Mixed Oligopoly: An Overview

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Abstract

In this paper we review various models that have been proposed for the study of mixed oligopoly, more precisely of those markets in which private and public firms compete on equal basis, that is, using only market instruments. The survey is preceded by a brief discussion of other regulatory mechanisms available to the public authority to improve social welfare in a sector.

Keywords: mixed oligopoly, public firm, social welfare, marginal cost pricing.
Table of Contents

1. Introduction ......................................................... 4

2. Public firms in Western economies .............................. 7

3. Public intervention within an industry ......................... 10
   3.1. Regulation ..................................................... 11
       3.1.1 .............................................................. 11
       3.1.2 .............................................................. 13
       3.1.3 .............................................................. 14
   3.2. Public monopoly ................................................ 16
       3.2.1 .............................................................. 16
       3.2.2 .............................................................. 18
   3.3. Public firm as an instrument for the internal
        regulation of an industry .................................... 20
       3.3.1 .............................................................. 20
       3.3.2 .............................................................. 22
       3.3.3 .............................................................. 25

4. The game theoretical approach to mixed oligopoly .......... 26
   4.1. The general setting ............................................ 26
       4.1.1 .............................................................. 27
       4.1.2 .............................................................. 31
   4.2. Objective functions for the public firms .................. 32
   4.3. Technologies and cost structures ........................... 35
   4.4. The move order of the game .................................. 36

5. Summary of the main results ...................................... 38
   5.1. Nationalization ................................................. 38
   5.2. Pricing rules ................................................ 39
   5.3. Cournot competition ......................................... 42

6. Concluding remarks and further extensions .................... 45

7. Some related literature ........................................... 48

References ............................................................. 50
1. Introduction.

A **mixed oligopoly** is a market where a homogeneous or differentiated good is supplied by a "small" number of firms and the objective function of at least one of them differs from that of the other firms.

We have chosen the term mixed in the above definition in analogy with the current use of the term "mixed economies", which refers to the simultaneous presence of private and public enterprises in the economic system.

Indeed, in this paper, we deal with a case of mixed oligopoly particularly interesting from the point of view of economic and industrial policy: a market where at least one publicly owned firm cohabits with at least one private firm. We use the term "owned", because it is the notion which comes across the mind when one talks of private and public firms. Of course we are aware of the fact that the ownership does not always identify completely the actual pattern of behaviour of modern corporations in the market. An analogous ambiguity would have arisen if we had used control instead of ownership. In this paper with the term "owner" we mean the subject who takes the relevant decisions, given the institutional setting in which the firm operates. A market where there are both private and public firms is then a mixed oligopoly as the
firms owned by private agents aim at maximizing profits, whereas the publicly owned firm is interested in optimizing social targets. Here we do not take sides on the issue of whether a public firm should behave like a private one: this case, when a public firm has the aim of maximizing profit for the sake of profit maximization, is obviously not covered by our definition, and it is of lesser interest here, as it can simply be analysed within the traditional oligopoly framework.

If the opinion that a public firm should not try to maximize profit is not unanimously held, it is however part of the traditional wisdom in economic culture of the Western world. The book by Posner and Woolf (1967) is an extremely useful reference on this subject: they mention various laws relative to the aims of public firms. For instance, in Italy, the law instituting the Ministero per le Partecipazioni Statali (Ministry for the State-Owned Enterprises) (Posner and Woolf 1967, p. 47), or, in Britain, various Nationalisation Acts, dated from 1945 to 1950 (p. 44). A recent influential textbook of public sector economics states that the public firm is one of the instruments used by the public authority to correct market failures and to reach an improvement in social welfare (Stiglitz 1987, pp. 156-7).

In this paper, besides the discussion of the role of a public firm in mixed oligopolies, we will briefly survey some instruments (regulatory mechanisms) which might have similar
effects on the markets under consideration. There may be circumstances under which the imposition of constraints is almost indistinguishable from the consequences of having a public firm in the market; however, both from a theoretical and an operational viewpoint, it is useful to maintain the distinction between the cases in which the public authority issues some rules designed to regulate the behaviour of firms that preserve their private nature, from the case in which the public authority does directly control an enterprise.

This survey is organized as follows. In Section 2 we report the reasons that have been proposed to motivate the presence of government owned firms in Western Europe. In Section 3 a brief outline of the various kinds of public intervention within an industry is provided. Our taxonomy identifies three groups: the regulation of privately owned firms, the nationalization of a whole sector, and the creation of a mixed oligopoly. Sections 4 and 5 review the models that examine the behaviour of a public firm in a mixed oligopoly. In these models, the main emphasis is on the strategic interaction between private and public enterprises, and the solution concepts of the theory of games are used extensively. The last two Sections present concluding remarks and a note on some related literature.
2. Public firms in Western economies.

In two recent and very interesting books, Floyd, Gray and Short (1984) and Parris, Pestieau and Saynor (1987), provide a very detailed account of the economic role of public enterprises in the world, the former, and in nine European countries, the latter.

Table 1

<table>
<thead>
<tr>
<th>Output and investment shares of public enterprises.</th>
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<tr>
<td><strong>AVERAGES</strong></td>
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<td>77 Mixed economies</td>
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<tr>
<td>Industrialised countries</td>
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<td>Developing countries:</td>
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<td>Africa</td>
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<td>Asia</td>
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<td>Latin Am. and Caribbean.</td>
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<td>Individual countries in Western Europe:</td>
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<td>Austria</td>
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<td>West Germany</td>
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Table 1 gives the shares of output and investment of the public enterprises, defined, as far as possible, as firms directly owned or controlled by the government (Short 1984, Section 1).

The extent of government production shown by this table prompts the question of the justifications which are usually put forward to motivate the existence of public firms. A detailed account of these motives can be found in the excellent book by Rees (1984, pp. 2-9); without considering them in great depth, it is worth summarizing them in the following three groups.

