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Deciding fast and slow

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Abstract

Empirical evidence suggests that choices are affected by the amount of time available to the decision maker. Time pressure or a cooling-off period (mandatory delay of choice) changes how choices are determined. Yet, few models are able to account for the role of available time on decisions. This paper proposes a dual-self model in which a fast and a slow self bargain to decide: the longer is the decision process, the higher is the bargaining power of the slow self when deciding. A large variety of behaviors observed under time pressure or cooling-off can be explained by our model. Quantitative predictions concerning the effect of nudging through time manipulation are also provided. We characterize the model imposing testable conditions on revealed preferences combined with non-choice data.

KEYWORDS: Time pressure, dual-self, decision time, nudging, cooling-off
JEL CLASSIFICATION: D01, D03, D11, D81

1 Introduction

Many decisions are made under time pressure, others only after a given amount of time has elapsed (for example, after a cooling-off period, Camerer et al., 2003). Intuitione and experimental evidence[1] suggest that the decision process and preferences are affected by the amount of time available to the decision maker. Yet, few theoretical models are able to take into account the effect of decision time on choices. To fill this gap, we propose a model integrate the length of the decision process into a choice rule. The main ingredient is the *contemplation time* (CT). It measures the amount of time spent facing a menu of options before a choice is made. The CT is stochastic and cannot be controlled by the individual.[2]

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[1]The experimental evidence of the effect of time pressure on choices is large, for example time pressure affects: choice under risk (Zur and Breznitz, 1981), purchasing behavior (Park et al., 1989), financial decisions (Nursimulu and Bossaerts, 2013), intertemporal choice (Lindner and Rose, 2016).

[2]Uncertainty of the length of contemplation time is a natural assumption in many choice situations: the time spent queuing at the checkout, the time waiting for a bus, are all random. Moreover, random contemplation times

Differently from other non-choice data (e.g. eye or mouse movement) the contemplation time is cheap and easy-to-measure and it can be readily manipulated in both field and laboratory experiments. In our model, a choice is the outcome of a process in which two selves, the "fast" and the "slow", bargain to decide. The length of the contemplation time acts as a shock to the bargaining power between them: as time passes, the weight of the "slow" self becomes larger. The model can be applied to situations where other "static" models cannot, above all, choice under time pressure. The model offers novel predictions concerning the effect of nudging through time manipulations (e.g. cooling-off periods or time pressure) providing quantitative bounds to the effectiveness of such policies. Third, the model can be used to refine the estimation of preference parameters, such as risk attitude or discount rates, in experiments. Indeed, most experiments allow a limited amount of time to decide and the estimated parameters may be affected by such limitation. Moreover, the model gives new flavor to a large class of dual-self models, assigning a causal role to a non-choice datum, for example:

S1-S2. The slow and the fast self may represent System 2 and System 1 of Kahneman (2011). In our model, S2 increases its weight on the final choice as the contemplation time grows. This interpretation encompasses many alternative labeling of dual-self models: intuitive/contemplative (Rubinstein, 2013; Cerigioni, 2016), doer/planner (Thaler and Shefrin, 1981), affective/deliberative (Loewenstein and O'Donoghue, 2004), selfish/altruistic (Rand et al., 2012), implicit/explicit preferences (Friese et al., 2006).

Temptation. The fast self can be interpreted as the normative, while the slow self represents temptation in a dual-self model in the spirit of Gul and Pesendorfer (2001); Fudenberg and Levine (2006). Indeed, the model is observationally equivalent (see Section 2) to a costly temptation model, where the length of the contemplation time increases the cost of resisting temptation. This can explain the empirical findings of Houser et al. (2008). They study the behavior of customers queuing at the checkout of supermarkets and found that each additional minute spent queuing raised the probability of buying tempting items (snacks) of 0.17.

Household choice. In the collective model of household choice of Chiappori (1988) and Cherchye et al. (2007), choices depend on the bargaining power within the household. Our model may be interpreted as the collective model where the bargaining power of

can arise as response times. Indeed, when the individual has an unlimited amount of time to perform a choice, contemplation times are indeed *response times*. Response times are typically stochastic (see Luce, 1986; Webb, 2015).

a member is directly proportional to the length of the decision process. The "slow member" increases her bargaining power as the contemplation time passes.

In the first part of the work, our observables are choices *and* the time at which the choice is made. For example, the modeler observes an individual entering a queue at the checkout of a supermarket and selecting an item after 30 seconds (as in Houser et al., 2008). Two rationality requirements, a form of WARP and a No-cycle condition, imposed to the extended revealed preferences, characterize our model: In the second part of the work, we assume that individual choice are observed only at the aggregate level, hence without knowing the time at which they are made. For example, the modeler observes scanner data and he knows the distribution of queue at the checkout of a supermarket. In this case, we exploit the stochastic nature of choice: indeed, ex-ante uncertainty about the length of the contemplation time implies stochastic choice from menus. We assume that observables are choice probabilities *and* the distribution of contemplation time. We characterize our model through observable conditions on the choice probabilities. In particular, our stochastic model belongs to the class of Single-Crossing Random Utility model (SCRUM), introduced by Apesteguia et al. (2016). We identify a sufficient condition characterizing our model within the SCRUM. Section 5 focuses on the main application of our model: choice under time manipulations. We can explain the behavior observed under time pressure in laboratory or field experiments, as well as, the effect of cooling-off periods (Camerer et al., 2003). For example, we can explain the differential behavior observed under time pressure in different domains: risky choices (Zur and Breznitz, 1981), financial decisions (d'Acremont and Bossaerts, 2008), altruism and selfishness (Rand et al., 2012), purchasing behavior (Friese et al., 2006) and intertemporal choice (Lindner and Rose, 2016). In the context of temptation, our model can explain a class of behaviors related to waiting when facing temptation. These include rehab enrollment (Redko et al., 2006), participation to Supplemental Nutrition Assistance Program (SNAP) (Gennetian and Shafir, 2015) and the effect of time limited promotion as a form of nudging (Duflo et al., 2011). Lastly, we apply our model to intertemporal choice and we show how dynamic preference reversal can occur without assuming hyperbolic discounting. Indeed, when elements in the menu are dated outcomes, i.e. (p, T) representing a reward p at time T, at the beginning of the contemplation time, a later/larger option may be preferred to an earlier/smaller one, however, as contemplation time goes by, the earlier/smaller option becomes more attractive. Differently from hyperbolic discounting, preference reversal follows from the increasing cost of temptation. Despite the rationality content of the conditions characterizing the model, apparently non-rational behaviors, such as dynamic preference reversal or varying risk attitude, are compatible with our model.

1.1 Related literature

Experiments on the effect of time pressure on choice are popular (see Klapproth, 2008, for a review). The present work provides a simple and flexible model that can explain the behaviors observed in those experiments. Approaches to choice under time pressure assume that time pressure changes the decision process (Payne et al., 1988; Gigerenzer and Goldstein, 1996). For example, according to Payne et al. (1988), under time pressure choice is more "attribute based" rather than "alternative based".[3] In our model, the choice process is not affected by time pressure and it is alternative based. Time pressure affects the balance between intuitive and contemplative preferences. Models based on neural activity can also account for decision under time pressure (Diederich, 2003). Differently from our model, in these models time pressure increases the rates of "mistake". Our paper contributes to the literature studying the *process* of decision making through the use of non-standard data. Recent papers focus on response times (Chabris et al., 2009; Clithero, 2016) or eye-tracking (Reutskaja et al., 2011). Others approaches include non-choice data such as: the time spend learning a stochastic signal (Natenzon, 2015), state-dependent choice (Caplin and Dean, 2015) and intermediate choices (Caplin and Dean, 2011; Caplin et al., 2011). Our model contributes to such literature proposing novel testable restrictions on extended revealed preferences and a model that gives a causal role to a non-choice datum. Our work is also connected to the work of Koida (2016), who studied decision time as a function of the magnitude of the trade-offs in the choice set. Larger trade-offs imply longer decision times. Our approach is symmetric, we consider decision time as exogenously determined, while choice varies as a function of it. It is also possible to imagine a feedback model where large trade-offs increase the decision time that, in turn, affects the final choice. On a different vein, Cerigioni (2016) studied how to elicit preferences when some choices are intuitive and may not reflect pure preferences. In his model, intuitive choices are made when the choice situation is sufficiently similar to previous choice situations. When situations are dissimilar, the individual decides according to her pure preference. Our model has a static nature while Cerigioni (2016) is dynamic. Therefore, the two approaches are complementary. Concerning

[3] "In alternative-based processing, multiple attributes of a single alternative are considered before information about a second alternative is processed. In attribute-based processing, the values of several alternatives on a single attribute are processed before information about a second attribute is processed." Payne et al. (1988).

the revealed preference restrictions, his model predicts identical choices from the same menu, if the individual chooses from the same menu in a row (hence our axiom No-cycle is trivially satisfied). Our model allows for different choices from the same menu as the decision time varies, hence the two models have different observable predictions.

