Solving the Milk Addiction Paradox

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Abstract

The milk addiction paradox refers to an empirical finding in which commodities that are typically considered to be non addictive, such as milk, appear instead to be addictive. This result seems more likely when there is persistence in consumption and when using aggregate data, and it suggests that the AR(2) model typically used in the addiction literature is prone to produce spurious result in favor of rational addiction. Using both simulated and real data, we show that the milk addiction paradox disappears when estimating the data using an AR(1) linear specification that describes the saddle-path solution of the rational addiction model. The AR(1) specification is able to correctly discriminate between rational addiction and simple persistence in the data, to test for the main features of rational addiction, and to produce unbiased estimates of the short and long-run elasticity of demand. These results hold both with individual and aggregated data, and they suggest that, for testing rational addiction, the AR(1) model is a better empirical alternative than the canonical AR(2) model.

Keywords: Adjacent complementarity, Forward-looking behavior, Milk addiction, Rational addiction, Spurious correlation

JEL codes: D11, D12, I12, L66
Non-Technical Summary

Becker and Murphy (1988)’s theory of rational addiction is the reference model to study addictive behavior in the economic literature. According to the theory, addictive consumption is predicted to be positively correlated with past consumption. Addiction is considered to be rational if the current consumption is affected by future events such as expected price changes, or announcements of future tax increases or smoking bans.

Since the pioneering papers of Becker et al. (1990, 1994); Chaloupka (1991), the empirical model used to test whether consumption of a good is consistent with rational addiction is a linear AR(2) model where present consumption depends on past and future consumption. When used to estimate the demand of addictive goods, such as cigarettes, the aforementioned model performs reasonably well, although the estimates are sometimes unstable and often sensitive to the instruments. Auld and Grootendorst (2004), however, observe that the model performs surprisingly ”well” also on goods that are typically considered to be habit forming, but not addictive. In particular, when applying the model to Canadian data of consumption of milk, oranges, eggs and cigarettes, Auld and Grootendorst (2004) find that milk consumption is consistent with rational addiction, and even more addictive than cigarettes. This milk addiction puzzle suggests that the AR(2) model typically used in the addiction literature is prone to produce false positives in favor of rational addiction. This result seems more likely to emerge when there is persistence in consumption and when using aggregated data.

In this paper we claim that there exists a way to reliably discriminate between addictive and non-addictive goods, and to estimate the associated elasticity of demand, provided one tests the theory of rational addiction using a different empirical model. In particular, we consider the AR(1) specification derived in Dragone and Raggi (2018). Such model has desirable theoretical and econometric properties, and it is simpler to estimate with respect to the canonical AR(2) model.

To investigate the performance of the AR(1) model, we first work with simulated data. We consider two different data generation processes, one featuring myopic habit formation, and one featuring rational addiction. We estimate the corresponding consumption trajectories using the AR(1) model and measure the corresponding bias. The results show that the AR(1) model correctly distinguishes between addictive and non-addictive consumption, that no false positives are
systematically found, and that the estimated short and long-run demand elasticities are unbiased. Notably, these findings are robust when aggregated data, instead of individual consumption series, are considered. Moreover, the results are robust even when considering the possible endogeneity of past consumption, or of prices.

Then, we replicate Auld and Grootendorst (2004)’s exercise, using the same Canadian data, but estimating the AR(1) model, instead of the canonical AR(2) model. We find that the milk addiction puzzle disappears. More precisely, milk consumption is consistent with habit-forming consumption, but not with forward-looking behavior. On the contrary, cigarette consumption is consistent with rational addiction, as one would expect. We conclude that the milk addiction paradox is an artifact of using the AR(2) model, and not a general result of the theory of rational addiction.

Keywords: Adjacent complementarity, Forward-looking behavior, Milk addiction, Rational addiction, Spurious correlation

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1 Introduction

The milk addiction paradox refers to an empirical finding in which commodities that are typically considered to be non addictive, such as milk or oranges, appear to be addictive (Auld and Grootendorst, 2004). This puzzling result seems to be more likely when the data are aggregated and serially correlated, two common features of time series, and it suggests that the canonical AR(2) model used to test addiction can produce spurious results in favor of rational addiction. This finding, and the additional observation that the canonical AR(2) model features an explosive root, has raised the question of whether the rational addiction model can be estimated altogether (Laporte et al., 2017).

In this paper we show that the rational addiction theory can be tested, provided one considers the AR(1) addiction model derived in Dragone and Raggi (2018), instead of the canonical AR(2) specification. Using both simulated and real data, we show that the AR(1) model is able to correctly discriminate rational addiction from simple persistence in the data, and to provide unbiased estimates of the short and long-run elasticity of demand, irrespective of whether individual or aggregated data are used. These results are likely due to the fact that the AR(1) model is stationary (while the AR(2) model is explosive) and that it does not suffer of the endogeneity concerns that arise when including lead consumption in the estimating equation.

To introduce the reader to the theoretical background, in section 2 we briefly present the rational addition model and the two specifications used in the empirical literature: the AR(1) equation representing the saddle path solution of the model (Dragone and Raggi, 2018), and the canonical AR(2) equation. The latter equation represents the Euler equation of the rational addiction model and it is the empirical specification typically used to test the theory on the data (Becker et al., 1991; Chaloupka, 1991; Grossman, 1993; Chaloupka, 1996; Chaloupka and Warner, 2000; Cawley and Ruhm, 2012). In section 3 we perform a battery of Monte Carlo experiments and we show no tendency of the AR(1) addiction model to detect addiction when there is just spurious correlation in the data. We first generate trajectories that feature no addiction by construction, then we estimate the corresponding parameters using the AR(1) addiction model. The results correctly show that the original trajectories are not consistent with rational addiction. Notably, the estimation exercise produces unbiased estimates of the short and long-run elasticity of demand. We also check whether the AR(1) is able to correctly detect rational addiction when, in fact, the simulated consumption trajectories feature rational addiction. As shown in Laporte et al. (2017), an analog validation exercise can produce unreliable estimates if one generates and estimates
addiction trajectories using the canonical AR(2) model. When using the AR(1) model, instead, the results are reliable and unbiased.