(i) To facilitate macroeconomic targets, for instance, to sustain employment, to control prices, to act on the commercial trade balance. A clear example of the use of public enterprises to sustain employment has been the Italian industrial policy in the early ’70s, when the policy makers kept on taking over and operating inefficient firms rather than letting them be shut down.

(ii) To carry on redistributive policies or to alter the composition of the aggregate output. Here again the experience of the Italian policy sets an example: in the
'70s public firms would grant wage increases more easily and sooner than private ones, thus dragging up the private sector wages. There are numerous examples of the use of public firms to achieve a higher level of production of some goods considered particularly valuable to social welfare.

(iii) To correct market failures. This is the kind of our main concern here; a further subdivision, perhaps arbitrary, could be the following.

(a) Imperfect or distorted competition: situations where only one or a limited number of producers are active and the threat of potential entry does not, by itself, lead to a Pareto optimum.

(b) Externalities and public goods.

(c) The various phenomena that go under the heading of "information failures" (Stiglitz 1987, pp. 90-91), for instance what are known as moral hazard and adverse selection, or, with a terminology due to Arrow (1986), which we find clearer and prefer, "hidden action" and "hidden information", respectively.
The fairly vague legal principles which we have mentioned above have been interpreted very freely by the industrial policy makers when they had to apply these directions to instruct managers of actual public firms. In this paper we shall focus only on the third reason for intervention, namely the use of public firms as an instrument for the correction of the inefficiencies which stem from the non-correspondence of actual industries with the paradigm of perfectly competitive markets.

3. Public intervention within an industry

The existence of market failures motivates some public intervention within a sector. One of the instruments that, in some circumstances, can be used by the public authority is the creation of a mixed oligopoly. This can be obtained either by nationalizing or taking over a private firm, or by opening a new one.

In this Section we briefly discuss the other instruments that can be put in action by the public authority when the building of a mixed oligopoly is also available.

The situations in which a mixed oligopoly is a possibility loosely corresponds to the cases grouped in (iii-a) in the previous Section.

A mixed oligopoly can be viewed as an intermediate
situation between the two extreme cases of "complete government ownership and control, and private ownership restricted by close government supervision in the form of regulation and anti-trust laws." (Merrill and Schneider 1966, p 400).

3.1. Regulation

3.1.1.

"Regulation involves government intervention in the market in response to some combinations of normative objectives and private interests reflected through politics. Whatever objective regulation is intended to achieve, the regulator must choose policies tailored to the particular regulatory setting and to characteristics of the firms subject to its authority. In choosing those policies, the regulator must take into account the strategies the firms might employ in response to those policies" (Baron 1987, p. 1).

In the most recent literature on regulatory mechanisms, private firms are thus represented as profit maximizers, and are assumed to take some actions that are permitted within the regulatory framework. The regulator, on the other hand, has different objectives, so that the relationship between the private firms and the public authority involves a conflict of objectives that is resolved endogenously through the interaction between the strategies chosen by the regulator and those
chosen by the regulated firms.

Early research on regulatory mechanisms was based on models representing stylized descriptions of actual regulatory processes. Most of these processes took the form of exogenously specified regulatory mechanisms: a famous example is due to Averch and Johnson (1962) (see Baron (1987) for a discussion of the main shortcomings of this model and the background which further research moved from).

The analysis of endogenous regulatory mechanisms has emphasized the importance of asymmetric information, namely of circumstances in which the private firm and the regulator do not have the same information about demand and especially costs. If the regulator has complete information about demand and costs, and it is willing to use any form of taxation, then the first best social optimum can be reached in a straightforward manner (e.g., the public authority could tell the private firms that if the imposed rule is not followed, then all the profit will be taken away in the form of taxation).

Given the asymmetry in the information sets, the regulator faces a problem of incentive compatibility: it has to introduce a mechanism which, by creating a further constraint in the maximization problem of the firms, induces them to choose a set of actions which the regulator prefers to those that firms would have chosen without this mechanism.
The literature has distinguished between delegation and revelation regulatory mechanisms (see Baron 1987, p.10-11). As a simple example of the former, one can think of the regulation of a monopolist whose production costs are unknown to the regulatory agency (Baron and Myerson 1982). The mechanism can consist of a function \( t(p) \), where \( p \in \mathbb{R}_+ \), is the market price charged by the monopolist, and \( t \) is a lump-sum tax. "By making \( t(p) \) a decreasing function, the firm can be induced to choose a price below the monopoly price" (Baron 1987, p.11).

Both the revelation and the delegation mechanisms are self-selection mechanisms, with minimal informational requirements. Moreover, they both result in first best efficiency, as long as "the objective of the regulator is the maximization of the total surplus" (p. 11). A limit of this last assumption is that "it does not appear to be descriptive of the actual objectives of regulators, nor does it reflect the cost associated with implementing regulatory policies" (p. 11).

3.1.2.

Regulatory mechanisms like the one described in the example above are rarely found in the real world. The actual practice of regulation can be usefully divided into the following types. Firstly, the imposition of constraints on some of the strategic variables or some of the magnitudes pertinent to the regulated firm. Examples of this kind are the imposi-
tion on constraints on the level of output, on market price, on profits, on the rate of profit, on productive capacity, on the market share, on the product quality standard. Of the same nature are entry fees and licenses. Various models have been proposed to study the different effects of rules of this kind; a seminal paper is Averch and Johnson (1962); see Baron 1987, and Böss 1986 for up-to-date bibliographies.

The second group of public interventions includes various types of taxation, such as profit taxes, or indirect taxes: a very extensive treatment of taxation is Atkinson and Stiglitz (1980).

3.1.3.

