AN ECONOMY OF INDUSTRIES AND
ITS AGGREGATE REPRESENTATION *

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1 *Purpose*

It is customary in many textbooks of macroeconomics, representing how the economic system as a whole works through the fiction of the one good economy. The purpose of the present paper, in contrast, is to show that Keynes, in *The General Theory*,[1] thought in more complex terms, referring to an economy made of industries in the Marshallian tradition. Such a procedure led him to anticipate a number of problems that were debated later, in connection with the issue of microfoundations. In the following essay I would like to trace the mentioned problems, and to show how they are hidden, but not ignored, in the Keynesian aggregate analysis.[2]

In chapter 4 of the G.T., "The choice of units", Keynes discusses the possibility of making employment homogeneous by taking an hour of ordinary labour as a unit and weighing an hour's employment of special labour in proportion to its remuneration. In a footnote he points out that "interesting complications obviously arise when we are dealing with particular supply curves since their shape will depend on the demand for suitable labour in other directions. To ignore these complications would, as I have said, be unrealistic. But we need not consider them when we are dealing with employment as a whole, provided we assume that a given volume of effective demand has a particular distribution of this demand associated with it." My contention in the present paper is that, "if a given volume of effective demand has a particular distribution of this demand associated with it", a Marshallian model of an economy made of industries may be adequatly represented through the Keynesian aggregate demand and supply functions.

1 In the following G.T.

2 Some years ago I already wrote a note on this subject (D'Adda 1981). At present I think I did not express all my ideas in a satisfactory way. In addition I hope to be able to go somewhat further.

2 *Individual industry output demand and supply price*

The economy we are going to describe is characterized by as many markets as there are industrial outputs. With regard to labour viceversa we assume a given hourly money wage and infinite elasticity of supply until full employment. This is a standard Keynesian assumption.

The final demand function for output of industry r, (qf_r), may be thought as

$$qf_r = qf_r \quad (wN, k, s_r, pd_r)$$

where the symbols have the following meaning: w money wage, N aggregate employment (so that wN is the wage bill), k autonomous expenditure in money terms, s_r is the rth component of a vector (s) of coefficients or functions associating the volume of effective demand with a particular distribution of final demands in money terms for the outputs of the various industries and pd_r demand price of output of industry r. As a practical example I am thinking of the qf_r function as one belonging to the linear expenditure system originarily introduced by Stone (1954), that might lead to the simple specification

$$qf_r = \frac{1}{pd_r} s_r \quad (cwN + k)$$

where the sum of the components of s amounts to one, c represents the Keynesian propensity to consume and the expression $(cwN + k)$ indicates the money value of the effective demand.

In my opinion this is a representation of the final demand function for the output of an individual industry completely respectful of the Keynesian view. In comparison with the demand functions ordinarily obtained by utility maximization in modern general equilibrium analysis the present representation may not be too restrictive, provided we consider first that the lack of a labour market makes labour endowments uninfluential and that therefore some version of the dual decision hypothesis according to the tradition of Keynes and Clower is in order;[3] second that at an industrial level of analysis making the final demand for output dependent only on the own price, rather than on the whole set of output prices and therefore assuming a world only of complements, may be reasonable as a first approximation.[4]

In order to describe the total demand function for output of the rth industry, (qd_r), an intermediate demand component must be added to the final demand.[5] For the sake of simplicity we are assuming that such a component may be described by the expression

$$_rA \ \ f(n)$$

where the symbol $_rA$ represents the rth row of the A input-output matrix and $f(n)$ represents a vector having the industrial production function $f_h(N_h)$ for hth component[6]. The total demand function for output of industry r may therefore be expressed by

$$qd_r = qd_r(wN, k, s_r, pd_r) + {}_rAf(n)$$

In an explicit specification

$$qd_r = \frac{1}{pd_r} s_r \ \ (cwN + k) + {}_rAf(n)$$

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3 The expression dual decision hypothesis was firstly introduced by Clower (1965) to connect individual demand with income generated by other people's decisions, rather than with the notional value of individual resources; but such a concept goes clearly back to Keynes, G.T., chap. 3, sec. I "The principle of effective demand".

