Confidence Crises and Public Debt Management

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Revised, March 1989

We thank Daniel Cohen, Vittorio Grilli, Torsten Persson, Luigi Spaventa and Guido Tabellini for insightful comments and discussions. Seminar participants at the London School of Economics (Financial Markets Group), at the Centre for Economic Policy Research (London) and at the Séminaire d'Economie Internationale (Paris) also provided useful suggestions.
1. Introduction

Which maturity structure should the Treasury choose for public debt? When is it advisable to issue indexed bonds? And, in general, is there an "optimal menu" of debt instruments to be issued by governments? These questions reflect everyday concerns of Central Bankers and Treasury Ministers. Still, until lately economists have provided little help in answering them. The last systematic treatment of the choice of the maturity structure dates back to Tobin's 1963 essay on debt management. Although more work has been done on the choice of debt instruments (chiefly on indexed bonds), as late as 1983 Fischer concluded that "there is as yet no satisfactory theory of what types of assets governments should issue" (p. 243).

Recently, however, research interest in this area has been revived by the game—theoretic view of the interaction between government and private sector. The insight is that debt management issues — such as the choice of maturity structure or the decision on debt indexation — can be seen as a way of altering the set of incentives faced by the government and thus the strategies that the private sector expects the government to play.

For instance, Lucas and Stokey (1983) and Persson and Svensson (1984, 1987) have shown that a government can choose the maturity structure of debt so as to tie the hands of successive governments, eliminating any future incentive to depart from the policy it has optimally chosen (i.e. ensuring time consistency). Thus, for a particular choice of the maturity structure, the public discounts that subsequent governments will not deviate from the-initial plan, and the latter is not only (ex ante) optimal but also credible.

Similarly, a game—theoretic framework has been used by Calvo (1987, 1988) to argue that public debt should be indexed. He produces examples where a government that issues nominal debt is forced to monetize it by the self-fulfilling expectations of the private sector. If debt-holders start fearing debt monetization, they will require a higher nominal rate of interest. To the extent that the government is not willing to pay this premium and accept the implied increase in the future tax burden, it may go for the monetization
anticipated by the public. Indexation eliminates the incentive to monetize and thus also the expectations that induce the government to monetize.

The novel element in Calvo's models is that the game between government and private sector may have multiple equilibria, and that some debt management decisions can rule out "bad" equilibria that imply monetization or repudiation. He captures a phenomenon that economists and business analysts have traditionally called a "confidence crisis". A confidence crisis is a critical change in expectations about the behavior of policy-makers, capable by itself of precipitating a policy regime shift or at least of increasing the chances that it will take place.

The question that we ask in this paper is whether the danger of a confidence crisis can be reduced by acting on the choice of the maturity structure (rather than on the degree of indexation) of public debt. Another novelty of our model is that it relies on the assumption that the public is imperfectly informed about the government's preferences or opportunities, and therefore does not know for sure what is going to happen if a confidence crisis occurs. In Calvo's models, once a confidence crisis explodes, a regime shift occurs — in fact, one can say that a confidence crisis is the regime shift (the new regime entailing repudiation). In our setup, instead, a confidence crisis does not necessarily lead to a regime shift, but just to a higher probability of it occurring. In other words, there are chances that the policy-maker will be able to withstand the crisis. The interesting question then becomes which factors determine the policy-maker's chances of resisting the crisis, and if debt management can affect some of these factors.

What the model reveals is that the probability that the authorities will withstand a confidence crisis is critically affected by the extent to which they have to appeal to the market at each given date to roll over public debt. This depends on three factors: the amount of debt outstanding, its average maturity and the time pattern of maturing debt. In a situation where the stock of debt is high, the average maturity is short and maturing debt is concentrated at few dates, the Treasury has to borrow huge amounts from the
market at those dates. If on one of those dates a confidence crisis occurs, the Treasury finds itself in the critical situation of refinancing a large portion of its debt at unfavourable terms. This leads the public to expect a higher probability of a regime shift if a confidence crisis arises.

2. Overview and motivation of the model

The model is set in an open economy context, where the Central Bank has embarked on a fixed exchange regime with free capital mobility. The danger of a confidence crisis is compounded in this context, since a crisis can occur not only if the public is afraid that the government might not honour its debt (monetization or default), but also if it fears that the Central Bank might abandon the fixed parity (devaluation). Here we concentrate on the danger of devaluation and rule out debt monetization or default, although it should be obvious that the model can be easily recast in terms of a closed economy where the regime shift concerns government solvency (as in Calvo). Before motivating this modeling strategy, it is worth offering a quick overview of the analysis to explain how a speculative attack on the currency links up with debt management problems.