In Section 3.1.1 we considered the need for regulation called for by a particular structure of the information sets of the agents. Another situation in which the content of information sets could lead to losses in social welfare is the phenomenon known as "cream skimming". In the business jargon by cream skimming is meant "a policy of entering into a market at a high price and then, later on if necessary, lowering the price to gain acceptance in other price segment. Thus only a small section (the "cream") of the market is aimed at initially, but the brand name is established" (Stiegerler (1985), pp. 258-59).

This term has been transferred in the debate about
regulation because of the analogy of the concept as cited above with the following situations.

Think of a market in which the production cost of the good offered varies widely across potential customers: this is the case with transportation linkages (motorways, railways), and with domestic services like telephone, electricity, and so on.

Profit maximization requires the setting of different prices for different customers: the connection of an isolated farm would cost much more than the connection of a flat in a block where the other flats are already connected. Therefore the isolated farm would end up without services. There may be reasons why the public authority prefers all the customers to be charged the same price for a service which is the same from their point of view; AT&T is "required to provide telephones to everyone who is willing to pay the fees set by by the government" (Stiglitz 1987, p.156), in Italy monopolies established via public franchising must set the same conditions for all customers (art. 2597 Codice Civile). Thus, the customers in the "cheap" area subsidize those who live in isolated farms. The need for regulation here stems from the necessity of preventing a potential entrant from entering and serving only the "creamy" market.

The regulatory commissions ought to forbid this kind of competition on the ground that competitors who go after the
creamy markets, and ignore the "skim milk" ones upset the balance of internal discrimination or subsidy of the full-fledged utility, and make it impossible for it to serve the skim milk markets also. Entry of new competitive services in the high rated markets can only benefit consumers there, by creating alternatives and/or lowering rates. For example, "Microwave Communication, Inc., should not be allowed to enter the lucrative Chicago-St.Louis trunkline market for microwave transmissions, because it does not plan to extend communication service to East Overshoe, Illinois, whereas AT&T, its target, does and must continue that service as well as the trunkline service which subsidizes it" (McKie 1970, p. 23).

A similar problem arises in the market for medical services, where clinics and/or insurers, while charging the same price to healthy and ill customers, would prefer to get rid of the latter and, possessing the information required to tell the two groups apart, can do so. How the regulator could solve this delicate problem is still a question for debate (see Pauli 1984 and Newhouse 1984).

3.2. Public monopoly

3.2.1.

The existence of public monopolies is generally due to such technological features of a sector that require that
there be only one firm in operation for the production to be efficient (in a one good market, this is the case when the average cost curve is decreasing when it meets the demand curve). Such a market is a natural monopoly. The social necessity for regulating a natural monopoly is easy to explain: were there a private monopolist it would price at monopoly price, bringing about the well-known dead-weight loss in social welfare. The public intervention can follow two routes: either regulating the private monopolist, or nationalizing the sector.

The marginal cost pricing rule for public utilities dates as far back as Dupuit (1844), and was put forcefully by Hotelling in a classical work (1938).

If this rule maximizes social welfare, it implies that the public utility incurs a loss. The problem is trivial in a single good market: price is set at average cost. Another classic paper that deals precisely with the optimal behaviour of a multiproduct public firm subject to a revenue constraint (such as a break-even rule) is Boiteux (1956). He derived "a formula for optimal pricing that appears to be virtually identical to those derived in the treatment of the Ramsey tax problem, e.g., that there should be an equiproportionate reduction in consumption, along the compensated curve from the level that would have ruled if price had been equal to marginal cost" (Atkinson and Stiglitz 1980, p. 459).
3.2.2.

The theoretical framework described above does not always fit into the actual operating of a public utility. In the real world, opportunity considerations could induce the public decision makers to adopt pricing policies which favour the consumption of some goods with respect to others. In a multiproduct firm this is done by cross-subsidizing some goods. "The cross-subsidization problem of a multiproduct enterprise refers to the relation between the revenues and the costs that are attributed to the individual goods or to combinations among them" (Böls 1986, p. 192).

Consider a public firm producing \( N \) goods and consider \( n \) goods \( (n \leq N) \). Let \( z \in \mathbb{R}_+^N \) be the vector currently produced by the public utility. Let \( s \in \mathbb{R}_+^N \) \( s = (s_1, \ldots, s_N) \) be the following vector:

\[
\begin{align*}
    s_i &= z_i & \text{if } i \leq n \\
    s_i &= 0 & \text{otherwise}
\end{align*}
\]

Let \( R \) and \( C \) be the revenues and cost functions of the public enterprise whose behaviour is constrained by a break-even rule:

\[
R(s(p)) = C(s(p))
\]

18
where \( p \in \mathbb{R}^N \) is the price vector. We can identify the sub-
subsidizer (subsidized) subsets of goods according to whether
\( R(s(p)) \) is greater (smaller) than \( C(s(p)) \).

It is worth noting that this definition is independent
of the form of the cost function. In other words, a good or a
subset of goods is a cross-subsidizer if it can "go it alone"
and make positive profits. Moreover, the break-even rule case
is a particular case of Ramsey pricing, so that there may well
be cross-subsidization even if the above equality does not
hold (see Bös 1986, pp. 193-4 for this extension, and Faul-
haber 1975 for a game-theoretic formulation).

Another interesting question arises when the public firm
interacts with private firms in a competitive markets and mo-
nopolizes another market: under what conditions is it socially
optimal for the public firm to undercut the competitor in the
first market? We know from Arrow (1983) that there is cross-
subsidization form a monopoly to a competitive market if
prices are higher in the former than they would be if the lat-
ter were abandoned by the public firm. Indeed public firms
tend sometimes to undercut competition, making up the losses
in the competitive markets by increasing prices in the monopo-
lized industry. The argument about cross-subsidization for
profit constrained regulated public enterprises is based on
the fact that under regulation there are usually unexploited
opportunities for monopoly profit. Therefore, a loss in the competitive market will lead to recoupment in the monopolized market.