4 Notice in addition that there is no need to conceive the final demand function for the output of industry r as the allocation of an aggregate variable. We can conceive of individual agents having all the same c and the same s_r. This seemingly naif assumption may be made without loss of generality for the following argument.

5 In my 1981 paper I forgot to stress this point.

6 Througout this paper fixed rather than variable coefficients of intermediate inputs may be assumed without loss of generality.

Alternatively the demand function for the output of industry r may be expressed by carrying the (demand) price on the left hand side

$$pd_r = \frac{s_r(cwN + k)}{qd_r - {}_rAf(n)}$$

The individual industry supply price may be represented by

$$ps_r = \frac{w}{f_r'(N_r)} + u_r'(p^e)$$

where the symbol $f_r'(N_r)$ means marginal product of labour in industry r, and $u_r'(p^e)$ indicates the marginal user cost in industry r as a function of the vector of the expected prices (p^e). In other words the individual industry supply price is given by the marginal cost of labour per unit of output, plus the marginal user cost per unit of output. This is the exact Keynesian assumption in chapter 4, section IV of the G.T. As it is usual the short run industrial production function [fr(Nr)] is characterized by diminishing marginal product of labour. Within the pricing procedure labour is imputed at its marginal cost per unit of output, whereas it is remunerated at its average cost, the difference representing the source of the industrial profit.

In what follows for the sake of simplicity I shall assume that

$$u_r' = P^e A_r$$

where A_r represents the rth column of the A input-output matrix accounting for current material "ingredients" and fixed capital replacement. Consequently we shall assume

$$ps_r = \frac{w}{f_r'(N_r)} + p^e A_r$$

We are so confronted with the demand and supply functions for the output of an individual industry. Provided that both industrial employment and expected prices are given, these two functions may be represented in the ordinary Marshallian diagram. Every market for the output of any industry may consequently be fully described.

3 *The working of an economy made of industries*

Our description begins with the decision, made by individual industries, about the daily (or weekly, according to the lenth of the production process) rate of output. Within a Marshallian context such a decision requires reference to short run expectations.[7] These expectations are essentially expectations as to the prices that will prevail in the course of the production period; prices at which inputs may be bought and outputs may be sold. Following a suggestion of Keynes,[8] we will assume that individual industries price expectations are normally based on the prices prevailing at the end of the previous period

$$p^e = p_{-1}$$

where the symbol p_{-1} represents the vector of such prices. Subsequently industries will utilize these expectations to make decisions about their individual rates of output. Marshallian perfectly competitive industries will decide employment and output levels by equating their suppy prices (which are functions of employment levels) to the expected prices

$$\left[w\frac{1}{f_r'(N_r)}\right] + p_{-1} \quad A = p_{-1}$$

where the expression in square brakets represents the vector of the unit labour costs of the individual industries. It must be noticed that the previous system is one of independent equations, each one solvable, so to speak, by the corresponding industry. It determins, equation by equation, every industrial level of employment (N_r) as a function of $[(1/w)p_{-1}]$. Let us

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7 G.T., ch. 5, sec. I

8 G.T., ch. 5, sec. II

indicate the vector of such industrial levels of employment by the symbol n. Since the industrial production functions $f_r(N_r)$ are given, the vector of the industrial outputs, call it (q), may also be computed. In addition the total employment N may be obtained by summation of the industrial records

$$N = ni$$

where the symbol i represents a vector having components all equal to one.

Until expected and realized variables do not coincide, n and q represent "tentative" solutions within a sequence; therefore we shall indicate them, for the time being, by the symbols n_1 and q_1. At the end of the production period the industrial outputs flow on their individual markets and output prices fluctuate so as to make every output demand equal to the corresponding supply, as it appears in the system

$$qd(wN, k, pd, Aq_1) = q_1$$

where the symbol $qd(\ldots)$ represents the vector of total demand functions for outputs of the various industries and pd represents the vector of the demand prices for industrial outputs. In explicit form the system may be expressed as

$$\hat{p}d^{-1}s \quad (cwN + k) = (I - A)q_1$$

where the script $\hat{p}d^{-1}$ indicates the diagonal matrix having the inverses of the components of the vector pd as non zero elements. Such a system is again made of independent equations, each one corresponding to the two sides of a particular market.