- Consider the following scenario. Suppose that current and expected monetary policy is consistent with a fixed exchange rate at the current parity, but that occasionally investors launch a speculative attack on the Central Bank (driven by sunspot-type beliefs). The attack raises the conditional probability of a devaluation and thus leads to an increase in the nominal rate of interest through the arbitrage condition between assets denominated in domestic and foreign currency. We assume that the level of reserves is such that the attack can be resisted by the Central Bank, provided the Treasury does not create
monetary base at the time of the attack. Consider now what happens if instead the Treasury is entitled to finance part of its current borrowing requirement by money creation. If at the time of the crisis the Treasury has to refinance a large chunk of debt on the market, it has a strong incentive to do so largely by money creation, given that the rate of interest is so high. However, the larger is this injection of monetary base, the lower are the chances that the Central Bank will be able to defend the initial parity.

Investors, on the other hand, know that the higher the refinancing needs of the Treasury at each instant, the higher the probability that a confidence crisis will end in a devaluation. Thus, when a confidence crisis occurs, they require a higher jump in interest rates, and this in turn raises the probability that the crisis will end in a devaluation. In equilibrium, the probability that the Central Bank will resist to a speculative attack can be enhanced either by lengthening the average maturity of public debt, so as to diminish the average recourse of the Treasury to the market, or by spreading the recourse to the market as uniformly as possible over time, so as to reduce the variance of new debt issues by the Treasury. These debt management policies are all the more needed the larger the size of public debt outstanding and the smaller the foreign exchange reserves owned by the Central Bank.

Other factors that can mitigate the effects of a confidence crisis are the possibility of issuing debt denominated in foreign currencies and the ability to borrow from other Central Banks in the event of a crisis. This would enable the public sector to have equal access to world capital markets as the private sector, eliminating the asymmetry between the two implied by our model. Limited access by the public sector to world capital markets is a key prerequisite for confidence crises to materialize: as noted by Obstfeld (1986) in the context of a similar model (where self-fulfilling beliefs can lead to the collapse of viable fixed exchange rate regimes), a regime collapse "presupposes the existence of some lower

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1In Italy, for instance, the Treasury can draw from an account at the Central Bank up to 14% of public spending in the current fiscal year.
bound on central bank reserves, ... though there is no reason in principle why a central bank facing a perfect international capital market cannot borrow indefinitely to support the exchange rate, provided it raises taxes to service the external debt it incurs" (p. 33). In practice, international lending to the Central Bank has the same role as a loan by the "lender of last resort" to a commercial bank in models of bank runs — namely enforcing the "good" Nash equilibrium by stabilizing expectations.

One may ask why we concentrate on the interaction between debt management and the danger of collapse of fixed exchange rates under capital mobility. We do so because this interaction is likely to become very important for a number of European countries in the near future. It is often suggested that EEC countries with high public debt will face unsustainable speculative attacks on their currencies if they attempt to liberalize capital movements while maintaining fixed or quasi-fixed exchange rates. Our model suggests that, while this concern is well-founded, it is not only the level of public debt that matters for the viability of fixed rates, but also the way one manages this debt. In a high-debt country, the viability of a fixed rate regime can be enhanced by lengthening the average maturity of debt, spreading maturing issues as uniformly as possible and developing a market for public debt denominated in foreign currency. A relevant historical precedent can be found in the experience of France and Italy in the late 1920s: in both countries, upon returning to full convertibility with the dollar and the pound, the government embarked on a drastic restructuring of public debt. Between 1926 and 1929, the proportion of short-term debt fell from 32% to 12% in France and from 30% to 3% in Italy2.

On the other hand, if the model’s prescriptions were to be applied today in the high-debt countries of the EEC, the overhaul in debt management policy would not be less drastic. As shown by Table 1, high-debt countries such as Italy, Greece and Portugal stand out for their relatively short debt maturity (columns 2 and 5) and greater reliance on