The fact that the simulation results obtained with the AR(1) model hold irrespective of whether we consider individual or aggregate consumption data, mitigates the concerns about testing rational addiction using aggregate data. To further explore the sensitivity of the AR(1) model, and to address some endogeneity concerns that have been raised in the literature, Section 4 performs two additional sets of experiments as robustness checks. In the first set, the price trajectories have both an exogenous and an endogenous component. In the second set of experiments, lagged consumption is explicitly considered to be endogenous. In the literature these endogeneity concerns have been addressed using further leads and lags of prices, or taxes, as instruments (see, for example, Becker et al., 1994; Chaloupka, 1991; Gruber and Köszegi, 2001; Baltagi and Geishecker, 2006). We proceed along the same lines and we find that the IV estimates are still unbiased (although, as expected, the IV estimates are less efficient). We conclude that the AR(1) correctly discriminates between persistence and rational addiction, and that aggregation, endogeneity of prices and endogeneity of lagged consumption pose no significant threat to testing the theory of rational addiction.

In section 5 we move to considering real data and directly addressing the milk addiction paradox. To compare the performance of the AR(1) addiction model with the results of Auld and Grootendorst (2004), we estimate the demand for milk, oranges, eggs and cigarettes using the same Canadian dataset analyzed in Auld and Grootendorst (2004). Instead of using the canonical AR(2) model, however, we estimate using the AR(1) addiction equation. The results show no evidence of milk being addictive, and we conclude that the milk addiction paradox is an artifact of using the Euler equation rather than the AR(1) solution of the model. In fact, in our estimations the consumption of milk, oranges and eggs is not consistent with the theory of rational addiction, while smoking, as expected, is addictive. Section 6 concludes.

2 The rational addiction model

Consider an intertemporal problem in which an agent allocates income between an addictive good $c$ and a numeraire good $q$. Consumption of the addictive good increases the stock $A$ of addiction according to $A(t) = c(t-1) + (1-\delta)A(t-1)$, where $\delta \in (0,1]$ describes the degree of persistence of the state of addiction and $t$ is time. Becker and Murphy (1988)’s model of rational addiction assumes that the marginal utility of current consumption is higher, the higher the consumption stock ($U_{cA} > 0$). This property, called reinforcement, represents the effect of a
learning-by-consuming process in which the more an agent consumes, the more she (marginally) appreciates the good. The marginal utility of addiction is negative if the addictive commodity is harmful, and positive if it is beneficial. As usual, the per-period utility is increasing in the two consumption goods, and it is concave.

While reinforcement describes the effect of past choices on current preferences, the second main feature of rational addiction, forward-looking behavior, implies that current choices take into account expectations about future events and how current behavior will affect future preferences. This property is in stark contrast with myopic models, where current behavior only depends on past events and choices, and not on future events and expectations (see, for instance, the habit formation model presented in Pollak, 1970, or in Gilleskie and Strumpf, 2005). Finally, agents are assumed to be time consistent. Accordingly, unless some new information arrives, any optimal plan will be faithfully implemented and no self-control failure should be observed. This property is explicitly required by Becker and Murphy (1988) and is formally obtained assuming the existence of a constant discount factor \( \beta \in (0, 1) \) to weight future utilities.\(^1\)

Under the above assumptions, the rational addiction model can be formalized as the following intertemporal problem

\[
\max_{c, q} \sum_{y=0}^{\infty} \beta^t U(c(t), q(t), A(t)) \tag{1}
\]

\[
s.t. A(t) = c(t-1) + (1 - \delta) A(t-1) \tag{2}
\]

\[
M(t) = p(t)c(t) + q(t) \tag{3}
\]

where \( p(t) \) is the price of the addictive good at time \( t \), \( M(t) \) is income and \( A(0) = A_0. \)^2

Dragone and Raggi (2018) show that the solution of the rational addiction model is the saddle path leading to a steady state of consumption and addiction. Using a quadratic utility function, the saddle path can be conveniently written as the following linear AR(1) equation,

\[
c(t) = \lambda c(t-1) + \varphi_1 p(t-1) + \varphi_2 p(t) + \sum_{s=1}^{\infty} \varphi_3(s) p(t+s) + \varphi_0 \tag{4}
\]

\(^1\)To address self-control in a rational addiction context, Gruber and Kőszegi (2001) augment the Becker and Murphy (1988) model and allow for time-inconsistent preferences through quasi-hyperbolic discounting. They show that forward-looking behavior and the effect of announced tax changes can still be tested, but since the time-consistent and the time-inconsistent solutions are isomorphic, it is not possible to derive a sharp empirical test that distinguishes the augmented model from the original one.

