PUBLIC INPUTS AS SIGNALLING DEVICES

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Abstract

A two stage model is built where the first period strategic variable (public input production) is used to update the information available on some private characteristics of the rival firms. Expressing the firms' decision rules as linear functions of both the signal and the second stage strategic variable, explicit comparisons between equilibrium outputs of the public input are possible when different second stage strategic variables (prices or quantities) and different agreements between the firms (to share or not the private information). An interesting analogy is shown between the role of the degree of appropriability on the public inputs in full information and the degree of correlation between the stochastic variables that generate the private information in incomplete information. The signalling role of the public input is also shown to vary according to different competitive structures and settings of the game.

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1 Introduction

When incomplete information is considered in oligopolistic markets firms can exchange informations about their own characteristics through the decisions that they take in early stages of the competitive process[1]. This strategic manipulation of information has been intensively investigated in two stage games where the strategic variables (prices or outputs) are the same in both stages[2]. However, a similar analysis can be applied to contexts in which the first period strategic variable (public input) is different from the one in second stage and the former can be used to update the information available on some private characteristics of the rival firms. In such a context the production of the public inputs has three effects on the rival firms' behaviour: the standard strategic and cost reducing effects and a signalling effect[3]. To separate these different influences use is made of a solution to Bayesian games for quadratic objective functions firstly derived by Radner (1962)[4]. Adopting this solution it is possible to express the firms' decision rules as linear generic functions of both the signal and the second stage strategic variable (either price or output). Since the public input is also the choice variable in the first stage of the game, this technique allows for an explicit solution in terms of its equilibrium level. Moreover, in this analytical framework comparisons between equilibrium quantities of the public input are possible when different second stage strategic variables (prices or quantities) and different agreements between the firms (to share or not the private information) are assumed.

Two firms that compete in a stochastic environment are considered. Each has access to private information either related to their own cost function or to the cost reducing effectiveness of an intermediate good whose spill-overs affect the rival firm. In the first stage of the game the firms choose the production level of the public input and in the second period they compete either on prices or quantities. Since the first period actions are assumed to become common knowledge, firms can use them in order to make inferences about the private information of the rival. The processing of the information in the second period causes what in the literature are known as non-zero intertemporal conjectural variations. This means that when a firm takes the first stage decision it considers how that choice affects the information set of the rival firm and therefore the rival firm's second stage decisions. A Bayesian revision of information on the nature of the firm or of the cost reducing properties of the intermediate good takes place on the basis of the rival firm choice in the first stage. This gives rise to a signalling game where the public input is produced also to distort the information available to the rival firms. As for the setting of the signalling game, a distinction between private value and common value games is drawn throughout the paper. For private value game we mean a situation in which uncertainty is about a cost parameter unrelated across firms. In a common value game uncertainty is assumed to be about a parameter that affects both profit functions in the same way, such as the cost reducing effects of the public input. As it will be seen, the nature of the uncertainty has important implications on the equilibrium outcomes[5].

In Cournot competition and with a private value game, the signalling role of the public inputs leads to its overproduction with respect to the sharing information case. On the contrary, when a common value game is considered, the Cournot equilibrium level of the public input is lower than in the sharing information case because its production signals low costs that is the target of Cournot competitors. In this respect an important analogy can be drawn with the models of private provision of inputs that generate externalities. In full information settings when the inputs are completely public underproduction results are obtained and viceversa for the case of completely private inputs. Here the role of the degree of appropriability is taken by the degree of correlation between the stochastic variables that generate the private information available to the firms. When such correlation is complete (common value games) underproduction with respect to the pooling information solution is obtained and viceversa for the no correlation case (private value games).

In Bertrand competition with private value games the signalling effect of g leads to its underproduction with respect to the sharing information case. This is so because a high production of g signals low costs and the latter have an unambiguous negative effect on profits. In common value games a high value of g signals a high cost reducing effect of such input and the latter has a negative impact on profits. Therefore, even in this case the Bertrand equilibrium production of g is smaller than in the pooling information solution. The analogy between the degree of appropriability in full information and the degree of correlation in incomplete information holds even here. In fact, it is well known that Bertrand equilibria imply underproduction irrespective of the degree of appropriability of the public inputs.

This analogy is easily explained by the similarities in the relations between the public input production (full information) or the mechanism of updating the information (incomplete information) and the reaction function of the rival firm. In the case of completely public input its cost reducing effect shifts the rival firm's reaction function in such a way to induce strategic underproduction. In the common value game the updating of the information on the firm's private characteristic by the rival again shifts the reaction function of the latter reducing the incentives for the production of g. When appropriability is complete or a private value game is analyzed there is no longer a direct effect on the other firm's objective function. Accordingly a greater production of the public input in the first stage does not move the rival firm's reaction function, but shifts only the own reaction function (the rival firm's perception of it in the incomplete information case).

The chapter is organised as follows: in section 2 we deal with the private value game, with either Cournot or Bertrand competition. In section 3 the presence of a common shock is examined and in the following section some conclusions are drawn.