Our paper contributes also to the rich literature of multiple selves modeling of individual behavior. In these models (Thaler and Shefrin, 1981; Laibson, 1997; Gul and Pesendorfer, 2001; Fudenberg and Levine, 2006), different selves of the individual compete to make a the final decision. Differently from these approaches, in our model, the inner battle among selves is mediated by an external device, the contemplation time. Our model focuses on "short" contemplation times. When "longer" time scales are considered, applying the model requires an extra assumption that may be too strong. We assumed that u and v are the same throughout the contemplation time, a plausible assumption for short periods (e.g. minutes, or even days). When the contemplation time is on a different scale (e.g. months), it is plausible to observe variations in u and v. Indeed, Dai and Fishbach (2013) found experimentally that waiting before a choice between delayed rewards increases patience. Their explanation is exactly based on a changing perception of the value of later rewards while waiting. Therefore, an application of the model to longer contemplation time scale, should take into account time-varying utilities.

2 The model: time, selves and choice

In this section we formalize our model. Let Z be a finite set of prizes, a menu A is a non-empty subset of $\Delta(Z)$, the lotteries on Z. We denote $\mathcal{X}$ the family of all menus. Let τ a random time, i.e. a random variable assuming positive values and let $F_\tau(t) = Prob(\tau \leq t)$, its cumulative distribution function. The main actor of our model is defined as

Definition 1. *The Contemplation Time (CT) is the amount of time going from the availability of the menu of options to the choice.*

For example, let A be the menu of snacks surrounding the checkout, in this case the contemplation time begins when the individual enters the queue and terminates when a choice is made or the queue ends. Notice that the time of a choice (selecting a snack surrounding the checkout) may be different from consumption (once decided to buy a snack you cannot consume it until you pay). This distinction is important when the choice set involves delayed options or consumption streams. To clarify, let consider a menu A containing (a lottery over) a consumption stream $c = (c_0, c_1, \ldots)$. Consumption at time 0 is interpreted as occurring after the contemplation time ends. We assume that the CT is stochastic and cannot be controlled by the individual. The contemplation time can be identified with response times when it is not determined by an external device. This identification requires the assumption that response times are not decided by the individual.[4] We implicitly assume that during the contemplation time the individual actually "contemplates" the options. Without additional non-choice data, such as eye tracking, we cannot rule out the case of an individual not focusing on the menu. This is a common problem to all studies using response times, for example.

According to the Contemplation Time Rule, an individual selects an option from a menu A by:

$$C(A, \hat{\tau}) = \underset{p \in A}{\text{argmax}}[u(p) + \phi(\hat{\tau})v(p)] \tag{CTR}$$

Contingent to a realization $\hat{\tau}$ of τ, the contemplation time, the individual maximizes a combination of the functions u and v. $u : \Delta(Z) \to \mathbb{R}$ is interpreted as the "fast" ranking, $u(p) \geq u(q)$ if p is instinctively superior to q. The function $v : \Delta(Z) \to \mathbb{R}$ represents the "slow" ranking, i.e. $v(p) \geq v(q)$, if p is contemplatively superior to q. $\phi : [0, \infty) \to [0, \infty)$ is such that $\phi(0) = 0$ and $\phi' \geq 0$. The assumption that $\phi' \geq 0$ is the main driver of the model. Longer contemplation times increase the weight of the slow self at the moment of choice. The function ϕ accounts for the possibility of subjective time perception, meaning a difference between the physical time and the perceived one.[5] Examples of the function ϕ are $\phi(t) = t$, $\phi(t) = 1 - e^{-t}$ or $\phi(t) = \ln(1 + t)$. Figure 1 illustrates the model in a binary menu and with $\phi(\tau) = \tau$. Let $A = \{p, q\}$, the fact that $u(q) > u(p)$ implies that q is superior to p according to the fast preference, $v(p) > v(q)$ (the slopes of the two lines) implies that p is superior to q according to the slow self. For example p is a chocolate bar and q is a sugar-free chewing gum. If the realization of the contemplation time is short enough, for example $\hat{\tau}$, the individual will choose the sugar-free chewing gum, $C(\{p, q\}, \hat{\tau}) = q$. Differently, a long enough contemplation time, for example $\hat{\tau}'$, implies an excessive cost of resisting the

[4]This is the case for standard formal models of response time, such as those arising from neural activity: the Bounded Accumulation Model (Ratcliff, 1978; Webb, 2015) and Decision Field Theory (Busemeyer and Townsend, 1993). Moreover, it is often assumed in the literature (e.g. Natenzon, 2015). An exception is Fudenberg et al. (2015). Relaxing this assumption would introduce an optimal stopping problem and a dose of sophistication that is difficult to justify. The experiment of Oud et al. (2016), for example, supports the assumption that response times are not chosen optimally. Under a condition of time pressure, subjects spent too much time to make decisions involving small stakes. Allowing for optimal stopping of the response time is an interesting path for future research.

[5]For example, while doing a boring task, time may seem to pass slowly (Block et al., 1980). See also Wittmann and Paulus (2008); Zauberman et al. (2009); Takahashi et al. (2008).

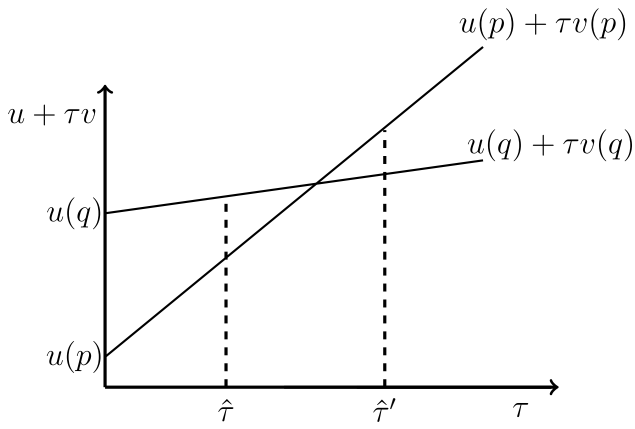

Figure 1: Visualization of the CT rule.

temptation and the most tempting option, the chocolate bar, is selected: $C(\{p, q\}, \hat{\tau}') = p$. It is clear from the previous example why the standard revealed preference analysis may not be able to distinguish slow from fast preferences. Without the time dimension, pure choice data say that p and q are both selected from A. This can be interpreted, for example, as p and q being indifferent for the decision maker. For a given realization of τ, the CT rule can be rewritten as:

$$C(A, \hat{\tau}) = \underset{p \in A}{\text{argmax}}[u(p) - \phi(\hat{\tau})(\max_{q \in A} v(q) - v(p))]$$

the latter is equivalent to the second-period choice of an individual facing temptation according to the model of Gul and Pesendorfer (2001), in which the contemplation time increases the cost of resisting temptation, $\max_{q \in A} v(q) - v(p)$. This provides a clearer explanation for the empirical findings of Houser et al. (2008) in the checkout experiment. However, without observing choice among menus, as in Gul and Pesendorfer (2001), this is only an interpretation. In this paper the focus on the observational restrictions of the model remaining agnostic about the interpretation of the selves.