General equilibrium systems do not usually include independent but rather interdependent market clearing equations. It must be emphasized that the reason for independent market clearing equations like ours is semplicity and perhaps in the very short run more realism, but in no way such an independence is a necessary condition, either for the solution of the whole system or for the representation of the keynesian system at the industrial level.

A system of market clearing equations, where supplies are given and demands for the outputs of the various industries are functions only of the own prices, lends itself to represent the coordination process between supply and demand on the various markets in a very simple way. There is in fact no need of a Walrasian auctioneer who explores in a timeless way which price vector to select within an infinite variety in order to clear at one time the whole set of interdependent markets. We may in fact think of a more modest middleman in every market, exploring the demand function for the corresponding output and permitting perfect balance between demand and supply.[9]

Coordination within our economy made of industries is consequently easily achieved and once market equilibrium prices have been established, a new production round may take place starting from new (revised) price expectations and leading to new industrial outputs q_2. Provided that the autonomous expenditure in money terms (k) flows regularly over time and provided the price revision process converges to a stable price vector $p(k)$ within a sufficiently short time, a situation of short run equilibrium employment $n(k)$ and industrial production $q(k)$ is reached, which may be considered to last until the autonomous expenditure in money terms is not changed.

4 *Perfect foresight and Aggregation*

If the sequence of daily outputs and prices converges quickly to short run equilibrium and/or agents are quick to learn how the system works, backward looking expectations may be replaced by perfect foresight or its modern equivalent,

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9 Alternatively, since income distribution is irrelevant to the industrial distribution of demand, transactions out of equilibrium maight be allowed for, provided the average transaction price times the quantity of output on sale in every market were equal to the share of effective demand assigned to that particular market.

rational expectations. One symbol (p) may be used to represent the equilibrium price vector and one symbol $[f(n)]$ to represent the equilibrium output vector, every output being obviously a function of employment in the corresponding industry. Recourse to a time sequence is no longer necessary. The supply side of the system may be expressed by

$$p = w\left[\frac{1}{f_r'(N_r)}\right] + pA$$

and the demand side, in explicit form, by

$$f(n) = \hat{p}^{-1}s \quad (cwN + k) + Af(N)$$

Solving for the price vector, setting in column form and equating to the previous expression for the supply side gives

$$(I - A^T)^{-1} w\left[\frac{1}{f_r'(N_r)}\right] = diag[(I - A)f(n)]^{-1}s \quad (cwN + k)$$

where the script $(I - A^T)^{-1}$ indicates the inverse of $(I - A)$, A^T represents the transpose of A and $diag[\ldots]$ indicates the diagonal matrix having the components of the vector in square brakets as non zero elements. The previous one is obviously an interdependent equation system, but far-looking agents now expect the price vector that shall be realized, and therefore the help of middlemen is not needed anymore. The present symultaneous system admits a vector of industrial levels of employment (n) as solution. We expect that for any k (or better k/w) the solution is unique. When n is known, be it $n(k)$, also a vector of industrial outputs and a vector of industrial prices may be computed, that must coincide with the vectors $q(k)$ and $p(k)$ obtained as limiting elements of the converging sequences mentioned in the previous section.

According to the suggestion of Keynes the "industrial" system may be synthesized into the aggregate demand function and aggregate supply function. In order to perform the aggregation process let us concentrate on the supply side first.

By using the equilibrium equation of the whole system that connects the solution vector $n(k)$, as well as the vectors $p(k)$, $q(k)$ and the scalar N to any value of k, it is possible to define

$$Z = p(k)(I - A)q(k) = \tilde{Z}(k)$$

as the aggregate supply function or value of the whole industrial output net of user cost (i.e. in modern terminology value added[10]) is generated in a continuous way as long as we keep increasing the value of k between zero and some maximum level entraining the full use of the available labour force. Since a value of N may be connected to every value of k, the aggregate supply function may also be defined as