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2See Eichengreen and Giavazzi (1984), Tables 2–3. Short-term debt is defined as debt with 1 year to maturity or less.
<table>
<thead>
<tr>
<th></th>
<th>Total debt as % of GDP</th>
<th>T-Bills and other short-term debt as % of domestic market debt</th>
<th>Central Bank loans to the Treasury as % of total debt</th>
<th>Foreign currency debt as % of total debt</th>
<th>Average residual maturity (years)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Belgium</td>
<td>122.2</td>
<td>21.8</td>
<td>2.8</td>
<td>16.5</td>
<td>3.6</td>
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<tr>
<td>Denmark</td>
<td>59.6</td>
<td>14.5</td>
<td>-13.7</td>
<td>30.9</td>
<td>3.6</td>
</tr>
<tr>
<td>Germany</td>
<td>42.2</td>
<td>1.8</td>
<td>0.0</td>
<td>N.A.</td>
<td>N.A.</td>
</tr>
<tr>
<td>Greece</td>
<td>69.0</td>
<td>92.5</td>
<td>14.6</td>
<td>33.5</td>
<td>N.A.</td>
</tr>
<tr>
<td>Spain</td>
<td>48.5</td>
<td>60.8</td>
<td>7.2</td>
<td>2.5</td>
<td>1.5(^b)</td>
</tr>
<tr>
<td>France</td>
<td>24.5</td>
<td>45.3</td>
<td>-5.3</td>
<td>3.3</td>
<td>4.0</td>
</tr>
<tr>
<td>Ireland</td>
<td>124.5</td>
<td>6.5</td>
<td>0.0(^c)</td>
<td>39.7</td>
<td>7.8(^c)</td>
</tr>
<tr>
<td>Italy</td>
<td>92.6</td>
<td>30.3</td>
<td>7.2</td>
<td>2.5</td>
<td>3.5</td>
</tr>
<tr>
<td>Netherl.</td>
<td>72.9</td>
<td>9.1</td>
<td>N.A.</td>
<td>N.A.</td>
<td>5.9</td>
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<tr>
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<td>62.5</td>
<td>32.8</td>
<td>22.4</td>
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<tr>
<td>U.K.</td>
<td>48.0</td>
<td>30.3</td>
<td>N.A.</td>
<td>9.2</td>
<td>8.2 - 10.9(^d,e)</td>
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<tr>
<td>U.S.A.</td>
<td>53.1</td>
<td>59.8</td>
<td>0.0</td>
<td>0.2</td>
<td>5.7</td>
</tr>
</tbody>
</table>


\(^b\) "Domestic market debt" is the sum of T-Bills, other short-term debt, medium and long-term debt. It excludes non-marketable debt, foreign currency debt and other debt.

\(^c\) *Central Bank of Ireland Quarterly Bulletin*, Winter 1988, Tables D1, D2 and D4. The estimate of average maturity in col. 5 excludes undated government stock (consols).


\(^e\) Lowest and highest bound for the estimate of average maturity. The highest bound refers only to dated domestic government stock (that excludes consols), while the lowest bound is based on the extreme assumption that all other debt (mostly national savings certificates) has zero residual maturity.
Maturing Public Debt by Year, Italy

As of August 31, 1986, in LIR.

FIG. 7
Central Bank loans to finance public debt\(^3\) (column 3). Italy is also characterized by very limited recourse to foreign currency debt (column 4). Spain, instead, has a relatively small debt–income ratio, but has the shortest maturity in Europe and uses foreign currency–denominated debt as little as Italy. The case of Ireland is symmetric and opposite to that of Spain, since Ireland has the highest debt–income ratio in Europe, but is characterized by relatively long maturity, no reliance on Central Bank loans and substantial recourse to foreign markets.

As explained above, another qualifying feature of debt management policy is how evenly maturing issues are distributed in the future. Cross–country data on this point are harder to obtain. However, at least as far as Italy is concerned, Figure 1 suggests that huge chunks of public debt are going to mature simultaneously at a few dates, especially 1990 and 1991. This suggests that scarce attention has been so far devoted to the objective of smoothing out the future pattern of debt issues.

3. The model

There are three agents in the economy: the private sector, the Treasury and the Central Bank. The private sector simply decides what is the probability of a devaluation at each date, and requires an interest rate that compensates debt–holders accordingly. The Treasury decides on the mix of debt and temporary money creation to be used to finance the deficit. It does so by weighting two objectives: (i) minimize the burden of debt servicing (and thus the deadweight loss associated with future taxes), and (ii) avoid

\(^3\)Recall that the easy access of the Treasury to Central Bank financing is crucial in determining the chances of success for a speculative attack on the currency.
impairing the ability of the Central Bank to defend the exchange rate. As shown below, there is a natural tradeoff between these two objectives. It is assumed that different governments choose different points along this tradeoff, depending on their preferences, and that the private sector is imperfectly informed about the preferences of the current government. The Central Bank tries to defend the existing nominal exchange rate parity, and pursues a monetary policy consistent with indefinite exchange rate stability. Although fundamentals are consistent with the existing parity, in some occurrences the interaction between the private sector and the Treasury can lead to a successful attack on the Central Bank and force it to devalue.

3.1 The private sector
There are two possible states of nature, N (for "normal") and C (for "confidence crisis"): their realizations are independent drawings from the same distribution and occur with exogenous probabilities 1−p and p respectively. State N is the set of situations in which debt-holders believe that in the current period there will be no devaluation, i.e. the probability of a devaluation r = 0. State C is instead that in which a confidence crisis occurs: investors become convinced that there is a positive probability (r > 0) that the Central Bank will devalue the domestic currency by x percent in the current period (x being for convenience a fixed amount), and stage a speculative attack on the Central Bank. At that point, the Central Bank can devalue or resist. How its move is determined will be analyzed below.

