DYNAMIC MONOPOLIST'S BEHAVIOR AND CONSUMER LEARNING

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1. INTRODUCTION

1.1 This paper studies the optimal behavior of a monopolist in the presence of asymmetric information concerning the characteristics of the good he produces. The monopolist knows his product, while consumers can only learn its quality (in the widest possible sense) through experience.

In particular, we will address the question of the optimal pricing decisions when current sales help the process of product diffusion: it will be proven that this policy very often requires a low introductory price that increases gradually as consumers acquire full information. This implies that the existence of uncertainty need not increase the market power of a monopolist, whose profit will be reduced. Furthermore, it is shown that consumers could even benefit from uncertainty, in terms either of a larger consumer surplus, or of a higher quality level relative to the full information level.

1.2 The usual treatment of this kind of problem rests on the fundamental distinction drawn by Nelson (1970) between *search goods* (whose quality can be discovered before the purchase, through a search cost) and *experience goods* (whose quality can be ascertained only after the purchase). Several recent articles explore these two possibilities, showing that in both kinds of models firms have greater monopoly power, and uncertainty has a negative effect on consumer welfare. In this sense, we may say that the literature tends to emphasize the similarities between them, rather than their differences.

In our paper, we want to show that when there is uncertainty about the quality of an experience good, consumers could end up being better off, while the only welfare loss would be suffered by the producer. In quality level will be lower than with full information: therefore consumers will be worse off because of their imperfect information.

These very clear-cut and strong results emphasize the monopoly power a firm acquires in such situations. However, they do not highlight an essential difference between search goods and experience goods: in the latter case, one should also take into account the fact that the necessity to make consumers know the product (supposedly of good quality) can be an important limitation to the firms' power.

One aim of this paper is to show that this necessity is indeed a very relevant incentive for a firm, so that the existence of uncertainty could make consumers better off. In many cases sales will decrease over time, eventually converging to the full information monopoly level: as long as there is uncertainty, consumers will be supplied a larger quantity at a lower price, so that the diffusion of the good in the market will be easier.

1.3 This result is closely related to another problem we would like to tackle: the modeling of *consumer behavior*. The very restrictive assumptions that are usually introduced can only be considered a first approximation that calls for further analysis[2].

One possible approach is to study the consequences of different assumptions on the information that consumers use, and then to check to what extent different cases give rise to different conclusions. One step in this direction is taken by Bagwell and Riordan (1986) who, like Wolinsky (1983) and Milgrom and Roberts (1986), assume that consumers look at price as a signal. Unlike Milgrom and Roberts (1986), they conclude that, under the informational assumptions they consider, the price that the monopolist charges is higher than its full information order to give an adequate account of this result, it will be useful to offer a brief review of the previous work in the field.

Important contributions in the analysis of markets for search goods are, among others, Salop and Stiglitz (1977) and Wolinsky (1983). In the former paper, consumers can acquire full information by paying an initial fixed cost, while in the latter, consumers follow a strategy of sequential search. The results obtained in both papers are examples of the market power firms are thought to have as a consequence of consumers' uncertainty: even with a large number of competitors, the equilibrium price is above the competitive level since the firms exploit the lack of "transparency" of the market.

In markets for experience goods, the interaction between consumers and firms is essentially dynamic[1]: its crucial feature is the possibility of *repeated purchase*, and therefore producers have an incentive to build up a good *reputation* in order not to lose customers, and possibly to acquire new ones.

In spite of this fundamental difference between search models and experience models, there are many analogies among the results they provide. For instance, Klein and Leffler (1981) prove that in equilibrium only the firms which supply the lowest quality level do not get positive profits. Other examples of this market power are offered by Shapiro (1982) and (1983). In his work Shapiro deals with the problem of a monopolist facing a large number of perfectly rational consumers; a crucial assumption is that learning is immediate (just after the purchase) and purely personal (i.e. there is no exchange of information among consumers). One of the main results he gets is that both output and level. Although this approach has many merits, this kind of analysis has a shortcoming, namely that it is impossible to give an exhaustive list of all possible assumptions, and each of them is by itself inevitably arbitrary and hard to justify.

Here we want to investigate a second, simpler possibility, namely that in conditions of uncertainty consumption choices may depend in an important sense on the previous consumption experience.

One argument in support of this point is provided, for instance, by Grossman *et al.* (1977). They examine the behavior of "bayesian" consumers, who try to get information on a new good through repeated experiments and revise their beliefs on its quality accordingly: here the interdependence among choices made at different times takes the form of a "learning curve". Another important example is given by the possibility that information spread can occur through *word-of-mouth*, like in Rogerson (1983) and Glaister (1974): in this case, a quite standard result indicates that product diffusion typically follows a pattern given by a logistic, S-shaped curve[3].

In this perspective, however, these are only *possible* specifications of the rather general idea just proposed, and thus here we prefer to adopt a more general formulation, in which demand dynamics depends (positively) on the current level of demand and on cumulated sales. Although for the sake of clarity in the next sections we will often commit ourselves to a specific version, many interpretations could in principle be given to this model: among them, the existence of imitative consumption, habit formation, ... We will generically refer to these phenomena as "learning", in the sense of "changes in behavior induced by experience".

Therefore, with this setup the whole system of market incentives is dramatically changed: the returns from any increase in current sales are now higher because of a sort of multiplicative effect due to learning. This is the main reason why we observe significant differences relative to some results obtained by other authors: in particular, unlike Shapiro (1982) and (1983), both output and quality levels could well be set above their full information values.

Within this framework, we show that the monopolist will "invest" in output, increasing production always above its myopic profit - maximizing level. It is also proved that output dynamics is not necessarily parallel to demand shifts: supply may decrease even if the demand curve shifts outwards. This leads to an increase in price over time, and this occurs when demand dynamics depends on cumulated output. The intuition behind this conclusion is similar to the one given by Milgrom and Roberts (1986), who derive an analogous result in quite a different model; the firm must set a low initial price in order to favor the spread of information among consumers and thus to enjoy later the advantage of having a larger market.

