# Indivisibilities in R & D Investment and Transient Leadership in Oligopoly

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# INDIVISIBILITIES IN R & D INVESTMENT AND TRANSIENT LEADERSHIP IN OLIGOPOLY

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**Abstract**

In this paper we study a one-shot game of R & D between two price-setting firms that are asymmetrically placed as they produce at different cost levels. The R & D technology displays increasing returns in the form of indivisibilities. We show that there exists a unique equilibrium in pure strategies, and we prove that the incumbent (or current leader) has never a greater probability of winning the patent race than the rival.

## 1. Introduction

In this paper we analyse a game of Research and Development (R & D) for a cost reducing innovation between two technologically _asymmetric_ firms. Asymmetric R & D races have been analysed by Gilbert and Newbery (1982), Reinganum (1983), and the subsequent debate in the _American Economic Review_ [1]. This debate deals with the comparison of the incentives to obtain a patentable innovation of an incumbent firm and a potential entrant. The analysis aims at establishing which firm invests more in R & D.

These contributions have focused on two rather extreme cases. Either the race is modelled as a deterministic game, so that not only the type, but also the timing of the innovation is known at the outset (Gilbert and Newbery, 1982), or the innovation is assumed to be drastic, that is so dramatic as to give the winner monopoly power (Reinganum, 1983). It appears that these two extreme hypotheses lead to opposite results: if the patent race is deterministic, then the current monopolist will have a greater incentive to innovate than the challenger, whereas in an uncertain environment, with a drastic innovation (so that the loser of the race makes zero profits) the challenger invests more than the incumbent.

The reason why these hypotheses allow one to reach a definite conclusion may be explained in terms of what Fudenberg and Tirole (1986, p. 32) have christened _efficiency effect_ and _replacement effect_. The former operates in favour of the firm whith the greatest difference in payoffs between winning the race and letting the rival win it. The latter operates against the firm which is currently making positive profits, when there is uncertainty on the timing of innovations: the existence of current positive profits induces the leading firm to reduce its effort so that the time of successful completion of the new technology is postponed. Since in Gilbert and Newbery's model the timing of innovations is fixed, there is no replacement effect; moreover, the non drastic nature of the innovation brings about a stronger efficiency effect for the incumbent than for the entrant. On the other hand, the drastic character of the innovation implies that the efficiency effect in Reinganum's (1983) model is the same for the two firms, so that the replacement effect becomes decisive and operates in favour of the challenger.

In a more general setting, however, both effects must be taken into account. For instance, under Bertrand competition or monopoly in the product market the efficiency effect and the replacement effect go in opposite directions. In a companion paper (Delbono and Denicolò, 1988) we have shown that if the R & D technology displays smoothly decreasing returns, both effects may prevail according to the value of certain parameters, such as the discount rate or the productivity of R & D expenditure.

Casual empiricism, however, suggests that R & D activity is subject to strongly increasing returns [2]. A simple way of modelling increasing returns in the context of a patent race is to assume that R & D activity is completely indivisible; in other words, for each firm there is a single R & D project which may be undertaken or not. It turns out that under this all-or-nothing hypothesis the entrant (or initially high cost firm) invests in R & D at least as much as the incumbent (or initially low cost firm). More precisely, in equilibrium either the entrant has the same probability as the incumbent to win the patent, or the entrant wins with probability one. Thus, we reach a conclusion that parallels that of Reinganum (1983), but without assuming that the innovation is drastic.

In next section we present the model; our results are obtained in section 3. Section 4 contains some concluding remarks.

## 2. The model

In this section we model a R & D race between two price setting firms which are asymmetrically placed as far as the technology is concerned. We assume that firms compete in prices in a homogeneous product market, so that either the market is monopolised or a Bertrand equilibrium is established.

Production takes place under constant returns to scale. Let B be the low cost firm and A be the high cost firm. Denote by $c_A$ and $c_B$ the constant marginal and average costs of the two firms, with $c_B < c_A$. Let $\pi_B$ be B's current profit; obviously A's current profit is zero.

Let us denote by $p_M(c)$ the monopoly price associated with a constant marginal and average cost c. If

$$p_M(c_B) \leq c_A \tag{1}$$

then B is a monopolist, A is a potential entrant and the equilibrium price is $p_M(c_B)$. On the other hand, if

$$p_M(c_B) > c_A \tag{2}$$

then B is a Bertrand leader, A is the inactive firm in the asymmetric

Bertrand equilibrium, and the equilibrium price is $c_A$. Notice that in both cases A's profits are null, while B's profits are positive. Obviously, B's profits are larger in case (1) than in case (2).