Consider now a 1-period bond issued after the current state (N or C) has been revealed, but before the Central Bank has made its move. For instance, it is known that a confidence crisis is on (state C has been drawn), but the Bank has not announced yet whether it is devaluing or resisting. If the current state is N, free capital mobility ensures that the return on such a bond is 1 + r*, where r* is the world rate of interest. If instead the current state is C, by international arbitrage the return on a 1-period bond will be
\((1 + r^*)(1 + \pi x)\), to compensate debt-holders for the expected devaluation \(\pi x\) (\(\pi\) being the probability of devaluing by \(x\), conditional on being in state \(C\)).

Let us now turn to bonds of maturity \(T > 1\), issued at par and paying the same rate of interest in every period. Let us denote this rate of interest by \(r^N\) if the bond is issued in state \(N\) and by \(r^C\) if it is issued in state \(C\). On such bonds, investors will also want to be protected against the danger of future devaluations. The probability of a devaluation at any future period is \(px\) (the probability of the joint event of a confidence crisis and of a devaluation). Under risk-neutrality, the compound return required on such a bond must equal in expectation the compound return from rolling over a $1 investment for \(T\) periods by a sequence of \(1\)-period bonds:

\[
(1) \quad (1 + r^N)^T = (1 + r^\ast)^T (1 + p\pi x)^{T-1},
\]

\[
(2) \quad (1 + r^C)^T = (1 + r^\ast)^T (1 + \pi x)(1 + p\pi x)^{T-1}.
\]

With a logarithmic approximation, the rates of interest \(r^N\) and \(r^C\) can be written as:

\[
(1') \quad r^N \approx r^\ast + p\pi x - p\pi x/T.
\]

\[
(2') \quad r^C \approx r^\ast + p\pi x + (1 - p)\pi x/T.
\]

Comparing \(r^C\) with \(r^N\), one sees that lengthening the maturity \(T\) lowers \(r^C\) and raises \(r^N\), narrowing the spread between the two. A longer maturity lowers the interest burden to be borne when a confidence crisis explodes at the cost of a higher premium to be paid in normal times to insure against the possibility of a crisis. This reflects the fact that a longer maturity, implying a longer investment horizon, induces investors to place a smaller weight on the current situation, even if the latter presents a relatively high danger of devaluation.
(state C), and a larger weight on the danger of devaluations in future periods.

3.2 The Treasury

Public debt is in the form of B bonds with maturity T. Each of these bonds has face value of 1, is issued at par, and pays in each period $t^N$ if it has been issued in state N and $t^C$ if it has been issued in state C. Goods' prices are constant, so B denotes the number of outstanding bonds, as well as their real value. Consider a situation where for T periods a confidence crisis has not occurred, i.e. the economy has been in state N. The government has stabilized debt at a constant level B and has also achieved a uniform distribution of debt issues over time, so that the amount of debt that must be renewed in each period is $B/T$. The government budget constraint thus requires that the primary deficit $g - t$, plus debt servicing on outstanding bonds B, and repayment of maturing bonds $B/T$, equal new issues $B/T$:

$$g - t + t^N B + B/T = B/T,$$

so that the primary surplus must equal the interest payments on outstanding debt. Equation (3) rules out permanent revenue from money creation, i.e. seignorage. However, we assume that the Treasury can draw from a limited overdraft facility at the Central Bank to meet temporary financing needs. After withdrawing from this credit facility, the Treasury must eventually rebalance its account, so that it can create monetary base up to its overdraft limit but cannot consider money creation as a steady state source of revenue.

When a crisis (state C) occurs, the Treasury faces a jump in the rate of interest from $t^N$ to $t^C$. At this point, it has three options: (i) increasing current taxes or lowering current expenditures; (ii) issuing more debt at the more unfavourable terms offered by the market; (iii) turning to its emergency credit line at the Central Bank. We rule out option
(i), for two reasons. First, budgets cannot be revised quickly enough as to react timely to sudden events like confidence crises. Second, if taxes are distortionary, inflicting the entire burden of the distortion on current taxpayers is suboptimal, especially considering that the blip in the cost of debt servicing is temporary. It is instead preferable to finance this blip with debt, and spread the implied tax burden on all future periods (as in Barro 1979).