\(^2\)Becker and Murphy (1988) allow for saving and borrowing and consider that case in which the marginal utility of wealth is constant. This produces the same solution and the same Euler equation of the model presented here, in which the budget constraint is binding in each period (see Dragone and Raggi, 2018, for details).
or, when $\delta = 1$, 
\[
c(t) = \lambda c(t-1) + \varphi_2 p(t) + \sum_{s=1}^{\infty} \varphi_3(s) p(t+s) + \varphi_0.
\]  
(5)

Equation 4 (or 5) states that optimal current consumption depends on past consumption and on current and future prices. The significance and sign of the estimated coefficients allow to test the main properties of the rational addiction model. Reinforcement implies that $\lambda$ is expected to be positive. When this is the case, past and current consumption are positively correlated, a property called adjacent complementarity (Ryder and Heal, 1973). Saddle path stability further requires $\lambda < 1$. Forward-looking behavior implies that $\varphi_3 \neq 0$. A positive value of $\varphi_3$ implies that a future (expected) price increase triggers an increase in current consumption. This behavior is consistent with stockpiling today as a response to the announcement or expectation of a future price increase (see, for instance, Gruber and Köszegi, 2001). A negative value of $\varphi_3$, instead, reveals the opposite reaction in which consumption today decreases in expectation of a future price or tax increase.

The solution 4 of the rational addiction model clearly satisfies the necessary conditions for optimality. In particular, it satisfies the following Euler equation (Becker and Murphy, 1988)
\[
c(t) = \alpha_0 + \alpha_1 p(t-1) + \alpha_2 c(t-1) + \alpha_3 p(t) + \alpha_4 c(t+1) + \alpha_5 p(t+1)
\]  
(6)

which, if $\delta = 1$, simplifies to 
\[
c(t) = \alpha_0 + \alpha_2 c(t-1) + \alpha_3 p(t) + \alpha_4 c(t+1)
\]  
(7)

The AR(2) equation 6 (or 7) constitutes the canonical model used in the empirical literature to estimate the demand for addictive goods (see for example Chaloupka, 1991; Becker et al., 1991, 1994; Baltagi and Griffin, 2001). As shown in Becker et al. (1994), reinforcement implies $\alpha_2 > 0$, i.e. adjacent complementarity between past and current consumption, analogously to the role played by $\lambda > 0$ in the AR(1) solution. Forward-looking behavior is assessed from the coefficient $\alpha_4$ of lead consumption and it is expected to be positive. This contrasts with the AR(1) solution, in which forward-looking behavior implies that lead price can either have a positive or negative effect on current consumption.\(^3\)

\(^3\)In both the saddle path and the Euler equation, current consumption negatively depends on its current price, so that the (static) law of demand applies. The Euler equation has two roots. One root $\lambda$ is less than one if $\alpha_2 + \alpha_4 < 1$, which in turn implies the restriction $\alpha_2, \alpha_4 \in (0,1)$. (Note that the same $\lambda$ is directly estimated in the AR(1) equation 4 as the coefficient of lag consumption). In addition, $\beta = \alpha_4/\alpha_2$, a property that has sometimes been used as a restriction, or as a test of the validity of the rational addiction model (Auld and Grootendorst, 2004; Baltagi and Geishecker, 2006). Importantly, the second root of the Euler equation is always larger than one,
Despite the general consensus on the empirical validity of the rational addiction model, testing it using the AR(2) Euler equation can be problematic (see for example Auld and Grootendorst, 2004; Baltagi and Geishecker, 2006; Laporte et al., 2017). In particular, Auld and Grootendorst (2004) observe that the empirical model based on the AR(2) Euler equation tends to find rational addiction when in fact the commodity does not feature addiction. For example, when estimating the demand for Canadian milk, they find the puzzling result that milk would be more addictive than smoking. Auld and Grootendorst (2004) suggest that this puzzle and, more in general, the tendency of the Euler equation to produce false positives and to erroneously classify a non-addiction good as rationally addictive, can be due to the endogeneity arising from the presence of a lead and lag consumption term in the AR(2) model, and to the use of aggregate data. To explore these possible explanations, they generate simulated consumption trajectories that feature persistence, but not rational addiction, and they estimate the corresponding parameters using the Euler equation to check for possible biases. The results show that the estimates are often unstable and very sensitive to the choice of the instruments, with a tendency to produce false positives that is more likely when the data generating process exhibits high serial correlation. This finding is particularly problematic, since time series typically display high serial correlation, in particular when data are aggregated. Accordingly, Auld and Grootendorst (2004) conclude that "time-series data will often be insufficient to differentiate rational addiction from serial correlation in the consumption series". In the following sections we show that the above claims do not hold when the empirical model is based on the AR(1) equation describing the saddle path, instead of the AR(2) equation describing the Euler equation. We claim that the better performance of the AR(1) model over the AR(2) is likely due to the fact that the Euler equation is not the solution of the model, but an intertemporal necessary condition that the solution of the rational addiction model must satisfy. Importantly, the Euler equation is intrinsically unstable, because it has at least one root that is explosive, as shown by Laporte et al. (2017). This violates the basic assumptions needed to perform econometric analysis of time series and it could produce erroneous estimates. On the contrary, the AR(1) specification we propose is stationary. Moreover, since it does not contain the lead of consumption, the endogeneity concerns afflicting the AR(2) model are likely to be less severe.

which implies that the Euler equation is in general explosive, the only exception being the saddle path (Dragone and Raggi, 2018).
3 Monte Carlo experiments

In this section we run a set of Monte Carlo experiments to investigate whether the rational addiction model is able to correctly detect rational addiction, and to distinguish it from non-addiction models that feature persistence in consumption but no forward-looking behavior. Differently from the Monte Carlo experiments of Auld and Grootendorst (2004) and Laporte et al. (2017), who estimate the simulated trajectories using the AR(2) Euler equation, we use the AR(1) addiction model.