2 The private value game

2.1 Cournot equilibrium

In the Cournot case with uncertainty on the cost structure a linear deterministic demand is assumed such as:

$$p^i = a - b(q^i + q^j)q^i \quad \text{where} \quad a, b > 0 \quad i \neq j = 1,2$$

and a stochastic linear cost function:

$$C^i = [c^i - \gamma(g^i + \theta g^j) + u^i]q^i \quad \text{where} \quad \gamma > 0 \quad \text{and} \quad u^i \sim N(0, \sigma) \quad i = 1,2$$

The private signal x^i is defined as an unbiased estimator of the mean of the random process that generates u^i. The following distributional properties are assumed:

$$x^i = u^i + \epsilon^i \quad \text{where} \quad \epsilon^i \sim N(0, \mu).$$

It is assumed that the random processes generating u^i and u^j are independent. The posterior expected values are the following:

$$E(u^i \mid x^i, x^j) = E(u^i \mid x^i) = \frac{\sigma x^i}{\sigma + \mu} \quad i = 1,2$$

For firm i, the first period strategy is $g^i(x^i)$ and the second period strategy is $q^i(x^i, g^i, g^j)$. Those strategies are chosen to maximize:

$$\operatorname*{Max}_{g^i, q^i} \; k^i = E[(a - b(q^i + q^j) - (c - \gamma g^i - \gamma\theta g^j + u^i)q^i \mid x^i] + (\alpha - \beta g^i)g^i$$

where $(\alpha - \beta g^i)$ are the net, non-output related, effects of g on profits. The public input is, in fact, assumed not only to reduce the production costs when introduced in the productive process, but also to be potentially sold to firms in other sectors. The first term indicates the revenues from the marketing of g as a separate commodity less its linear costs of production. The second term represents either decreasing returns in the production of g itself or a negatively sloped demand in its own market. These terms are needed in order to insure an interior maximum and a positive level for the equilibrium production of the public input in the first stage game. Given the linear structure that we have assumed for both demand and cost functions the decision rules for g^i and q^i can be derived as linear functions such as[6]:

$$g^i = C_0^i + C_1^i x^i$$

and (1)

$$q^i = D_0^i + D_1^i x^i + D_2^i g^i + D_3^i g^j$$

At the second stage of the game firm i chooses q^i in order to:

$$\max_{q^i} \; \Pi^i = E[(a - b(q^i + q^j) - (c^i - \gamma g^i - \gamma\theta g^j) + u^i) q^i \mid x^i, g^i, g^j] + (\alpha - \beta g^i) g^i =$$

$$= [(a - b(q^i + q^j) - (c^i - \gamma g^i - \gamma\theta g^j) + E\left[u^i \mid x^i, \frac{g^j - C_0}{C_1} \right] q^i + (\alpha - \beta g^i) g^i$$

From which the first order conditions are:

$$a - 2bq^i - bq^j - (c^i - \gamma g^i - \gamma\theta g^j) + E\left[u^i \mid x^i, \frac{g^j - C_0}{C_1} \right] = 0$$

or equivalently (2)

$$\left[a - 2bq^i - bq^j - (c^i - \gamma g^i - \gamma\theta g^j) + \frac{\sigma x^i}{\sigma + \mu} \right] = 0$$

Substituting (1) into (2) and using the distributional properties of u^i yields:

$$a - 2b[D_0 + D_1 x^i + D_2 g^i + D_3 g^j] - b\left[D_0 + D_1 \frac{g^j - C_0}{C_1} + D_2 g^j + D_3 g^i \right] - c^i + \gamma g^i + \gamma\theta g^j - \frac{\sigma x^i}{\sigma + \mu} = 0$$

Since the decision rules specified above hold for any values of x^i, g^i and g^j, expression (2) can be used to get the following system of four equations:

$$D_0 = \frac{a-c}{3b} - \frac{\sigma C_0}{6b(\sigma+\mu)C_1}$$

$$D_1 = -\frac{\sigma}{2b(\sigma+\mu)} \qquad (3)$$

$$D_2 = \frac{\gamma(2-\theta)}{3b} - \frac{\sigma}{6b(\sigma+\mu)C_1}$$

$$D_3 = \frac{\gamma(2\theta-1)}{3b} + \frac{\sigma}{3b(\sigma+\mu)C_1}$$

In D_0 the first term indicates the non-input related amount of q^i that is produced in absence of uncertainty. It is directly related to the size of the market and inversely to the cost level and the slope of the demand curve. The second term is the quantity of q^i produced under uncertainty. It is positive since C_1 will be shown to be negative. It is also directly related to the elasticity of the demand curve and to the amount of public input that is produced under full information (C_0). Moreover it depends negatively on the degree of uncertainty and on the coefficient that associates g with the signal x (C_i). The output level is then negatively affected by the amount of strategic signalling that can take place. D_1 shows the negative effect of the signal, in our case an unbiased estimator of the cost function, on the equilibrium level of output. This effect is now inversely related to the elasticity of the demand curve because the output reducing effect of high costs (signalled by a high x^i) is lower if the demand curve is relatively inelastic.

The first term in D_2 gives the effect of g^i on the output level under certainty. It is positive because $0<\theta<1$ and b, $\gamma > 0$. Such effect is then directly related to the level of external effects of g on the rival's cost function (θ). In fact, if the firm producing g completely appropriates the reduction in costs, in the second stage its output grows reducing the share of the rival firm. On the other hand if g is completely public, even if the absolute value of the output of the firm producing g increases, this is not the case for its relative share. Further, this effect is directly related to the cost reducing effectiveness of g (γ) and to the elasticity of demand. The second term in D_2 indicates the effect of the own production of public input on the output due to its signalling role. This is again positive and, as before, positively related to the amount of g used to send signals to the rival firm. The first term in D_3 represents the effect of the rival production of g on its own output level under certainty. It can be either positive or negative depending on the value of θ. More precisely, when the latter is greater than ½, the cross effect of g is to increase the equilibrium level of output and viceversa. The share of the output market is however reduced unless $\theta = 1$. The second term defines the effect on output of that portion of the rival public input production that is realized in order to misinform on its real costs. This is negative because an increase of the production of g^j signals that the rival has lower costs (its reaction function has shifted outward) and the equilibrium output level is accordingly reduced.