Assume then that the government does not change taxes $t$ and expenditures $g$, and uses only option (ii), i.e. it issues $B/T$ bonds each yielding the high interest rate $r^C$. If the maturity of these bonds is $\tau$ (not necessarily equal to $T$), the increase in debt servicing is $(r^C - r^N)B/T$ in each of the following periods. Using expressions (1') and (2') for $r^C - r^N$, one can express the total increase in debt service payments over the subsequent $\tau$ periods as:

\[
(4) \quad \frac{B}{T} (r^C - r^N) \tau = \frac{B}{T} \frac{r x}{\tau} \tau = \frac{B}{T} r x .
\]

This increase in future debt service measures the increase in the future tax burden implied by resorting to option (ii) in the event of a confidence crisis. Notice, incidentally, that this expression for the additional tax burden is independent of the maturity $\tau$ chosen for the bonds issued during the crisis.

The Treasury, however, has still another option. It can turn to its credit line at the Central Bank to get over the temporary increase in the cost of borrowing from the market. By drawing from this credit facility, the Treasury can compress the increase in debt service below the value given by expression (4). Let $\alpha$ denote the fraction of the increase in debt service that is monetized: for instance, if it sets $\alpha = 1$, the Treasury effectively prevents any increase in debt service, while if it sets $\alpha = 0$ it lets debt service increase by the full amount indicated in (4).

How does the Treasury choose the value of $\alpha$? Its choice is determined partly by
the institutional constraints on the use of the overdraft facility\textsuperscript{4} and partly by its preferences. During a crisis the Treasury faces a tradeoff between the welfare of future taxpayers and the objective of exchange rate stability, to which the Central Bank is fully devoted. The greater the Treasury's withdrawal from the overdraft facility during a crisis, the smaller the subsequent increase in debt service and thus in future taxes, but also the higher the probability that the Central Bank will exhaust its reserves and will be forced to devalue.

We assume that some governments are more conscious of taxpayers' interest (high\(\alpha\)) and others are instead more willing to cooperate with the Central Bank (low\(\alpha\)). The distribution of the parameter \(\alpha\), \(F(\cdot)\), describes the frequency of each "type" of government. It seems reasonable to assume that the support of \(\alpha\) is bounded, and ranges between 0 and some \(\alpha_{\text{max}}\).

The private sector does not observe the type of government it faces, i.e. the realization of \(\alpha\) that has currently occurred. Due to this informational asymmetry, it regards the current \(\alpha\) as a random variable, and can only make a probabilistic assessment about how the Treasury will behave in a confidence crisis.

### 3.3 The Central Bank

The balance sheet of the Central Bank is elementary. In state N, when the Treasury does not create monetary base by drawing from its credit line, the Central Bank has foreign exchange reserves \(R\) on the asset side and currency \(M^N\) on the liability side. Thus currency is fully backed by reserves (\(M^N = R\)). The private sector's demand for currency is assumed

\textsuperscript{4}Obviously the two key institutional constraints are the overdraft limit and the time allowed to rebalance the account. The effect of these two constraints on the recourse to the overdraft facility will depend on the frequency of confidence crises. For instance, the shorter the time allowed to rebalance the account and the higher the frequency of confidence crises, the lower will be the optimal value of \(\alpha\), \textit{ceteris paribus}: the Treasury will not risk being caught by another crisis just at the time when it must rebalance a large overdraft at the Central Bank. In the model of this paper we do not provide a formal analysis of the effect of these constraints on the choice of \(\alpha\) by the Treasury.
to be exponential, with interest elasticity $\sigma$. Both these assumptions — the full backing of currency and the functional form of money demand — simplify the analysis, but are not necessary for the qualitative results of the model to go through (see discussion at the end of Section 4).

The rate of interest that is relevant for the money demand function is that on 1-period bonds, i.e. $r^*$ in state N and $r^* + \pi x$ in state C. Money demand in the two states is thus respectively:

$$M^N = Ae^{-\sigma r^*},$$

$$M^C = Ae^{-\sigma (r^* + \pi x)},$$

where $A > 0$, $\sigma > 0$. When the state of nature changes from N to C, i.e. a confidence crisis occurs, money demand falls by:

$$M^N - M^C = M^N(1 - e^{-\sigma x}) = R(1 - e^{-\sigma x})$$

At the same time, the Treasury injects into the economy an amount of liquidity $\alpha\pi x B/T$, by monetizing a fraction $\alpha$ of the increase in debt service in (4). Thus, the Central Bank is faced with an increase in the monetary base at the wrong moment — when it already has people lining up at its door to convert currency into foreign exchange. If the (algebraic) sum of the fall in money demand (6) and the Treasury's injection of liquidity is larger than reserves $R$, the Central Bank will have to abandon the current parity. The condition for devaluation therefore is:
(7) \[ R (1 - e^{-\sigma x}) + \alpha x \frac{B}{T} > R. \]