The paper is organized as follows. In section 2 we set up the basic model, which will be used in order to investigate the dynamics of output and price and to draw some welfare conclusions (section 3) and to analyze the choice of the quality level (section 4). The final section contains some concluding remarks on some possible extensions of this paper.

2. THE MODEL

2.1 The inverse demand function is $p_t = f(x_t, Z_t)$, where $p \geq 0$ is the price of the good whose output level is x, t is time and Z_t is a vector of parameters whose values change through time as a consequence of learning; f is continuous. At this stage, quality is exogenously given and known; this assumption will be removed in section 4.

Not all parameters of f necessarily change over time. The precise definition of Z_t may vary according to the learning process we consider. This vector may contain a shift parameter and other ones more directly related to the slope or the elasticity of the curve. We will concentrate our attention on unambiguous outward shifts of the demand curve: an increase in Z_t raises $f(x)$, possibly decreasing the absolute value of its slope. As we will see, this implies that the marginal revenue (MR) increases.

The reason is that it seems reasonable to assume that consumers are rather sensitive to price changes when they are uncertain of the good's quality, while small variations in price become less important in their choice when they know - and like - the product. An extension to the case of leftward shifts of the demand curve is straightforward, and in general we obtain symmetric results; considering the case of non monotonic shifts requires minor qualifications.

The behavior of individual consumers will be only sketched. In each instant every agent decides either to buy one unit of the good, or not to buy at all; he has some prior point expectations on the utility (u) he may get, and will buy only if his expected net benefit ($Eu - p$) is positive.

This expectation can be based both on previous, personal experience and on information obtained from other consumers; if the agent buys, he will get more information and will therefore revise his beliefs. Repeat purchase will occur only if the (new) expected net benefit is positive. The learning mechanism we are considering is (in general) neither purely personal, nor immediate, although these may be considered special cases of this framework[3].

The number of informed consumers and possibly, as we will see, the quality of their information are therefore the driving forces of learning and as a consequence of the dynamics of Z_t. In particular, we will consider

(1) $\dot{Z}_t = L(x_t, y_t, Z_t)$

where the dot indicates the derivative with respect to time and y is the level of cumulated output up to time t:

(2) $$y_t = \int_{s=0}^{t} x_s ds.$$

The vector Z_t shifts from its initial position Z_o to a given final position $\underline{Z}$: we will maintain an analogous notation also in the examples we will offer.

2.2 It seems now important to characterize more clearly the influence of these three variables on the spreading of information. Interpreting Z_t as an index of market size, that is of the number of consumers who know that the good exists, we may consider the three following situations.

(i) Lack of knowledge of the *existence* of the good. In this case Z_t itself is the crucial variable : direct experience of the good does not matter much, since once consumers are aware of the existence of the good there is no more uncertainty, and information can spread through *word-of-mouth*[4].

This situation is similar to epidemic diffusion models, where it is usually assumed that the effect of z_t on L changes over time. When the market is still small, an increase in the proportion of informed consumers makes information spread easier, while later a phenomenon similar to decreasing returns arises. Communication among consumers becomes less effective, since the share of unexploited potential market becomes increasingly small; this implies that it is more difficult to reach uninformed (potential) consumers. Thus, we may assume that at the beginning $L'z$ is positive, becoming negative as time goes on.

(ii) Uncertainty on the *variety* of the good (in "horizontal" sense). Even consumers who know of the existence of the good may be unaware of its features. Therefore the relevant information is the one regarding its variety, and not only its existence: in this case, the quantity actually sold can be thought as an appropriate index of the number of consumers who possess and can spread this information.

(iii) Uncertainty on the variety of goods, like before. In many cases, especially when learning is not immediate, the process does not depend just on the number of consumers who tried the good, but on the *number of units* they have consumed: in this situation, *cumulated* sales is a more appropriate index of consumers' information.

Several arguments may be put forward to support this hypothesis. In particular, Grossman *et al.* (1977) prove that the speed of learning of a perfectly rational, bayesian consumer increases with the cumulated quantity he has bought; a fortiori,this will be true if we admit word-of-mouth as a mechanism of information spread. Other interpretations are however also possible.

In cases (ii) and (iii), current market size does not affect the shift of the demand curve, and therefore the only effect of its increase is to reduce the likelihood of informing new potential consumers. As a consequence, under these conditions we assume $L'_z \leq 0$.

These considerations may lead to introduce also a formal distinction among three "polar" cases in which each of the three variables is the only determinant of demand dynamics. These may be considered particular cases of the general formulation (1), and can be useful in order to single out and analyze the different situations previously outlined.

2.3 We analyze the behavior of a non threatened monopolist, who maximizes his profit (π) with a time horizon T, that can possibly be infinite; his problem can therefore be formalized as a problem of optimal control, where x_t is the control variable and Z_t is the vector of state variables. The choice of x instead of p as choice variable does not affect the results. If $C(x_t)$ is the cost function, which is continuous for each positive value of x_t, and r is the discount rate, the maximization problem becomes:

$$\max \quad \pi = \int_0^T [x_t \, f(x_t, Z_t) - C(x_t)] e^{-rt} dt$$

(3)

$$\text{s.t.} \quad \dot{Z}_t = L[x_t, y(x_t), Z_t]$$

Therefore, the monopolist's problem is different from the usual one, since output may affect demand dynamics; the solution of (3) is the profit - maximizing output level, x_L, while we will refer to the maximization of <u>*current*</u> profit as a problem of "myopic" profit maximization. Throughout the paper, variables referring to problem (3) will be indexed with *L*, while the index *M* will be used in relation to the myopic case. Defining mt as the vector of positive current value Lagrange multipliers[5] associated with *L*, the Hamiltonian can be written as:

(4) $H = e^{-rt} \{x_t\ f(x_t,\ Z_t) - C(x_t) + m_t L[x_t,\ y(x_t),\ Z_t]\}$

A First Order (necessary) Condition for a maximum is $H'_x = 0$, which means:

(5) $MR - MC + G(x,t) = 0$

where *MC* is the marginal cost and $G(x,t) \equiv m_t L'_x$. When *G* is different from zero, the condition for profit maximization differs from $MR = MC$, that we have in the full information model.