$$Z = Z(N)$$

In order to justify why the function $Z(N)$ has been denominated aggregate supply function, even though every solution of the complete industrial system belongs to the supply and to the demand side at the same time, we have to complete the description of the aggregate system. To this purpose let us consider the autonomous expenditure (k) as given and take N as the independent variable. Looking at the demand side of the system, it is then possible to define the aggregate demand function as the value of the final demand for the whole industrial output,[11] that would become effective if N workers were employed. In explicit form

$$D_k = cwN + k = D_k(N)$$

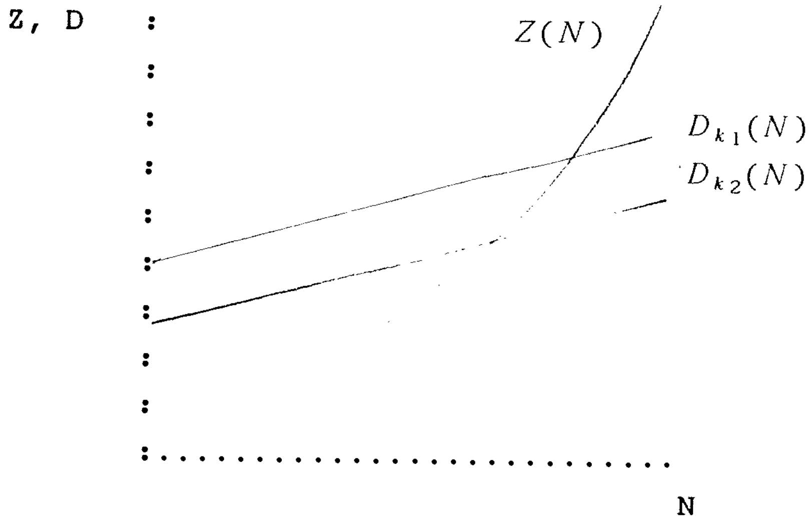
Such a function will obviously have only one point in common with the aggregate supply function.

The following chart depicts Z and D as functions of N. The second one is represented with respect to two levels of the autonomous expenditure k_1 and k_2

10 Notice that Z may also be expressed as

$$Z = \left[\frac{w}{f_r'(N_r)} \right] q(k)$$

11 i.e. demand for the whole industrial output net of user cost.



It is apparent that a whole family of aggregate demand curvs may be generated in connection with a variable level of autonomous expenditure. Along every such curve only one point may become effective; on the contrary, the whole succession of effective points depicts the aggregate supply curve. Qualitatively such a curve is similar to a Marshallian supply price curve of an industry. As a matter of fact the aggregate supply function may be interpreted as a weighted average of industrial supply prices (net of user costs) with weights given by the outputs of the various industries.

5 *Meaning of the present paper*

The third chapter of The General Theory has been criticized a number of times even by followers of Keynes. This was mainly because of the scarse attention to imperfect competition that represented the principal concern of his disciples when Keynes was intent on writing the G.T. My impression is that Keynes' recourse to Marshallian categories is not to be considered as disregard for the economics of imperfect competition. It has rather to do with his purpose of exploring the consequences of "injecting" his intuitions, like the principle of effective demand, the idea of ordinary ineffectiveness of the supply of labour and the distinction between short and long run expectations, into a comprehensive framework, such as the one depicted by Marshall in book V of the *Principles*. Yet the implications of general interdependence of such a scheme were not explored and Keynes, who was interested in the problem,[12] made them clear.

The present paper may be considered as an attempt to interpret chapter 3 of The General Theory in accordance with the vision of Keynes. Of course within the literature on the subject a Marshallian foundation of that chapter was already remarked, among others by E.R.Weintraub (1979)[13], Casarosa (1981) and also myself (1981). What has been contributed here is to show that the Keynesian system may be viewd simply as the synthetic representation of a complete industrial system in short run equilibrium, that does not require recourse to expectations about aggregate variables or to macroeconomic behavioural functions for its solution. In contrast with the opinions of several authors, including Patinkin (1979), who alleged the obscurity of the third chapter of The General Theory, this paper shows that there is no apparent inconsistency in the construction of Keynes.

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12 See especially ch. 5 of the G.T., on "expectations as determining output and employment".

13 See chap. 3.

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