We consider consumption trajectories generated using two alternative data generating processes (DGP). The first one corresponds to the process considered in Auld and Grootendorst (2004). It consists of a static demand model where prices and errors are autocorrelated. Specifically, consumption is assumed to depend on current price and errors,
\[ c_t = -\eta p_t + u_t, \]
where prices and errors are autocorrelated according to \( p_t = 0.9p_{t-1} + \nu_t \) and \( u_t = \rho u_{t-1} + \epsilon_t \). Parameter \( \rho \in (0,1) \), and \( \nu_t \), \( \epsilon_t \) and \( u_t \) are sequences of i.i.d. Gaussian shocks. Manipulating the above equations yields
\[ c_t = \rho c_{t-1} + \gamma_1 p_{t-1} + \gamma_2 p_t + \gamma_0 + \epsilon_t \] (8)
where \( \gamma_1 = \eta \rho \) and \( \gamma_2 = -\eta \). Since \( \rho \in (0,1) \), the trajectories generated by (8) are stationary and persistent (in the form of adjacent complementarity in consumption). This model, however, does not allow for forward-looking behavior. In fact, it formally resembles the solution of a myopic habit or taste formation model in which current consumption only depends on current and past variables (see, e.g. Pollak, 1970; Becker et al., 1994; Gilleskie and Strumpf, 2005; Dragone and Raggi, 2018). In the following we refer to (8) as to the non-addiction DGP.

The second DGP features rational addiction. Based on the saddle path solution (4), the consumption trajectories are generated according to the following process
\[ c_t = \rho c_{t-1} + \gamma_1 p_{t-1} + \gamma_2 p_t + \gamma_3 p_{t+1} + \gamma_0 + \epsilon_t, \] (9)
while the price and error dynamics follow the same autoregressive processes used for the non-addiction DGP. As emphasized in Becker et al. (1991), testing for the effects of future prices on current consumption distinguishes rational models of addiction from myopic models. Accordingly, the main difference with respect to the non-addiction DGP is that the addiction DGP features forward-looking behavior (\( \gamma_3 \neq 0 \)).

Equation (9) is the AR(1) addiction equation used to generate trajectories compatible with the theory of rational addiction, and it will also be used as the empirical model for testing rational

\footnote{With respect to equation (8), in equation (9) the coefficients \( \rho \), \( \gamma_1 \) and \( \gamma_2 \) are independent. The assumption of exogenous prices is relaxed in the robustness checks in Section 4.}
addiction. Given that the only difference between the two processes is the presence of the lead of price, we expect the estimated $\gamma_3$ to be non statistically significant when the trajectory is generated by the non-addiction process 8, and to be different from zero when it is generated by the addiction process 9.

For later reference, the short and long run response of consumption to a permanent price increase are, respectively,

$$C_S = \gamma_2 + \gamma_3 < 0,$$
$$C_L = \frac{\gamma_1 + \gamma_2 + \gamma_3}{1 - \rho} < 0. \tag{10}$$

where $\gamma_3 = 0$ when the DGP features non-addiction.

### 3.1 Experiment 1: Estimating non-addictive consumption

In the first set of experiments we generate individual consumption trajectories using the non-addiction DGP in equation 8. We randomly select $\rho$ and $\gamma_1$ from a uniform distribution $(0, 1)$, we assume that prices are strictly exogenous, and we generate 2,000 different sets of parameters $\alpha_i = (\rho, \gamma_1, \gamma_2)_i$. Each set of parameters represents an individual $i$ and determines the individual short and long-run elasticity according to equations 10.\(^5\) For each set, we generate 1000 different trajectories of length 500 of consumption and prices, which are meant to represent 'alternative life courses' of individual $i$, depending on the sequence of random shocks experienced by $i$ over her lifetime. Using the AR(1) addictive model, we use OLS and 2SLS to estimate $\hat{\alpha}_i$ over the 1000 alternative life courses of $i$. Since we know the true values of the DGP, we can compute the (relative) estimation bias $b_i$ for each $i$ using the formula $b_i = (\hat{\alpha}_i - \alpha_i)/(1 + \alpha_i)$. Aggregating these individual biases yields a measure of the average bias $\overline{b}_i = \sum b_i$ that results when using individual level data.

The left panel of Figure 1 reports the average bias $\overline{b}_i$ obtained using OLS.\(^6\) The results show no notable bias in the estimation of the true parameters of the data generating process. In particular, the AR(1) model correctly finds that the lead price coefficient is not statistically significant, which is indeed the case because the DGP features non-addiction. If anything, there is a slight downward bias in the estimation of the parameter of lag consumption, but its is negligible (less than 1%) with respect to the true value of $\rho$. The short and long-run elasticities estimated using the individual trajectories are unbiased. In contrast with the findings of Auld and Grootendorst (2004), these

\(^5\)Since the DGP is non-addictive, the short and long-run elasticities (eqs. 10) are computed setting $\gamma_3 = 0$. Parameter $\gamma_2$ is computed as $= -\gamma_1/\rho$.

\(^6\)The results obtained with IV estimation are presented in Section 4.
results suggests that the AR(1) empirical addiction model is not prone to wrongly detect rational addiction when the data display persistence, but no forward-looking behavior.