Besides competing in the product market, firms compete for a single patentable innovation; the winner of the patent race will get the exclusive right to produce forever at a cost level $c^* < c_B$. Since the case of drastic innovations has already been studied by Reinganum (1983), throughout the paper we assume that

$$p_M(c^*) > c_B \qquad (3)$$

that is, the innovation is non-drastic in the sense that if the initially less efficient firm wins the technological race, it will not monopolise the market. On the other hand, if B wins the race, three cases are possible, i.e.:

(i) (1) holds so that B <u>remains</u> a monopolist;

(ii) (2) holds, and $p_M(c^*) \leq c_A$, so that B <u>becomes</u> a monopolist;

(iii) (2) holds, and $p_M(c^*) > c_A$, so that B <u>remains</u> a Bertrand leader, while increasing its technological lead.

In the post-innovation equilibrium (monopoly or Bertrand equilibrium), if B has innovated its profit will be $\pi_B^*$ per unit of time and A will get nothing forever; if A has innovated, its profit will be $\pi_A^*$ per unit of time and B will receive nothing forever. In all the three cases distinguished above, it can be shown that $\pi_B^*$ >

$\pi_A^*$, provided that the marginal revenue curve is decreasing (see Appendix 1).

We assume that the timing of the innovation is uncertain, and that each firm's probability of innovating is an increasing exponential function of its R & D expenditure. As far as the R & D activity is concerned, the two firms have the same technology, displaying increasing returns. Specifically, we assume that there is one indivisible R & D project that may be undertaken by the two firms. The project costs g (a flow cost that is sustained until someone succeeds), and gives a probability

$$Pr(T \leq t) = 1 - \exp(-t) \qquad (4)$$

to innovate at or before time t, provided no one has innovated yet. Each firm has therefore a binary choice, as far as pure strategies are concerned. However, we also allow firms to randomise between pure startegies. Let us denote by x (resp., y) the probability that firm A (resp., B) undertakes the R & D project. Taking into account mixed strategies, the strategy space of both firms will be the interval [0,1].

Firms noncooperatively choose their strategies in order to maximise the discounted value of expected profits net of R & D costs. A's expected discounted profits are given by the sum of: (a) the probability that both firms undertake the project times the discounted expected profits of A in this event, i.e.:

$$\int_0^{\infty} \exp(-rt)\exp(-2t)\left(\frac{\pi_A^*}{r} - g\right) dt = \frac{\pi_A^*/r - g}{2 + r},$$

where r is the discount rate, and (b) the probability that only firm A undertakes the project times the discounted expected profits of A in this event, which is:

$$\int_0^{\infty} \exp(-rt) \exp(-t) \left(\frac{\pi_A^*}{r} - g\right) dt = \frac{\pi_A^*/r - g}{1 + r} .$$

In all other events A's payoff is zero. Thus, A's expected payoff will be

$$W_A = xy \frac{\pi_A^*/r - g}{2 + r} + x(1-y) \frac{\pi_A^*/r - g}{1 + r} \tag{5}$$

Analogously, B's payoff may be written as

$$W_B = xy \frac{\pi_B^*/r - g - \pi_B}{2 + r} + y(1-x) \frac{\pi_A^*/r - g - \pi_B}{1 + r} +$$

$$+ (1-x)(1-y) \pi_B/r + (1-y)x \pi_B/(1+r) \tag{6}$$

The expression for B's payoff differs from that of firm A as B is currently reaping positive profits $\pi_B$. Hence, if no firm undertakes the R & D project firm B will earn profits $\pi_B$ forever, while if only firm A invests, firm B will earn a positive flow of profits $\pi_B$ until the A succeeds.

## 3. Results

In this section we study the Nash equilibria of the R & D game described in the previous section. We shall show that the equilibrium is almost always unique and does not involve the use of strictly mixed strategies. We shall also show that it cannot be the case that the technological leader, i.e. firm B, undertakes the R & D project with probability one while the follower (firm A) gives up. Finally, a complete description of the range of parameters which sustain the various possible equilibria is provided.