The interpretation of this difference is straightforward by noticing that, when current output can shift future demand, it can affect also the future marginal revenue: therefore the "true" marginal revenue referred to the whole time horizon is no longer the one it is possible to get in the static model.

The additional element to take into account is $G(x,t)$, which is the product of two terms. The first one is m_t, the profitability of a shift in the function (1), which describes the dynamics of the demand curve, while L'_x describes the effect of an increase in output on the same function. Therefore, $G(x,t)$ expresses the effect of an increase in output on the present value of revenues (R) via the relaxation of the dynamic constraint (1):

$$(6) \qquad G(x,t) \equiv m_t L'_x = \frac{\partial R}{\partial L} \frac{\partial L}{\partial x} \geq 0$$

Thus $G(x,t)$ is an additional, "long run" component of marginal revenue, and this allows us to rewrite the *FOC* (5) as

$$(7) \qquad LRMR = MC$$

which is a straightforward extension of the myopic condition to a case in which long run effects of output on consumers' learning are considered.

We are now ready to examine more carefully the output choice of the firm in this situation; to this end, let $x_{M,t}$ be the path of profit maximizing output levels for the myopic monopolist and $x_{L,t}$ the solution of equation (5) (in the following exposition we will often ignore the index t for notational convenience)[6]. More formally, writing the marginal profit functions as $g(x) \equiv MR - MC$ in the myopic case and $h(x) \equiv LRMR - MC$ when learning is considered:

(8)

$x_{M,t} \equiv x_t$: $px - C(x)$ has a global (finite) maximum; this implies $g(x_M) = 0$ and $g'(x_M) \leq 0$.

$x_{L,t} \equiv x_t$ s.t. it is the solution of (3); this implies $g(x_L) = - m_t L'x \leq 0$ and $h'(x_L) \leq 0$.

It is now possible to prove that the output level will always be higher than the one produced in the absence of learning. Although we will prove it for each given position of the demand curve, the result holds _a fortiori_ if we consider the demand shifts induced by output.

PROPOSITION 1. If x_M and x_L exist, for each t $x_{L,t} \geq x_{M,t}$.

PROOF. Let us consider first the case in which x_L is the unique point such that $h(x) = 0$.

Since $g'(x_M) \leq 0$, then for each $x < x_M$, $MR > MC$, and for each $x > x_M$, $MR < MC$. By the definition of x_L, given in (8), $g(x_L) \leq 0$: this implies $x_L \geq x_M$.

However, in general there may be $n \geq 1$ local myopic maxima and $k \geq 1$ local "long run" maxima: w.l.o.g., let us consider $n = k = 2$. We call x_m the <u>local</u> myopic maximum and x_l the <u>local</u> "long run" maximum. From the continuity of the functions, it follows that in both cases we have a local minimum between the points of maximum; we call the minima x_{mm} and x_{lm}, respectively. Since $LRMR > MR$, $x_{lm} < x_{mm}$.

If $x_M < x_m$, the proposition holds quite obviously for the previous argument. The same is true if $h(x) = 0$ only when $x > x_M$.

If $x_M > x_m$, we have to show that we cannot have $x_m < x_L < x_M < x_l$. In fact, it is possible to show that assuming $x_L < x_M$ we get a contradiction. Since $LRMR > MR$, the following is true:

$$\int_{x_m}^{x_{mm}} g(x)dx < \int_{x_L}^{x_{lm}} h(x)dx < 0 \qquad \text{and}$$

$$0 < \int_{x_{mm}}^{x_M} g(x)dx < \int_{x_{lm}}^{x_l} h(x)dx$$

From here it follows that

$$\int_{x_L}^{x_l} h(x)dx > \int_{x_m}^{x_M} g(x)dx > 0.$$

This implies that $\pi(x_l) > \pi(x_L)$, against the definition of x_L. **Q.E.D.**

Analogously, when price is the control variable it is always optimal to set a price lower than the myopic monopoly level; therefore pricing will involve a sort of "discount".

A graphical illustration of the argument we have used is given in Figure 1. It is graphically evident that if $x_M > x_m$, then $x_L > x_l$. The shaded area is the integral of $h(x)$ between x_l and x_L: compared with the myopic case, the losses from the increase in output are smaller and gains are larger, because $LRMR > MR$.