Given the reaction functions in the second stage, in the previous stage firm i:

$$\max_{g^i} \; k^i = [a - b(q^i + E(q^j)) - c + \gamma g^i + \gamma\theta g^j - E(u^i \mid x^i)]q^i + (\alpha - \beta g^i)g^i \quad (4)$$

At the first stage firm i computes the expected value of its' rival strategy (g^j, q^j) since at that time the realization of x^j is unknown to i. The first order conditions, using the envelope theorem, are:

$$\left[\gamma - b\frac{dq^i}{dg^j}\right]q^i + \alpha - 2\beta g^i = 0 \tag{5}$$

Substituting (1) into (5) and using the distributional properties of x^j and u^i, the expression above can be written as:

$$(\gamma - bD_3)(D_0 + D_1 x^i + D_2 C_0 + D_2 C_1 x^i + D_3 C_0) + \alpha - 2\beta(C_0 + C_1 x^i) \tag{6}$$

Substituting for D_i i=0,1,2,3 from (3) into (6) and requiring that (6) holds for every realization of x^i, yields the following system of two equations:

$$\left[\frac{2\gamma(2-\theta)}{3} - \frac{\sigma}{3(\sigma+\mu)C_1}\right]\left[\frac{\alpha - c}{3b} + \frac{\gamma(1+\theta)C_0}{3b}\right] + \alpha - 2\beta C_0 = 0 \tag{7}$$

and $$2[\gamma^2(2-\theta)^2 - 9b\beta](\sigma+\mu)^2 C_1^2 - 5\gamma\sigma(2-\theta)(\sigma+\mu)C_1 + 2\sigma^2 = 0 \tag{8}$$

expression (8) is a quadratic equation with the following two roots:

$$C_1 = \frac{5\gamma(2-\theta) \pm \sqrt{9\gamma^2(2-\theta)^2 + 12^2 b\beta}}{4(\gamma^2(2-\theta)^2 - 9b\beta)}\frac{\sigma}{\sigma+\mu}$$

Differentiating (6) once again with respect to g^i we derive the second order condition:

$$2\gamma(2-\theta)L[\gamma(2-\theta)L - 4H] + 8H^2 - 18b\beta L^2 < 0$$

where:

$$H = [\gamma^2(2-\theta)^2 - 9b\beta]$$

and $$L = [5\gamma(2-\theta) \pm \sqrt{9\gamma^2(2-\theta)^2 + 12^2 b\beta}]$$

Substituting C_1 in (7) we can solve explicitly for C_0 obtaining:

$$C_0 = \frac{(2\gamma(2-\theta)L - 4H)(a-c) + 9b\alpha L}{18b\beta L - (2\gamma(2-\theta)L - 4H)\gamma(1+\theta)}$$

To insure that this expression is positive and that the second order condition holds we need that $H<0$ and $L>0$ so that the only acceptable solution of C_1 is:

$$C_1 = \frac{5\gamma(2-\theta)+\sqrt{9\gamma^2(2-\theta)^2+12^2 b\beta}}{4(\gamma^2(2-\theta)^2-9b\beta)}\frac{\sigma}{\sigma+\mu}$$

whose sign is negative. This is hardly surprising since the coefficient C_1 indicates the effect of the signal x^i, that is an unbiased estimator of the cost parameter, on g^i. The sign of C_1 allows one to draw some implications on the signalling role of g in modifying the second stage equilibrium. In fact, an increase in the production of g signals low costs of production and thus rises the private incentives for its production. Without the quadratic term in the cost function for g then, the sum of the strategic and the signalling effects would make it impossible to have an interior solution for the maximand. Now we can solve the system (3) and write the decision functions of the firm i as:

$$g^i = \frac{(2\gamma(2-\theta)L-4H)(a-c)+9b\alpha L}{18b\beta L-(2\gamma(2-\theta)L-4H)\gamma(1+\theta)} + \frac{L}{H}\frac{\sigma}{\sigma+\mu}x^i$$

and

$$q^i = \left\{\frac{a-c}{3b}-\frac{2HC_0}{3bL}\right\} - \left\{\frac{\sigma}{2b(\sigma+\mu)}\right\}x^i + \left\{\frac{\gamma(2-\theta)}{3b}-\frac{2H}{3bL}\right\}g^i + \left\{\frac{\gamma(2\theta-1)}{3b}+\frac{4H}{3bL}\right\}g^j \qquad (9)$$

where the last term indicates the dynamic conjectural variation term. The part of it that is due to the strategic effect of the production of the public input can be either positive or negative depending on the value of θ. The part that indicates the signalling effect is negative, because an increase of the rival firm's public input production signals that the latter is characterized by low costs.

Following the same procedure shown above, in the symmetric

Cournot equilibrium of the game with direct communication the strategies satisfy the following conditions:

$$g^s = \frac{2\gamma(2-\theta))(a-c)+9\alpha b}{18b\beta-(2\gamma(2-\theta))\gamma(1+\theta)} + \frac{\gamma(2-\theta)}{(\gamma^2(2-\theta)(1+\theta)-9\beta b)}\frac{\sigma}{\sigma+\mu}x^i$$

and

$$q^s = \left\{\frac{a-c}{3b}\right\} - \left\{\frac{2\sigma}{3b(\sigma+\mu)}\right\}x^i + \left\{\frac{\sigma}{3b(\sigma+\mu)}\right\}x^j + \left\{\frac{\gamma(2-\theta)}{3b}\right\}g^i + \left\{\frac{\gamma(2\theta-1)}{3b}\right\}g^j \qquad (10)$$

Using (10) we can now compare the two equilibrium level of g in the private and sharing information cases. We have the following inequality:

$$g^s \geq g^i \quad \Leftrightarrow \quad \frac{(2\gamma(2-\theta))(a-c)+9\alpha b}{18b\beta-(2\gamma(2-\theta))\gamma(1+\theta)} \geq \frac{(2\gamma(2-\theta)L-4H)(a-c)+9b\alpha L}{18b\beta L-(2\gamma(2-\theta)L-4H)\gamma(1+\theta)}$$

that can be simplified as:

$$-\alpha\gamma(1+\theta) \geq 2\beta(a-c)$$

The latter shows that $g^s<g^i$. The production of the public input is then greater when the two firms do not share informations about their cost structure. In the first stage the two firms want the rival to believe that their cost level is below the real level in order to persuade the latter to reduce its share in the second stage competition. To do so, more public input is produced than would be optimal under sharing information.