Differentiating (5) and (6) with respect to x and y, respectively, we get

$$\frac{dW_A}{dx} = \frac{\pi_A{*}/r - g}{1 + r}\,[1 - y/(2+r)] \tag{7}$$

and

$$\frac{dW_B}{dy} = x\left[\frac{\pi_B{*}/r - g - \pi_B}{2 + r} - \frac{\pi_B}{1 + r}\right] + (1-x)\left[\frac{\pi_B{*}/r - g - \pi_B}{1 + r} - \frac{\pi_B}{r}\right] \tag{8}$$

The existence of a Nash equilibrium follows from the linearity of the payoff functions of firm A and B in x and y, respectively. An inspection to (7) and (8) shows that generically an equilibrium does not involve randomisation of pure strategies. This also implies that (generically) the equilibrium is unique.

**Proposition 1.** Generically, there exists a unique equilibrium in pure strategies of the R & D game.

Proof. It suffices to note that the sign of (7) coincides with the sign of $\pi_A*/r - g$, which does not depend on x and y. Thus, in equilibrium either $x = 0$ (if $\pi_A*/r < g$) or $x = 1$ (if $\pi_A*/r > g$). Only if $\pi_A*/r = g$ firm A is indifferent between any admissible value of x. Given that generically either $x = 0$ or $x = 1$, it follows from (8) that generically $dW_B/dy$ (which does not depend on y) is different from 0. It follows that generically either $y = 0$ or $y = 1$. ∎

In view of Proposition 1, there are four candidate equilibria (all in pure strategies), i.e.:

(E.1) $x = 0$, $y = 0$;

(E.2) $x = 1$, $y = 1$;

(E.3) $x = 1$, $y = 0$;

(E.4) $x = 0$, $y = 1$.

We next show that (E.4) cannot be an equilibrium.

**Proposition 2.** In equilibrium, it cannot be the case that $x=0$ and $y=1$.

Proof. For $x = 0$ to be true in equilibrium, it must be $\pi_A*/r \leq g$. On the other hand, for $y = 1$ to be true in equilibrium, given $x = 0$, it must be

$$(\pi_B* - \pi_B - gr) - 2r\pi_B \geq 0.$$

But this is impossible because $\pi_A* \leq rg$, and

$\pi_A^* \geq \pi_B^* - \pi_B$.

(For a proof of this inequality see Appendix 2.) ∎

Finally, we study under what configurations of the parameters equilibria (E.1), (E.2) and (E.3) may occur. Intuitively, (E.1) is an equilibrium if the cost of the R & D project is very high, or if the prospective discounted profits from the innovation are very low (for instance, because the cost reduction is small or the discount rate $r$ is very large): then both firms will find it convenient not to undertake the project. On the other hand, (E.2) will be an equilibrium if the cost of the R & D project is very low compared to the expected discounted profits (this may happen if the discount rate $r$ is very low), so that both firms have an incentive to do R & D. One expects that (E.3) may be an equilibrium of the game for "intermediate" values of the parameters, implying that only the follower can profitably undertake the project.

This intuition is confirmed by the formal analysis of the model. First of all, notice that A's best strategy depends only on the sign of $(\pi_A^* - gr)$. We already know from the proof of Proposition 2 that if $(\pi_A^* - gr)$ is negative, then $dW_B/dy$ is negative as well, so that (E.1) is the unique equilibrium. On the other hand, when $(\pi_A^* - gr)$ is positive,

$$\text{sign } (dW_B/dy) = \text{sign } [(1+r)\pi_B^* - rg(1+r) - r(3+2r)\pi_B] \qquad (9)$$

If the sign of the term in square brackets is positive then $y = 1$, if it is negative then $y = 0$, while if it vanishes firm B is indifferent betwwen undertaking the R & D project or not. Figure 1 depicts, in the r-g space, the loci

$$\pi_A^* - gr = 0 \tag{10}$$

and

$$(1+r)\pi_B^* - rg(1+r) - r(3+2r)\pi_B = 0. \tag{11}$$

Clearly, (10) is a rectangular hyperbole. As far as (11) is concerned, it can be easily shown that the curve is decreasing in the g-r space; furthermore, g tends to 0 as r goes to infinite, and tends to infinite as r goes to 0. Also, since $\pi_B^* > \pi_A^*$, the two curves (10) and (11) intersect once in the positive orthant. The abscissa of the intersection point is the (unique) positive root of the following equation:

$$r(3+2r)/(1+r) = (\pi_B^* - \pi_A^*)/\pi_B \tag{12}$$

which we have denoted by $r^+$ in figure 1.