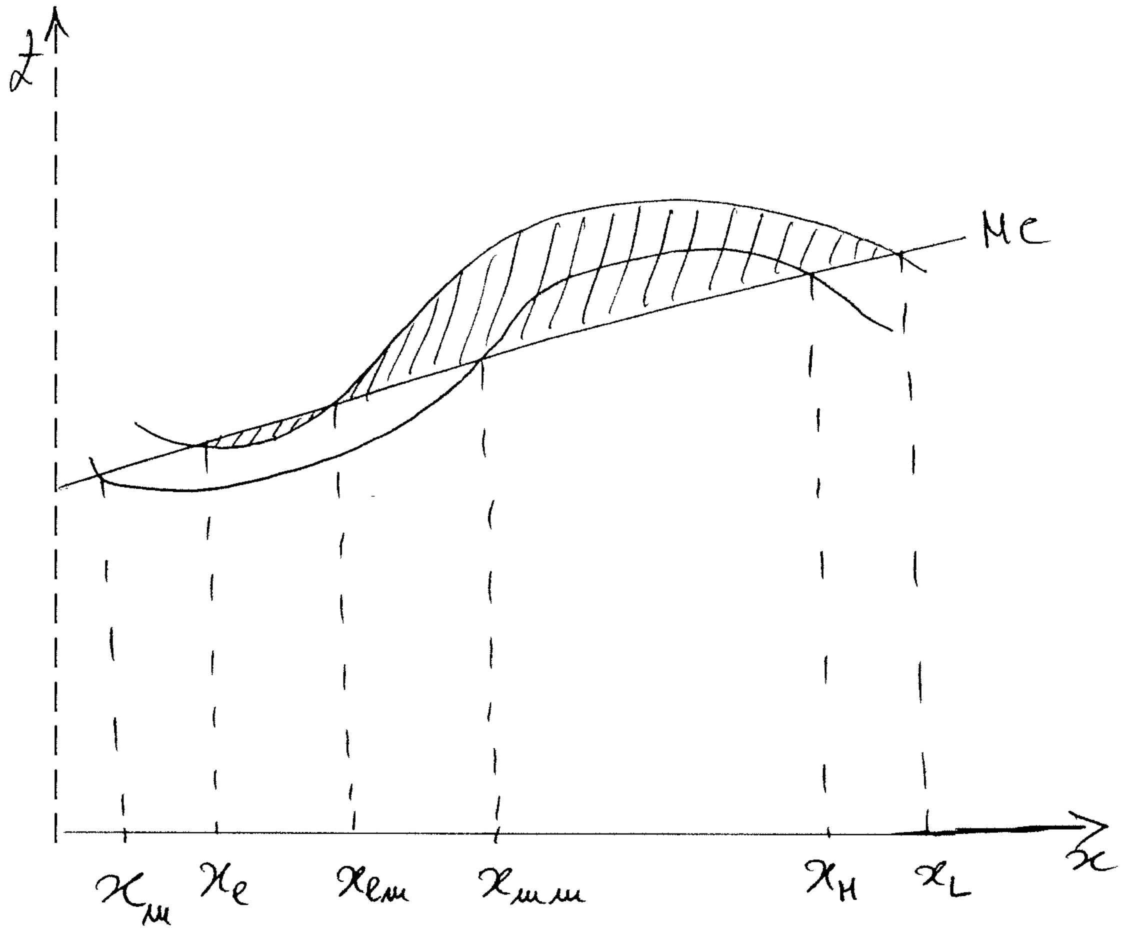

Figure 1

This very intuitive (almost obvious, perhaps) result is quite general, since it is independent of the specification of our general mechanism for information spread. In cases (ii) and (iii), L depends (more or less directly) on x, and this is enough for Proposition 1 to hold. In case (i), $L'_x = 0$ and the *FOC* becomes the standard marginal equality $MR = MC$: therefore, $x_L = x_M$, since the monopolist has no reasons for producing a different output level.

Moreover, the conditions for this result to hold are rather weak; in particular, when the profit function is concave, the result follows. This is anyway far from being necessary: neither the marginal cost nor the marginal revenue curve must necessarily be monotonic, and the same is true also for the marginal profit function, $g(x)$.

Under such conditions, therefore, the output level in each instant must be not less than x_M. At the limit, when Z_t tends to $\underline{Z}$, no increase in output can affect any more L, and therefore (5) reduces to the ordinary $MR = MC$ condition.

2.4 The previous remarks lead us to write the profit maximizing output level x_L as the sum of two components:

(9) $$x_L = x_M + x_I.$$

This relationship defines x_I, which represents the part of current output that is not linked to the maximization of current profit; x_I assumes instead the character of an *investment*, since it is produced with the aim of increasing future demand and profit. Given our assumptions on the mechanism of spread of information, the analogy with expenditure in advertising is very clear. The study of the conditions that induce a firm to substitute the sales level to advertising or other variable as means of raising future demand seems to be an interesting task of this kind of analysis: we will return to this point later.

Of course, the supply of an output level larger than x_M has a cost in terms of foregone current profit; this loss is the difference between the additional cost of production the monopolist has to incur and the additional revenue derived from the increase in sales. The expression for this "investment cost" is the following:

$$(10) \qquad IC(x) = C(x_L) - C(x_M) - [\ R(x_L) - R(x_M)\ ].$$

Since x_M is the (myopic) profit maximizing output level, $IC(x) > 0$. It seems rather obvious that x_I will be a decreasing function of its cost IC, and therefore will be lower:

- the steeper the cost function;
- the steeper (more rigid) the demand function.

The latter condition is easily understood if one considers that when the demand function is very steep, the reduction in price, and hence in revenue, following a given increase in output is larger.

An example can perhaps be useful in showing these effects: to this end we try to get an explicit expression for x_I. Equation (5) can be rewritten as

$$MC(x_L) + [\ MC(x_M) - MC(x_M)\ ] =$$

$$= MR(x_L) + [\ MR(x_M) - MR(x_M)\ ] + G(x_L)$$

where G is the function defined in (6). By the definition of x_M, this becomes

$$MC(x_L) - MC(x_M) = MR(x_L) - MR(x_M) + G(x_L).$$

Consider now $C = (c/2)x^2$ and $x = A - bp$. The previous expression becomes:

$$cx_I = -(2/b)\ x_I + G(x_L)$$

Rearranging,

$$(11)\qquad x_I = \frac{G(x_L)}{c + 2/b}$$

Therefore, x_I is higher, the lower c and the higher b, in line with our previous claim.

3. DYNAMIC FEATURES OF THE MODEL

Within this framework, it is now possible to investigate what determines the profit - maximizing path of output and prices; while it will be assumed that the demand curve shifts monotonically, it can be shown that we cannot be sure that either output or price will move in the same direction. Even if only outward shifts of demand are considered, the analysis of the case where demand shrinks is completely symmetric.

It seems useful to work on the (purely logical) distinction between the static profit maximizing component of output (x_M) and the "investment" part of it (x_I); the dynamic behavior of these variables is different, and keeping the analyses separate will help in understanding our results.

The first thing to notice is that the dynamics of the model is driven completely by the revenue side: the cost structure is given, and changes in the equilibrium position are uniquely due to shifts of the

LRMR curve along the given marginal cost function. Nonetheless, the shape of the cost function affects the output level in every single instant, and in general marginal cost will be relevant in the following analysis.

Given our definition of Z_t, the effects of shifts of this vector on x_M are rather standard: the outward shifts of the demand curve we are considering will also move outwards the *MR* curve. Therefore, the myopic profit maximizing output level will not decrease, and an increase in x_M is certain with positive marginal costs.