In order to see to which solution implies higher output and profits we substitute the first expression in (9) and (10) in the second one. After substituting, the average output level in the private information case becomes:

$$E(q^i) = \frac{a-c}{3b} + \frac{\gamma(1+\theta)}{3b}\;\frac{(2\gamma(2-\theta)L-4H)(a-c)+9\alpha b}{18b\beta L-(2\gamma(2-\theta)L-4H)\gamma(1+\theta)} \qquad (12)$$

and in the sharing information:

$$E(q^s) = \frac{a-c}{3b} + \frac{\gamma(1+\theta)}{3b}\frac{(2\gamma(2-\theta))(a-c)+9\alpha b}{18b\beta-(2\gamma(2-\theta))\gamma(1+\theta)} \tag{13}$$

The comparison of (12) and (13) shows that the output levels are always higher in the private information case because they directly depend on the amount of g produced. As for the profits, substituting (12) and (13) in (4) we end with the following inequalities:

$$k^i \geq k^s \quad \Leftrightarrow \quad (a-c)[q^i - q^s] + \gamma(1+\theta)[g^i q^i - g^s q^s] + 3\alpha[g^i - g^s] + 3\beta[g^{s2} - g^{i2}] \geq 0$$

In the last inequality the left hand side can either be positive or negative. In general, there are several conflicting influences of the production of g in the first stage on the profit level of the overall competition. The positive ones are the cost decreasing effect, the non output related benefits and the signalling of a lower than real level of costs. The negative influences are the external effect of reducing the rival costs and of shifting its reaction outward and the decreasing return in the afore mentioned benefits. In choosing if sharing information or not firms have to value which of these different effects of g are prevailing in the specific competitive setting. However, the more the public input is cost reducing and the less are its external effects on the other firm, the more the private information solution becomes attractive.

2.2 Bertrand equilibrium

In the Bertrand case, still in the private value game setting, it is assumed that the output is differentiated, to avoid problems of existence of the equilibrium that arise in such case with homogeneous outputs. The linear deterministic demand becomes now:

$$p^i = a - bq^i - dq^j \quad \text{where} \quad a,b,d > 0 \quad \text{and} \quad b > d \qquad i = 1,2$$

and accordingly the output levels traded in the market are:

$$q^i = \frac{a}{b+d} + \frac{dp^j - bp^i}{(b+d)(b-d)} \qquad i \neq j = 1,2$$

The stochastic linear cost function and the distributional properties of the signal are assumed to be the same as in the Cournot case. For firm i, the first period strategy is $g^i(x^i)$ and the second period strategy is $p^i(x^i, g^i, g^j)$. Those strategies are chosen to maximize:

$$\underset{g^i, p^i}{\text{Max}} \quad k^i = E\left[(p^i - c + \gamma g^i + \gamma\theta g^j + u^i)\left(\frac{a}{b+d} + \frac{dp^j - bp^i}{(b+d)(b-d)}\right) | x^i \right] + (\alpha - \beta g^i) g^i \qquad (14)$$

Having kept a linear structure for both the demand and cost functions the decision rules for g^i and p^i can be derived as linear functions as in the analogous expression (1):

$$g^i = C_0^i + C_1^i x^i$$

and (15)

$$p^i = D_0^i + D_1^i x^i + D_2^i g^i + D_3^i g^j$$

At the second stage of the game firm i chooses p^i in order to:

$$\max_{p^i} \Pi = E[(a - bq^i - dq^j - (c^i - \gamma g^i - \gamma\theta g^j) + u^i) q^i \,|\, x^i, g^i, g^j] + (\alpha - \beta g^i) g^i =$$

(16)

$$= a - bq^i - dq^j - (c^i - \gamma g^i - \gamma\theta g^j) + E\left(u^i \,|\, x^i, \frac{g^j - C_0}{C_1}\right) q^i + (\alpha - \beta g^i) g^i$$

From which, substituting (15) in (16), the first order conditions can be written as:

$$a(b-d) + d\left[D_0 + D_1 \frac{g^j - C_0}{C_1} + D_2 g^j + D_3 g^i\right] - 2b[D_0 + D_1 x^i + D_2 g^i + D_3 g^j] + c^i b - b\gamma g^i - b\gamma\theta g^j + \frac{\sigma x^i}{\sigma + \mu} = 0$$

(17)

Expression (17) can be used to get the following system of four equations, analogous to (3):

$$D_0 = \frac{a(d-b)-cb}{d-2b} + \frac{d\sigma C_0}{2(d-2b)(\sigma+\mu)C_1}$$

$$D_1 = \frac{\sigma}{2(\sigma+\mu)} \qquad (18)$$

$$D_2 = \frac{\gamma b(d\theta+2b)}{d^2-4b^2} - \frac{\sigma d^2}{2(d^2-4b^2)(\sigma+\mu)C_1}$$

$$D_3 = \frac{\gamma b(2b\theta+d)}{d^2-4b^2} - \frac{bd\sigma}{2(d^2-4b^2)(\sigma+\mu)C_1}$$