(figure 1 here)

We have therefore proved the following:

**Proposition 3.** If

$$\pi_A^* - rg < 0, \tag{13}$$

then (E.1) is the unique equilibrium. If

$$\pi_A^* - rg > 0 \quad \text{and} \quad (1+r)\pi_E^* - rg(1+r) - r(3+2r)\pi_E > 0 \tag{14}$$

then (E.2) is the unique equilibrium. If

$$\pi_A^* - rg > 0 \quad \text{and} \quad (1+r)\pi_E^* - rg(1+r) - r(3+2r)\pi_E < 0 \tag{15}$$

then (E.3) is the unique equilibrium. The three regions defined by inequalities (13), (14) and (15) are all non empty.

## 4. Concluding remarks

In this paper we have studied an asymmetric race of R & D between two technologically asymmetric price-setting firms. We have assumed that R & D activity is characterised by strongly increasing returns, which we have captured by considering a single, indivisible R & D project.

We have shown that the R & D game almost always admits a unique equilibrium in pure strategies. A priori, there are four candidate equilibria: two symmetric (both firms invest in R & D or do not invest) and two asymmetric (only one firm invests in R & D). Most of the existing literature on R & D games has focused on cases similar to our asymmetric equilibria (E.3) and (E.4). In our model it turns out that only the former can be an equilibrium of the game (Propositions 2 and 3).

This contrasts with Gilbert and Newbery's (1982) findings according to which the incumbent always has a stronger incentive to do R & D than the challenger. Intuitively, the reason why we get a different outcome lies in the presence of the replacement effect (which is ruled out in Gilbert and Newbery's deterministic model), and in our hypothesis on the R & D technology.

Our scenario (E.3) looks like the equilibrium in Reinganum (1983) where the current incumbent has always a lower probability of innovating than the challenger. (E.3) is an equilibrium in our model as well (actually, it is the only asymmetric equilibrium), although our innovation is non drastic. Since in our model the post-innovation profits are greater for the incumbent (or current leader) than for the challenger, our result does not follow from the absence of the efficiency effect, as in Reinganum (1983). Instead, it is driven by the hypothesis of increasing returns in the R & D activity.

## Appendix 1

In this Appendix we show that $\pi_B* > \pi_A*$. This is obvious when B is a monopolist in the post-innovation equilibrium (cases (i) and (ii)). If B remains a Bertrand leader (case (iii)), its profits will be

$$\pi_B* = (c_A - c*)\ q(c_A)$$

whereas A's profits in case A wins the race will be

$$\pi_A* = (c_B - c*)\ q(c_B)$$

where q(.) denotes the demand function. Now, if $p_M(c*) > c_A$, then B's profits if B wins are given by the difference between the areas of the regions ABC and CDE, while A's profits in case A wins are given by the difference between the areas of the regions ABC and CFG (see figure 2). If the marginal revenue curve is downward sloping, then the difference between the two areas is clearly positive. (Actually, it would suffice that the marginal revenue curve cuts the post-innovation marginal cost curve from above only once.)

(figure 2 here)

## Appendix 2

In this Appendix we show that $\pi_A^* > \pi_B^* - \pi_B$. We distinguish between cases (i), (ii), and (iii) (cf. section 2).

Case (i). In this case, the difference $\pi_A^* - (\pi_B^* - \pi_B)$ is the shaded area in figure 3.

(figure 3 here)

Case (ii). In this case B is a Bertrand leader before the innovation, but becomes a monopolist after the innovation. Hence, the following inequalities hold:

$p_M(c_B) > c_A$, (B is not a monopolist before the innovation)

$p_M(c^*) > c_B$, (the innovation is non-drastic)

$p_M(c^*) \leq c_A$. (B becomes a monopolist winning the patent race)

If $p_M(c^*) = c_A$, the result follows as in case (iii) below. Let us now suppose that $c^*$ falls. If the marginal revenue curve is downward sloping, then $p_M(c^*) < c_A$. Clearly, $\pi_B$ is independent of $c^*$. Then, the result follows if $\pi_A^*$ grows more quickly than $\pi_B^*$ as $c^*$ decreases. Now,

$$\frac{d\pi_A^*}{dc^*} = - q(c_B)$$

because A becomes a Bertrand leader in case he wins the patent, so that his profits will be $(c_B - c^*)\, q(c_B)$. Moreover,

$$\frac{d\pi_B^*}{dc^*} = - MR^{-1}(c^*)$$

where $MR^{-1}$ is the inverse marginal revenue function. Since by hypothesis $c_B < p_M(c^*)$, we have $q(c_B) > MR^{-1}(c^*)$. This completes the proof in case (ii).

Case (iii). In this case, the difference $\pi_A^* - (\pi_B^* - \pi_B)$ is the shaded area in figure 4.