2.1 Testable implication of the CT rule

In this section we give simple revealed preference conditions that characterize the CT rule. The conditions are stated in terms of observable restrictions on the choice function and the time of the choice. We assume that the modeler can observe choices from the menu and the time at which choices are made. For example, if $p \in \{p, q\}$ is selected at time $t = 3$, we denote it $p \in C(\{p, q\}, 3)$. In general, our revealed preferences are of the form $C : \mathcal{X} \times [0, \infty) \to \mathcal{X}$ and $p \in C(A, t)$ means that p is selected from A at time t. The advantages of focusing on contemplation time rely on the simplicity of its measurement and manipulation in experiments.

Given our extended revealed preferences, we propose the following axiom:

No-cycle For all $A \in \mathcal{X}$, if $p \in C(A, t-s)$ for some $s > 0$ and $p \notin C(A, t)$, then $p \notin C(A, t+s')$, for all $s' > 0$.

If an option is selected at time t and it is not selected at a point after t, there is no possibility that it will be selected again later on. No-cycle imposes acyclic preferences over time: preferences switch can happen at most once as the contemplation time goes by. No-cycle rules out, for example, the following behavior: if $A = \{c, s\}$ is the menu containing candies and sugar-free chewing gum, we cannot observe $(s, 30''), (c, 1'), (s, 2')$. In this choice, sugar free chewing gum s were preferred to candies for queues lasting 30 seconds, however, the preference is reversed after 1 minute and candies are selected. We may interpret this as "revealing" that candies are more tempting than chewing gums. No-cycle implies that a longer contemplation can only increase the cost of resisting temptation, hence going back to the initial preference is impossible and s cannot be selected later. No-cycle is related to the condition introduced in Caplin and Dean (2011), although their observables are different from our, indeed, they focus on intermediate choices. The next axiom synthesizes standard axioms.[6]

t-WARP For all $A \in \mathcal{X}$ and $p \in A$, $p \in C(A, t)$, if and only if, there exists a von-Neumann-Morgernstern expected utility $u_t : \Delta(Z) \to \mathbb{R}$ such that $u_t(p) \geq u_t(q)$ for all $q \in A$.

The t-WARP postulates the existence of a family of expected utilities that rationalizes the choice function conditional on each observed contemplation time, however, it says nothing concerning the relation among choices at different realization of the contemplation time. The following is the main result of this section:

Theorem 1. *$C(A, t)$ satisfies t-WARP and No-cycle, if and only if, there exist expected utilities $u, v : \Delta(Z) \to \mathbb{R}$ and a monotone $\phi : [0, \infty) \to \mathbb{R}_+$ such that*

$$C(A, t) = \operatorname*{argmax}_{p \in A} [u(p) + \phi(t) v(p)]$$

u is unique up to positive affine transformations, v is unique up to a positive rescaling and ϕ is unique with $\phi(0) = 0$.

[6] In particular, t-WARP is equivalent to: non-emptyness, independence (i.e. $p \in C(\{p, q\}, t)$, then $\alpha p + (1-\alpha) r \in C(\{\alpha p + (1-\alpha) r, \alpha q + (1-\alpha) r\}, t)$ for all $r \in \Delta(Z)$ and $\alpha \in [0, 1]$) and continuity of $C(\cdot, t)$ for all t.

The CT rule is characterized by two simple and rational axioms. A time-dependent WARP and a condition limiting the possibility of preference switches. Though based on extremely rational conditions, the model allows for "non-rational" behavior, for example, dynamic preference reversal (see Section 3). Notice that the No-cycle condition can be vacuously true. If there are no preference switches, i.e. for no $t, t' \in [0, \infty)$ and no $A \in \mathcal{X}$, $p \in C(A, t)$ and $p \notin C(A, t')$, then all u_t of t-WARP are ordinally equivalent. Being expected utilities they can be normalized to be all equal. In this case, $C(A, t) = \text{argmax}_{p \in A} u(p)$ for all t, the fully rational and contemplation time independent choice rule.

3 Preference reversal in the short and in the long

The contingent choice rule $C(\cdot, \hat{\tau})$ reproduces in the short term a phenomenon of dynamic preference reversal that is often ascribed to hyperbolic discounting. When elements in A are dated outcomes, i.e. (p, T) meaning a payoff $p \in \mathbb{R}_+$ is payed at time T, the individual may prefer a later/larger outcome over a sooner/smaller, but she may reverse her preference as time passes. This form of time inconsistency may arise under hyperbolic or quasi-hyperbolic discounting or when uncertainty affects future payoffs. Indeed, Dasgupta and Maskin (2005) proved that such dynamic preference reversal may be rational when there is uncertainty about both the possibility and the timing of *realization* of future payoffs. Consider two dated payments (p, T) and (q, T') with $p < q$ and $T < T'$ and assume uncertainty about their actual realization, for example they may disappear. If, in addition, the timing of their actual realization is uncertain (e.g. apples may ripen earlier), it is rational to have a preference for (q, T') at the beginning and to reverse it to a preference for (p, T), as time goes by. More precisely, Dasgupta and Maskin (2005) show the existence of a t^* such that, the later/larger payoff (q, T') is preferred to the smaller/earlier one (p, T) for all $t < t^*$ and the preference is reversed after t^*. We will show how this dynamic preference reversal happens in our model as well since the Dasgupta and Maskin (2005) model can be seen as a particular case of the CT rule.

In the model of Dasgupta and Maskin (2005), the value at time zero of a payment p made at time T is given by the following:

$$U_0(p, T) = \left(\int_0^T e^{-rs} \pi ds + (1 - \pi T) e^{-rT} \right) p$$

where r is the "hazard rate" of disappearance of outcomes and π the probability of early realization. As time goes by and conditional on the fact that no reward is materialized earlier, the same utility at time t becomes:

$$U_t(p,T) = \left(\int_t^T e^{-rs} \frac{q}{1-\pi t} ds + (1 - \frac{\pi}{1-\pi t}(T-t)) e^{-rT} \right) p$$

Now define a choice rule that selects the options in A that maximizes the previous utility at a given time $\hat{\tau}$:

$$C_{DM}(A,\hat{\tau}) = \operatorname*{argmax}_{(p,T)\in A} U_{\hat{\tau}}(p,T)$$

In the binary choice between (p,T) and (q,T'), the result of Dasgupta and Maskin (2005) implies the existence of t^* with $(q,T') = C_{DM}(A,\hat{\tau})$ if $\hat{\tau} < t^*$ and $(p,T) = C_{DM}(A,\hat{\tau})$ if $\hat{\tau} \geq t^*$. The next proposition shows that the C_{DM} can be written as a particular case of our model.

Theorem 2. *If π is small enough, the choice rule $C_{DM}(A,\hat{\tau})$ is a particular case of $C(A,\hat{\tau})$ with*

$$C(A,\hat{\tau}) = \operatorname*{argmax}_{(p,T)\in A} [u(p,T) + \phi(\hat{\tau})v(p,T)]$$

where $u(p,T) = U_0(p,T)$, $v(p,T) = -\frac{\pi p}{r}$ and $\phi(\tau) = 1 - e^{-r\tau}$.

The fast self u assigns to (p,T) its value at the beginning of the contemplation time whereas the slow self punishes high pure payoffs. If the realization of the contemplation time is large enough, the individual will prefer the earlier/smaller reward. Otherwise, she will select the later/larger one. It is interesting to note that the function $\phi(\tau) = 1 - e^{-r\tau}$ is a function introduced in Luce (1986, Pag. 150) as a model of response times.

4 Contemplation time and stochastic choice

In this section, we exploit the stochastic features of our model to extend our analysis and we study the prediction of the stochastic choice. Since we assumed that the length of the contemplation time is stochastic, at the beginning of the contemplation time the choice from menu is stochastic. The assumption of stochastic CT is more realistic since, in many cases, the amount of time available to decide is random and unknown to the decision maker. Moreover, as hinted in the introduction, random contemplation time arises as *response times*. When the contemplation time does not depend on external conditions, the amount of time spent before deciding is endogenous and corresponds to response times. This identification is possible if we assume that the response time is not decided by the individual, hence the length of the CT is not chosen consciously. This is the case for neural-based models of choice, such as, the Bounded Accumulation Model (Ratcliff, 1978; Webb, 2015) and Decision Field Theory (Busemeyer and Townsend, 1993).