In D_0 the first term indicates the constant part of p^i that is charged in absence of uncertainty. It is directly related to the size of the market and to the degree of product differentiation. The second term is the constant part of p^i charged in presence of uncertainty. It is positive because C_1 will be shown to be negative. Besides, as in Cournot, it depends positively on the ratio between the quantity of g produced in absence of uncertainty and the coefficient that relates g with the signal x. D_1 shows the negative effect of the signal, in our case an unbiased estimator of the cost function, on the equilibrium level of output. This effect is inversely related to the elasticity of the demand curve because the output reducing effect of high costs (signalled by a high x^i) is lower if the demand curve is relatively inelastic. The first term in D_2 gives the effect of g^i on p^i under certainty. It is negative because $d<b$. Such effect is further directly related to θ and γ. The second term in D_2 is again negative and inversely related to the "elasticity" of g to the signal x. In other words, the price tends to be lower when relatively more public input is required to have a given amount of signal sent to the other firm. In Cournot these terms have both

a positive value since in that context g has always a positive own effect on output, even if not always on the market share (for instance when $\theta = 1$). The first term in D_3 represents the effect, always negative, of the rival production of g on its own price under certainty. The same term in Cournot could take a positive or a negative value depending on the value of θ. As in the full information case, we have that in Bertrand the production of g shifts both the reaction functions inward, reducing the equilibrium price for both firms. The second term introduces another negative cross effect of g on the price level. A higher g^j in this case signals that the rival costs are lower and so doing shifts inwardly the reaction function of the latter, with the result of further lowering the price level.

In the first stage firm i maximizes:

$$\underset{g^i}{\operatorname{Max}} \Pi^i = E\left[(p^i - c + \gamma g^i + \gamma\theta g^j + u^i)\left(\frac{a}{b+d} + \frac{d p^j - b p^i}{(b+d)(b-d)}\right) \mid x^i \right] + (\alpha - \beta g^i) g^i$$

At the first stage firm i computes the expected value of its' rival firm strategy (g^j, p^j) since at that time the realization of x^j is unknown to i. The first order conditions, using the envelope theorem, are then:

$$\left[\gamma + \frac{d}{b}\frac{dq^i}{dg^j}\right]\left[\frac{a}{b+d} + \frac{d p^j - b p^i}{(b+d)(b-d)}\right] + (\alpha - 2\beta g^i) = 0 \tag{19}$$

Substituting (14) into (19) and using the distributional properties of x^j and u^i the expression above can be written as:

$$\left(\gamma - \frac{d}{b}D_3\right)\left\{\frac{a}{b+d} + \frac{d}{b^2-d^2}(D_0 + D_2C_0 + D_3C_1x^i + D_3C_0) - \frac{b}{b^2-d^2}(D_0 + D_1x^i + D_2C_0 + D_2C_1x^i + D_3C_0)\right\} + \alpha - 2\beta g^i = 0 \tag{20}$$

Substituting for D_i i=0,1,2,3 from (18) into (20) and requiring that (20) holds for every realization of x^i yields the following system of two equations:

$$\left\{\frac{2\gamma N}{d^2-4b^2}-\frac{4bd}{(d^2-4b^2)C_1}\right\}\left\{-\frac{(a-c)b(d+2b)}{(b+d)(d-2b)}-\frac{\gamma b(1+\theta)(d+2b)C_0}{(d^2-4b^2)(b+d)}\right\}+\alpha-2\beta=0$$

and

(21)

$$2[\gamma^2 bN^2-(d^2-4b^2)^2(b^2-d^2)\beta](\sigma+\mu)^2C_1^2+\gamma b\sigma N(4b^2-3d^2)(\sigma+\mu)C_1-b\sigma^2d^2(2b^2-d^2)=0$$

where $N=(d^2-2b^2+bd\theta)$

The second equation in (21) is a quadratic equation with the following two roots:

$$\frac{-\gamma bN(4b^2-3d^2)\pm\sqrt{(\gamma b\sigma)^2N^2(4b^2-3d^2)^2+8(b\sigma d)^2(2b^2-d^2)(b\gamma^2N^2-(d^2-4b^2)^2(b^2-d^2)\beta)}}{4[(b\gamma^2N^2-(d^2-4b^2)^2(b^2-d^2)\beta)]}\frac{\sigma}{\sigma+\mu}$$

Differentiating (21) once again with respect to g^i we have the second order condition:

$$[2\gamma(d^2-4b^2)L-d^2H](\gamma(1+\theta)b(2b+d)L-2(d+2b)dH)+2\beta(b+d)(d^2-4b^2)L^2>0$$

where again L is the numerator of C_1 and H is its denominator without the term $\frac{\sigma}{\sigma+\mu}$. Substituting for C_1 in (20) we can solve for C_0:

$$C_0=\frac{\alpha(d^2-4b^2)^2(b+d)L-[2\gamma NL-d^2H](a-c)b(d+2b)}{[2\gamma NL-d^2H]\gamma(1+\theta)b(d+2b)+2\beta(d^2-4b^2)^2(b+d)L}$$

To insure that this expression is positive and that the second order condition holds we need again that L>0 and that H<0, so that the only acceptable solution for C_1 is:

$$C_1=\frac{-\gamma bN(4b^2-3d^2)+\sqrt{(\gamma b\sigma)^2N^2(4b^2-3d^2)^2+8(b\sigma d)^2(2b^2-d^2)(b\gamma^2N^2-(d^2-4b^2)^2(b^2-d^2)\beta)}}{4[b\gamma^2N^2-(d^2-4b^2)^2(b^2-d^2)\beta]}\frac{\sigma}{\sigma+\mu}$$

whose sign is negative. Now we can explicitly solve the system (18) and write the reaction functions of the firm i as:

$$g^i = \frac{\alpha(d^2-4b^2)^2(b+d)L - [2\gamma N L d^2 H](a-c)b(d+2b)}{[2\gamma NL - d^2 H]\gamma(1+\theta)b(d+2b) + 2\beta(d^2-4b^2)^2(b+d)L} + \frac{L}{H}\frac{\sigma x^i}{\sigma+\mu} \qquad (22)$$

and (23)

$$p^i = \frac{a(d-b)-cb}{d-2b} + \frac{2dHC_0}{d-2b} + \frac{\sigma}{2(\sigma+\mu)}x^i + \left\{\frac{\gamma b(d\theta+2b)}{d^2-4b^2} - \frac{2d^2H}{(d^2-4b^2)L}\right\}g^i + \left\{\frac{\gamma b(2b\theta+d)}{d^2-4b^2} - \frac{4bd}{(d^2-4b^2)L}\right\}g^j$$

