ON THE OPTIMAL VARIANCE OF MONEY

Vincenzo Denicolo'

Dipartimento di Scienze Economiche

Piazza Scaravilli,2

40126 Bologna (Bo)

Italy

ABSTRACT. This paper analyses some monetary policy issues in a model of business cycle derived from Lucas (1972). We show that an autoregressive monetary policy rule may dominate a k-percent rule, and that the optimal monetary policy is characterised by an infinite amount of noise.

1. INTRODUCTION

This paper analyses some monetary policy issues in the context of a model of the business cycle closely related to that of Lucas (1972). It is widely mantained that Lucas' model provides support to a non activist monetary policy and that it implies that a k-percent rule would be socially optimal. However, a closer look at the model has shown that this claim is unwarranted.

The model of Lucas is a general equilibrium rational expectations model with continuous market clearing but incomplete markets. Agents react to real and monetary shocks which the economy is subject to by varying their labour supply. Because of the absence of forward markets for labour and consumption, there is no presumption that these variations be socially optimal. A reasonable conjecture is therefore that some kind of policy might improve upon the rational expectations equilibrium analysed by Lucas. The first best policy would clearly involve the creation of a complete set of forward markets giving all agents the opportunity to fully insure against shocks. When this is not possible, active macroeconomic policies may help. In this paper we shall focus on monetary policies.

In a model derived from that of Lucas (1972) by choosing a special functional form of the utility function, Polemarchakis and Weiss (1977) and Azariadis (1981) have shown that a totally random monetary policy may dominate a k-percent rule. More recently, using the same utility function but assuming that money transfers are not proportional to existing money balances (so that money is neutral but not superneutral), Canning (1989) has derived a log-linear model with firm microeconomic foundations which can be explicitly solved and has analysed the effects of a feed-back monetary policy rule. He has shown that such a rule, if properly devised, may be superior to a k-percent rule. A similar feed-back rule was first proposed by Marini (1985) in the context of an ad hoc macroeconomic model.

Both Canning's and Polemarchakis and Weiss' results depend in a crucial way on the ordinal and cardinal properties of the utility function. Specifically, they assume that the utility function is separable in leisure and consumption, and that agents are more risk averse in leisure than in consumption. This implies that it is socially desirable to stabilise labour supply. This can be done in two ways. The first one is to increase the variance of money growth so as to make current price information a less accurate signal of the true price of future consumption in terms of current leisure. This makes it more difficult for agents to react to real shocks; in the limit, as the variance of money growth tends to infinity, real output is perfectly stabilised. The second way is to use the inflation tax to counterbalance real shocks. The trick consists in calibrating the rate of growth of the money supply in such a way as to keep the real terms of trade between leisure and consumption constant. This completely insulates the economy from real shocks.

The point at issue is whether labour supply (i.e., real output) stabilisation is desirable or not. Generally speaking, real output stabilisation may have two costs: a) it may lower the average level of output; b) it may increase the variability of consumption. If agents are more risk averse in consumption than in leisure these costs may outweight the gain in terms of more stable leisure. This suggests that it may be worthwile to analyse the implications of other utility functions. This is the purpose of the present paper. Like the one used by Canning, our utility function allows us to perform an exact log-linearisation of the Lucas model. The model can therefore be explicitly solved, making it possible to determine the optimal monetary policy rule whithin any given class.

The main property of the utility function used in this paper is that it is strongly concave in consumption. As a result, real output stabilisation turns out to be no longer desirable. This does not imply, however, that a non activist monetary policy (e.g., a k-percent rule) is socially optimal. Indeed, monetary policy may be used to introduce additional shocks into the economy. Given that money is not superneutral, so that changes in the rate of growth of the money supply have real output effects, additional shocks increase the variability of real output, thereby increasing social welfare.

In addition to the different specification of the utility function, there are two main differences between the model analysed in this paper and that of Canning. First, we shall consider a multi-island economy. This has an important consequence on the way real output stabilisation can be performed. In Canning's model, real shocks are economy-wide, so that a deterministic monetary policy can completely offset them. When real shocks are island specific, however, this is no longer possible and the simplest way to stabilise output is to increase the variance of monetary shocks. For this reason, in this paper we shall focus on the issue of the optimal variance of money and we shall not consider feed-back rules. Second, we make a different hypothesis about the way in which money is injected into the economy. Whereas Canning assumes lump sum money transfers [1], in our model money is injected into the economy via a sort of "open market operations" [2].