Rearranging (7), the probability of a devaluation \( \pi \) can be written as:

(8) \[ \pi = P(\alpha g(\pi) > 1), \text{ where } g(\pi) = e^{\sigma x} x x \frac{B}{T}. \]

Notice that, if the Treasury always abstained from creating monetary base, the Central Bank would always be able to resist the speculative attack (with \( x \) finite, if \( \alpha = 0 \) the inequality \( \alpha g(\pi) > 1 \) never holds). In this case, then, the probability of a devaluation would always be zero (\( \pi = 0 \)). What generates the danger of devaluation is precisely the institutional setup that allows the Treasury to create monetary base (i.e. set \( \alpha > 0 \)) in response to a speculative attack: this prevents it from precommitting to set \( \alpha = 0 \).

4. Confidence crises and average debt maturity

Equation (8) is the condition for a rational expectation equilibrium. It says that the probability of devaluation entering the decisions of investors, \( \pi \), must be equal to the probability of devaluation resulting from the interaction of the three agents in the economy, \( P(\cdot) \). This implies that private sector expectations are consistent with the actual working of the economy.

Analyzing equation (8), one can establish that, under certain parameter configurations, for the public it is equally rational to assign a zero probability or a (specific) positive probability to a devaluation in the current period: in the first case, the devaluation will not occur, whereas in the second case it will occur with the probability chosen by the public. Thus both state \( N \) and state \( C \) correspond to rational expectations
Proposition 1 (Number of equilibria)

There is always an equilibrium where the probability of devaluation is zero ($\pi = 0$). There can also be other equilibria where the probability of devaluation is positive ($\pi > 0$).

To prove Proposition 1, recall that equilibria are values of $\pi$ that solve equation (8). Define a value $\overline{\pi} = g^{-1}(1/\alpha_{\max})$. Intuitively, $\overline{\pi}$ can be interpreted as follows: if the private sector "plays" a value of $\pi$ below $\overline{\pi}$, the increase in the burden of debt servicing is so small that even if the government "plays" the highest possible value of $\alpha$ (i.e. $\alpha_{\max}$), a devaluation cannot occur (i.e. the actual probability $P(\alpha g(\pi) > 1) = 0$ for $\pi < \overline{\pi}$). We then consider separately whether a solution to (8) exists (i) for $\pi < \overline{\pi}$ and (ii) for $\pi \geq \overline{\pi}$.

(i) For $\pi < \overline{\pi}$, the function $\alpha g(\pi) < 1$ for every possible $\alpha$, so that $P(\alpha g(\pi) > 1) = 0$. Thus if $\pi < \overline{\pi}$, then $\pi = 0$ is the only solution to (8).

(ii) For $\pi \geq \overline{\pi}$, one can show that the function $P(\alpha g(\pi) > 1)$ is increasing, starts at 0 for $\pi = \overline{\pi}$ and approaches asymptotically 1 for $\pi$ large (although in equilibrium $\pi$ cannot exceed 1, of course). Figure 2 displays the function $P(\alpha g(\pi) > 1)$ as the locus $P$, in the

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5To show that $P$ starts at 0 for $\pi = \overline{\pi}$, notice that $\alpha g(\overline{\pi}) = \alpha/\alpha_{\max} \leq 1$, so that $P(\alpha g(\overline{\pi}) > 1) = 0$. Similarly, to prove that $P$ has an asymptote at 1 for $\pi$ large, notice that $\lim_{\pi \to \infty} \alpha g(\pi) = \infty$, implying $\lim_{\pi \to \infty} P(\alpha g(\pi) > 1) = 1$. Finally, the proof that $P$ is increasing goes as follows: the probability $P(\alpha g(\pi) > 1)$ is $1 - F(1/g(\pi))$, and its first derivative with respect to $\pi$ is:

$$P'(\pi) = F'(1/g(\pi)) \frac{[\alpha \pi + 1/\pi]}{g(\pi)} > 0.$$  

The second derivative of $P(\alpha g(\pi) > 1)$ is:

$$P''(\pi) = -F''(1/g(\pi)) \frac{[\alpha \pi + 1/\pi]}{g(\pi)}^2 - F'(1/g(\pi)) \frac{1/\pi + (\alpha \pi + 1/\pi)^2}{g(\pi)} g(\pi),$$

whose sign is in general ambiguous because $F''(\cdot)$, the slope of the density function, can be either positive or negative. $P''(\cdot)$ is however unambiguously negative for $\pi$ large, when $F''(\cdot)$ becomes positive. In the special case of a uniform distribution for $\alpha$, $F''(\cdot) = 0$, so that $P''(\pi)$ is negative for all values of $\pi$, so that the function $P(\pi)$ is concave.
special case of a uniform distribution for \( \alpha \); in this case \( P(\cdot) \) is concave, so that there are at most two intersections between \( P \) and a 45° line from the origin. These intersections are equilibria with \( \pi > 0 \); in the figure they are indicated as \( \tau_1 \) and \( \tau_2 \). In general, there will be either no such equilibria or multiple equilibria with \( \pi > 0 \). The possibility of one equilibrium with \( \pi > 0 \) occurs only if the \( P \) locus is tangential to the 45° line, i.e. for a unique combination of parameters.