To assess whether the results above are robust to using aggregate data, we consider a second set of experiments. We consider 2000 villages $k$, each composed of 100 individuals $j$. Each individual is characterized by a different set of parameters $\alpha^j_k = (\rho, \gamma_1, \gamma_2)^j_k$, $j = 1, 2, \ldots 100$, $k = 1, 2, \ldots 2000$, which are used to generate individual consumption trajectories of length 500. For a given village $k$, we aggregate the corresponding 100 individual trajectories to obtain a village-specific trajectory with average parameters $\bar{\alpha}_k = \sum \alpha^j_k$ and average short and long-run elasticity $\bar{C}_{S,k}$ and $\bar{C}_{L,k}$. Generating 1000 alternative life courses for each individual, we generate 1000 alternative life courses of village $k$. Using OLS and IV on the aggregate trajectories we estimate $\hat{\alpha}_k$ and compute $\hat{C}_{S,k}$ and $\hat{C}_{L,k}$, which we compare to the true values to measure the village-specific relative bias $b_k = (\hat{\alpha}_k - \bar{\alpha}_k)/(1 + \bar{\alpha}_k)$. (The same formula is used to measure the relative bias for elasticity). We define $\bar{b}_k = \sum b_k$ as the average bias over the 2000 villages, and we report it with 99% confidence intervals in the right panel of Figure 1.

The results show that the AR(1) model correctly detects that the aggregated data feature persistence but no forward-looking behavior, as with individual data. Hence it does not erroneously detect addiction when there is no addiction in the data. Considering the estimated coefficients, we find an upward bias for the lagged variables, a result that is not surprising because aggregation tends to increase the persistence in time series (Granger and Morris, 1976; Havranek et al.,
Importantly, despite the overestimation in the persistence of consumption, the estimated coefficient of future price is unbiased and equal to zero, which is indeed correct because the non-addiction DGP features no forward-looking behavior. Moreover, the estimates of short and long-run elasticity are unbiased. Similar results hold also when we run additional regressions using 2SLS and prices as instruments for lagged consumption (see Section 4.2). We can therefore conclude that, even with aggregated data, the AR(1) model is able to properly distinguish pure autocorrelation from rational addiction, and to correctly detect persistence in consumption and the absence of forward-looking behavior in the data.

3.2 Experiment 2: Estimating addictive consumption

Laporte et al. (2017) show that generating and estimating consumption series using the AR(2) Euler equation can produce unreliable estimates. In this subsection, we show that this is not the case when using the AR(1) model. We run additional Monte Carlo experiments with individual and aggregate trajectories generated and estimated as in the previous subsection. The difference is that we now generate trajectories that display forward-looking behavior (with $\gamma_2$ and $\gamma_3$ selected from a uniform distribution with support $(-1,0)$ and $(-1,1)$, respectively) and that we use the AR(1) model both as the DGP and as the empirical model.

**Figure 2: Estimation bias when consumption is addictive**

Notes: Left panel: estimation of the relative bias $\bar{b}_i$ on individual trajectories. Right panel: estimation of the relative bias $\bar{b}_k$ on aggregate trajectories. Addiction consumption trajectories are generated according to 9, and estimated using 9 and OLS. The vertical bars represent the 95% confidence intervals for the estimated bias.

The estimated biases using individual and aggregate trajectories and OLS are reported in Figure 2 (left and right panel, respectively). The results show that the AR(1) model produces unbiased
estimates of the parameters and of the corresponding elasticities, both when using individual and aggregate data. With respect to the previous Monte Carlo experiments, here the coefficient of lead price is estimated to be different from zero, which is correct because the DGP is addictive, and the lagged variables are precisely estimated. Given that the estimates of the coefficients are unbiased, the estimation of the short and long-run elasticity is also unbiased. We conclude that the AR(1) model is able to correctly detect rational addiction when the data truly feature rational addiction, and that using aggregated data pose no particular threat for the empirical estimation.

4 Robustness checks

4.1 Endogenous prices

In the previous Section we have shown that the AR(1) empirical model is suitable to test rational addiction, both with individual and aggregate data. The results were obtained assuming that prices are exogenous. As a robustness check, in this section we investigate the performance of the AR(1) model when prices are endogenous. Specifically, we consider the case in which the observed price can be decomposed as follows

\[ p_t = a\tau_t + (1 - a)\pi_t \]

where \( a \in [0, 1] \). The term \( \tau_t \) is exogenous, and it can be interpreted as taxes, or as the effect of a regulation that affects the opportunity cost of consuming the good (e.g. smoking bans). The term \( \pi_t \) is endogenous and is assumed to be negatively correlated with contemporaneous consumption. Parameter \( a \) describes the relative weight of the exogenous with respect to the endogenous component of price. Accordingly, the limit case where \( a = 1 \) implies that prices are fully exogenous. The case where \( a < 1 \) seems to be more common in the empirical applications, with various degrees depending on the specific application. For example, Gruber and Köszegi (2001) observe that about 80 percent of the within-state-year variation in the price of US cigarettes can be attributed to tax changes, while Blanchette et al. (2019) report that for alcohol the median specific excise taxes account for about one fifth of state alcohol taxes.

In the following we consider two different values of \( a \): \( a = 0.8 \) (‘almost exogeneity’), and \( a = 0.2 \) (‘mild exogeneity’). For each value of \( a \) we generate price trajectories and we perform the same exercise of the previous experiments, both with individual and aggregate data, with the

\[ \pi_t = 0.7\pi_{t-1} + \xi_t \quad \text{and} \quad \text{corr}(\xi_t, \epsilon_t) = -0.5, \] where \( \epsilon_t \) is the error used in Section 3 to determine consumption.
Figure 3: Estimation bias when prices are endogenous

Scenario 1

DGP: non-addictive consumption

DGP: addictive consumption

Scenario 2

DGP: non-addictive consumption

DGP: addictive consumption

Notes: Scenario 1: prices are mildly exogenous ($\alpha = 0.2$). Scenario 2: prices are almost exogenous ($\alpha = 0.8$). The left panel reports the results obtained with a non-addictive data generating process. The right panel reports results obtained from trajectories featuring addiction. All individual trajectories are estimated using 9 and 2SLS. The vertical bars represent the 95% confidence intervals for the estimated bias.