(figure 4 here)

## FOOTNOTES

1. Asymmetric games have also been investigated in models with a sequence of R & D races (e.g., Reinganum 1985, Vickers 1986, Beath *et al.* 1987). Since a race is a contest in which it is possible to distinguish sharply between the winner and the loser, after the first race firms will be in an asymmetric position even if they were symmetrically placed at the outset. In other words, the second race of a multistage model will necessarely be one in which there is a high cost and a low cost firm.

2. See Kamien and Schwartz (1982), pp. 64-70. Notice also that some of the most representative game theoretic models of R & D, as Loury (1979) and Lee and Wilde (1980), assume that the hazard function exhibits initial increasing returns even if decreasing returns must eventually prevail.

## References


Beath, J., Katsoulacos, Y., and D. Ulph (1987), "Sequential Product Innovation and Industry Evolution", Economic Journal (Conference Papers), vol. 97, pp. 32-43.

Delbono, F. and V. Denicolò (1988), "Asymmetric Races of Research and Development", mimeo, University of Bologna.

Fudenberg, D. and J. Tirole (1986), Dynamic Models of Oligopoly, London, Harwood.

Gilbert, R. and D. Newbery (1982), Preemptive Patenting and the Persistence of Monopoly", American Economic Review, vol. 72, pp. 514-26.

Gilbert, R. and D. Newbery (1984), Preemptive Patenting and the Persistence of Monopoly: Comment", American Economic Review, vol. 74, pp. 238-42.

Kamien, M. and N. Schwartz (1982), Market Structure and Innovation, Cambridge, Cambridge University Press.

Lee, T. and L. Wilde (1980), Market Structure and Innovation: A Reformulation, Quarterly Journal of Economics, vol. 94, pp. 429-36.

Loury, G. (1979), Market Structure and Innovation, Quarterly Journal of Economics, vol. 93, pp. 395-410.

Reinganum, J. (1983), "Uncertain Innovation and the Persistence of Monopoly", American economic review, vol. 73, pp. 741-8.

Reinganum, J. (1985), "Innovation and Industry Evolution", Quarterly Journal of Economics, vol. 99, pp. 82-99.

Vickers, J. (1986), "The Evolution of Market Structure When There Is a Sequence of Innovations", Journal of Industrial Economics, vol. 35, pp. 1-12.

