Optimal Participation Income and Negative Income Tax in Poverty Alleviation Programs

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Abstract: We compare two different redistributive policies specifically designed to alleviate poverty. The first one is inspired to the idea of participation income, the second focused on the introduction of a negative income tax. Briefly characterizing an economy with workers and non-workers and using income as evaluation space for poverty, we determine optimality conditions of both measures under a flat rate labor income tax. Finally, we emphasize a necessary condition for these measures to mitigate poverty.

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1 Introduction

Nowadays, “as actual trends in economic development increase the likelihood that wealth will be generated by a smaller proportion of total population [...] the redistributive function of the public sector is likely to become more important than ever” (Hobsbawm (1996)). Moreover, such a function seems primarily important in front of increasing poverty and inequality in both developed and developing countries. Hence, as social tensions have recently testified, the design of poverty alleviation programs and newly-arranged redistributive policies will become a central issue in the architecture of sustainable welfare systems. These policies may be arranged in at least two different ways.

On the one hand, policy makers may decide to implement reforms of existing redistributive tools. Re-designing taxation in order to make traditional social security schemes more inclusive may be an example. In this case, the policy alleviation effort is undertaken re-allocating social costs and benefits through a new structure of traditional taxation schemes. From this perspective, Makdissi and Wolon (2002) have discussed a possible reform of commodities taxation regimes and its impact on aggregate income poverty.

On the other, policy makers may design a targeted poverty alleviation program. We define it as a supplementary poverty alleviation intervention independently financed through an anchored tax and accompanied to well-established social security systems or mean-tested benefits. Two fairly famous tools for redistributing income inside a poverty alleviation program are: a participation income (PI) (Atkinson (1996)) defined as a basic income paid conditional on the participation to the social contract (through some form of recognized social contribution3) and a negative income tax (NIT) (Friedman (1962)) that may be seen as a proportional subsidy targeted to poor income-earners.

Surprisingly so far, few contributions have tried to give insights on the optimal architecture of these two redistributive tools. Atkinson (1995) has introduced a model in which it is possible to determine the optimal basic income and the welfare maximizing anchored flat rate tax in an economy with infinitely many well-endowed workers and disable or sick citizens. Groot and Peeters (1997) have, for instance, compared in terms of incentives to efficiency a negative income tax and a basic income in a efficiency-wages model with workers and firms. Nevertheless, no attempts have been undertaken to derive optimality conditions for a PI or a NIT inside economies where poverty is present. Hence, using income as reference space, we characterize an economy with poors and non-poors and we derive some conditions, in the case of flat-rate taxation, under which the above redistributive tools are optimally designed. As it will be clearer below, our set up is a two-sided extension of the well-known model of basic income proposed by Atkinson (1995). First of all, it does include poverty

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3 As stressed by many authors a wide definition of social contribution may be accepted. It may include engaging approved forms of education, training, caring for others, voluntary activities etc... .
considering poor and non-poor workers. Secondly, it is able to deal with the similar case of the introduction of a negative income tax.

Furthermore, we deduce effectiveness conditions for both instruments using a well-defined poverty measure: the "Hagenaars's index" (Hagenaars (1987)). Several reasons are for such a measure instead of a simpler one (e.g. a poverty gap ratio or some headcount ratio based measures). On one hand, if the introduction of a PI/NIT reduces poverty such a mitigation is obtained through progressive transfers. Thus, we need a poverty measure, like the Hagenaars’ one, which satisfies a progressive transfer axiom\(^4\) being sensitive to the redistribution of income within poors and non-poors. On the other, a too high tax rate applied on non-poors’ incomes for financing the introduction of a PI or a NIT may induce some agents to cross the poverty line thus changing the number of the poors even with the same aggregate poverty gap. As known any poverty gap-related index does not satisfy neither a non-poverty growth axiom nor a poverty growth axiom (Kundu and Smith (1983)) contrary to the Hagenaars’ Measure. Finally as stressed by Zheng (1997), the Hagenaars’ Index is one of the few poverty indices which does satisfy almost any nice property for an aggregate poverty measure.

The format of the remainder of the paper is as follows: section 2 introduces our set up. Optimality conditions are derived and discussed for a PI and a NIT in section 3. Section 4 provides a necessary condition for their effectiveness in poverty reduction interventions. Section 5 concludes.

2 The Set Up

Let us consider a finite population of participative agents and normalize, without loss of generality, it at one. A share \(\eta\) of individuals have no current job (i.e. sick, unemployed, disables) and \(1 - \eta\) agents actually are hired in the labor market.

Agents’ preferences are represented by an indirect utility function symmetric for any agent

\[
v = v_i(p, w_i, M) = v(w_i, M)
\]

where \(p\) are fixed market prices, \(w_i\) is the nominal earned wage and \(M\) are non-labor incomes. As usual, such a function is traditionally assumed strictly increasing and quasi-convex. Any worker supplies labor according to a supply function \(L = L(w_i, M)\) with \(L = 0\) if \(i\) belongs to the fraction \(\eta\) of the population.