This can be shown from the condition for myopic profit maximization:

$$MR(Z, x) = MC(x)$$

This expression defines the (unique) value of x_M. Differentiating, we get:

$$MR'_Z \, dZ + MR'_x \, dx = MC'_x \, dx$$

From here, we obtain

$$(12) \qquad \frac{dx}{dZ} = - \frac{MR'_Z}{MR'_x - MC'_x} = - \frac{MR'_Z}{g'(x)}$$

In a neighborhood of x_M, the sign of (17) determines the effect of demand shifts on x_M itself. Since $g'(x_M) < 0$, expression (17) has the same sign as MR'_Z, whose expression is the following:

$$(13) \qquad MR'_Z = f'_Z(Z,x) + xf''_{x,Z}(Z,x).$$

We defined Z so that its increases imply an outward shift of the demand curve ($f'_Z > 0$), and possibly an increase in its steepness

($f''_{x,Z} > 0$), and therefore $MR'_Z > 0$. From here $\frac{dx_M}{dZ} > 0$ follows.

The reaction of the "investment" component of output depends instead on various factors. When Z_t approaches $\underline{Z}$ the share of potential market that has not yet been reached becomes smaller and this will reduce the effectiveness of an increase in output.

One should also consider the effect of the shifts in Z_t on m_t, and here we assume that m_t must not increase over time and, therefore, that in general it is a non-increasing function of Z_t. This assumption relies on the interpretation of the multipliers as shadow prices of the relaxation of the dynamic constraint [equation (1)]. From the Hamiltonian (4), a necessary condition for a maximum is $H'_Z = -\dot{m}_t + rm_t$; it follows that

$$\dot{m}_t = -xf'_Z(Z,x) - m_t L'_Z + m_t r \tag{14}$$

On one hand, when demand shifts, further increases become less and less profitable: since an upward shift of demand raises profit in every following instant, the earlier the constraint is relaxed, the higher the value of its shift.

On the other hand, we find two forces. The higher the (negative) effect of Z_t on its own dynamics, the higher the value of m_t: at the limit, if a (late) increase in Z_t reduces "too much" the value of L, a late relaxation of the constraint can be more important to the monopolist than an early one. The second factor is obviously the rate of interest, that tends to postpone payments.

Assuming $m_t \leq 0$ implies therefore that the interest rate cannot be "too high", and that the effect of Z_t on its own dynamics cannot be "excessively" negative. Under these conditions, the price the monopolist would be willing to pay for relaxing the dynamic constraint does not increase over time. We must point out that in the polar cases (ii) and (iii), when Z_t does not affect demand dynamics, our assumption imposes a restriction only on the interest rate.

The conclusion is therefore that an increase in Z_t will raise x_M while reducing x_I. This seems to be fairly intuitive: the shorter the time horizon remaining, the lower the incentive to invest (in this case, in output). In general, it does not seem possible to say anything on the global effect on output, and hence on the dynamics of x_L. Formally, we have

$$(15) \qquad \dot{x}_L = \dot{Z}_t \left\{ \frac{\partial x_M}{\partial Z_t} + \frac{\partial x_I}{\partial Z_t} \right\}$$

The two terms within brackets have opposite signs, and the global result will depend on the relative strength of these competing effects.

<u>**The dynamics of price.**</u>

Given our demand function, once we are able to determine the path of Z_t and x_t we can also say how the dynamic behavior of price will be. We have, in fact:

$$\dot{p}_t = f'_Z(x, Z)\dot{Z}_t + f'_x(x,Z)\dot{x}_t$$

Substituting from (20), we get

$$(16) \quad \dot{p}_t = \dot{Z}_t \left\{ f'_Z + f'_x \left[\frac{\partial x_M}{\partial Z_t} + \frac{\partial x_I}{\partial Z_t} \right] \right\}$$

The first, rather intuitive conclusion is that, as long as $L > 0$, $\dot{x}_t \leq 0$ is sufficient for $\dot{p}_t > 0$. This denotes a situation in which market conditions make it worthwhile to charge an initial "strategic" price in order to *launch the new product*.

On the other hand, if $\dot{x}_t$ is sufficiently high relative to $\dot{z}_t$, the opposite situation may arise: price decreases over time, like in a case of *intertemporal price discrimination*[8]. There are at least two interesting cases in which one might observe this kind of behavior.

The first one occurs when the decrease in the incentive to invest is by its nature particularly slow. This happens, essentially, when it is impossible to move the demand function fast enough because of a slow spread of information. In such a situation, producing a very high output level in the early periods may not be a good strategy, since the effect on the demand curve will be rather small.

The second situation that may be favourable to this kind of intertemporal price discrimination is linked to a very high cost of investing in output: rapidly increasing marginal costs and a very steep demand function are the key factors that may drive the results towards this outcome. With a linear demand function $x_t = A - b_t p_t$, for instance, when b decreases over time, the expression for price dynamics is

$$(17) \qquad \dot{p}_t = - \frac{\dot{x}_t + \dot{b}_t p_t}{b_t}$$

As we have already shown, $\dot{x}_t < 0$ is sufficent for p_t to increase over time, while if $\dot{x}_t > 0$ the sign of $\dot{p}_t$ remains uncertain, and may also change through time.

In an important case it is possible to show that the initial price will be lower than its final level. This is case (ii), when demand shifts do not depend directly on current sales, but rather on y_t, cumulated output; in this case, the problem becomes:

$$\max \quad \pi = \int_0^T [x_t\, f(x_t,\, Z_t) - C(x_t)]e^{-rt}dt$$

(18)

$$\text{s.t.} \quad \dot{Z}_t = L(y_t,\, Z_t)$$

In this case, which is very similar to the one of learning-by-doing on the cost side[9], we can prove the following result.

PROPOSITION 2. When demand shifts depend on cumulated output, and $r = 0$, price increases over time.