An answer to the initial question is contained in the following proposition (Sheshinsky 1986, p. 1261): "A sufficient condition for undercutting competition to be socially desirable is that revenues in the competitive markets cover incremental costs when output in the monopolized market is at the 'stand alone' level". This is true for a Ramsey firm (i.e., a firm which maximizes social welfare), but similar results hold also for profit maximizing monopoly and they help to test whether regulated firm diverges from the (second-best) social optimum or engages in forms of predatory pricing.

3.3. Public firm as an instrument for the internal regulation of an industry.

3.3.1. When the correction of market failures is pursued through the operating of a public firm two main groups of issues seem worth mentioning.

The first group can be summarized under the heading of "controlling the public managers' behaviour".

"At first glance, the principal-agent distinction can readily be applied to public enterprises: the government is
the principal, and the managers of public enterprises its agents. The goal of government is to use the enterprise to maximize social benefit by a social welfare function which defines the goal to be pursued and the relative weights to be given in trading one goal off against another. Enterprise managers may have their own set of goals, but the way for government to resolve this potential conflict parallels precisely the solution that has been identified for private principals who contend with managers with distinctive interests of their own: provide a set of incentives that, insofar as it is possible given the costs of monitoring and enforcing, aligns the interests of managers with the goals of government" (Levy 1987, p. 77).

The principal-agent approach is certainly fruitfully applicable the to relation between public authority and managers of a public enterprise (an analysis conducted along the principal-agent approach is in Chamley, Marchand and Pestieau 1986 paper), but, as also Levy points out, it presents a non negligible shortcoming: "the notion that the government or the minister is the principal and that the enterprise is the agent is misleading.... The state is not a person, not even a single organization. It acts through a variety of ministers, legislators and civil servants, who are themselves agents of the general public. These different agents invariably see their mission as different from one another. Their goals are rarely,
if ever, stated explicitly, and trade offs among them are not agreed. Thus different agents give the enterprise conflicting parallel commands" (Aharoni 1982, p. 68-69).

In the survey contained in the next Sections this problem is ignored; in the models presented there are no principal-agent complications: no problem arises in implementing the public objective function at the managerial level, or alternatively, public managers do have the same objectives as the public authority.

3.3.2.

The second group of issues deals with the efficiency of public with respect to private production. There is a widespread opinion that incentives for efficiency are greater with private enterprises, even if regulated, than with public ones.

The reason leading to this belief are various. In this place, Aharoni's point seems relevant: if what he says is true, then we can imagine the managers of a public firm fed with a multitude of instructions different and often conflicting with one another; if they keep changing the policy of their firm according to the instructions they receive, inefficiency and waste are the inevitable outcome. The ensuing organizational inefficiency is but augmented by changes of the government following a general election.
A second factor put forward when claiming that public firms are less efficient is that they are subject to less intensive competition from rival firms. This observation misses the point, in that, if there is a reduced efficiency, then, in our view, it is due to the absence of competition in itself, as underlined by Selten (1986), and not by the fact that the firm is a public or a private one. In other words, given that there are, at least in Western Europe, certainly more public than private monopolies, then one does observe poorer performances in the public sectors.

A third fallacious argument in the direction of public inefficiency relies upon the observation of public firms' budget losses. If a perfectly efficient public firm is instructed to price at marginal cost, there are cases when this requires pricing below average costs, as it has been mentioned in Section 3.2.1, and operating at a loss. A more subtle development of this point is the following: since public managers can be allowed, in certain circumstances, to make negative profits, a budget loss in a public firm could be attributed to the application of the marginal cost pricing rule rather than to real inefficiencies.

Although we are aware that industrial performances experienced in some public enterprises in the past (the cited Italian industrial policy in the '70s) support the claim of lesser efficiency of public firms, we believe that this claim
is weak both from a theoretical and from an empirical point of view.

The arguments for inefficiency are not convincing, and the empirical evidence is far from being uncontroversial.

Borchering, Pommerenehne and Schneider (1982) show that production by private enterprises in five countries is more cost efficient than production by public enterprises. On the other hand, Pescatrice and Trapani (1985) obtain results that "indicate that publicly owned electric utilities perform better than their privately owned regulated counterparts.... A cost differential of 24-33% was estimated for the sample employed" (p. 274). A similar study in the US electric utilities by Atkinson and Alvorsen (1986) reaches the conclusion that private and public firms are equally cost inefficient. Färe, Grosskopf and Logan (1985) "find that the publicly and the privately owned utilities are not significantly different in terms of the overall allocative and overall technical efficiency measures. On the other hand [they] find that publicly owned utilities have better ratings in terms of purely technical efficiency, but are worse than privately owned utilities in terms of congestion and scale efficiency" (p. 100). An opposite result of greater efficiency by private firms in the water industry is found by Bruggink (1982).
In the next Section we review the recent developments in the analysis of mixed oligopolies. Table 2 provides an example of the relevance of mixed oligopolies in the French economy. The situation is similar in other Western European countries.

In our survey we consider the contributions where the emphasis is on the strategic interaction in oligopoly markets. Some of the problems described in the two Subsections above should however be considered in evaluating the performance of public enterprises, that is the extent to which they achieve the objectives which have been set for them. Assessments on the performance of publicly owned firms have received a great deal of attention in recent debates about the social advantages of (de)nationalization, privatization, and so on. Leaving aside measures of consumers' satisfaction, two main indicators have been used to assess public enterprises: financial performance and partial productivity statistics. The former may be captured by means of the rate of profit; the latter by means of the ratio of output to labour. However, the meaningfulness of these indicators is highly disputable; furthermore, as the empirical evidence shows, comparisons between the public and private firms' performance have not provided robust answers on their relative efficiency and capability to achieve the goals which are asked from them.
Table 2


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<td>EXCL.TAX</td>
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<tr>
<td>Iron, steel, minerals, and metals</td>
<td>45.5</td>
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<tr>
<td>Mechanical and electrical engineering (incl. armaments, automobiles and shipbuilding)</td>
<td>37.3</td>
</tr>
<tr>
<td>Chemical, glass and pharmaceuticals</td>
<td>31.6</td>
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Drawn from Parris, Pestieau and Saynor 1987, Table 2.8, p. 41.