When the two firms do not share information about their cost, they can increase their pay-offs above the level reached in the static model if they produce less public input than under certainty. In this case, in fact, an increase in the rival firm's production of g shifts both reaction functions inward therefore reducing the equilibrium price level for both firm.

At the symmetric Bertrand equilibria of the game with direct communication the strategies satisfy the following:

$$g^s = \frac{\alpha(d^2-4b^2)^2(b+d) - [2\gamma N](a-c)b(d+2b)}{[2\gamma N - d^2]\gamma(1+\theta)b(d+2b) + 2\beta(d^2-4b^2)^2(b+d)} \qquad (24)$$

and (25)

$$p^s = \frac{a(d-b)-cb}{d-2b} + \frac{2b^2\sigma}{(4b^2-d^2)(\sigma+\mu)}x^i + \frac{bd\sigma}{(4b^2-d^2)(\sigma+\mu)}x^j + \frac{\gamma b(d\theta+2b)}{d^2-4b^2}g^i + \frac{\gamma b(2b\theta+d)}{d^2-4b^2}g^j$$

From (22) and (24) we can now compare the two equilibrium level of g in the private and sharing information cases. After some algebraic manipulations we have the following:

$$g^s \geq g^i \qquad \Leftrightarrow \qquad 2\gamma L(2b^2 - d^2 - db\theta) \geq d^2 H$$

The left hand side of the last inequality is always greater than the right hand side because H<0. The production of the public input is then greater when the two firms share informations about their cost structure. In the first stage the two firms want the rival to believe that their costs are above the real ones. This would lead to an increase in the equilibrium price in the second stage. To do so they have to reduce their production of public input below the level that would be optimal under full information. In Bertrand competition the signalling role of the public input reduces its production respect to its level under certainty. If this is the outcome that the private firm choose, however, can only be seen looking at the profits in the two cases of private information and of information sharing. After substituting (22) into (23) we can derive the price level in the private information case as:

$$E(p^i) = \frac{a(d-b)-cb}{d-2b} + \frac{\gamma b(1+\theta)(2b+d)}{(d^2-4b^2)} E(g^i) \qquad (26)$$

and in the sharing information one:

$$E(p^s) = \frac{a(d-b)-cb}{d-2b} + \frac{\gamma b(1+\theta)(2b+d)}{(d^2-4b^2)} E(g^s) \qquad (27)$$

so that price is always lower under sharing information than with private information because the term for which E(g) is multiplied is negative. As for the profits, substituting (26) and (27) in (14) we can define the following inequality:

$$k^i \geq k^d \quad \Leftrightarrow \quad a\gamma(1+\theta)(g^i - g^d) + \frac{\gamma b(1+\theta)(2b+d)}{(d^2-4b^2)}(g^{i2} - g^{d2}) + \alpha(g^i + g^d) + \beta(g^{d2} - g^{i2}) \geq 0$$

The right hand side of the last inequality can again be either positive or negative. If profits are higher when firms have private information, the positive impact of g on the cost structure is not such to overcome both negative effect of g on prices: its negative role in signalling low costs and its strategic effect of shifting inward the reaction function. Even in Bertrand we have that the choice between sharing information or using the uncertainty to exploit strategic signalling depends on the specific setting of the competition between firms. The role played by the public input in the signalling game are however the opposite than in Cournot because opposite is the "direction" in which the firms want to misinform the rival.

3 The common value game

3.1 Cournot equilibrium

In the common value game, uncertainty is about a parameter that is common to both firms. More specifically, we assume that such uncertainty affects the cost reducing parameter of the public input (γ). In the Cournot case we continue to assume a non differentiated output market in which the linear deterministic demand function is assumed of the form:

$$p^i = a - bq^i \quad \text{where} \quad a,b > 0 \quad i = 1,2$$

The stochastic linear cost function is:

$$C^i = [c^i - \gamma(g^i + \theta g^j) + u]q^i \quad \text{where} \quad \gamma > 0 \quad i = 1,2$$

where now the distributional properties of u^i are the following:

$$x^i = u^i + \epsilon^i \quad \text{where} \quad u^i \sim N(0,\sigma), \quad \epsilon^i \sim N(0,\mu)$$

and $\qquad i \neq j = 1,2$

$$u = \left(\frac{u^i + u^j}{2}\right)$$

The above properties mean that both firms observe a private signal on the cost-reducing effectiveness of the public input. That is an unbiased estimator of the parameter γ. The random processes that determine this cost effectiveness in the two firms are independently distributed. The expected effectiveness is derived by averaging over the private signals. In terms of conditional expectations the latter can be written as:

$$E(u \mid x^i) = \frac{\sigma x^i}{\sigma + \mu} \quad \text{and} \quad E(u \mid x^i, x^j) = \frac{\sigma(x^i + x^j)}{2(\sigma + \mu)}$$