The optimality criterion we adopt is also slightly different from that of Canning. Specifically, we adopt an ex-ante point of view in the sense that we shall look at the expected utility of a representative agent. However, we evaluate expected utility after pre-birth uncertainty has been resolved. For a more detailed discussion of the reasons why it may be appropriate to neglect pre-birth uncertainty the reader is referred to Canning (1989).

The rest of the paper is organized as follows. In section 2 we set up and solve the model. In section 3 we consider optimal monetary policy rules when the rate of growth of the money supply is a white noise, establishing the optimality of a k-percent rule within this class. Section 4 contains an analysis of autoregressive monetary policy rules. It is shown that they will dominate a k-percent rule, and that in this case the optimal variance of the rate of growth of money supply is infinite. Section 5 concludes the paper.

2. THE MODEL

Consider an overlapping generations economy. Each agent lives for two periods, works only in the first period and consumes only in the second period of his life, and leaves no bequests. The von Neumann - Morgenstern utility function for an agent born in period t is

$$W_t = -(1+\alpha) N_t n_t + N_t + \alpha N_t c_{t+1} \tag{1}$$

where N_t is the amount of labour supplied in period t, and C_{t+1} is consumption in period $t+1$; α is a positive parameter. (Throughout the paper, natural logs are denoted by lowercase letters.) It can be easily checked that the utility function is concave in N and C, which means that agents are risk-averse both in leisure and in consumption. The utility function has been chosen so as to obtain a log-linear supply function.

We assume that labour can be transformed into a homogeneous, non storable consumption good on a one-to-one basis. Thus, the equilibrium real wage rate is equal to one. Agents hold money so as to transfer purchasing power from youth to old age. As of time t, the supply of money is HM_t, where H is the number of markets and

$$m_t = m_{t-1} + x_t \tag{2}$$

In this section and the following one we assume that $\{x_t\}$ is a sequence of i.i.d. normal variates with mean μ and variance σ^2. Money is injected into the economy so as to finance public expenditure. More precisely, we assume that the government buys or sells money in exchange for the consumption good which is produced in the economy. (We assume that the government can store the consumption good, but private agents can not).

The budget constraint of a representative agent is $C_{t+1}P_{t+1} = N_t P_t$, where P_t is both the money price of the consumption good and the money wage rate. In logs:

$$c_{t+1} = n_t + p_t - p_{t+1} \tag{3}$$

Maximising the expected value of (1), subject to the budget constraint (3), we get the individual supply of labour (or, equivalently, the supply of the consumption good):

$$n_t = a(p_t - E p_{t+1}) \tag{4}$$

Each period, H markets (or Phelpsian islands) are open, indexed by $z = 1,2,\ldots,H$. The supply of money is M_t on each market. This requires that old agents and public expenditure be distributed across markets in an appropriate way. As far as young agents are concerned, it is assumed that Lv_t^z agents populate market z in period t, where $\{v_t^z\}$ is a sequence of i.i.d. normal variates, independent of $\{x_t\}$, with mean 0 and variance τ^2, and L is a scale factor.

It is assumed that each young agent knows the past history of the economy and observes the current price of the consumption good on the market where he operates. He does not know, however, the current prices on the other markets, the current average price p_t nor the current value of the supply of money. He can only make inferences on these variables, using his information set I_t^z, where:

$$I_t^z = \{p_t^z, m_{t-1}, p_{t-1}, \ldots\} \tag{5}$$

I_t^z is the set of information which is used to calculate the expected value which appears in (4).

Aggregate supply on market z is $Y_t^z = V_t^z L N_t^z$, that is, in view of (5),

$$y_t^z = l + a(p_t - E p_{t+1} | I_t^z) + v_t^z \tag{6}$$

Aggregate demand, on the other hand, is simply [3]:

$$y_t^z = m_t - p_t^z \tag{7}$$

Equating demand and supply, and keeping in mind equation (2), the equilibrium condition on market z may be written as follows:

$$l + a(p_t - E p_{t+1} | I_t^z) + v_t^z = m_{t-1} + x_t - p_t^z \tag{8}$$

The model can be easily solved (for instance, using the method of undetermined coefficients). A solution of the model is:

$$p_t^z = a\mu + \frac{a(1-b)}{(1+a)}\mu - l + m_{t-1} + \frac{(1+ab)}{(1+a)}(x_t - v_t^z) \tag{9}$$

where $b = \sigma^2/(\sigma^2+\tau^2)$ is a measure of the relative volatility of real and monetary shocks. (All other variables may be easily obtained from (9)). The reduced form of our model is exactly identical to that of Canning (1989). We refer the reader to Canning's paper for a comparison with Lucas' model.