4. The game theoretical approach to mixed oligopoly

4.1. The general setting

In this section we survey the papers dealing with the problems arising in a market where both privately and publicly owned firms are active on the supply side. There are not many of them, as Merrill and Schneider's seminal paper (1966) has only recently received the attention which it deserves.

Their paper was characterized by particular assumptions, and by results which seem neither general nor robust with respect to variations in the hypotheses. Despite these flaws, Merrill and Schneider's contribution is an important one, for the novelty of the idea of letting private and public firms
operate in the same market. The works reviewed below all consider situations analogous to that studied in their paper, and the general setting in which the analysis is carried over in these works can be described in the following way.

4.1.1.

There are $m$ private firms, and $(n-m)$ public firms, which are engaged in the production of a good sold in a market and bought by consumers who have no market power.

Let $A_j$, $i=1,\ldots,m$ be the set in which private firm $i$ can choose its actions, and let $A_j$, $j=m+1,\ldots,n$ be the correspondent for the public firm $j$. The payoff functions of the private firms are assumed to be the profit functions, defined as functions from the product space of actions into $\mathbb{R}$, the set of real numbers:

$$\Pi_k : \prod_{i=1}^{m} A_i \times \prod_{j=m+1}^{n} A_j \rightarrow \mathbb{R} \quad k = 1,\ldots,n$$

where $\Pi_k(a_1,\ldots,a_m,a_{m+1},\ldots,a_n)$ is the profit obtained by the $k$-th private firm when the actions chosen by the $n$ firms are given by $a_1,\ldots,a_n$, $a_h \in A_h$, $h=1,\ldots,n$.

The definition of the payoff functions of the public firms allows us to introduce a hypothesis which has been assumed in all the papers that deal with mixed oligopolies: we refer to the absence of any "incentive problem" as far as the
choice of the actions of the public firms is concerned: public firms do not have payoff functions of their own. As we said in an earlier Section, public managers accept any order which they receive from the public authority that entrusted to them the management of the public firm; if they do have different objectives from those of the public authority they are not able to undertake any action towards these ends. This simplifying assumption aims at separating the issue of the principal-agent problem from the analysis of mixed oligopolies: the problem dealt with by the public authority is that there are some private firms in the market, and not the fact that the actions of public managers are not wholly visible by the public authority itself.

Given this hypothesis, it is meaningless to talk about the objective function of the public firms: it is more appropriate to consider instead the payoff function of the public authority.

\[ W: \bigotimes_{i=1}^{m} A_i \times \bigotimes_{j=m+1}^{n} A_j \rightarrow R \]

where the choice set of the public authority is \( A_j \), \( j=m+1, \ldots, n \).

Most of the papers model a market where only one a public firm is active, in which case, of course, one can safely speak of the payoff function of the public firm, and
given the absence of incentive problems, it coincide with the public authority’s payoff function. This can no longer be done when there is more than one public firm, and the framework we are presenting is general to allow for the possibility of handling the case of a plurality of public enterprises as well.

We are therefore left with an \((m+1)\)-person game, the players of which can be divided into two groups: \(m\) private firms, aiming at the maximization of their profit, and one public authority, aiming at the optimization of some social target, who, towards this target, chooses the strategic actions of the \((n-m)\) public firms.

Another common feature of most of the papers we are considering is the nature of the strategic variable for the players: it is assumed that agent choose quantities rather than prices. One may object that this assumption does not always catch the observed behaviour of actual firms, but it is a widespread assumption in the analysis of homogeneous oligopolistic markets. This is probably due to the fact that the assumption of Cournot competition allows a simple modelling of the interaction between oligopolists, which does not result in the unpalatable conclusion one gets using Bertrand competition, namely, the same outcome as in a perfectly competitive market. Moreover, Kreps and Scheinkman (1983, see also Osborne and Pitchick 1986) have shown how Cournot com-
petition leads to the same outcome as a more sophisticated game where price is the final strategic variable of the firms (a related result is obtained by Boyer and Moreau 1985).

There is another reason why the analysis is preferably carried over in the framework of quantity competition; there is a simple result in De Fraja and Delbono's paper (1986 a), which shows that, if firms' marginal cost is constant, although possibly different across firms, the maximum welfare can be obtained by the public authority by simply imposing on the the public firm the rule of pricing at marginal cost. This is the best thing to do independently of the relative efficiency of the private and public firms. As we show in Section 4.5, therefore, the problem of the optimal behaviour for a public firm in a mixed oligopoly for a homogeneous good when firms compete in prices is solved for these fairly general conditions.

On the other hand, to the best of our knowledge, no paper examines whether and how the analysis changes if product differentiation is introduced (the effect of product differentiation being that of eliminating the discontinuities in the payoff functions which lead to Bertrand's result). This observation suggests what could be an interesting line for future research in this area.
4.1.2.

The general framework outlined above can thus be narrowed in the following way.

Let a market for a homogeneous good be given, the inverse demand for which is:

\[ p = p(Q) \]
\[ Q = q_1 + \ldots + q_m + q_{m+1} + \ldots + q_n \]

where \( p \) is the price of the good as a function of \( Q \), the total quantity supplied in the market; \( q_h \) is the quantity supplied by firm \( h, h = 1, \ldots, n \).