We restrict our attention to a finite set of options Z and we define $\mathcal{Z}$ the family of finite and non-empty subsets of Z.[7] For all $A \in \mathcal{Z}$ and a given random time τ with law $F_\tau(t) = P(\tau \leq t)$, the Random Contemplation Time (RCT) choice rule is a family of functions $\mathbb{P}_\tau(p|A) : A \to [0,1]$ with $\sum_{p\in A} \mathbb{P}_\tau(p|A) = 1$ given by:

$$\mathbb{P}_\tau(p|A) = P\left(u(p) + \phi(\tau)v(p) \geq \max_{q\in A} u(q) + \phi(\tau)v(q)\right) \tag{RCT}$$

represents the probability of choosing p from A when the contemplation time is τ. If there are two elements $p, p_1 \in A$ with $u(p) = u(p_1)$ and $v(p) = v(p_1)$, we call p and p_1 *duplicates* and we need a tie-breaking rule. We consider the following: if, for some $p_1, \dots, p_n \in A \setminus p$, $u(p) = u(p_1) = \dots = u(p_n)$ and $v(p) = v(p_1) = \dots = v(p_n)$, then

$$\mathbb{P}_\tau(p|A) = \frac{1}{n+1}\mathbb{P}_\tau(p|A \setminus \{p_1, \dots, p_n\}) \tag{tb}$$

For example, let $A = \{p, q, p_1\}$, with $u(q) \neq u(p) = u(p_1)$ and $v(q) \neq v(p) = v(p_1)$, then $\mathbb{P}_\tau(p|A) = \frac{1}{2}\mathbb{P}_\tau(p|p,q)$. All the results in the paper hold under a more general specification of the tie-breaking rule. We assume the previous form due to its simplicity. A second form of tie can result from the realization $\hat{\tau}$ of τ at a point such that $u(p) + \phi(\hat{\tau})v(p) = u(q) + \phi(\hat{\tau})v(q) = \max_{r\in A} u(r) + \phi(\hat{\tau})v(r)$. To impose probability zero to these ties, we assume that the distribution of τ has a density $f_\tau = F'_\tau$. If $A = \{p, q\}$ is a binary menu, we write $\mathbb{P}_\tau(p|p,q)$ and similarly for ternary menus. To gain intuition, consider a binary menu $A = \{p, q\}$ with $u(p) \neq u(q)$ and $v(p) \neq v(q)$ and assume that ϕ is strictly monotone, then

$$\mathbb{P}_\tau(p|p,q) = P(u(p) + \phi(\tau)v(p) \geq u(q) + \phi(\tau)v(q)) = P\left(\tau \geq \phi^{-1}\left(\frac{u(q)-u(p)}{v(p)-v(q)}\right)\right)$$

The probability of choosing p over q is given by the probability that the realization of τ is larger than $\phi^{-1}\left(\frac{u(q)-u(p)}{v(p)-v(q)}\right)$. Knowing the distribution of the contemplation time, as we assumed, allows to give an explicit form to choice probabilities. For example:

[7] The framework has two differences with respect to the one in the previous section: elements of choice are not necessarily lotteries and we restrict our attention to finite menus. We modify the choice framework to place our model into the class of Single-crossing Random Utility models of (Apesteguia et al., 2016), since SCRUM is characterized on finite choice set.

Example 1.

Exp. *If τ is distributed as a negative exponential, i.e. $P(\tau \geq k) = e^{-\lambda k}$ for some $\lambda > 0$.*

$$\mathbb{P}_\tau(p|p,q) = e^{-\lambda\phi^{-1}\left(\frac{u(q)-u(p)}{v(p)-v(q)}\right)}$$

Weibull. *If τ is distributed as a Weibull, i.e. $P(\tau \geq k) = e^{-\lambda k^\theta}$ for some $\theta, \lambda > 0$,*

$$\mathbb{P}_\tau(p|p,q) = e^{-\lambda\left(\phi^{-1}\left(\frac{u(q)-u(p)}{v(p)-v(q)}\right)\right)^\theta}$$

Loglogistic. *If τ is distributed as a Loglogistic, i.e. $P(\tau \geq k) = 1 - \frac{1}{1+(\lambda k)^\theta}$ for some $\lambda, \theta > 0$,*

$$\mathbb{P}_\tau(p|p,q) = 1 - \frac{1}{1 + \left(\lambda\phi^{-1}\left(\frac{u(q)-u(p)}{v(p)-v(q)}\right)\right)^\theta}$$

Knowing the distribution of contemplation time allows us to recover preferences. For example, in the exponential case, is easy to see that $\phi^{-1}\left(\frac{u(q)-u(p)}{v(p)-v(q)}\right) = -\frac{1}{\lambda}\ln \mathbb{P}_\tau(p|p,q)$. We will exploit such possibility in Section 6.

Since the contemplation time is a random variable we can consider first-order stochastic dominance:

Definition 2 (FOSD)**.** *Let τ and τ' be two random times, we define $\tau \geq_{FOSD} \tau'$, if and only if, $P(\tau \geq t) \geq P(\tau' \geq t)$, for all $t \geq 0$, with strict inequality for some t.*

Let define an element p of A to be *dominated* by q if $u(p) \leq u(q)$ and $v(p) \leq v(q)$, with at least a strict inequality. We say that p is dominated if it is dominated by some $q \in A$.

We now focus on a particularly important set of items in a menu: those with, $v(p) = \max_{q\in A} v(q)$, the subset of maximally "contemplative" options in A. For example, these may represent the most tempting items (if v represents the temptation ranking). We have the following immediate result:

Proposition 1. *Suppose that ϕ is unbounded above, p is undominated and $v(p) = \max_{q\in A} v(q)$,*

1. *If $\tau \geq_{FOSD} \tau'$, then $\mathbb{P}_\tau(p|A) \geq \mathbb{P}_{\tau'}(p|A)$.*
2. *If $P(\tau \geq t) \geq P(\tau' \geq t)$ for all $t \geq \tau^*$ and $\tau^* \leq \tau_p$, then $\mathbb{P}_\tau(p|A) \geq \mathbb{P}_{\tau'}(p|A)$.*

The proof follows directly from the definitions. The previous results shows that, a longer (in a stochastic sense) contemplation time increases the probability of choosing the most contemplative options. As for the interpretation, we can imagine τ and τ' as two different shopping occasions of the same individual at the same supermarket, one with shorter queue and the other with a longer queue.

Proposition 2. *Suppose that ϕ is unbounded above, p is undominated and $v(p) = \max_{q\in A} v(q)$. Then, there always exists a finite $\tau_p \geq 0$ such that $u(p) + \phi(\tau_p)v(p) \geq \max_{q\in A}[u(q) + \phi(\tau_p)v(q)]$. Moreover,*

$$\mathbb{P}_\tau(p|A) = P(\tau \geq \phi^{-1}(\tau_p)) \tag{1}$$

if p has no duplicates in A and $\mathbb{P}_\tau(p|A) = \frac{1}{1+n}P(\tau \geq \phi^{-1}(\tau_p))$, if p has n duplicates in A.

There is always a long enough contemplation time for which choosing (one of) the most contemplative option is optimal. Using the Markov's inequality, we can give an upper bound for the probability of choosing (one of) the most contemplative option in A. For simplicity, assume there are no duplicates. By Markov's inequality:

$$\mathbb{P}_\tau(p|A) \leq \frac{\mathbb{E}[\tau]}{\phi^{-1}(\tau_p)} \tag{2}$$

The previous upper bound is binding if $\mathbb{E}[\tau] < \phi^{-1}(\tau_p)$. Therefore, our model predicts a bound to the probability of choosing a particularly important class of items in a menu. Notice that $\mathbb{E}[\tau]$ is known to the modeler or experimenter and can be easily manipulated. $\phi^{-1}(\tau_p)$ is also known to the modeler due to equality (1). Varying the expected value of the contemplation time, the upper bound on the choice probability of the most contemplative option varies, since the denominator in Eq. (2) depends on u and v but it is independent of the distribution of τ. Hence, inequality (2) can be used to estimate the effect of different distribution of the contemplation time.

