) could be considered a short run link between home produced goods demand and import demand. Some values of \( d \) might be associated with balance of trade, some might not. Indeed what we are working on is a short run model of specialization, which is compatible with balance of trade disequilibrium and deviations from PPP, which is usually required in equilibrium models. Trade balance would require

\[
(18.1) \quad (M_{a2} + M_{b2}) + (M_{a3} + M_{b3}) = (x_{2b} + x_{3b} + x_{2a} + x_{3a})
\]

In addition to what said above, we should introduce trade balance between country 2 and 3, whose reciprocal trade does not have to entertain us. Hence without this equilibrium condition, equilibrium between home and country 2 and 3 is not needed.

Using a procedure similar to the one used for exports we obtain

\[
(19) \quad E(\pi^o) = \pi \left( \frac{M_{a2}}{a2} E(e^o_{a2}) + \frac{M_{a3}}{a3} E(e^o_{a3}) \right)
\]

\[
(19') \quad E(\pi^o) = \pi \left( \frac{M_{b2}}{b2} E(e^o_{b2}) + \frac{M_{b3}}{b3} E(e^o_{b3}) \right)
\]

\[
(20) \quad V(\pi^o) = \pi \left( \frac{M_{a2}}{a2} V(e^o_{a2}) + \frac{M_{a3}}{a3} V(e^o_{a3}) + 2 \text{cov}(e^o_{a2}, e^o_{a3}) \right)
\]

\[
(20') \quad V(\pi^o) = \pi \left( \frac{M_{b2}}{b2} V(e^o_{b2}) + \frac{M_{b3}}{b3} V(e^o_{b3}) + 2 \text{cov}(e^o_{b3}, e^o_{b2}) \right)
\]
We maximize a utility function for industry a and b as we did with (8). Hence we get, by defining $\beta^R_M$, relative risk aversion of importers,

\[(21)\]  
\[
M_{a2} = 1/2AP_M \left( \frac{E(e_{a2})}{V(e_{a2})} \right) - \frac{2\ M_{a3} \ \text{cov}(e_{2a} e_{3a})}{V(e_{a2})}
\]

\[(22)\]  
\[
M_{a3} = 1/2AP_M \left( \frac{E(e_{a3})}{V(e_{a3})} \right) - \frac{2M_{a2} \ \text{cov}(e_{2a} e_{3a})}{V(e_{a3})}
\]

\[(21')\]  
\[
M_{b2} = 1/2AP_M \left( \frac{E(e_{b2})}{V(e_{b2})} \right) - \frac{2M_{b3} \ \text{cov}(e_{2b} e_{3b})}{V(e_{b2})}
\]

\[(22')\]  
\[
M_{b3} = 1/2AP_M \left( \frac{E(e_{b3})}{V(e_{b3})} \right) - \frac{2M_{b2} \ \text{cov}(e_{2b} e_{3b})}{V(e_{b3})}
\]
4. International specialization and Intraindustry Trade

It might be interesting to calculate for the home country different Grubel-Lloyd (Grubel-Lloyd) indices of Intraindustry Trade (IIT) or trade specialization, according to the hypotheses we make about prices home and foreign covariances, relative risk aversion of exporters and importers. We calculate the IIT index for home country trade with country 2, we call IIT$_2$.

\[
\begin{align*}
\text{IIT}_2 &= (1 - \frac{M_{a2} - X_{a2} + M_{b2} - X_{b2}}{(M_{a2} + X_{a2}) + (M_{b2} + X_{b2})}) \\
\end{align*}
\]

from which we obtain:

\[
\begin{align*}
\text{IIT}_2 &= 1 - \left\{ \frac{1}{2\text{AP}} \left( \frac{M(E(e^0_{a2})/V(e^0_{a2}))}{V(e^0_{2a})} - \frac{2M_{a3} \text{cov}(e^0_{a2}, e^0_{a3})}{a3_{a2}a3_{a2}} - \frac{2(E(e^0_{2a}))(E(e^0_{a2}) - E(e^0_{a2})^2)}{V(e^0_{2a})^2} \right) \right. \\
&\quad \quad \quad + \left. \frac{2(x_{1a} \text{cov}(e^0_{1a}, e^0_{a2}) + x_{3a} \text{cov}(e^0_{a2}, e^0_{3a}))}{V(e^0_{2a})^2} \right) \\
&\quad \quad \quad + \frac{1}{2\text{AP}} \left( \frac{M(E(e^0_{b2})/V(e^0_{b2}))}{V(e^0_{2b})} - \frac{2M_{b3} \text{cov}(e^0_{b2}, e^0_{b3})}{b3_{b2}b3_{b2}} - \frac{2(E(e^0_{2b}))(E(e^0_{b2}) - E(e^0_{b2})^2)}{V(e^0_{2b})^2} \right) \right. \\
&\quad \quad \quad + \left. \frac{2(x_{1b} \text{cov}(e^0_{1b}, e^0_{2b}) + x_{3b} \text{cov}(e^0_{2b}, e^0_{3b}))}{V(e^0_{2b})^2} \right) \\
&\quad \quad \quad \frac{1}{V(e^0_{b2})} \\
\end{align*}
\]
\[
\begin{align*}
\left\{ \begin{array}{c}
E(e^o_{2a}) - \frac{2(x_{1a} \text{cov}(e^o_{2a} p^o_1) + x_{3a} \text{cov}(e^o_{3a} e^o_{2a}))}{V(e^o_{2a})} + \\
\frac{1}{2} \text{AP} \left( \frac{\text{V}(e^o_{2a})}{\text{V}(e^o_{2a})} \right) \\
E(e^o_{2b}) - \frac{2(x_{2b} \text{cov}(e^o_{2b} p^o_{2b}) + x_{3b} \text{cov}(e^o_{3b} e^o_{2b}))}{V(e^o_{2b})} + \\
\frac{1}{2} \text{AP} \left( \frac{\text{V}(e^o_{2b})}{\text{V}(e^o_{2b})} \right) \\
E(e^o_{a2}) - \frac{2M_{a3} \text{cov}(e^o_{a2} e^o_{a3})}{V(e^o_{a2})} + \\
\frac{1}{2} \text{AP} \left( \frac{\text{V}(e^o_{a2})}{\text{V}(e^o_{a2})} \right) \\
E(e^o_{b2}) - \frac{2M_{b3} \text{cov}(e^o_{b2} e^o_{b3})}{V(e^o_{b2})} + \\
\frac{1}{2} \text{AP} \left( \frac{\text{V}(e^o_{b2})}{\text{V}(e^o_{b2})} \right)
\end{array} \right.
\end{align*}
\]

If we assume that in the short run \(^7\)

\(25\) \(\text{cov}(e^o_{2a} p^o_1) = \text{cov}(e^o_{2a} e^o_{3a}) = \text{cov}(e^o_{3a} p^o) = \text{cov}(e^o_{2b} p^o_{2b}) = \text{cov}(e^o_{2b} e^o_{3b}) = 0\)

and

\(26\) \(\text{AP}_M = \text{AP}\)
IIT will equal 1. This means that there is no specialization of interindustry sort. Even if countries are alike in all respects they trade (volume of trade is determined by (15-16-17-21-21'-22-22')) and all trade is intraindustrial, as a result of geographic diversification of sources of export demand in a world in which prices are not known with certainty.

If (25) holds but (26) does not we shall obtain

\[
\begin{vmatrix}
E(e^o_{2a}) & E(e^o_{2b}) & 1 & 1 \\
\frac{1}{V(e^o_{2a})} + \frac{1}{V(e^o_{2b})} & \frac{1}{V(e^o_{2b})} & \frac{1}{AP_{M}} & \frac{1}{AP}
\end{vmatrix}
\]

(27) \[ IIT = 1 - \frac{E(e^o_{2a})}{V(e^o_{2a})} \frac{E(e^o_{2b})}{V(e^o_{2b})} \left( \frac{1}{AP} + \frac{1}{AP_{M}} \right) \]

from which we get

\[
\begin{vmatrix}
1 & 1 \\
\frac{1}{AP_{M}} & \frac{1}{AP}
\end{vmatrix}
\]

(28) \[ IIT = 1 - \frac{1}{AP_{M}} \frac{1}{AP} \left( \frac{1}{AP} + \frac{1}{AP_{M}} \right) \]
The above equation shows that under assumption (25) the higher is the difference in relative risk aversion between importers and exporters the lower will be IIT, i.e. the higher is specialization.