The results using individual data are reported in Figure 3. They show no significant bias, both when estimating the non-addictive and the addictive trajectories. A possible exception is
the bias in the coefficient of lagged consumption when prices are almost exogenous and the DGP features non-addiction (bottom-left panel). This bias, however, is negligible (less than 1%) and, importantly, it does not bias the estimation of lead terms, nor the short and long-run elasticity of consumption. When using aggregate data, instead of individual trajectories, the results (not reported) are similar to those obtained in the previous simulations with exogenous prices. We can therefore conclude that, even when prices are endogenous, the AR(1) model is able to correctly distinguish between pure persistence and rational addiction, and to provide unbiased estimates of the elasticity of demand.

4.2 Endogenous lagged consumption

When estimating rational addiction using the AR(2) model (eq. 7), Becker et al. (1994) observe that past and future consumption can be endogenous, and they suggest using past and future prices as instruments. The AR(1) model does not contain future consumption, but the presence of past consumption can still give rise to endogeneity concerns. To address them, in this subsection we re-run the analysis presented in Section 3, with the only difference that we use 2SLS, and $p_{t-2}$ and $p_{t+2}$ as instruments (overidentification), rather than OLS.

**Figure 4: Estimation bias when lagged consumption is endogenous**

![Graph showing estimation bias](image)

**Notes:** Left panel: estimation on individual trajectories. Right panel: estimation on aggregate trajectories. Consumption trajectories are generated according to the non-addiction process 8, and estimated using 9 and 2SLS. The vertical bars represent the 95% confidence intervals for the estimated bias.

In Figure 4 we report the estimation bias obtained when the DGP features non-addiction and the estimating model is the AR(1) equation 9. This experiment is analog to the first one reported in Section 3, which uses OLS estimation, and the results are similar. We find (i) no significant
bias when considering individual trajectories, (ii) a positive bias on the lag coefficients when using aggregate trajectories, and (iii) unbiased estimates of the short and long-run elasticity, both when using individual and aggregated consumption series. As expected, since IV estimation is less efficient than OLS estimation, the confidence intervals are wider.

As a final check, we also re-run the experiment when the DGP features rational addiction, as in the second experiment in Section 3. The IV estimates (not reported here and available upon request) are similar to those obtained when estimating with OLS and show not significant bias. We can therefore conclude that the endogeneity concerns due to presence of lag consumption in the estimating equation does not pose a relevant threat for the empirical estimation.

5 Is Milk really addictive? No

In this section we use real data to investigate the Auld and Grootendorst (2004)’s result about milk addiction. To allow for a direct comparison, we consider the same Canadian dataset containing annual aggregate national data on consumption and prices for milk, oranges, eggs and cigarettes (Auld and Grootendorst, 2004).\textsuperscript{8} Instead of the AR(2) equation 7 used by Auld and Grootendorst (2004), here we use the AR(1) empirical model (eq. 9). Accordingly, rational addiction predicts the coefficient of \(c_{t-1}\) to be positive and less than one, the coefficient of \(p_t\) to be negative, and the coefficient of \(p_{t+1}\) to be significantly different from zero. The first prediction reveals adjacent complementarity and is consistent with reinforcement in preferences, the second one shows consistency with the law of demand, and the third one reveals forward-looking behavior and is the main test to distinguish between rational addiction and simple persistence in consumption.

As a preliminary analysis, we test whether the commodities are stationary, a necessary condition for both the rational addiction theoretical model and for the empirical estimation. A battery of stationarity tests (the Augmented Dickey-Fuller, the GLS Dickey-Fuller, and the Zivot-Andrews tests) consistently rejects the unit-root hypothesis only for oranges. For milk, eggs and cigarettes, we are unable to reject the null hypothesis of non-stationarity, both for consumption and prices. Additional testing shows that there exists a cointegration relationship between consumption and prices for milk, eggs and cigarettes. Hence, when estimating the parameters for milk, eggs and

\textsuperscript{8}The data were obtained from the Statistics Canada’s CANSIM database. Oranges are observed starting from 1960, Eggs and milk starting from 1961, cigarettes starting from 1968. Prices are expressed in real terms by adjusting by all-items CPI (1992 = 100). All quantities (liters for milk, dozens for eggs, kilos for oranges) are in per-capita terms. As in Auld and Grootendorst (2004), cigarette consumption includes cigars and is computed as the sum of domestic and export sales to account for smuggling between Canada and the USA. Real per-capita outlays on consumer non-durables are used as a proxy for permanent income. See Auld and Grootendorst (2004) for details.
cigarettes, we follow the two-step Engel-Granger procedure for cointegration modeling. We consider the Error Correction Mechanism (ECM) representation of the AR(1) model and we apply Dynamic OLS for estimation (Stock and Watson, 1993). As shown in the Appendix A.2, this is relatively easy to implement using our linear AR(1) model. For oranges, we simply use OLS.9

A potential concern for the empirical analysis is that the AR(1) model can suffer of endogeneity, due to the inclusion of a lag consumption term (Gilleskie and Strumpf, 2005). This concern is also present (and actually more pervasive) when using the AR(2) Euler equation, due to the existence of both a lead and lag term, and it has been addressed in the literature using instrumental variables and GMM (Becker et al., 1991; Chaloupka, 1991; Baltagi and Geishecker, 2006; Auld and Grootendorst, 2004). The post-estimation analysis (available upon request) shows that the model’s residuals are uncorrelated, which corroborates the exogeneity assumption of lag consumption. In addiction, the results of the Monte Carlo simulations reported in the previous sections show that endogeneity is a minor concern when testing the rational addiction model using the AR(1) model. According to these preliminary considerations, we consider $c_{t-1}$ to be exogenous and we run OLS (or 2SLS) when stationarity is satisfied (oranges), and the two-step procedure described in Engle and Granger (1987) when the data are non-stationary but cointegrated (cigarettes, milk and eggs).