PROOF. By definition, the derivative of y_t with respect to time is x_t. Thus, setting $r = 0$, (18) can be solved as a problem of calculus of variations; the Euler equation, necessary for a maximum, is the following:

$$xf'_y(x,y) = \frac{d}{dt}\{\, f(x,y) + xf'_x(x,y) - MC(x) \,\}$$

Differentiating the terms in brackets with respect to t and rearranging, we get

$$\dot{x}_t\,[\, 2f'_x - MC'_x + xf''_{x,x} \,] = -x^2 f''_{x,y}$$

and, finally,

$$(19) \qquad \dot{x}_t = -\frac{x^2 f''_{x,y}}{2f'_x - MC'_x + xf''_{x,x}} = -\frac{x^2 f''_{x,y}}{g'(x)}$$

By the definition of Z_t, an increase in cumulated output will not decrease the steepness of the demand function, and therefore the numerator is at least non-positive.

Given our definition of x_L, in a neighborhood of x_L the denominator is also negative, and therefore we observe that, *whatever the speed of demand shifts*, output will not increase through time. We have already shown that this is *sufficient* for observing an increase in price over time, and therefore the proposition follows. **Q.E.D.**

If we allow for discounting, the (19) is modified as follows:

$$(19') \qquad \dot{x}_t = \frac{r(MC - MR) - x^2 f''_{x,y}}{g'(x)}$$

Since $MR \leq MC$, Proposition 2 holds whenever r is not "too large": in order to avoid unnecessary complications that would add little to the following analysis, we will maintain the assumption of no discounting. We must however point out that the conditions we have given are sufficient and are likely to be far too restrictive than necessary: this gives our result a greater generality.

The intuition behind it is fairly clear. Cumulated output, which is the crucial variable for shifting Z_t, rises over time, and therefore it becomes more and more difficult to increase the value of L through *current* output, the only variable the firm can use in each instant. Therefore the incentive to invest in output decreases very rapidly, and the final outcome is that the firm will set a very high output level at the beginning (possibly even with $p = 0$), reducing it in later periods.

A completely symmetric result holds when $L < 0$, and therefore $f''_{x,y} \geq 0$: output increases over time, while price will be reduced.

An immediate consequence of Proposition 2 is:

COROLLARY. When demand shifts depend on cumulated output and f'_x is constant, we have that:

(a) the optimal policy is to supply in each t the full information output level $x_t = \underline{x}$;

(b) price dynamics is directly proportional to L.

PROOF. Part (a) follows from (19), setting $f''_{x,y} = 0$. As a consequence, from (16) $\partial p/\partial t = f'_Z L$. **Q.E.D.**

In other words, when we have parallel upward shifts of the demand curve the reduction in x_I is perfectly matched by the increase in x_M, and the only effect of demand shifts is a proportional increase in price. In such a case, defining appropriately Z_t, we may write the demand function as

(20) $$f(x_t, Z_t) = f^*(x_t) + Z_t$$

This allows us to add some interesting considerations on the welfare aspects of the situation we are analyzing. In particular, we prove the following

PROPOSITION 3. When demand shifts depend on cumulated output and f'_x is constant, consumer surplus (CS) is not affected by the initial uncertainty.

PROOF. We have already shown that it is legitimate to write the demand function as (31). In each t, the CS is

$$CS_{L,t} = \int_0^x F(x)\mathrm{d}x + Z_t \int_0^x \mathrm{d}x - p_t x = \int_0^x F(x)\mathrm{d}x + (Z_t - p_t)x$$

Given part (b) of the Corollary, $(Z_t - p_t)$ is constant in each t, and therefore also when $Z_t = \underline{Z}$ (when full information is attained). As a consequence, in each t,

$$CS_{L,t} = CS_{F,t}$$ **Q.E.D.**

Some remarks are necessary, in order to make precise the meaning of this result.

In situations where the current position of the demand curve is affected by the imperfect information that consumers have on the the characteristics of the good, one has to make a particularly cautious use of the consumer surplus for any welfare conclusions. The usual, now standard answer to this problem is that the *full information* demand curve is the one which is relevant for such judgements, since, for each given quantity of the good, it represents the marginal utility that is actually obtained.

In general, we find that consumers suffer a loss due to uncertainty, since, given the price, they buy a "wrong" quantity of the good[10]. However, this answer would not be correct in the present context because x_t is fixed, while the price is proportional to Z, and thus it depends directly on the position of the demand curve. Hence, the *coeteris paribus* assumption does not hold, since the initial price is low *just* *because* consumers are imperfectly informed: the previous corollary shows that in the case of full information the same quantity would be bought, but at a higher price, and Proposition 3 proves that this leaves consumers indifferent between being uncertain (with a low price) and having perfect information (with a high price).

Only the level of current profit changes over time, increasing up to the full information level. This means that the firm bears entirely the consequence of consumers' uncertainty: the consumer surplus is unaffected by uncertainty, while, when $L > 0$, profit is obviously lower. If demand shifts are not parallel it is also possible that $CS_L > CS_F$: this quite paradoxical result indicates therefore that increasing consumers' information will not necessarily increase their welfare.

On the other hand, it is quite obvious that π_L is *always* smaller than π_F: in early periods the monopolist faces a lower demand curve and, moreover, he must make an investment (in output) which is costly and which is not incurred with full information. Therefore, it seems that the monopolist who supplies a product of good quality does not benefit at all from consumers uncertainty, while the opposite is clearly true when consumers' initial expectations are too optimistic. This could also explain why producers may sometimes deliberately decide to supply a good of low quality: however a more complete analysis of this point is beyond the scope of this paper.

4. QUALITY LEVEL AS A CHOICE VARIABLE

So far, we have analyzed uncertainty on the horizontal dimension of product differentiation, while we assumed exogenously given quality; now we want to remove this quite implausible assumption in order to examine what the quality level (q) will be in such a situation, to compare it to its full information level (q_F) and to study the relationship between quality and output as alternative strategic variables.