The precise location of the \( P \) locus depends on the value of \( g(\pi) \), that is in turn proportional to \( (x \frac{B}{R} T) \), as shown by equation (8). Thus the \( P \) locus shifts down if there is a ceteris paribus increase in \( T \) or decrease in \( B/R \) and \( x \): for instance, it may shift as shown in Fig. 3, where the new locus is displayed as \( P' \) (the figure is again drawn for the case of a uniform distribution). It is apparent that, whatever the values of the other parameters, there is always a value of \( T \) high enough as to rule out any intersection between the \( P \) locus and the 45° line, leaving no equilibria with \( \pi > 0 \). This leads to:

**Proposition 2 (Conditions for the existence of a unique equilibrium with \( \pi = 0 \))**

For each given value of \( B/R \) (the ratio of public debt to reserves) and of \( x \) (the size of the expected devaluation), there is a critical maturity \( T^* \), such that for \( T > T^* \) only the equilibrium where \( \pi = 0 \) exists, i.e. confidence crises never occur. This critical maturity \( T^* \) is larger the higher \( B/R \) and \( x \).

Thus, if the average maturity of public debt is higher than the critical value \( T^* \), a devaluation cannot occur: if people expected it to occur, they would be systematically proven wrong by facts. In other words, state \( C \) is no longer a rational expectations equilibrium. The critical maturity \( T^* \) is obviously the value of \( T \) for which the locus \( P \) is tangential to the 45° line, and thus has a slope of 1. Formally, this implies that \( T^* \) satisfies both the equilibrium condition (8) and the tangency condition:
(9) \[ dP(\alpha g(x) > 1)/dx = 1. \]

Solving equations (8) and (9) for \( T \) and \( x \), one finds the critical maturity \( T^* \) and the associated probability of devaluation \( x^* \). Differentiating the probability \( P(\cdot) \) at \( x = x^* \), it is easy to verify the claim that \( T^* \) is larger the higher the debt–reserves ratio \( B/R \) and the expected devaluation \( x \):

\[
\begin{align*}
(10a) \quad & \frac{dT^*}{d(B/R)} \bigg|_{x=x^*} = T^* / (B/R) > 0, \\
(10b) \quad & \frac{dT^*}{dx} \bigg|_{x=x^*} = T^* (\sigma x^* + 1/x) > 0.
\end{align*}
\]

Example. Suppose that \( F(\alpha) \) is a uniform distribution: \( \alpha \sim U(0, \alpha_{\text{max}}) \). Then the equilibrium condition (8) specializes to:

\[ x = 1 - \frac{1}{\alpha_{\text{max}} g(x)}, \]

and the tangency condition (9) becomes:

\[ \frac{1}{x} + \sigma x - \frac{1}{\alpha_{\text{max}} g(x)} = 1. \]

These can be solved to yield

\[ x^* = \frac{1}{2} - \frac{1}{\sigma x} + \left[ \frac{1}{4} + \left( \frac{1}{\sigma x} \right)^2 \right]^{1/2} \]

and
\[ T^* = (1 - \tau^*) \tau^* \sigma x^* B \frac{\alpha}{R} \alpha_{\text{max}}. \]

Setting the elasticity of money demand \( \sigma \) at 0.05, the expected devaluation at 10\%, the debt–reserve ratio \( B/R \) at 39 (the actual value for Italy) and \( \alpha_{\text{max}} \) at 1 (so that at most the Treasury monetizes the whole increase in the interest burden), one finds that \( T^* = 0.98 \). If the time unit (implicit in the choice of \( x \)) is 1 year, in this example the confidence crisis equilibrium can be ruled out if the average maturity of public debt is at least 1 year.

The results in Proposition 1 and 2 generalize in at least two directions. First, they easily extend to the case in which currency is not fully backed by foreign exchange reserves. Predictably, it turns out that the lower the share of currency backed by foreign exchange rather than by domestic credit, the larger is the critical maturity \( T^* \): the lower are foreign exchange reserves relative to currency, the less likely is the Central Bank to withstand speculative attacks, so it is even more important to avoid conflicting behavior by the Treasury in such occurrences. Second, the results do not hinge on the exponential form assumed for the money demand function: they go through also with a linear demand function (provided money demand is restricted to be non–negative).