5 Nudging: time pressure and cooling-off

The main application of our model is to choice under time manipulation. If a choice has to be made within a deterministic time deadline τ', the subject can choose at any time before τ'. Hence, the contemplation time is the minimum between the realization $\hat{\tau}$ of a random time τ, representing the response time and τ', the choice deadline. Our model becomes:

$$C(A, \hat{\tau} \wedge \tau') = \underset{p\in A}{\text{argmax}}[u(p) + \phi\left(\hat{\tau} \wedge \tau'\right) v(p)]$$

The following result is immediate:

Lemma 1. *Suppose p is undominated and it is such that $v(p) = \max_{q \in A} v(q)$, then*

$$\mathbb{P}_{\tau}(p|A) \geq \mathbb{P}_{\tau \wedge \tau'}(p|A)$$

Suppose that u is more risk averse than v. Then, the individual will be more risk averse under time pressure with respect to a condition of no time pressure, as found in Zur and Breznitz (1981). Let u be a mean-variance utility and v a more general expected utility. Under time pressure individuals use a heuristic mean-variance criterion rather than a more sophisticated expected utility. Such model has been proposed by d'Acremont and Bossaerts (2008) to explain their experimental finding on choice under risk and time pressure. Related to choice under risk, our model may be used to calibrate the estimation of risk attitude in experiments. Indeed, in most experiments, the time to decide is limited and the estimation of risk preferences (or time preferences) does not take into account the effect of such limitation. If u and v have different degree or risk aversion, the estimated parameters may be distorted depending on the length of the decision process. A similar reasoning applies to experiments on intertemporal choice.

If we interpret u and v as the "altruistic" and "selfish" selves respectively, our model can account for the result of Rand et al. (2012). They found a monotone decreasing relation between time pressure and contribution in a public good game. Higher time pressure (deciding within 10 seconds) corresponded to higher contribution, with respect to a no-time pressure condition. In turn, the contribution under the no-time pressure condition was higher than the contribution under forced choice delay (decide after 10 seconds). If we interpret u as the implicit and v as the explicit preference[8], we can explain the result of Friese et al. (2006). Under time pressure, individuals purchase their implicitly preferred items more often than in the condition of no time pressure, where explicit preferences are more prominent.

In a binary choice, decreasing the probability of choosing the most contemplative option clearly increases that of choosing the instinctive option. Indeed, let $A = \{p, q\}$ with $u(q) > u(p)$ and $v(q) < v(p)$ and suppose that τ' is a deterministic choice deadline. Then

$$\mathbb{P}_{\tau}(q|p, q) \leq \mathbb{P}_{\tau \wedge \tau'}(q|p, q)$$

a choice deadline increases the probability of choosing the best option according to the slow

[8]In the psychological literature (see Wilson et al., 2000; Friese et al., 2006), explicit preferences (attitudes) are those expressed directly by individuals (stated preferences). Implicit preferences (attitudes) are defined as: (a) having unknown origins, (b) being activated automatically and (c) influence uncontrolled responses.

preference. This result can explain the findings of Duflo et al. (2011). They showed that time-limited promotions (free delivery) nudge farmers in rural Kenya to buy fertilizers, when they are tempted to procrastinate the purchase. Differently from time-unlimited promotions, imposing a choice deadline lowers the contemplation time, diminishing the intensity of the temptation of procrastinate and increasing the probability of behaving normatively. Duflo et al. (2011) explain their findings with naivety of the farmers that are uncertain about the present bias they will face. Let assume that A contains streams of consumption, $c \in A$ implies $c = (c_0, \ldots, c_T)$. The interpretation of c_0 is consumption happening immediately after a choice is made, hence at the end of the contemplation time, rather than at the beginning. Assume the following form of the CT rule:

$$C(A, \hat{\tau}) = \underset{c \in A}{\operatorname{argmax}} \left[\sum_{t=0}^{T} \delta^t U(c_t) + \phi(\hat{\tau}) U(c_0) \right]$$

the fast (normative) self $u(c) = \sum_{t=0}^{T} \delta^t U(c_t)$ has an additively separable utility and discounts the future geometrically with factor δ. The slow (temptation) utility $v(c) = U(c_0)$ is completely impatient, it only cares about consumption at time zero (the end of the contemplation time). It is immediate to see that $C(A, \hat{\tau})$ is equivalent to the following:

$$C(A, \hat{\tau}) = \underset{c \in A}{\operatorname{argmax}} \left[U(c_0) + \frac{1}{1 + \phi(\hat{\tau})} \sum_{t=1}^{T} \delta^t U(c_t) \right]$$

a β-δ discounting model with $\beta_{\hat{\tau}} = \frac{1}{1+\phi(\hat{\tau})}$. The longer the contemplation time, the higher is the degree of present bias when making the final choice. The stochastic choice at the beginning of the contemplation time $\mathbb{P}_\tau(c|A)$ is:

$$\mathbb{P}_\tau(c|A) = P\left(U(c_0) + \beta_\tau \sum_{t=1}^{T} \delta^t U(c_t) \geq \max_{c' \in A} U(c'_0) + \beta_\tau \sum_{t=1}^{T} \delta^t U(c'_t) \right)$$

that is equivalent to the Random Quasi-hyperbolic Discounting (RQD) model used in Duflo et al. (2011). Uncertainty concerning future present bias follows from uncertainty about the length of the contemplation time, in this case a response time. Proposing time-limited promotions nudges farmers because it reduces the length of the contemplation time. When procrastination is tempting, limiting the contemplation time decreases the cost of temptation hence increasing the probability of behaving normatively. A similar explanation can be applied to the findings of Lindner and Rose (2016). They estimated time preferences under time pressure and they found a lower present-bias when the time pressure was higher. The previous model can exactly account for such behavior, since an upper time limit τ' on contemplation time implies $\beta_{\hat{\tau}} = \frac{1}{1+\phi(\hat{\tau})} \geq \frac{1}{1+\phi(\hat{\tau}\wedge\tau')} = \beta_{\hat{\tau}\wedge\tau'}$ for all the possible realizations of τ. An additional experimental finding that is compatible with our model comes from Nordgren and Chou (2012). They asked hungry dieters to select among a variety of snacks. Dieters under time pressure (choice within 30 seconds) selected less tempting snacks with respect to subjects deciding with lower time pressure (choice within 3 minutes).

The previous results suggest a new form of nudging when individual are tempted: *limiting the contemplation time to increase the probability to choose normatively optimal options.* This is an example of "libertarian paternalism" (Thaler and Sunstein, 2003), since the set of available options in not changed, but the shortening of the contemplation time nudges individuals to behave normatively. An additional conclusion of our model, concerns the correct quantification of the "cost of waiting". Waiting time is often considered as a barrier to participation in public programs. For example, enrollment of substance users in rehab programs is affected by the "waiting time". According to Redko et al. (2006) "... the longer substance users have to wait to be admitted to treatment, the more likely they are to not follow through with treatment". While on a waiting list, substance users are tempted to quit and the longer the waiting time, the higher is the probability succumb the temptation. Similarly, Gennetian and Shafir (2015) claimed that reducing waiting time can increase participation in the Supplemental Nutrition Assistance Program (SNAP), where participation rates are well-known to be low. Going to the designed office and waiting for hours has a cost that can be too large with respect to the gains of being enrolled in the SNAP. Beyond the opportunity cost of waiting, if an individual is tempted to quit while waiting for applying, the actual probability to give in and quit is higher than predicted by a pure opportunity cost analysis. Therefore, the speedup of the application process can be more effective than expected in terms of participation rates. In general, our model calls for a redefinition of the cost of waiting when facing temptation. Any intervention aiming at lowering transaction costs, for example in program enrollment, should take into account the implicit cost of temptation.

5.1 Cooling-off

Opposite to choice deadlines, the imposition of cooling-off periods represents a common form of nudging (Camerer et al., 2003). In this case, a choice cannot be made before a given amount of time or there is a mandatory period during which the choice is reversible.