The empirical results shown in Table 1 suggest that only cigarettes are consistent with the Becker and Murphy (1988)’s theory of rational addiction. As predicted by the model, the coefficients of lagged consumption and of current price are negative and significant, and the coefficient of lead price is different from zero, which implies that law of demand holds and that the demand for cigarettes features adjacent complementarity and forward-looking behavior.10 On the contrary, consumption of milk, eggs and oranges is not forward-looking. Hence it is not consistent with the rational addiction model, and it is instead compatible with a habit formation (or myopic addiction) model.

The result that neither milk, or oranges, or eggs are addictive is in sharp contrast with Auld and Grootendorst (2004)’s puzzling finding. We suspect a main reason for this discrepancy is

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9In both models deterministic trends have not been included, because of their irrelevant impact on the results. As in Auld and Grootendorst (2004), we add outlays as a control variate. Additional information on the unit-root and cointegration tests is reported in Table 2 in Appendix A.1. Details on the ECM representation and on the estimation procedure of the AR(1) model are in Appendix A.2.

10Although our goal is not to provide new estimates for the elasticity of demand, note that estimated values of the short and long-run elasticity are $-0.23$ and $-0.59$, respectively. These values are compatible with those found by Gruber et al. (2003), who report an elasticity of the demand for Canadian cigarettes in the range from $-0.45$ to $-0.47$, as well as with the estimates obtained using US data (Chaloupka and Warner, 2000; Gruber and Köszegi, 2001; Callison and Kaestner, 2014; Zheng et al., 2017).
Table 1

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Cigarettes</th>
<th>Milk</th>
<th>Eggs</th>
<th>Oranges</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$ ($c_{t-1}$)</td>
<td>0.879*** (0.100)</td>
<td>0.772*** (0.160)</td>
<td>0.832*** (0.088)</td>
<td>0.751*** (0.134)</td>
</tr>
<tr>
<td>$\gamma_2$ ($p_t$)</td>
<td>-0.093** (0.045)</td>
<td>-0.478** (0.175)</td>
<td>-0.087 (0.052)</td>
<td>-0.634*** (0.142)</td>
</tr>
<tr>
<td>$\gamma_3$ ($p_{t+1}$)</td>
<td>-0.140*** (0.015)</td>
<td>0.285 (0.169)</td>
<td>0.021 (0.050)</td>
<td>0.136 (0.120)</td>
</tr>
</tbody>
</table>

Rational Addiction?  Yes  No  No  No

Notes: Estimation of the demand for cigarettes, milk, eggs and oranges using the AR(1) model and national annual data from the Statistics Canada’s CANSIM database. Dependent Variable = $c_t$; * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$; standard errors in parentheses. Prices are expressed in real terms by adjusting by all-items CPI (1992 = 100). All quantities are in per-capita terms. Cigarette consumption includes cigars and is the sum of domestic and export sales to account for smuggling between Canada and the USA. Rational addiction predicts the coefficient of $c_{t-1}$ to be positive and less than one, the coefficient of $p_t$ to be negative, and the coefficient of $p_{t+1}$ to be significantly different from zero. Only cigarettes are consistent with rational addiction. Milk, eggs and oranges are consistent with persistence in consumption, but do not display forward-looking behavior.

An additional reason for the discrepancy between our empirical results and Auld and Grootendorst (2004) is that some of the time series under examination are non stationary. This may have produced unreliable estimates, even in absence of endogeneity. To address this concern, before running our analysis we have tested the data for stationarity and cointegration. This has allowed to distinguish between cases in which stationarity holds and OLS can be used (oranges), and cases in which one should follow a different route, such as the two-step procedure of Engle and Granger (1987).
6 Conclusion

The evidence on milk addiction found by Auld and Grootendorst (2004) has raised the question of whether the AR(2) model typically used to test the model of rational addiction is an appropriate empirical specification. The AR(2) model tends to find spurious evidence for rational addiction and is very sensitive to the choice of the instrumental variable estimators, a result that is more likely when the consumption series display high persistence (Auld and Grootendorst, 2004). In addition, Laporte et al. (2017) show that the AR(2) model is intrinsically explosive, which makes estimating and testing the rational addiction model problematic. In this paper we have shown that the above results do not hold when, instead of the AR(2) equation, a linear AR(1) model is used. This specification describes the saddle path solution of the rational addiction model, it retains the main theoretical predictions that have been investigated in the literature using the canonical AR(2) model, and it is empirically simpler to estimate.

Using Monte Carlo simulations, we first show that the AR(1) model does not produce false positives and it is able to correctly detect rational addiction. Moreover, it produces unbiased estimates of the short and long-run elasticity of consumption, and it does not suffer of the endogeneity concerns that may arise when using lag consumption in the estimating equation. These results hold both with individual and aggregate data, a finding that is particularly appealing because consumption series are typically available as aggregate data.