If we drop assumption (25), the evaluation of (24) becomes pretty awkward even if some considerations could be proposed if we assume (26) and (29)

\[
(29) \text{cov}(e^o_{a2}, e^o_{a3}) = \text{cov}(e^o_{b2}, e^o_{b3});
\]

\[
\text{cov}(e^o_{a2}, p^o_a) = \text{cov}(e^o_{b2}, p^o_b);
\]

\[
V(e^o_{a2}) = V(e^o_{b2})
\]

which means that relative prices (between country a and b) stay constant.

We can then obtain

\[
(30) \text{IIT} = 1 - \frac{\begin{vmatrix}
(M_{a3} + M_{b3})e_{32} & -(X_{a3} + X_{b3})c_{32} & -(x_{1b} + x_{1a})c_{21} \\
M_{a3} & -(X_{a3} + X_{b3})c_{32} & -(x_{1b} + x_{1a})c_{21} \\
0 & 0 & 0
\end{vmatrix}}{E(e^o_{2a}) + E(e^o_{b2}) - (M_{a3} + M_{b3})e_{32} - (X_{a3} + X_{b3})c_{32} - (x_{1b} + x_{1a})c_{21}}
\]

where \( e_{32} = \text{cov}(e^o_{a2}, e^o_{a3}) \)

\( c_{21} = \text{cov}(e^o_{a2}, p^o_a) \)
Since $0 \leq \text{IIT} \leq 1$ and numerator and denominator of (30) are not equal, $\text{IIT} < 1$, showing that with assumptions displayed in (26) and (29), some kind of specialization comes out. This sort of specialization is the output of different risks (variances) and risk correlations (co-variances) evidenced by two countries. Those countries who show no difference but for the expected values and covariances of home and foreign prices will specialize somehow, even if there will be a great deal of IIT.
5. Conclusions

Some partial conclusions can be drawn from the model presented, where two industries have been analyzed in a 3-country model.

First: IIT appears to be simply the outcome of geographic diversification of demand. This is due to the sort of choices firms make when they face price uncertainty. IIT does not need any sort of market imperfection (economies of scale, concentration, product differentiation) to play a role. When countries are similar or even alike, uncertainty makes countries trade and determines specialization, making the difference:

Second: in the short run, when financial determinants of exchange rates are predominant we can assume zero covariance between home and foreign prices. Under this assumption IIT is a function of the difference between relative risk aversion of importers and exporters. The higher is the divergence the higher interindustry specialization of the traditional nature will be and the lower will be IIT. This means either that less risk averse exporters in face of more risk averse importers make a difference, or that the habit-currency with which exports and imports are invoiced can change the pattern of specialization of a country through their influence on risks undertaken by importers and exporters. Other insights can be gained by evaluations of IIT indices under different assumptions about exchange rates behaviours and by empirical estimations of imports and exports equations country-by-country seen in section 3.
6. Footnotes


4. When there is price uncertainty and risk aversion, the usual problem of the determination of the size of the firm disappears, since the non-concavity of production functions is taken over by concavity of utility.

5. We can obtain

\[
\begin{align*}
(15') \quad x_{1b} = & 1/2AP(\frac{\text{E}(p^o)_{b}}{\text{V}(p^o)_{b}}) - \frac{2(\text{cov}(p^o, e^o)_{b} + x_{3b} \text{cov}(p^o, e^o)_{3b})}{\text{V}(p^o)_{b}} \\
(16') \quad x_{2b} = & 1/2AP(\frac{\text{E}(e^o)_{2b}}{\text{V}(e^o)_{2b}}) - \frac{2(\text{cov}(e^o, p^o)_{1b} + x_{2b} \text{cov}(e^o, e^o)_{3b})}{\text{V}(e^o)_{2b}} \\
(17') \quad x_{3b} = & 1/2AP(\frac{\text{E}(e^o)_{3b}}{\text{V}(e^o)_{3b}}) - \frac{2(\text{cov}(e^o, p^o)_{1b} + x_{2b} \text{cov}(e^o, e^o)_{3b})}{\text{V}(e^o)_{3b}}
\end{align*}
\]


7. This means that we allow for zero covariance in the short run between home and foreign prices. If one assumes that in the short run exchange rates are determined mainly by monetary and financial variables, as most of the recent literature on exchange rates determination has done, the above assumption is not so heroic.

6. References


G. Rossini, "Intraindustry Trade and Tariffs: New Wave or Old Fashion?", *Economia Internazionale*, forthcoming.