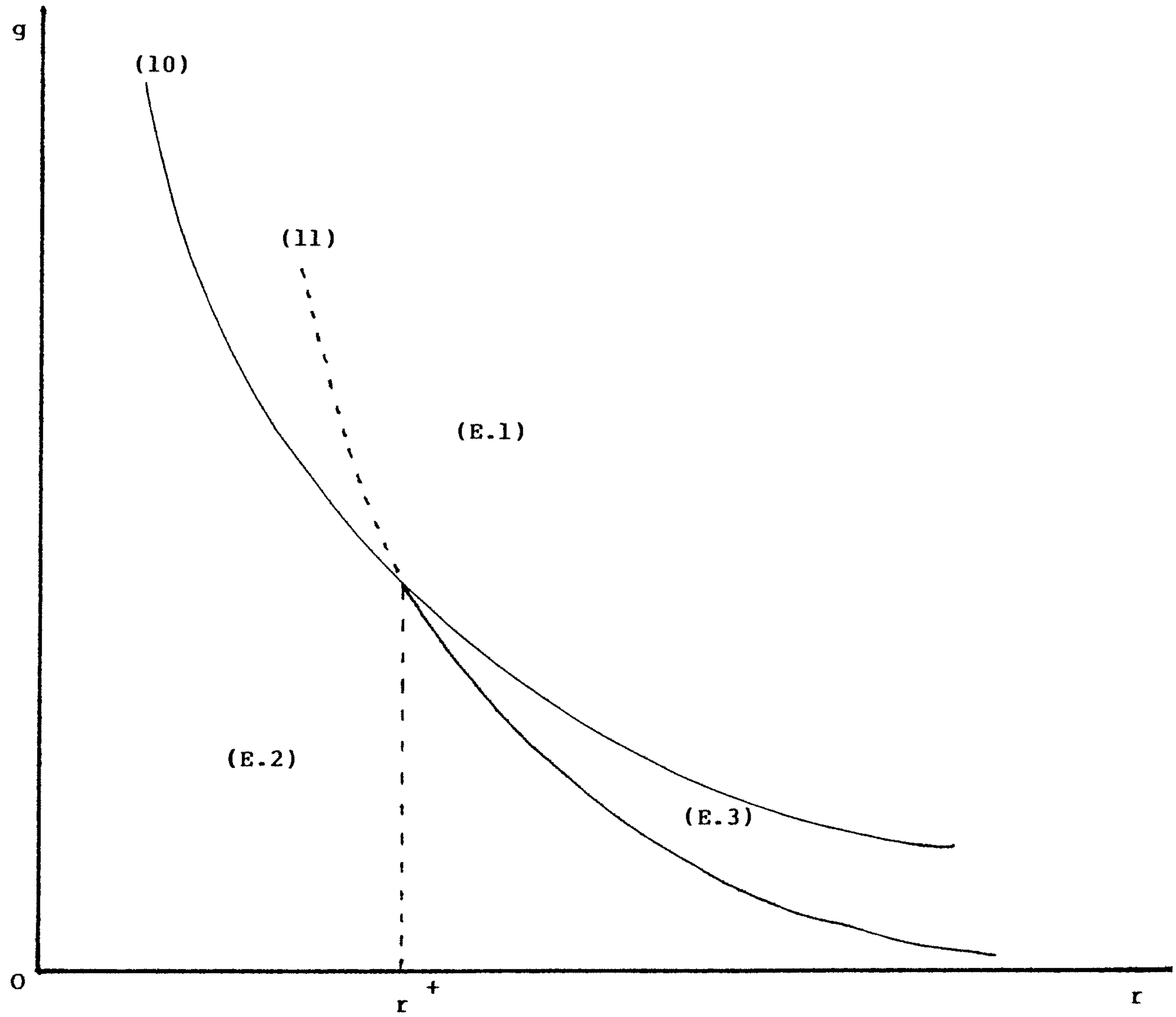


FIGURE 1