The idea is to cool-off the decision maker who entered a "hot state". For example, after buying a car, the customer has to wait one day before buying a car insurance. Letting τ' be a deterministic time threshold, choice with a cooling-off period can be represented in our model by:

$$C(A, \hat{\tau} \vee \tau') = \underset{p \in A}{\operatorname{argmax}}[u(p) + \phi\left(\hat{\tau} \vee \tau'\right) v(p)]$$

where $\hat{\tau} \vee \tau'$ is the maximum between $\hat{\tau}$ (the realization of the response time) and τ'. Suppose that $u(q) > u(p)$ and $v(p) > u(p)$, by Markov's inequality

$$\mathbb{P}_{\tau}(p|p,q) \leq \mathbb{P}_{\tau \vee \tau'}(p|p,q) \leq \frac{\mathbb{E}[\tau \vee \tau']}{\phi^{-1}(\tau_p)}$$

Setting a cooling-off period increases the probability of selecting the most contemplative option. However, its effect can be bounded if the threshold $\phi^{-1}(\tau_p)$ is very large respect to the mean duration of the contemplation time. The bound provided by the Markov's inequality can be used to estimate the effect of different length of cooling-off periods as argued at the end of Section 4. $\phi^{-1}(\tau_p)$ can be identified observing the choice probability of p, hence, we can use the previous inequality to perform comparative statics on the upper bound to choice probability.

6 Characterization of choice probabilities

Suppose that the modeler observes aggregate choices at the individual level, without knowing the timing at which each choice is made, but knowing the distribution of the CT. For example, the modeler has scanner data concerning the snacks surrounding the checkout, hence, without information about the time of each decision and he knows the distribution of the length of the queue at the checkout. In this case, our primitives are choice probabilities and a distribution of the CT. We show in this section, how to recover the fast and slow utility from such observables imposing conditions on the choice probabilities.

It can be seen[9] that choice probabilities given by the RCT rule belong to the family of choice probabilities generated by the Single-Crossing Random utility model of Apesteguia et al. (2016). They provide a set of axioms characterizing the general Single-Crossing RUM, therefore, we need to identify the testable restrictions that characterize the RCT within such class. As in Apesteguia et al. (2016), we assume that there exists a linear order over

[9] I thank Ryota Iijima for pointing out such connection.

the elements of Z denoted $\succ$. They also show how to define a linear order directly from observed choice probabilities, hence, providing a foundation of their model based only on observables. To focus on the main innovation of the present work with respect to Apesteguia et al. (2016), we assume directly the existence of $\succ$ and a function v representing it. The interested reader can refer to their paper for details. The Single-Crossing Random utility model is characterized by the following axioms:

Positivity. $\mathbb{P}_\tau(p|A) > 0$ for all $p \in A$ and all $A \in \mathcal{Z}$.

Monotonicity. $\mathbb{P}_\tau(p|A) \geq \mathbb{P}_\tau(p|B)$ if $A \subseteq B$.

Centrality. If $p \succ q \succ r$, $\mathbb{P}_\tau(p|p,q,r) = \mathbb{P}_\tau(p|p,q)$ and $\mathbb{P}_\tau(r,|p,q,r) = \mathbb{P}_\tau(r|q,r)$.

Positivity implies that no options are dominated. Monotonicity that choice probabilities cannot increase in larger sets. Centrality is the main axiom in Apesteguia et al. (2016). Figure 2 illustrates centrality and one of its consequence with $\phi(\tau) = \tau$. Given a triplet

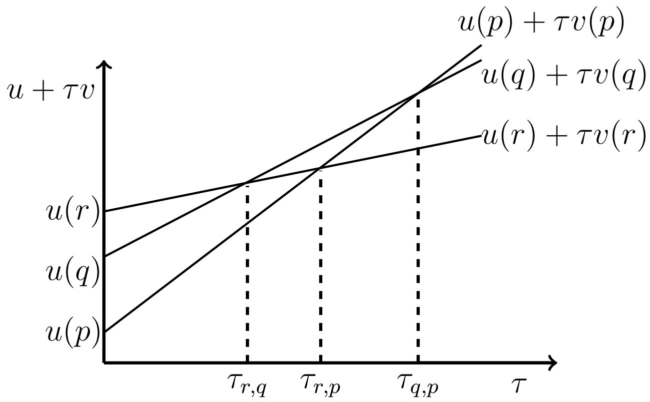

Figure 2: Illustration of the Centrality axiom.

$p \succ q \succ r$, the probability of choosing r from $\{r, q, p\}$ is the probability of observing a realization of τ smaller than τ_{rq}. This is the same probability of choosing r from $\{r, q\}$, the presence of p has no effect. A similar fact holds for p. Another consequence of centrality (see Apesteguia et al., 2016, Th. 1) is that, given a triplet $p \succ q \succ r$,

$$\mathbb{P}_\tau(r|r,q) \leq \mathbb{P}_\tau(r|r,p) \leq \mathbb{P}_\tau(q|p,q) \tag{3}$$

It follows that, for some $\alpha \in [0,1]$ that may depend on p, q, r, $\mathbb{P}_\tau(r|r,p) = \alpha\mathbb{P}_\tau(r|r,q) + (1-\alpha)\mathbb{P}_\tau(q|p,q)$. The probability of choosing r over q, $\mathbb{P}_\tau(r|r,q)$, is equal to the probability that τ is realized before $t_{r,q}$. This is smaller than the probability of selecting r over p, $\mathbb{P}_\tau(r|r,p)$, corresponding to the probability of observing a realization of τ before $t_{r,p}$. Lastly, the latter is smaller than the probability of selecting q over p, $\mathbb{P}_\tau(q|p,q)$. The next axiom characterizes a particular form of the RCT rule within the class of single-crossing random utilities imposing particular form to the α's of the previous example. Let order the $p \in Z$ according to $\succ$ and write $p_{|Z|} \succ p_{|Z|-1} \succ \ldots \succ p_1 \succ p_0$.

Averaging. Let v a utility representing $\succ$, then for all p_i, p_j with $p_j \succ p_i$,

$$\mathbb{P}_\tau(p_i|p_i,p_j) = \sum_{k=i}^{j-1} \mathbb{P}_\tau(p_k|p_k,p_{k+1}) \frac{v(p_{k+1}) - v(p_k)}{v(p_j) - v(p_i)}$$

Averaging imposes that binary choice probabilities can be written as a weighted average of choice probabilities of intermediate options in the grand set Z and the weight depends on the ranking $\succ$. To clarify the geometric meaning of Averaging, let assume $v(p_k) = k$ and consider $\mathbb{P}_\tau(p_1|p_1,p_3)$. By Averaging,

$$\begin{aligned}\mathbb{P}_\tau(p_1|p_1,p_3) =& \mathbb{P}_\tau(p_1|p_1,p_2)\frac{v(p_2)-v(p_1)}{v(p_3)-v(p_1)} + \mathbb{P}_\tau(p_2|p_2,p_3)\frac{v(p_3)-v(p_2)}{v(p_3)-v(p_1)} \\ =& \mathbb{P}_\tau(p_1|p_1,p_2)\frac{1}{2} + \mathbb{P}_\tau(p_2|p_2,p_3)\frac{1}{2}\end{aligned}$$

Averaging is a geometric requirement and it is not immediately transparent from a behavioral point of view, however, it is necessary and sufficient to characterize a particular case of the RCT rule within the SCRUM class. Averaging characterizes an RCT rule, with uniformly distributed contemplation time and linear time perception (i.e. $\phi(\tau) = \tau$).

Theorem 3. *$\succ$ is a linear order, choice probabilities satisfies Monotonicity, Centrality and Averaging, if and only if,*

$$\mathbb{P}_\tau(p|A) = \bar{P}\left(u(p) + \tau v(p) \geq u(q) + \tau v(q), \forall q \in A\right)$$

where $\bar{P}$ is the Lebesgue measure and $u, v : Z \to \mathbb{R}$ are such that $p \succ q$ implies $u(q) > u(p)$ and $v(p) > v(q)$.