3. OPTIMAL MONETARY POLICY

In the model of section 2 a monetary policy rule is completely described by two parameters, μ and σ^2. We now determine how these parameters must be chosen so as to maximise social welfare.

In dealing with this issue, we adopt an ex-ante point of view. This means that we maximise the expected utility enjoyed in equilibrium by a representative agent. However, there are many different optimality criteria within this broad class. One criterion is the Equal Treatment Pareto Optimum (ETPO) adopted by Polemarchakis and Weiss (1977), Azariadis (1981), and Canning (1989, section 4); this amounts to maximising the unconditional (i.e., pre-birth) expected utility of a representative agent. Another criterion is the Conditional Pareto Optimum (CPO) discussed by Muench (1977), Peled (1982), and Canning (1989, section 5); in this case, expected utility is calculated when some uncertainty has already been realised.

Clearly, there is a whole family of optimality criteria of the CPO type, depending on the size of the information set expected utility is conditioned on. Lucas' (1972) ex-post optimality is at one extreme of the spectrum, since it considers actual utility levels and thus requires that all uncertainty has been realised. ETPO lies at the other extreme. In this paper, we assume that the expected utility level of the representative agent is calculated prior to knowing the monetary shock, the island-specific real shock and the current price on market z. However, expected utility is calculated after pre-birth uncertainty has been resolved, when the agent knows that he has been born and assigned to market z. Thus, the only difference between our criterion and the ETPO is that we neglect the fact that, before birth, a representative agent has a greater probability of being assigned to an island with a negative real shock (i.e., an island more populated by young agents than on average) than to an island with a positive real shock.

We therefore postulate that μ and σ^2 are determined so as to maximise $EW_t|I_t$, where $I_t = I_t^z - \{p_t^z\}$. We shall evaluate $EW_t|I_t$ through an indirect but intuitive method. First of all, notice that $EW_t|I_t = E(EW_t|I_t^z)|I_t$. Let us evaluate $EW_t|I_t^z$ first. From (4) and

(3) it follows:

$$E c_{t+1} | I_t^z = \frac{1+a}{a} n_t^z \tag{10}$$

Substituting (10) into (1), and noting that, conditional on I_t^z, N_t^z is no longer a random variable, we get $E W_t | I_t^z = N_t^z$. Hence

$$E W_t | I_t = E N_t^z | I_t \tag{11}$$

Since

$$n_t^z = -a\mu + \frac{a(1-b)}{(1+a)} (x_t - \mu - v_t^z) \tag{12}$$

we have $E n_t^z | I_t = -a\mu$ and

$$Var(n_t^z) = \frac{a^2}{(1+a)^2} \frac{(\tau^2)^2}{(\sigma^2+\tau^2)} \tag{13}$$

It follows

$$E W_t | I_t = \exp\left[-a\mu + \frac{1}{2} \frac{a^2}{(1+a)^2} \frac{(\tau^2)^2}{(\sigma^2+\tau^2)} \right] \tag{14}$$

From (14) it is clear that a large negative value of μ would be desirable. However, this policy is not be feasible since the government would run out of its reserves of the consumption good in finite time. It is therefore necessary to impose the additional feasibility constraint $\mu \geq 0$. Under this constraint, it is optimal to set $\mu = 0$ and $\sigma^2 = 0$.

This result contradicts that of Polemarchakis and Weiss (1977) and Azariadis (1981). The reason is that their utility function implies that real output stabilisation is socially desirable. This objective can indeed be reached through a totally random monetary policy. With our utility function, on the contrary, the stabilisation of the rate of money growth is preferred to the stabilisation of real income. The point is that, as is apparent from (11), in our model the expected value of real output is a correct index of social welfare. By Jensen's inequality, if the log of real output is stabilised around its mean, the expected value of the level of real output is reduced. In other words, a representative agent would not be prepared to pay the cost, in terms of foregone output, of a policy of real output stabilisation.

4. AUTOREGRESSIVE MONETARY POLICY

In section 3 we have shown that a k-percent rule is optimal within the class of monetary policy rules satisfying the condition that $\{x_t\}$ is a white noise. Let us now consider autoregressive monetary policy rules of the type

$$x_t = \theta x_{t-1} + \epsilon_t \tag{15}$$

where $\{\epsilon_t\}$ is a sequence of i.i.d. normal variates, with mean 0 and variance σ^2, and θ is a parameter. Notice that we have assumed a zero mean rate of growth of the money supply, as we have seen that a positive mean rate of money growth is bound to be suboptimal and a negative mean rate is not feasible. The two parameters which completely describe monetary policy are now θ and σ^2.