The technologies available to the firm are represented by their cost functions:

\[ c_h = c_h(q_h) \quad h = 1, \ldots, n \]

\( c_h \) being the total cost incurred by the \( h \)-th firm in the production of \( q_h \).

The following hypotheses on the shape of the function \( c_h \) can safely be assumed:

for all \( h = 1, \ldots, n \):
$c_h : \mathbb{R}_{++} \to \mathbb{R}_{++}$ is twice continuously differentiable

\[
\frac{dc_h(q_h)}{dq_h} \geq 0 \quad \text{for all } q_h \in \mathbb{R}_{++}
\]

e.g., marginal cost is non-negative. The payoff to the private firm $i$ is given by its profits:

\[
\Pi_i(q) = p(\sum_{j=1}^{n} q_j)q_i - c_i(q_i) \quad i = 1, \ldots, m
\]

The payoff to the public authority is a function $W$,

\[
W = W(q_1, \ldots, q_n)
\]

the features of which are going to be discussed in the next section.

4.2. Objective functions for the public firms

As the analysis is a partial equilibrium one, the most immediate assumption, as far as the arguments of the public authority's payoff function are concerned is that the welfare is a function of consumers' and producers' surpluses.

To define $W$ more precisely, consider the following
function:

\[ U : \mathbb{R}^{n+1} \rightarrow \mathbb{R} \]

where

\[ U(\pi_1, \ldots, \pi_m, \pi_{m+1}, \ldots, \pi_n, \int_0^Q (p(t) - p(Q))dt) \]

is the social welfare when firm \( h \)'s profit is \( \pi_h \), \( h = 1, \ldots, n \) and the definite integral is consumers' surplus. Now, \( W \), the payoff function of the player public authority as described abstractly in Subsection 4.1.1, can be defined as the composition of \( U \) with the function that sends the vector of the strategic choices of the players into the vector of profits and consumers surplus:

\[ \Pi : \prod_{i=1}^m A_i \times \prod_{j=m+1}^n A_j \rightarrow \mathbb{R}^n \times \mathbb{R}_+ \]

\[ \Pi : (q_1, \ldots, q_n) \mapsto (\pi_1(q), \ldots, \pi_n(q), \int_0^Q (p(t) - p(Q))dt) \]

\[ W = U \]

It seems natural to us that the following two assumptions on the shape of the function \( U \) should be introduced:

a) \( U \) is an increasing function in all its arguments, and

b) \( \lim_{\Pi_j \rightarrow -\infty} U(.) = -\infty \)
Assumption (b) simply states that there is no bound in the loss of welfare due to a budget loss of a public firm increasing without limit. If (b) does not hold, but (a) does, the public authority could increase social welfare by means of huge deficits of the public firms.

This representation of the social welfare is the more general one that can be thought within a partial equilibrium setting. In all the models surveyed, the function \( U \) is given a precise shape consisting in the sum of producers' and consumers' surpluses.

The only exception to the rule of the maximization of the sum of the surpluses is in Merrill and Schneider's paper: they assume that the objective of the public authority is the maximization of the total amount of the commodity produced. Obviously enough, they impose a budget constraint on the public firm, in order to avoid that it produces an infinite amount of that good.

Merrill and Schneider do not give any rationale for their choice of such an objective, and, indeed, it seems to us that there is not a compelling one; it could however be argued that the quantity of the commodity supplied is a proxy for employment, at least if the production coefficients are relatively fixed in the short run. But it would be employment in that particular productive sector, and there could be more
beneficial effects if the public authority tried to push the production up in some different sectors. In any case, the analysis of the effect of public action on unemployment cannot be studied in a partial equilibrium analysis; for this kind of research the use of a general equilibrium approach really seems to be a necessity.

4.3. Technologies and cost structures.

The main interest of the papers surveyed here lies in the nature and the effects of the competition between private and public firms, and the technological aspects of the market become somehow of lesser importance. The hypotheses on the cost functions tend therefore to be as simple as possible in order for the analysis not to become overburdened.

Hence, most papers assume that all firms are endowed with a technology which imposes a fixed cost and a constant marginal cost, not necessarily the same across firms. In De Fraja and Delbono (1986 a,b) it is considered a cost function, the same for all firms, such that the marginal cost is increasing with respect to the quantity produced. Beato and Mas-Colell (1984) consider a more general approach, and obtain different results in correspondence of the various shapes of the cost function.

We feel that the issue of the technology available to
the producers is an important one, and that, as it is shown by Beato and Mas-Colell, the results may vary considerably according to the hypotheses considered. The analysis of the effects of alternative cost functions should be carried over, and the kind of public intervention which is more likely to be beneficial should be ascertained in the different cases. Finding out whether the public firms are doing the right thing in the right market will then be a question of empirical research.

As far as this topic is concerned, it can safely be claimed that more research is needed to develop the analysis for the cases of less simple and more realistic cost functions.

Another hypothesis which seems to be lurking behind some of the papers is that of the relative inefficiency of the public firm(s) with respect to the private ones. As we have said, there does not seem to be enough empirical evidence to take this inefficiency for granted.

4.4. The move order of the game

If the papers present a fairly similar set of hypotheses as far as the objective of the public firm and the cost structure are concerned, they display a wide range of possibilities for the move order of the game. All the following pos-
sibilities have been considered in the literature: private and public firms play a Cournot-Nash game, that is, they assume each other’s strategic choice as a datum in taking their decisions; the public firm is a Stackelberg leader, and the private firms take its decision as given; and, vice-versa, the public firm is a Stackelberg follower, and take as a datum the strategic decision of the assumed unique private firm. The comparison between the cases in which the public firm is a Stackelberg leader and a Stackelberg follower is carried on by Beato and Mas-Colell, and leads to interesting conclusions (see below, 5.2).