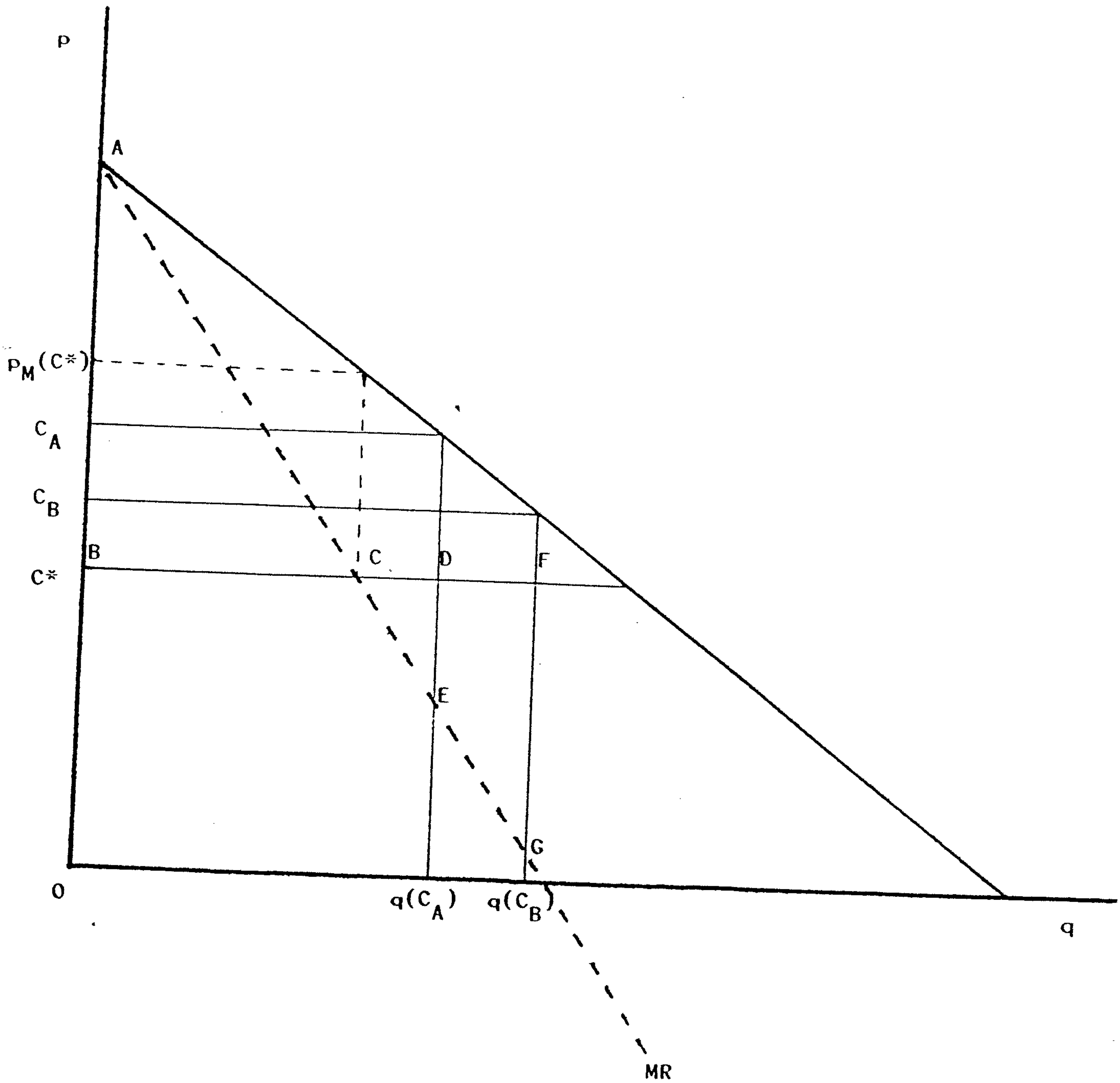


FIGURE 2

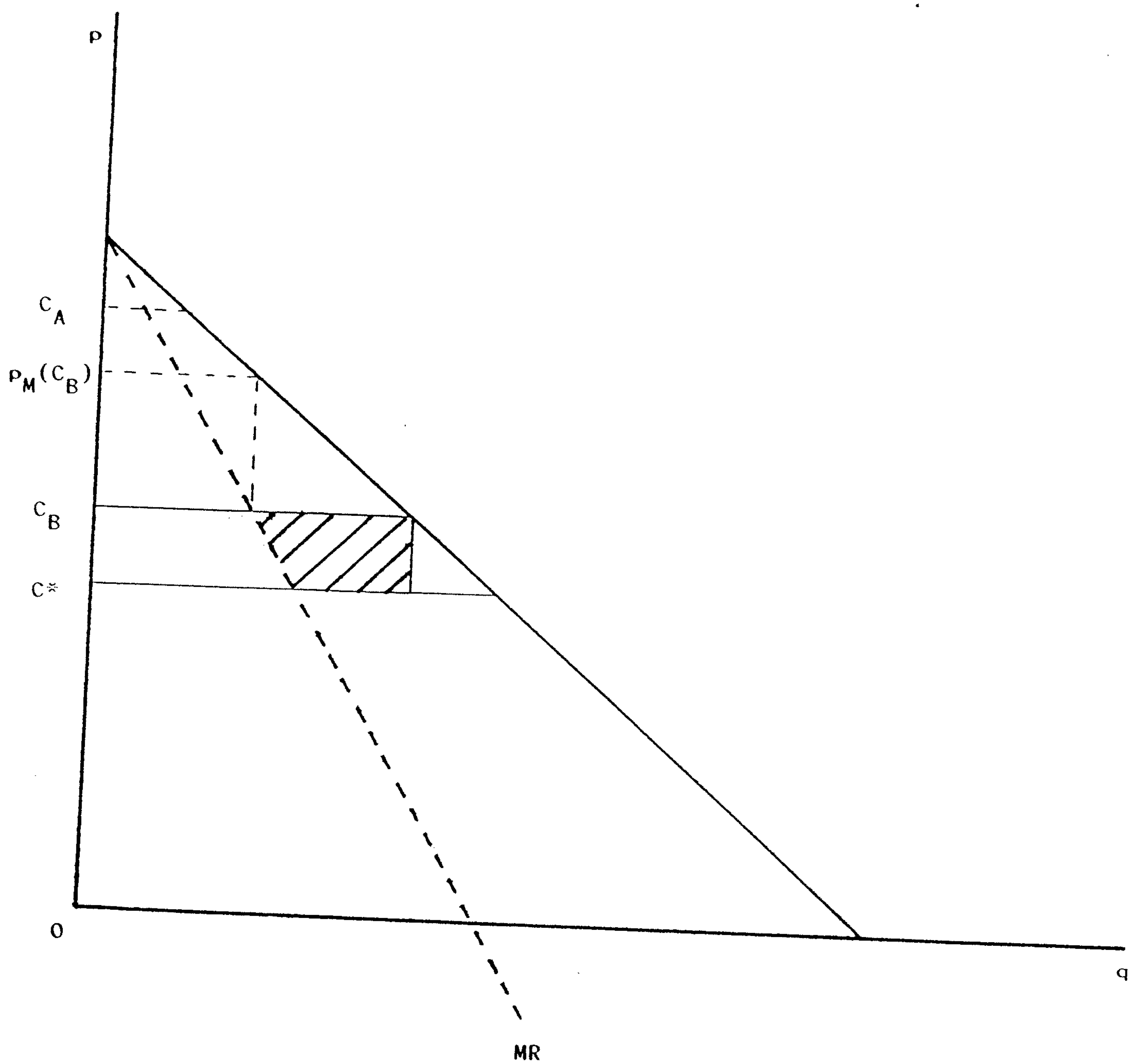