To directly address the milk addiction paradox, we then consider the same Canadian data used by Auld and Grootendorst (2004) and we proceed with the empirical analysis using the AR(1) model, instead of the AR(2) model. This allows to show that the milk addiction paradox is only apparent. In fact, using the AR(1) addiction model, we show that milk is not compatible with the rational addiction model, while cigarettes are, as expected.

These results allow us to conclude that the AR(1) model is a good candidate to test the theory of rational addiction. We claim that the better performance of the AR(1) model over the AR(2) is likely due to the fact that the Euler equation is not the solution of the model, but an intertemporal necessary condition that the solution of the rational addiction model must satisfy. Moreover, the Euler equation is intrinsically unstable, because it has at least one root that is explosive (Laporte et al., 2017). This violates the basic assumptions needed to perform econometric analysis of time series and it could produce erroneous estimates. On the contrary, the AR(1) specification we propose is stationary and, since it does not contain the lead of consumption, the endogeneity concerns afflicting the AR(2) model are likely to be less severe.
Bibliography


Appendices

A Empirical analysis: Supplementary material

A.1 Unit-root, cointegration tests

Table 2 shows the results of the Augmented Dickey-Fuller test (ADF, Dickey and Fuller, 1979), the GLS Dickey-Fuller test (DF-GLS, Cheung and Lai, 1995; Elliot et al., 1996), and the Zivot-Andrews tests (ZA, Zivot and Andrews, 1992), to check for unit-roots in the data analyzed in the empirical analysis of Section 5. The optimal number of lags is automatically selected. Since trends appear to be not relevant, in the ZA test we allow for structural breaks only for the intercept.

Table 2: Unit-root and cointegration tests

<table>
<thead>
<tr>
<th></th>
<th>Cigarettes</th>
<th>Milk</th>
<th>Eggs</th>
<th>Oranges</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>price</td>
<td>cons.</td>
<td>price</td>
<td>cons.</td>
</tr>
<tr>
<td>ADF test</td>
<td>-1.208</td>
<td>0.355</td>
<td>-1.745</td>
<td>-1.791</td>
</tr>
<tr>
<td>DF-GLS test</td>
<td>-2.008</td>
<td>-1.107</td>
<td>-1.329</td>
<td>-0.349</td>
</tr>
<tr>
<td>Stationary?</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>GH/EG test</td>
<td>-5.39</td>
<td>-3.651</td>
<td>-3.816</td>
<td></td>
</tr>
<tr>
<td>z crit. value</td>
<td>-4.99</td>
<td>-3.497</td>
<td>-3.497</td>
<td></td>
</tr>
<tr>
<td>Cointegrated?</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td></td>
</tr>
</tbody>
</table>

Notes: Unit-root and cointegration tests for the Canadian data analyzed in the empirical analysis of Section 5. z is the critical value of the corresponding test (10% for the unit-root tests, 5% for the cointegration tests).

For cigarettes we use the Gregory-Hansen cointegration test (GH, Gregory and Hansen, 1996) to account for potential structural breaks due to the antismuggling policies implemented in the early 1990s (Gruber et al., 2003). The p-values are smaller than 10% and allow rejecting the
no-cointegration hypothesis. Similarly, for milk and eggs the Engle-Granger test (EG, Engle and Granger, 1987) allows rejecting no-cointegration at the 5% significance level.

A.2 AR(1) model and error correction mechanism

Based on the results presented in Table 2, consumption and prices of cigarettes, milk and eggs are cointegrated. Accordingly, we follow the two-step Engel-Granger procedure for cointegration modeling. We consider the Error Correction Mechanism (ECM) representation of the AR(1) model and we apply Dynamic OLS for estimation (Stock and Watson, 1993), as explained below.

As a starting point, consider the AR(1) model

\[ c_t = \rho c_{t-1} + \gamma_1 p_{t-1} + \gamma_2 p_t + \gamma_3 p_{t+1} + \gamma_0 + \xi_t. \]  

(12)

Subtracting \( c_{t-1} \) and manipulating the above equation allows for the following Error Correction Mechanism representation

\[ \Delta c_t = \mu (c_{t-1} - \gamma_0 + \omega p_{t-1}) + \gamma_0 \rho + \gamma_2 \Delta p_t + \gamma_3 \Delta_2 p_{t+1} + \epsilon_t. \]  

(13)

where \( \mu \equiv \rho - 1, \omega \equiv \frac{\gamma_1 + \gamma_2 + \gamma_3}{\mu} \), \( \Delta \) is the difference operator, and \( \Delta_2 p_{t+1} \equiv p_{t+1} - p_{t-1} \). This representation allows to describe consumption as a combination of a long run relationship \( (c_{t-1} = \gamma_0 - \omega p_{t-1}) \), and a short run relationship \( (\gamma_0 \rho + \gamma_2 \Delta p_t + \gamma_3 \Delta_2 p_{t+1}) \) between consumption and prices.

Considering equation 13, estimates and inference for the parameters can be derived through the two-step procedure described in Engle and Granger (1987). First, the long run relationship \( c_{t-1} = \gamma_0 - \omega p_{t-1} \) is estimated using the Dynamic OLS procedure of Stock and Watson (1993), which produces super-consistent estimators of \( \gamma_0 \) and \( \omega \). The lagged residuals are then plugged into equation 13 to obtain estimates of \( \gamma_2, \gamma_3 \) and \( \rho \). Finally, using the definition of \( \omega \) and \( \mu \), the point estimate of \( \gamma_1 \) can be computed as \( \gamma_1 = - (\mu \omega + \gamma_2 + \gamma_3) \).