Notice that in Theorem 3, we "created" contemplation time. All the axioms are timeless, however, it is possible to construct the flow of time using the linear order $\succ$. If we assume that the modeler knows the distribution of the contemplation time, for example, she knows the distribution of waiting time at the checkout of a supermarket, we can construct the time perception function ϕ.

Theorem 4. *Let τ be distributed according to P and $\succ$ a linear order: if choice probabilities satisfy Positivity, Monotonicity, Centrality and Averaging, then*

$$\mathbb{P}_\tau(p|A) = P\left(u(p) + \phi(\tau)v(p) \geq u(q) + \phi(\tau)v(q), \forall q \in A\right)$$

for some $\phi : \mathbb{R}_+ \to \mathbb{R}_+$, $u, v : Z \to \mathbb{R}$, such that $p \succ q$ implies $u(q) > u(p)$ and $v(p) > v(q)$.

We provided a sufficient condition to identify the RCT within the SCRUM class when observables are choice probabilities and the distribution of contemplation time (plus a linear order that can be defined from choice probabilities).

7 Conclusion

We introduce a dual-self model of choice that gives a causal role to the length of the decision process on determining choice. We characterize the model combining choice and non-choice data. Two rational conditions, a form of WARP and a No-cycle requirement characterize the model. It can explain many patterns of choice observed under time pressure or time dilation.

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Appendix. Proofs

Proof. Of Theorem 1. We prove necessity first. If $\phi(t) = k$ for all t there is nothing to prove, both axioms hold. Suppose ϕ is non-constant, then t-WARP holds. To see No-cycle, if $p \in C(A, t-s)$ then, $u(p) + \phi(t-s)v(p) \geq u(q) + \phi(t-s)v(q)$ for all $q \in A$, whereas $p \notin C(A,t)$, implies $u(q) + \phi(t)v(q) > u(p) + \phi(t)v(p)$, for some $q \in A$. Together, they imply that $\phi(t-s)(v(q)-v(p)) \leq u(q)-u(p) < \phi(t)(v(q)-v(p))$. By monotonicity of ϕ, this implies that $v(q) - v(p) > 0$ and $\phi(t)$ must be different from zero. Then, monotonicity of ϕ implies, $u(q) + \phi(t+s')v(q) > u(p) + \phi(t+s')v(p)$ for all $s' \geq 0$. Then $p \notin C(A, t+s')$, hence the conclusion. For sufficiency, we first show that the family of expected utilities $(u_t)_t$ satisfies the "one-switch" condition of Abbas and Bell (2012). If for some $t < s$, $u_t(p) > u_t(q)$ and $u_s(p) < u_s(q)$, then $p \in C(\{p,q\}, t)$, $p \notin C(\{p,q\}, s)$, by No-cycle, $p \notin C(\{p,q\}, s')$ for all

$s' \geq s$, hence $u_{s'}(q) > u_{s'}(p)$ for all $s' \geq s$. Then, there can be at most one preference switch as t grows. Let $u_t(p) = \sum p(x_i)U(x_i, t)$, let $w, b \in Z$ such that $u_t(w) \leq u_t(b)$ for all t and, w.l.o.g. normalize $U(w, t) = 0$ for all t and $U(b, t) = 1$ for all t, by Abbas and Bell (2012, Prop. 3.1), there exist $g_0, g_1, \phi : [0, \infty) \to \mathbb{R}$ and $f_1, f_2 : Z \to \mathbb{R}$ such that: $U(x, t) = g_0(t) + g_1(t)[f_1(x) + f_2(x)\phi(t)]$. Moreover, $g_0(t) = U(w, t) = 0$, $g_1(t) = [U(b, t) - U(w, t)] = 1$. A look at the proof allows us to choose $f_1(x) = \frac{U(x,0)-U(w,0)}{U(b,0)-U(w,0)} = U(x, 0)$, $f_2(x) = \frac{U(x,1)-U(w,1)}{U(b,1)-U(w,1)} = (U(x, 1)-U(x, 0))$ and $\phi(t) = \frac{U(x,t)-U(x,0)}{U(x,1)-U(x,0)}$ is independent of $x \in Z$ and monotone. Therefore, $u_t(p) = \sum p(x_i)[f_1(x_i) + f_2(x_i)\phi(t)] = u_0(p) + \phi(t)(u_1(p) - u_0(p))$. Defining $u(p) = u_0(p)$ and $v(p) = u_1(p) - u_0(p)$, we have $C(A, t) = \text{argmax}_{p \in A}\, u(p) + \phi(t)v(p)$. □

Proof. Of Proposition 2. Let $v(p) = \max_{q \in A} v(q)$. To prove the result we need to show that there exists a finite realization τ_p of τ, such that, $u(q)+\phi(\tau_p)v(q) \geq \max_{p \in A} u(p)+\phi(\tau_p)v(p)$ and that $u(q) + \phi(\tau')v(q) \geq \max_{p \in A} u(p) + \phi(\tau')v(p)$ for all $\tau' \geq \tau_p$. Assume first that p has no duplicates in A. Consider $u(p) + \phi(\tau)v(p) \geq \max_{r \in A} u(r) + \phi(\tau)v(r)$. By the property of p, $u(p) + \phi(\tau) \max_{q \in A} v(q) \geq \max_{r \in A} u(r) + \phi(\tau)v(r)$, for all $\phi(\tau) \geq 0$. Equivalently, $u(p) \geq \max_{r \in A} u(r) - \phi(\tau)(\max_{q \in A} v(q) - v(r))$. If $\max_{q \in A} v(q) - v(r) = k > 0$, the fact that $u(p) \leq \max_{r \in A} u(r)$ and continuity in $\phi(\tau)$ implies the existence of some τ_p big enough, with $u(p) = \max_{r \in A} u(r) - \phi(\tau_p)k$. Moreover, for all $\tau' \geq \tau_p$, $u(p) \geq \max_{r \in A} u(r) - \phi(\tau')k$. If $\max_{q \in A} v(q) - v(r) = 0$, then there exists $r \in A$ with $r \neq p$ and $v(r) = v(p)$. Since p has no duplicates and it is not dominated, $u(r) < u(p)$. Hence r is dominated by p. It follows that $\max_{q \in A} u(q) + \phi(\tau)v(q) = \max_{q \in A \setminus r} u(q) + \phi(\tau)v(q)$, for all $\tau \geq 0$. Applying the same reasoning of above to the set $A \setminus r$ gives the result. Then $\mathbb{P}(p, A, \phi(\tau)) = P(\tau \geq \phi^{-1}(\tau_p))$. Now, assume that p is not dominated and it is one of the most contemplative option in A and it has n duplicates. It is easy to see that τ_{p_i} is the same for all duplicates p_i of p and p itself. By the tie-breaking rule $\mathbb{P}(p, A, \phi(\tau)) = \frac{1}{1+n} P(\tau \geq \phi^{-1}(\tau_p))$. □

Proof. Of Theorem 2. $U_0(p, T) = \pi p \int_0^T e^{-rs} ds + (1 - \pi T)e^{-rT}p$, that is equivalent to $U_0(p, T) = \pi p \left[-\frac{e^{-rT}}{r} + \frac{1}{r}\right] + (1 - \pi T)e^{-rT}p$. Now consider $U_t(p, T) = \frac{\pi p}{1-\pi t} \int_t^T e^{-rs} ds + (1 - \frac{\pi}{1-\pi t}(T - t))e^{-rs}p$, that is equivalent to

$$U_t(p, T) = \frac{\pi p}{1 - \pi t} \left[-\frac{e^{-rT}}{r} + \frac{e^{-rt}}{r}\right] + (1 - \frac{\pi}{1 - \pi t}(T - t))e^{-rT}p$$