For the firm i the first period strategy is $g^i(x^i)$ and the second period strategy is $q^i(x^i, g^i, g^j)$. Those strategies are chosen to maximize:

$$\underset{g^i, q^i}{\text{Max}} \quad k^i = E[(a - b(q^i + q^j) - (c - \gamma g^i - \gamma\theta g^j + u)q^i \mid x^i, x^j] + (\alpha - \beta g^i)g^i \quad (28)$$

where $(\alpha - \beta g^i)$ is defined as before. At the second stage of the game firm i chooses q^i in order to:

$$\max_{q^i} \quad \Pi^i = [(a - b(q^i + q^j) - (c^i - \gamma g^i - \gamma\theta g^j) + E\left[u^i \mid x^i, \frac{g^j - C_0}{C_1}\right] q^i + (\alpha + \beta g^i)g^i$$

From which the first order conditions are:

$$\left[a - 2bq^i - bq^j - (c^i - \gamma g^i - \gamma\theta g^j) + \frac{\sigma x^i}{\sigma + \mu} + \frac{\sigma}{\sigma + \mu}\frac{g^j - C_0}{C_1}\right] = 0$$

Then, following the procedure shown in previous sections it is possible to derive that at the Cournot equilibrium, firm i decision rules are:

$$g^i = \frac{(2\gamma(2-\theta)\hat{L} - 4\hat{H})(a-c) + 9b\alpha L}{18b\beta L - (2\gamma(2-\theta)\hat{L} - 4\hat{H})\gamma(1+\theta)} + \frac{\hat{L}}{\hat{H}}\frac{\sigma}{\sigma + \mu}x^i$$

and (29)

$$q^i = \left\{\frac{a-c}{3b} - \frac{2\hat{H}}{3b\hat{L}}\right\} + \left\{\frac{\sigma}{4b(\sigma + \mu)}\right\}x^i + \left\{\frac{\gamma(2-\theta)}{3b} - \frac{2\hat{H}}{3b\hat{L}}\right\}g^i + \left\{\frac{\gamma(2\theta - 1)}{3b} + \frac{4\hat{H}}{3b\hat{L}}\right\}g^j$$

where:

$$\hat{L} = -\gamma\sigma(2-\theta) - \sqrt{9\gamma^2(2-\theta)^2 - 72b\beta}$$

and

$$\hat{H} = 8[\gamma^2(2-\theta)^2 - 9b\beta]$$

are both negative. In the first expression of (29) the coefficient that relates the signal to the production of the public input is positive. This is due to the fact that in the present setting a high x signals low production costs due to the high cost reducing effects of g. It follows that if the firms want to signal low costs as in the Cournot case they produces a positive amount of public good specifically aimed at misinforming the rival firm. In the second expression the second term that relates the signal to the output is positive since now it indicates the average cost effectiveness of the public input on the equilibrium output. The term that gives the cross output effect of the public input production is made of two component. The one that indicates the effect under certainty depends, as expected, on the value taken by θ. On the contrary, the effect of g due to its signalling role is negative because in Cournot the more public input is produced by the rival firm, the higher is its expected cost reducing effectiveness, and so the more outward is shifted the reaction function of the latter. As we know if θ is not such to make g completely public, a shift only in the rival firm's reaction function is such to lower the own share in the output market.

The symmetric Cournot equilibria of the game with direct communication is such that the strategies satisfy the following conditions:

$$g^s = \frac{2\gamma(2-\theta))(a-c)+9\alpha b}{18b\beta-(2\gamma(2-\theta))\gamma(1+\theta)} + \frac{\gamma(2-\theta)}{(9b\beta-\gamma^2(2-\theta)(1+\theta))}\ \frac{\sigma}{\sigma+\mu}x^i$$

and

$$q^s = \left\{\frac{a-c}{3b}\right\} - \left\{\frac{\sigma}{6b(\sigma+\mu)}\right\}x^i + \left\{\frac{\sigma}{6b(\sigma+\mu)}\right\}x^j + \left\{\frac{\gamma(2-\theta)}{3b}\right\}g^i + \left\{\frac{\gamma(2\theta-1)}{3b}\right\}g^j \tag{30}$$

From (29) and (30) we can now compare the two equilibrium level of g in the private and sharing information cases. We have the following inequality:

$$g^s \geq g^i \quad \Leftrightarrow \quad 2\beta(a-c) \geq -\alpha\gamma(1+\theta)$$

that shows that always $g^s > g^i$. Unlike the private value game, the production of the public input is then greater when the two firms share informations about their cost structure. In the first stage of the Cournot competition the two firms want the rival to believe that their cost level is below the real level in order to persuade the latter to reduce its share in the second stage competition. To do so in this setting less public input is produced than would be optimal under full information. In fact, the less g is produced the lower is its expected effectiveness and the higher is thought to be the cost level. That is indeed the target of the signalling game.

Following the discussion of the private value game it is easy to see that when both firms decide not to share information they produce less public input in order to deceive the rival on their real cost structure. As a consequence they produce less output. The effect of this on profits can be either negative or positive depending on the actual cost reducing effects of g, on the demand curve and on the non output related benefits from the production of g.

3.2 Bertrand equilibrium

In the Bertrand case it is still assumed that the output is differentiated for the above mentioned reasons. The same linear demand structure is also assumed. The stochastic linear cost function and the distributional properties of the signal are instead taken to be the same as in the Cournot case in the common value game setting. With the same procedure followed in the previous section the decision functions of the firm i can be derived as follows:

$$g^i = \frac{\alpha \hat{M} \hat{L} - [2\gamma \hat{N} \hat{L} + 4d\hat{H}(2b+d)](a-c)b(d+2b)}{2\beta \hat{M} \hat{L} + [2\gamma \hat{N} \hat{L} + 4d\hat{H}(2b+d)]\gamma(1+\theta)b(d+2b)} + \frac{\hat{L}}{\hat{H}} \frac{\sigma x^i}{\sigma+\mu}$$

and

(31)