4. Smoothing out maturing debt and other policy implications

The policy prescriptions that can be drawn from this simple model extend beyond those on the average maturity of public debt. There are three additional ways to increase the Central Bank’s ability to withstand a confidence crisis.

First, the Treasury should avoid concentrating bulges of maturing debt at specific
moments in time, and play instead with debt maturities so as to smooth out the time pattern of maturing debt. So far this point has not emerged from the analysis because because the Treasury has been assumed to inherit from the past a constant amount B/T of maturing debt per unit time. If instead we suppose that the amount of debt that matures per unit time changes over time in a known fashion, the equilibrium probability of devaluation (conditional on a confidence crisis) will also shift around, rising and falling together with the amount of maturing debt. Formally, this is because in the function g(π) of equation (8) the constant amount of maturing debt B/T would be replaced by a time-varying magnitude. Graphically, this translates in the locus P shifting up when maturing debt is high and down when it is low. Intuitively, the private sector anticipates the Central Bank to be weakest if the confidence crisis takes place at a time when the Treasury is forced to re-issue a large portion of public debt, and in equilibrium this is indeed the case.

Second, the Treasury can further improve the system’s resilience to confidence crises by developing a well-functioning market for public debt denominated in foreign currencies. This in fact provides the government with another way to bridge the gap of a speculative attack on the currency without paying abnormally high interest rates. To be able to absorb large amounts of debt in such circumstances, however, the market for foreign-denominated debt must be deep enough, which requires that the Treasury must feed it with substantial issues at regular intervals. This prescription is particularly relevant for countries such as Italy and Spain, that are characterized by negligible amounts of foreign currency debt (see Table 1, col. 4).

Third, the Central Bank can itself strengthen its position against speculators by ensuring that foreign Central Banks will cooperate by lending foreign exchange in the event of a crisis. If, for instance, foreign Banks open an emergency credit line in foreign currency up to a maximum amount L, the condition for devaluation (7) must be rewritten by adding L to the RHS, and correspondingly the equilibrium condition (8) becomes:
(10') \[ r = P(\alpha g(x) - e^{\sigma x} \frac{L}{R} > 1). \]

This implies a downward shift in the P locus relative to the case with no cooperation from foreign Central Banks (\( L = 0 \)), and thus also a fall in \( T^* \), the minimal average debt maturity required to avoid confidence crises.

The latter two prescriptions both go in the direction of eliminating the asymmetry between the private and the public sector in the access to international financial markets. As discussed in the introduction (p.4–5), such an asymmetry is crucial for speculative attacks to have a chance of success. Thus it is not surprising that reducing this asymmetry also reduces the danger of successful attacks.

A final word must be spent on transitional problems. Up to this point, we have maintained inflation out of the picture, assuming essentially that all debt is real debt. In reality, very few countries issue substantial amounts of indexed debt (the U.K. being one of the few exceptions). In fact, the reason why in high-debt countries the authorities are reluctant to lengthen the average maturity of debt is precisely that they expect the private sector to demand very high returns on long-term nominal debt, because of the implied risk of real capital losses due to inflation and because of the incentive that the government has to inflict such losses on the private sector. This problem can obviously be overcome by issuing long-term debt indexed to the price level or denominated in a foreign currency. Thus the choice from the debt menu — the mix of nominal, real and foreign currency debt — is crucial in making the transition to a more balanced maturity structure.

In this sense, the results of this paper connect with those obtained by Calvo on the desirability of indexed versus nominal debt. Indeed our model provides the opportunity of performing another comparison, that between indexed and foreign currency debt. While both types of debt can in fact ease the transition towards a longer average maturity, foreign currency debt turns out to have an additional advantage — that its real return is totally insensitive to expected exchange rate changes, and thus to confidence crises.
5. Conclusions

In several European countries, there is growing concern over the combination of free capital mobility and fixed exchange rates that EEC treaties have planned for the 1990s. This concern is often motivated by remarking that some member countries are entering the process of liberalization with a comparatively high level of public debt. It is feared that defending a fixed parity from speculative attacks may prove particularly arduous for high-debt countries, and may eventually force them to restore capital controls or to opt out of the fixed exchange rates system.

In this paper we argue that, although there is reason to worry, there are steps that high-debt countries in the EEC can take to face the odds of the 1990s in a better position. These steps concern the way in which these countries should manage their debt, and are summarized by three simple rules: (i) lengthen the average maturity, (ii) smooth out the time pattern of maturing debt and (iii) develop a well-functioning market for foreign-currency debt. For Italy, Spain, Portugal and Greece, the adoption of these policies would imply a sharp turnaround relative to the practice so far prevailing in their debt management.
References