It can be rewritten as

$$\begin{aligned}
U_t(p, T) =& \frac{1}{1 - \pi t}\left[\pi p[-\frac{e^{-rT}}{r} + \frac{e^{-rt}}{r}] + (1 - \pi t - \pi(T - t))e^{-rT}p\right] \\
=& \frac{1}{1 - \pi t}\left[\pi p[-\frac{e^{-rT}}{r} - \frac{1}{r} + \frac{1}{r} + \frac{e^{-rt}}{r}] + (1 - \pi T)e^{-rT}p\right] \\
& \frac{1}{1 - \pi t}\left[\pi p[-\frac{e^{-rT}}{r} + \frac{1}{r}] + (1 - \pi T)e^{-rT}k + (1 - e^{-rt})\frac{(-\pi p)}{r}\right]
\end{aligned}$$

Now, consider

$$C_{DM}(A, \hat{\tau}) = \underset{(p,T) \in A}{\text{argmax}}\, U_{\hat{\tau}}(p, T)$$

Given a realization $\hat{\tau}$ of τ and for a π such that $1 - \pi\hat{\tau} > 0$ (it is sufficient that $\pi < \frac{1}{T'}$), $C_{DM}(A, \hat{\tau})$ is unchanged if $U_{\hat{\tau}}$ is multiplied by $1 - \pi\hat{\tau} > 0$, hence C_{DM} can be written as

$$C_{DM}(A, \hat{\tau}) = \left[\pi p[-\frac{e^{-rT}}{r} + \frac{1}{r}] + (1 - \pi T)e^{-rT}k + (1 - e^{-rt})\frac{(-\pi p)}{r}\right]$$

which is equivalent to a CT rule

$$C(A, \hat{\tau}) = \underset{(p,T) \in A}{\text{argmax}}\, u(p, T) + \phi(\hat{\tau})v(p, T)$$

where $u(p,T) = U_0(p,T)$, $v(p,T) = -\frac{\pi p}{r}$ and $\phi(t) = 1 - e^{-rt}$. □

Proof. Of Theorem 3. Necessity is straightforward. Let order $p_n \succ p_{n-1} \succ \ldots \succ p_0$ and define $u(p_0) = 0$ and $v(p_0) = 0$. Averaging implies that all binary probabilities are strictly positive. Let define $v : Z \to \mathbb{R}_+$ as $v(p_j) = \mathbb{P}_\tau(p_0|p_0,p_i)$ for all $p_j \succ p_0$. v represents $\succ$ since Averaging implies $\mathbb{P}_\tau(p_0|p_0,p_j) > \mathbb{P}_\tau(p_0|p_0,p_i,p_j) = \mathbb{P}_\tau(p_0|p_0,p_i)$ by Centrality, when $p_j \succ p_i \succ p_0$. Now let $t_1 = \mathbb{P}_\tau(p_0|p_0,p_1)v(p_1)$, by positivity and the fact that v represents $\succ$, $t_1 > 0$. Let define $u(p_1) = -t_1$. By definition, $u(p_0) + tv(p_0) = 0 \geq -t_1 + tv(p_1) = u(p_1) + tv(p_1)$ for all $t \in [0, t_1]$. Define $t_2 > 0$ as $t_2 = t_1 + \mathbb{P}(p_1|p_1,p_2)(v(p_2) - v(p_1))$, by positivity $t_2 > t_1$ and define $u(p_2) = -t_2$. By definition, $u(p_1)+tv(p_1) = -t_1+tv(p_1) \geq -t_2+tv(p_2) = u(p_2) + tv(p_2)$ for all $t \in [t_1, t_2]$, and $\mathbb{P}_\tau(p_1|p_1,p_2) = t_2 - t_1$. Repeating the argument for p_k with $t_{k+1} = t_k + \mathbb{P}_\tau(p_k|p_k,p_{k+1})(v(p_{k+1}) - v(p_k))$ and $u(p_k) = -t_k$. Consider now p_i, p_j with $p_j \succ p_i$. By definition, $u(p_i) + tv(p_i) \geq u(p_j) + tv(p_j)$ for all $t > 0$ such that $t_j - t_i \geq t(v(p_j) - v(p_i))$. By the definition of t_j and t_i, $t_j - t_i = \sum_{k=i}^{j-1} \mathbb{P}_\tau(p_k|p_k,p_{k+1})(v(p_{k+1}) - v(p_k))$, by Averaging, $t_j - t_i = \mathbb{P}_\tau(p_i|p_j,p_i)(v(p_j) - v(p_i))$, therefore, $u(p_i) + tv(p_i) \geq u(p_j) + tv(p_j)$ for all $t \leq \mathbb{P}_\tau(p_i|p_i,p_j)$. To conclude we need to show that for each $A \in \mathcal{X}$, defining $p_0 \prec p_1 \prec \ldots \prec p_{|A|}$, $\mathbb{P}_\tau(p_0|A) = \mathbb{P}_\tau(p_0|p_0,p_1)$ and $\mathbb{P}_\tau(p_i|A) = \mathbb{P}_\tau(p_i|p_{i-1},p_i,p_{i+1})$ for all $i < |A|$ and $\mathbb{P}_\tau(p_{|A|}|A) = \mathbb{P}_\tau(p_{|A|}|p_{|A|-1},p_{|A|})$, but this follows from Step 4 of the proof of Th. 1 in Apesteguia et al. (2016). Lastly, notice that $p_i \succ p_j$, if and only if, $u(p_i) < u(p_j)$ and $\bar{P}$ is the Lebesgue measure. □

Proof. Of Theorem 4. By Theorem 3, Positivity, Monotonicity, Centrality and Averaging imply the existence of u, v and $\bar{P}$ such that $\mathbb{P}_\tau(p|A) = \bar{P}(u(p) + \tau v(p) \geq u(q) + \tau v(q),\ \forall q \in A)$. Consider p_0, then $\mathbb{P}_\tau(p_0|p_0,p_1) = \bar{P}\left(t \leq \frac{u(p_0)-u(p_1)}{v(p_1)-v(p_0)}\right)$, now define τ_{01} such that $\mathbb{P}_\tau(p_0|p_0,p_1) = P(t \leq \tau_{01})$ and define ϕ such that, $\phi(\tau_{01}) = \frac{u(p_0)-u(p_1)}{v(p_1)-v(p_0)}$, then $\mathbb{P}_\tau(p_0|p_0,p_1) = P\left(t \leq \phi^{-1}\left(\frac{u(p_0)-u(p_1)}{v(p_1)-v(p_0)}\right)\right) = P(u(p_0) + \phi(t)v(p_1) \geq u(p_1) + \phi(t)v(p_1))$. Consider now a central element $p_{i-1} \succ p_i \succ p_{i+1}$. By Theorem 3, $\mathbb{P}_\tau(p_i|p_{i-1},p_i,p_{i+1}) = \bar{P}\left(\frac{u(p_{i-1})-u(p_i)}{v(p_i)-v(p_{i-1})} \leq t \leq \frac{u(p_i)-u(p_{i+1})}{v(p_{i+1})-v(p_i)}\right)$. Now define $\tau_{i-1,i}$ and $\tau_{i,i+1}$ such that $\mathbb{P}_\tau(p_i|p_{i-1},p_i,p_{i+1}) = P(\tau_{i-1,i} \leq t \leq \tau_{i,i+1})$ and define $\phi(\tau_{i-1,i}) = \frac{u(p_{i-1})-u(p_i)}{v(p_i)-v(p_{i-1})}$ and $\phi(\tau_{i,i+1}) = \frac{u(p_i)-u(p_{i+1})}{v(p_{i+1})-v(p_i)}$. Then, $\mathbb{P}_\tau(p_i|p_{i-1},p_i,p_{i+1}) = P(\tau_{i-1,i} \leq t \leq \tau_{i,i+1}) = P\left(\phi^{-1}\left(\frac{u(p_{i-1})-u(p_i)}{v(p_i)-v(p_{i-1})}\right) \leq t \leq \phi^{-1}\left(\frac{u(p_i)-u(p_{i+1})}{v(p_{i+1})-v(p_i)}\right)\right)$ and the latter is equal to $P(u(p_{i-1})+\phi(t)v(p_{i-1}) \leq u(p_i)+\phi(t)v(p_i) \leq u(p_{i+1})+\phi(t)v(p_{i+1}))$. The argument can be extended to all elements as in the proof of Theorem 3. □

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