The new solution of the model is

$$p_t^z = -l + m_{t-1} + \lambda x_{t-1} + \left[\frac{1 + ab(1+\lambda)}{(1+a)}\right](x_t - v_t^z) \tag{16}$$

where $\lambda = [\theta(1+a)]/[1+a(1-\theta)]$

Notice that in the model of section 2 the expected utility level of a representative agent of generation t was not affected by the past history of the economy: ex-ante, all generations had the same real opportunities, notwithstanding the fluctuations in the rate of growth of the money supply. But now things are different. If x_{t-1} and θ have the same sign, the generation born in period t expects an inflation tax higher than on average, and hence a lower utility level, while the opposite is true if x_{t-1} and θ are opposite in sign. In what follows, we shall consider the (after birth) expected utility level of an agent of a "representative" generation; thus, we shall condition on $x_{t-1} = 0$.

Proceeding as in section 3, the function to be maximised reduces to $EN_t^z|I_t$. Now we have:

$$n_t^z = \frac{a}{(1+a)}[1-b(1+\lambda)](\epsilon_t - v_t^z) \tag{17}$$

from which it follows $En_t^z|I_t = 0$ and

$$Var(n_t^z) = \frac{a^2}{(1+a)^2}\frac{(\tau^2)^2 - 2\sigma^2\tau^2\lambda + (\sigma^2)^2\lambda^2}{(\sigma^2+\tau^2)} \tag{18}$$

In order to maximise expected utility it is necessary to maximise the variance of n_t^z. An inspection of (18) reveals that $\sigma^2 = 0$ is not the optimal policy. By choosing a value of θ different from 0 and a sufficiently large σ^2, one may devise a monetary policy rule that dominates the k-percent rule (defined by $\theta = 0$ and $\sigma^2 = 0$). It is also clear that the optimal policy would be obtained letting σ^2 go to $+\infty$, and therefore would involve an infinite amount of noise.

The intuition behind this result may be explained as follows. When agents infer, from the observation of the current price, a rate of money growth different from zero, under an autoregressive monetary policy rule they are led to anticipate a positive (or negative) inflation tax for the next period, and react accordingly by reducing (or increasing) their labour supply. Hence, real output variability is an increasing function of σ^2. Since with our utility function real output stabilisation is not desirable, adding noise in the process of money creation yields a social gain. In this case, aiming at 'full information output' would be sub-optimal.

5. CONCLUDING REMARKS

In this paper we have performed an exact log-linearization of the model of Lucas (1972). Though the reduced form of the model is identical to that of Canning (1989), the policy implications are quite different. In Canning's model, it is socially desirable to stabilise real output. This can be done through a feed-back monetary policy rule or, assuming that the rate of growth of the money supply is a white noise, through an infinite variance of money.

In our model, on the contrary, agents prefer to have a larger expected real income than a more stable but, on average, lower one. Thus, if the rate of growth of the money supply follows a white noise process, a zero variance is socially optimal. This does not imply, however, the optimality of a k-percent rule. Indeed, an autoregressive monetary policy involving a very high amount of noise would dominate a k-percent rule.

REFERENCES

Azariadis, C., (1981), "A Reexamination of the Natural Rate Theory", American Economic Review, 71, 946-60.

Canning, D., (1989), "Optimal Monetary Policy over the Real Business Cycle", Economica, forthcoming.

Lucas, R.E., (1972), "Expectations and the Neutrality of Money", Journal of Economic Theory, 4, 103-24.

Marini, G., (1985), "Intertemporal Substitution and the Role of Monetary Policy", Economic Journal, 95, 87-100.

Muench, T.J., (1977), "Optimality, the Interaction of Spot and Futures Markets, and the Nonneutrality of Money in the Lucas Model", Journal of Economic Theory, 15, 325-344.

Peled, D., (1982), "Informational Diversity Over Time and the Optimality of Monetary Equilibria", Journal of Economic Theory, 20, 255-274.

Polemarchakis, H.M. and L. Weiss, (1977), "On the Desirability of a 'Totally Random' Monetary Policy", Journal of Economic Theory, 15, 345-350.

Notes

1. The lump sum nature of money transfers is responsible for the lack of superneutrality of money in Canning's model. Recall that Lucas postulated that money transfers are proportional to the money balances that old agents already hold. This hypothesis eliminates any inflation tax effect and makes money superneutral.

2. Given our utility function, this hypothesis seems to be necessary in order to derive a log-linear supply curve.

3. Aggregate demand is made up of two components: private demand of old agents and public expenditure. Both components are unit elastic. The distribution of old agents and public expenditure across markets is such that there are no island-specific demand shocks.