In all the above cases the number of firms operating in the market is fixed: the range of possibilities is widened by allowing entry and exit of new firms, either public or private. The usual warnings about careful modelling when entry by new firms is allowed hold here as well, given that all the models proposed are static; care must be taken, particularly if policy conclusions are to be drawn from the theoretical analysis.

The most insightful game theoretic way of studying entry by new firms in traditional oligopoly markets is the use of multistage games. Sertel’s paper (1987) considers the problem of entry, but in his approach, a single period game is modelled. In Ware’s paper (1986), on the contrary, entry is modelled explicitly as a two-stage, two-player game; both
cases are considered: public firm as incumbent and private firm as potential entrant, and vice-versa.

Another kind of behaviour deserving consideration is the possible formation of collusive agreements or cartels among the private producers: this problem is tackled by Sertel, and, as intuition could suggest, the results obtained are similar to those obtained when there is only one private firm.

5. Summary of the main results

5.1. Nationalization.

A sketch of the results obtained in the papers dealing with the problem of the public firms' behaviour in a mixed oligopoly is given in this Section.

As we have said, the seminal contribution is Merrill and Schneider's: they study a market where the price of the commodity produced may be different across firms, and capacity constraint are imposed upon the firms. The goal of the public authority is the maximization of total output, and they show that there is only one way to achieve industry full capacity production: the nationalization of the whole sector.

This result is confirmed by all the other works: the first best social optimum is reached when the whole industry is under government control. In such a situation, price equals
marginal cost, and the number of active productive units is determined by the government following the rule of equating the market price to the minimum average cost. This conclusion parallels the results obtained in the contestable markets literature (see, for instance, Baumol and Fisher 1978).

This conclusion seems to be contradicted in the paper by Cremer, Marchand and Thisse; however, the relative inefficiency of the nationalized industry depends upon the hypotheses that the public firms pay a premium to their workers, and moreover that this premium does not enter into the social welfare. We think that both these hypotheses need justifying.

That nationalization is in theory the most efficient outcome is no novelty: the need for an analysis of mixed markets lies in the fact that it is often impossible, for political or economic reasons, to nationalize an entire sector, or, simply, the government does not wish to do so.

5.2. Pricing rules.

Although most papers consider Cournot competition, there are results available for the case in which the public firm sets the price.

Sheshinsky (1986) studies a public firm producing a
quantity \( z \) and one private firm producing a quantity \( y \) of a homogeneous good: thus, \( x = z + y \) is the industry output. A decision rule for the public enterprise is a function \( f: \mathbb{R} \rightarrow \mathbb{R}_+ \), belonging to an admissible set \( F \), assigning a public output \( f(y) \) to every private output \( y \).

A first question deals with the set \( F \); examples of \( f \) belonging to it are the marginal cost pricing rule and the constant public output rule. If no restriction is imposed on \( F \), and the public firm has complete information, then the first best solution of the public firm's problem can be attained by the following rule:

\[
f(y) = \max(x^* - y, 0)
\]

where \( x^* \) is the first best level of output. This rule results in the private firm producing the socially optimal level of output if it turns out to be credible, irrespective of possible losses of the public firm.

An analogous conclusion is reached in De Fraja and Delbono (1986 a), where the situation in which two firms each producing at constant marginal cost with the private one producing at a lower marginal cost is studied. Here the public authority can reach the first best efficiency by announcing that it will sell at a price which maximizes the social welfare (\( p^* \) in Figure 1); this price allows the private firm to
make positive profit. In Figure 1, below $p_1$ the most efficient firm does not produce: the monopoly price is $P_m$. If the welfare function depends continuously on profits, it is discontinuous in $p_1$: below $p_1$ only the public firm is producing and it makes a "big" negative profit. Above $p_1$ the most efficient private firm makes a small positive unit profit, and, since its total profit is increasing with the quantity sold, it wants to satisfy the whole demand, therefore the public firm makes zero profit. Thus, the optimal strategy for the public firm is to announce that it will sell at the price which maximizes the social welfare, $p^*$. The resulting outcome will be the most efficient firm satisfying the entire demand.

![Figure 1](image-url)

*Private firm's profit and welfare as functions of price.*
Sheshinsky goes on to discuss two second best decision rules which can be adopted when the first best is not attainable, i.e., the marginal cost pricing and the constant public output. The comparison between the social welfare obtained in the two cases yields ambiguous results.

The objective of Beato and Mas-Colell (see also Gilsanz 1987) is to find out if there are circumstances in which the simple rule of pricing at marginal cost could be fruitfully followed by the public firm(s). They consider two firms and they obtain that "under constant public marginal cost, price regulation is superior to quantity regulation" (p. 91); this is the same conclusion shown above in the discussion of Figure 1.

Another finding of theirs regards the comparison between the cases in which the public firm plays the role of Stackelberg leader and follower. If the public firm has no capacity limit, and its cost function is concave, then welfare is higher when the public enterprise is the follower; its optimal strategy is to price at marginal cost (result 9, p. 93).

5.3. Cournot competition.

In the De Fraja and Delbono (1986 a), in a rather general setting, it is proved that with quantity competition
is always better for the public firm to be a Stackelberg leader than to play a Nash game with the private firms. They then consider a more specific model with linear market demand, increasing marginal cost, and a given number of firms. The main results are the following:

(i) If the number of firms is sufficiently large, if the public firm cannot get the leadership in a Stackelberg game, - which would be socially preferable - but has to play a Cournot-Nash game, then a higher welfare can be obtained if the public firm is instructed to maximize profits instead of worrying about social welfare.

(ii) In the Stackelberg equilibrium of the socially preferred game - public firm as leader - the price is set at a level higher than the public firm's marginal cost.