$$p^i = \left\{ \frac{a(d-b)-cb}{d-2b} - \frac{2(2b+d)\hat{H}C_0}{d-2b} \right\} - \left\{ \frac{\sigma}{4(\sigma+\mu)} \right\} x^i + \left\{ \frac{\gamma b(d\theta+2b)}{d^2-4b^2} + \frac{2d\hat{H}(2b+d)}{(d^2-4b^2)\hat{L}} \right\} g^i + \left\{ \frac{\gamma b(2b\theta+d)}{d^2-4b^2} + \frac{4\hat{H}b(d+2b)}{(d^2-4b^2)\hat{L}} \right\} g^j$$

where:

$$\hat{L} = (-\gamma(d+2b)\hat{N}(4d-4b-db) - \sqrt{\gamma^2 \hat{N}^2 (d+2b)^2 (4d-4b-bd) + 2d(b-d)(d+2b)\hat{H}})$$

$$\hat{H} = 8[b\gamma^2 \hat{N}^2 - (d^2-4b^2)^2 (b^2-d^2)\beta]$$

$$\hat{N} = (d^2 - 2b^2 + bd\theta)$$

and

$$\hat{M} = ((b+d)(d^2-4b^2))$$

In the second expression in (31) the coefficient that relates the signal to the equilibrium price level is negative as in the Cournot case when common value game and for the same reasons. Both the strategic and the "signalling" component of the own effect coefficient of g on the equilibrium price are negative. The sign of the first is due to the Bertrand type of second stage competition, whereas the sign of the second is due to the common value setting of the signalling game. The decision functions for the firm i when there is information pooling are the following:

$$g^s = \frac{\alpha \hat{M} - [2\gamma \hat{N} + 4d(2b+d)](a-c)b(d+2b)}{2\beta \hat{M} + [2\gamma \hat{N} + 4d(2b+d)]\gamma(1+\theta)b(d+2b)} + \frac{\sigma x^i}{\sigma+\mu}$$

and

(32)

$$p^s = \left\{ \frac{a(d-b)-cb}{d-2b} \right\} - \left\{ \frac{\sigma}{2(\sigma+\mu)} \right\} x^i - \left\{ \frac{\sigma}{2(\sigma+\mu)} \right\} x^j + \left\{ \frac{\gamma b(d\theta+2b)}{d^2-4b^2} \right\} g^i + \left\{ \frac{\gamma b(2b\theta+d)}{d^2-4b^2} \right\} g^j$$

From (31) and (32) the following inequalities are derived:

$$g^i \geq g^s \qquad \Leftrightarrow \qquad \alpha\gamma(1+\theta) \geq -2\beta(a-c)$$

from which $g^i<g^s$. The potential signalling role of g is such to reduce its production below the optimal level under certainty. In the first stage the two firms want the rival to believe that their costs are above the real ones because this leads to an increase in the equilibrium price in the second stage. Therefore, in Bertrand the reversal of results between the private and the common game, shown in Cournot, does not take place. In Bertrand competition, in fact, the possibility of using g to misinform the rival firms always implies an under-production of the latter with respect to the full information solution.

4 Conclusions

The most important result of the analysis pursued so far is that the public input production can indeed be used in order to misinform the competitors on specific characteristics of the firm itself or of the market. This influence can go either way on the equilibrium production of the public input depending on the different specifications adopted in terms of competitive structure and setting of the game played. More specifically, we have investigated two different polar types of interdependence between the private information available to the firms. In the private value game the expectation on the uncertain parameter does not depend on the signal sent by the other firm. In the common value game, on the contrary, Bayesian estimates of the value of the uncertain parameter are drawn using the signal sent by the other firm.

The results obtained in the Cournot framework depend on the nature of the signalling game played by the firms. More precisely, when a private value setting is analyzed, an overproduction of the public input is used in order to signal low costs and therefore to raise the relative share in the second stage competition. When a common value game is examined the opposite is true because a firm wants the rival to believe that its own costs are high. In Bertrand competition the results in terms of under or overproduction with respect to the pooling information solution do not depend on the existence of correlation between the private information available to the firms. The cost reducing property of the public input, implies that its role in the signalling game is such to further reduce its equilibrium private production. This observation seems to strengthen what already noticed in full information models where the Bertrand type of oligopolistic competition brings about a general under-production of the public input with respect to several welfare standards.

A final note should be added in order to recall what we see as the major limitations of the analysis pursued so far. First, no explicit selection between sharing and private information equilibria in terms of profits has been obtained. This means that which of the two solutions will be chosen by the private firms depends on the specific features of the demand and cost structures, but no specific indications can be offered. Second, here the strategic interaction is limited to an exogenously fixed duopoly. What happens to the equilibrium level of the public input if new firms can enter the market is not clear. These are two of the directions along which further research is much needed.

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Endnotes

1. Strategies of misinformation have been recently introduced in different settings mainly concerned with the problem of entry deterrence. Examples of this tendency are predation pricing models like Kreps and Wilson (1982) and limit pricing models as Milgrom and Roberts (1982).

2. Models that include strategic manipulation of information can be traced back to the Riordan (1985) paper. After those contributions some papers have tried to generalize earlier results on the effect of misinformation on competition (Gal-Or (1988)).

3. See Shapiro (1987) for an introduction to the first two effects.

4. According to Radner (1962) it is possible to restrict attention to decision rules of a very generic form since with linear demand and cost structures the decision rules have to be affine in the vector of observations.

5. This a widely known property of models including asymmetric information between firms. The degree of correlation between the shocks affecting the private information available to the firms is likely to modify the results obtained. More precisely, in the polar cases of no correlation and perfect correlation, as the private and common value game can be seen, the switching from one setting to another reverses the sign of most conclusions. See Shapiro (1987).

6. See Gal-Or (1986)