FIGURE 3

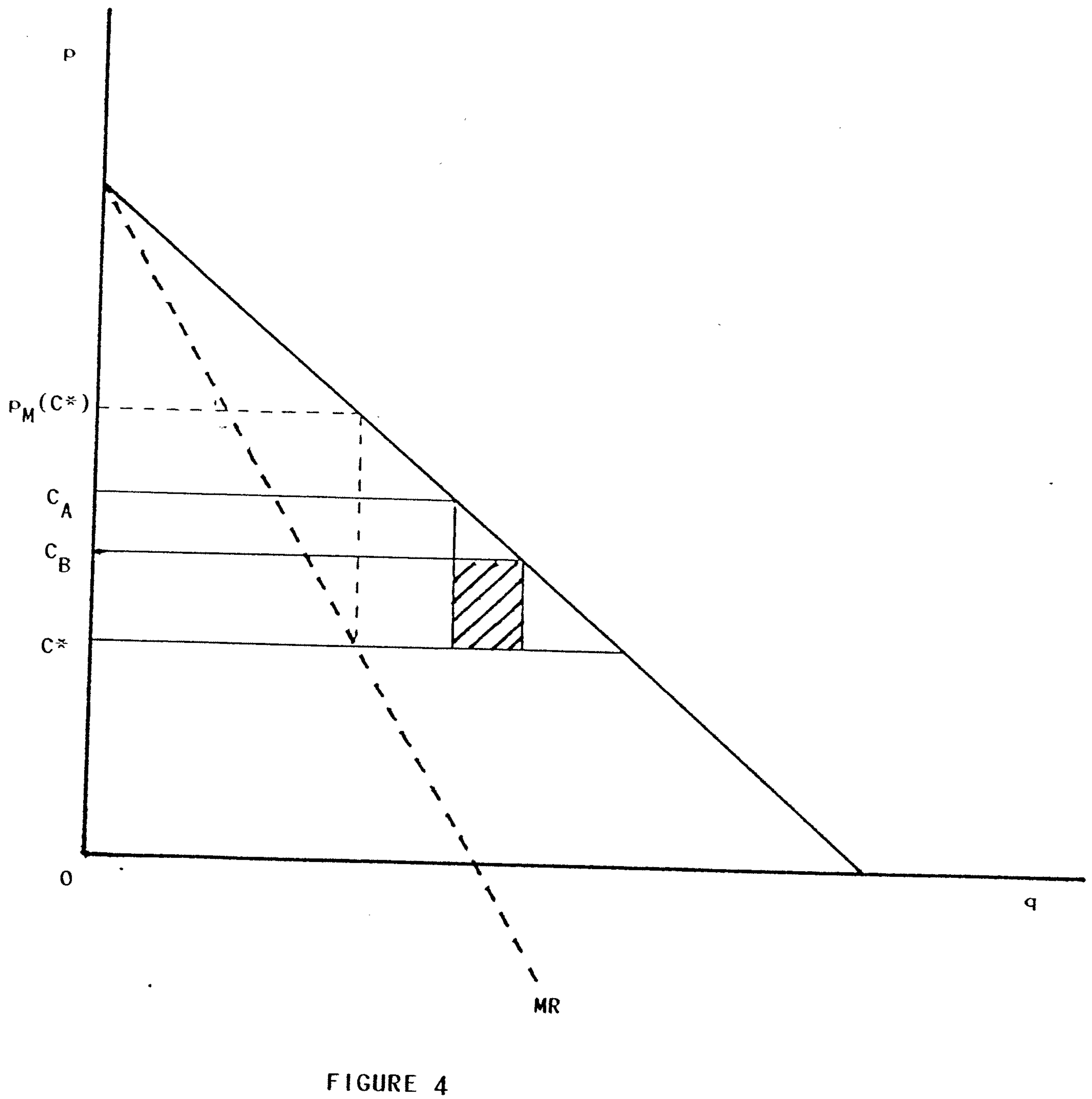


FIGURE 4