The approach taken by Ware (1986) is slightly different. He considers the case of a market, which, from being at first a natural monopoly, can accommodate for entry of another firm, because of an increase in market demand. Both cases are considered: an incumbent public firm, and a private one. An interesting conclusion is that, in non-pathological circumstances, if the public firm is committed to social welfare maximization, then it cannot induce a private firm to enter
the market, which would lead to higher social welfare, and which would be permitted by a profit-maximizing public firm. In such circumstances, one may think that the privatization of the public firm would increase welfare; this is not true if capacity is fixed: privatization can only reduce social welfare.

The paper by Cremer, Marchand and Thisse considers again a market with a given number of firms and in which the technology determines a fixed cost and constant marginal costs. This case has the trivial solution for the public firm of pricing at marginal cost, but in this case there would be a loss given by the fixed cost. To avoid this conclusion, a budget constraint is imposed. In their model, for most parameters, social welfare maximization is obtained through the nationalization of a single firm; in the remaining cases through the nationalization of the whole industry. The existence of situations in which social welfare is lower in a nationalized sector than in a mixed oligopoly is due to the assumption of theirs, which we mentioned before, of a premium paid by public firms to their employees; this premium is not included in the computation of social welfare. The authors do not give an explanation for this premium, and without it nationalization is always better.

Finally, Sertel's paper assumes throughout that the public firm is a Stackelberg leader in a quantity setting.
game, where technology exhibits constant marginal costs, and where the public firms are less efficient than the private ones. Sertel compares the results for three different kinds of behaviour of the private sector: (a) Cournot-Nash behaviour with free entry from outside, (b) collusive behaviour, i.e., the formation of an explicit cartel, and (c) limit pricing behaviour, in which case only one private firm is active.

In this last case there is no room for a mixed oligopoly: either the public firm keeps on threatening entry, or it does enter, prices at marginal cost, and pushes the private firm out of the market. Building up a mixed oligopoly is instead the best thing to do — from a social welfare point of view — if private producers form a cartel. When private firms play a Cournot-Nash game among them, nationalizing one of them is the more likely to have beneficial effects the fewer are the private firms operating in the market.

6. Concluding remarks and further extensions.

In the light of the above discussion, we may argue that a general theory of mixed oligopolies is not yet available. The various models proposed in the literature differ in the basic hypotheses: there is not yet a unified and accepted framework in which the analysis is carried over. In our opinion this is due mainly to the following reasons. Firstly,
the coexistence of private and public firms in the same industry has captured the attention of economic theorists only very recently, and none of the approaches proposed seems truly pathbreaking. Secondly, we still miss an adequate set of empirical findings indicating the most appropriate theoretical framework to tackle this problem. However, the lack of a general theory does not mean that the literature has not provide any answer. There are indeed some interesting results which have been reviewed in Sections 4 and 5.

A general comment on the various models is that their conclusions are rather sensitive to the different assumptions on technology, demand, move order, and so on. Thus, in some sense, it is not possible to compare them; each of them considers a particular set of questions and the markets analysed in the various papers are in fact different markets.

There is however a broad indication which seems to emerge from the models considered above: in general, it does not seem to be optimal for the public authority to instruct the public firms to take decisions that result in the equality between price and marginal cost. There are two sets of reasons that lead to this conclusion.

The first is the presence of a budget constraint for the public firm: for some plausible technologies, pricing at marginal cost would imply negative profits for the public firms. Secondly, in some models where there is no budget constraint
to be fulfilled by the public firms, it still turns out that the maximum social welfare is reached in a situation in which the price is higher than public marginal cost. This is shown in De Fraja and Delbono (1986 a), but we suspect that this conclusion holds also in more general settings. This is a line of research towards which future efforts should be directed.

Keeping in mind the main shortcomings of the models reviewed in Sections 4 and 5, one is able to identify other routes for future research in the field. Without a budget constraint imposed by the public authority, the problem arises of financing the loss. This question cannot obviously be dealt with in a partial equilibrium approach. A general equilibrium framework is the most appropriate setting within which the question may be addressed. Such a question would also allow one to take into account the cross-effects among the various markets. A very ambitious project would therefore be a general equilibrium model where goods are produced by private and public firms and the public authority maximizes total social welfare by deciding the optimal behaviour of its production units.

Another important question which cannot be ignored, and which can seriously alter the conclusions, is the informational requirement needed by the public authority for the implementation of its strategies. These requirements ultimately amount to the knowledge of firms' technologies and market
demand functions. It is usually the case that private entrepreneurs have no incentive to truthfully reveal the information they possess; this arises the following question: what should a public firm do when it knows only approximately the features of the market in which it operates?

7. Some related literature

Excellent introductions to the public enterprise economics are Böls (1986) and Rees (1984); for a broader treatment of public sector economics see Stiglitz (1987).

An interesting review of second-best pricing rules in the Boiteux tradition is provided by Guesnerie (1980).

Some interesting papers dealing with optimal pricing decisions in imperfectly competitive markets are the following.

Hagen (1979): he shows that piecemeal policy recommendations based on the marginal cost pricing rule may be socially inefficient. This paper might be seen as an extension of Baumol and Bradford (1970).

Spencer and Brander (1983): they take the same question as Hagen for publicly produced inputs when there is a profit constraint and suggest a measure of downstream industry distortions for the purpose of efficient pricing.

Ware and Winter (1986): they model the public pricing
decision in a game theoretic framework when there exists some form of market power outside the regulated sector. They provide a model which unifies two separate lines of inquiring: the Ramsey problem for a multiproduct firm and the game models investigated in Sections 4 and 5 above.

Finally, three interesting papers which deal with incentives and control in public firms are Gravell (1982), Vogelsang (1983) and Levy (1987). The first one is a helpful survey of previous contributions. The second suggests a performance index and a reward structure according to which public managers should be treated as if they were private entrepreneurs if the welfare optimum is the public authority goal. The last paper discuss the validity of a principal-agent framework to analyse the internal organization of public enterprises.
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