Quality is potentially a very important dimension of our problem. It affects the position of the full information demand curve and the speed of diffusion of the new good: it is reasonable to think that if the observed quality is far from the "usual" standard, information on the good will spread more easily[11]. Now the problem becomes:

$$\text{(3')} \qquad \begin{aligned} &\max \quad \pi = \int_0^T [x_t\, f(x_t,\, Z_t,\, q_t) - C(x_t,\, q_t)]e^{-rt}dt \\ &\text{s.t.} \quad \dot{Z}_t = L[x_t,\, y(x_t),\, Z_t,\, q_t] \end{aligned}$$

The choice of q can be either sequential and variable in each moment like the output level, or initial and invariable; we will examine the two cases separately.

Sequential quality choice

The idea of learning (at least, in strict sense) requires that the characteristics to be learnt is fixed or, in some sense, stable. Although this is not explicitly taken into account in some previous work (like

Shapiro, 1982) an analysis which involves learning and a variable quality level is by its nature admissible only in particular situations.

For instance, if the quality level which is initially perceived is higher than the expected one, it can be thought that learning is consistent with a variable quality level only if quality may only improve: in other terms, learning is unlikely to take place if quality may vary at random or if consumers can be fooled. The conditions under which these possibilities can be ruled out seem to be quite restricitve.

In this case, anyway, the quality level is an additional *control* variable in the problem of profit maximization (3'). This requires quality to be easily adjustable and flexible, and resembles a situation in which the production plant is the same for every level of q, while quality improvements are substantially based on those variable inputs that can be changed without relevant adjustment costs.

The current value Hamiltonian is a straightforward extension of the previous formulation (4):

(4') $H = x_t f(x, Z, q) - C(x, q) + m_t L(x, y, Z, q)$

Differentiating with respect to the control variables x and q, we get the following necessary conditions for a maximum:

(21) $LRMR_x = MC_x \quad \Rightarrow x_L$

(22) $LRMR_q = MC_q \quad \Rightarrow q_L$

where, with a slight change in notation, $LRMR_i$ and MC_i are the first derivatives of the (long run) revenue function and the cost function with respect to the variable i ($i = x, q$).

In general, we will have a simultaneous solution to these equations, and x_L and q_L will depend on one another. In order to investigate this relationship, we may consider the effect of quality on the marginal conditions that determine the output level.

On the cost side, since quality improvements consist mainly of improvements in variable inputs, we expect $MC'_{x,q}$ (the second cross derivative of the cost function) to be positive: a high quality level requires a higher marginal cost of production. On the revenue side, quality will help information diffusion for each level of output, and therefore will also increase the marginal revenue in each moment; hence, $LRMR'_{x,q} \geq 0$.

The relationship between output and quality as strategic variables depends on the relative magnitude of $LRMR'_{x,q}$ and $MC'_{x,q}$. When $LRMR'_{x,q} > MC'_{x,q}$, quality raises marginal revenue more than marginal cost, and it will stimulate the production of an output level higher than the previous one (found disregarding quality as a choice variable). Thus, we will observe a positive correlation between output and quality: we may say that the two instruments are *complements* in the firm's policy[12].

On the other hand, if $LRMR'_{x,q} < MC'_{x,q}$, we have a negative correlation between x and q: in order to maximize profit, the monopolist trades off output and the quality level, which are now *substitutes* from the firm's viewpoint.

In general, this kind of strategic complementarity between variables arises every time a larger amount of one of them raises the marginal profitability of using the other one. Therefore, if an increase in the value of a certain variable does not affect the marginal cost of using other variables, we cannot have strategic substitutability between them.

Consider as an example advertising. It seems quite reasonable to assume that expenditure in ads will increase MR_q, since it will shift outwards the demand curve, possibly making it steeper. On the other side, the cost of ads is independent of the marginal cost of quality, and hence only the revenue side of the marginal condition that determines q_L will be (positively) affected. This implies that in our model we have a positive correlation between advertising and quality, a finding that supports Nelson's (1974) intuition.

Initial, invariable quality choice

In many situations, especially for new goods, quality improvements are typically borne by some fixed cost, like R&D expenditure or the kind of machinery employed. If this is the case (or if anyway $MC'_{x,q}$ is small) we already know that output and quality are complementary variables in the firm's strategy: in fact, quality has no negative effects on the marginal conditions that determine the output level.

However, in this situation it is reasonable to think that the quality level is decided at the beginning, and that adjustment costs will be so high to discourage frequent changes in quality. The analytical solution to the problem is different, since quality is a choice variable, but not a control variable.

Now we have to solve (3') with respect to x_t only, finding $x_{L,t}$ as a function of the parameters; this determines the maximum profit $\pi_L(Z_o, \underline{Z}, L, \theta, r)$, where θ represents the parameters of the cost function, defined so that $C'_\theta > 0$. Since some of the parameters depend on quality, we find its optimal level maximizing π_L with respect to q. This value, q_L, will satisfy the following expression:

$$(23)\quad \frac{\partial \pi_L}{\partial \underline{z}} \frac{\partial \underline{z}}{\partial q} + \frac{\partial \pi_L}{\partial L} \frac{\partial L}{\partial q} = - \frac{\partial \pi_L}{\partial \theta} \frac{\partial \theta}{\partial q}$$

In the full information model, quality does not lead to any shifts in the demand curve, and the analogous marginal condition will be:

$$(24)\quad \frac{\partial \pi_F}{\partial \underline{z}} \frac{\partial \underline{z}}{\partial q} = \frac{\partial \pi_F}{\partial \theta} \frac{\partial \theta}{\partial q}$$

Unlike Shapiro (1982), we have no reasons to conclude that quality level will be lower than the full information one. One reason for this difference may be found in the different scheme of information diffusion we are considering, that gives a greater importance to the mechanism of repeat purchase and to the reputation the monopolist is able to establish. A second, and perhaps the main reason is the fact that we deal with uncertainty on both dimensions of product differentiation: here the existence of full information on *quality* leaves an incentive to raise q over q_F in order to speed up the "horizontal" learning process.

In general, we are more likely to observe $q_L > q_F$, the lower the discount rate, the longer the time horizon, and the greater the effectiveness of quality as a mean for shifting L. If these conditions are met, initial imperfect information of consumers will work as an incentive to increase quality.

In the opposite case, i.e. if the time horizon is short, the firm may have too low an incentive to produce high quality goods, since its return from quality in this short period would not justify the initial investment. This is one reason why, if the firm expects its product to be in any case replaced by another one within a short period, it may be optimal for it to supply a quality level inferior to q_F .

5. CONCLUDING REMARKS

In our analysis we have tried to shed some light on general characteristics of the optimal behavior of a monopolist when consumers acquire information through time. The absence of initial full information has important consequences, since the demand curve shifts and the firm is able to affect the speed of these shifts, acting on output level and quality.

In every case in which current output increases the speed of diffusion of information, the firm has an incentive to supply a larger quantity than in a myopic framework. Moreover, when the crucial variable is cumulated output the optimal strategy is to set an initially low price and to raise it over time.

When quality changes are borne mainly by an increase in the fixed cost, as it is often assumed, quality and quantity of output are complementary strategic variables. This positive correlation exists also between quality and the expenditure in advertising.

Other possibilities are the following:

- to consider many firms with competing products: this would require either dynamic (differential) games, or a multiperiod model (possibly of duopoly);
- to consider the possibility of entry: in this case the output decision is likely to be similar in its effect to advertising, leading not only to a higher demand curve, but also to a greater possibility of entry[13].

NOTES

1) Recent important contributions include Klein and Leffler (1981), Shapiro (1982) and (1983), Rogerson (1983), Allen (1984), Farrell (1986) and Riordan (1986).

2) On this point, also see Shapiro (1982).

3) Cfr. Glaister (1974).

4) A possible example of Z_t in this situation may be the following: given $x = A - bp$, we consider $Z \equiv A$, number of consumers who would buy the good at $p = 0$.

5) See Seierstad and Sydsaeter (1987). The assumption of $m_t \geq 0$ is made because Lagrange multipliers are the shadow prices of relaxing our constraint: $m_t = \partial\pi/\partial L_t$. In particular, $m_T = 0$.

6) As second order sufficient condition, we will assume that the Arrow concavity condition holds, that is that H is concave in Z. Cfr. Seierstad and Sydsaeter (1987), theorem 2.5.

7) Here, since a favourable shift in the parameter corresponds to a decrease in b_t, the Lagrange multipliers should be negative and increasing over time; in order to preserve the previous notation and the assumptions on m_t, we use $(-m_t)$ as Lagrange multipliers.

8) Of course, the sign of $\dot{p}$ may also change over time, in contrast with Shapiro (1983), where a two step pricing regime results from the model.

9) See Spence (1981).

10) Cfr. Friedman (1985) for a good account of this point. However, the use itself of aggregate consumer surplus in conditions of uncertainty is highly objectionable, as the same author correctly emphasizes, and this is an obvious limitation also to the present analysis.

11) An analogous assumption can be found in Shapiro (1982).

12) These "intra-firm" relationships are analogous to the inter-firm ones, considered by Bulow <u>et al.</u> (1985), who analyze strategic complementarity and substitutability between variables set by competing firms.

13) Models along these lines are Schmalensee (1983) and Farrell (1986).

BIBLIOGRAPHY

F. Allen (1984), Reputation and Product Quality, *Rand Journal of Economics*, vol.15, pp.311-27.

K. Bagwell and M. Riordan (1986), Equilibrium Price Dynamics for an Experience Good, Discussion Paper n.705, Northwestern University.

J. Bulow, J. Geanakoplos, P. Klemperer (1985), Multimarket Oligopoly: Strategic Substitutes and Complements, *Journal of Political Economy*, vol.93, pp. 488-511.

J. Farrell (1986), Moral Hazard as an Entry Barrier, *Rand Journal of Economics*, vol.17, pp.440-49.

L. Friedman (1985), *Microeconomic Policy Analysis*, McGraw Hill, New York.

S. Glaister (1974), Advertising Policy and Returns to Scale in Markets where Information is Passed Between Individuals, *Economica*, vol.41, pp.139-58.

S. Grossman, R. Kihlstrom and L. Mirman (1977), A Bayesian Approach to the Production of Information and Learning By Doing, *Review of Economic Studies*, vol.44, pp.533-47.

B. Klein and K. Leffler (1981), The Role of Market Forces in Assuring Contractual Performance, *Journal of Political Economy*, vol.89, pp.615-41.

P. Milgrom and J. Roberts (1986), Price and Advertising Signals of Product Quality, *Journal of Political Economy*, vol.94, pp.796-821.

P. Nelson (1970), Information and Consumer Behavior, *Journal of Political Economy*, vol.78, pp. 311-29.

P. Nelson (1974), Advertising as Information, *Journal of Political Economy*, vol.81, pp. 729-54.

M. Riordan (1986), Monopolistic Competition with Experience Goods, *Quarterly Journal of Economics*, vol.101, pp.265-79.

W. Rogerson (1983), Reputation and Product Quality, *Rand Journal of Economics*, vol.14, pp.508-16.

S. Salop and J. Stiglitz (1977), Bargains and Ripoffs: A Model of Monopolistically Competitive Price Dispersion, *Review of Economic Studies*, vol.44, pp.493-510.

R. Schmalensee (1983), Advertising and Entry Deterrence: An Exploratory Model, *Journal of Political Economy*, vol.91, pp. 636-53.

A. Seierstad and K. Sydsaeter (1987), *Optimal Control Theory with Economic Applications*, North Holland, Amsterdam.

C. Shapiro (1982), Consumers Information, Product Quality and Seller Reputation, *Bell Journal of Economics*, vol.13, pp.20-35.

C. Shapiro (1983), Optimal Pricing of Experience Goods, *Bell Journal of Economics*, vol.14, pp.497-507.

M. Spence (1981), The Learning Curve and Competition, *Bell Journal of Economics*, vol. 12, pp.534-44.

A. Wolinsky (1983), Prices as Signals of Product Quality, *Review of Economic Studies*, vol.50, pp.647-58.