\(^4\)For a discussion of transfer axioms see Zheng (1997).
Only non-workers receive a non-labor income equal to $\tilde{M}$ from existing social security systems. All other agents do not have any non-labor income.

Denote with $l_i$ the hours of labor supplied by a worker receiving a net market wage equal to $w_i$. Workers are identical except for their earned wages and they are distributed with respect to $w_i$ according to a distribution function $F(w_i)$. Furthermore, let us suppose that a lower and an upper bound are institutionally fixed for wages i.e. $w_i \in [w_l; w_u]$.

In order to assess poverty, a government sets an expenditure-oriented poverty line $z$ (with $z > \tilde{M}$) aimed to identify working poors and to allow the calculation of poverty rates or poverty gaps. Then, the set of income poors is given by any member of the portion $\eta$ of the population plus any worker with $w_i l_i \leq z$ or alternatively with $w_i \leq u_0 = z/l_i$.

Hence, it is possible to define:

$$
(1 - \eta) = \sum_{i: w_l \leq w_i < w_0} p(w_i) + \sum_{i: w_0 \leq w_i \leq w_u} f(w_i)
$$

(2)

denoting with $p$ the mass function of $F(w_i)$ with respect to working poors and $f$ the corresponding function for non-poor workers.

Consistently, the social welfare in our economy is given by:

$$
W = \eta \Psi\left(v\left(\tilde{M}\right)\right) + \sum_{i: w_l \leq w_i < w_0} \Psi\left(v(w_i, 0)\right) p(w_i) + \sum_{i: w_0 \leq w_i \leq w_u} \Psi\left(v(w_i, 0)\right) f(w_i)
$$

(3)

where $\Psi$ is a not decreasing and non-convex transformation of individual utilities.

In absence of any poverty alleviation program, the individual poverty gap is equal to $g(w_i) = z - w_i l_i$ for $w_i < w_0$ and the aggregate poverty gap is given by:

$$
G = \eta \left(z - \tilde{M}\right) + \sum_{i: w_l \leq w_i < w_0} \left(z - w_i l_i\right) p(w_i)
$$

(4)

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5 For nice surveys on poverty lines see Lanjouw (2000) and Bjorkw (2000).

6 We are assuming that such an income is determined after any direct or indirect form of taxation.

7 In expression (2) a weak definition of poverty is used. A worker is poor whereas his wage is strictly lower than the poverty line-related value of earned wage.
where the second term on the right hand side indicates the aggregate poverty gap referred to working poors (henceforth $\tilde{G}$). Finally, the Hagenaars’ Index slightly modified consistently with our set up is equal to:

$$H = \left(1 - \frac{\ln(\tilde{M})}{\ln z}\right) + \frac{1}{\mu} \sum_{i: w_0 \leq w_i < w_0} \left(1 - \frac{\ln(w_i^l)}{\ln z}\right)$$

where $\mu$ is the portion of working poors in our economy.

3 Optimal PI and NIT in presence of Poverty

In order to reduce aggregate poverty, a policy maker decides to introduce a participatory income ($b$). Such an income is given to any participative agent, independently by its current position into the job-market and it is financed by flat-rate labor income tax ($t$) levied on non-poors’ labor income. Consistently, the government budget constraint is given by:

$$b = t \sum_{i: w_0 \leq w_i \leq \tilde{M}} w_i L(w_i, M) f(w_i)$$

Optimality conditions for the introduction of a participatory income may be obtained maximizing (3) with respect to $b$, taking into account (6) and that now $M = b > 0$ for any worker in our economy.

**Proposition 1** A Participation Income is optimal if the aggregate expected marginal benefit for poors and non-poors is equal to the marginal cost of public funds.

**Proof.** Maximize the following Lagrangian wrt $b$ and $t$:

$$L = \eta \Psi \left( v \left( \tilde{M} + b \right) \right) + \sum_{i: w_0 \leq w_i < w_0} \Psi \left( v(w_i, b) \right) p(w_i) +$$

$$+ \sum_{i: w_0 \leq w_i \leq \tilde{M}} \Psi \left( v \left( (1 - t)w_i, b \right) \right) f(w_i) - \lambda \left[ b - t \sum_{i: w_0 \leq w_i \leq \tilde{M}} w_i L(w_i, b) f(w_i) \right]$$

85 Given our normalisation is straightforward to see that the aggregate participation income is equal to $B = b$. 

5
where $\lambda \geq 0$ is the marginal cost of public funds. With $\Psi_0 = \partial \Psi / \partial v > 0$, 
$\gamma = \partial v / \partial b > 0$ and $\gamma_0 = \partial v (f_M) / \partial b > 0$ we get the following foc:

$$
\eta \Psi_0 \gamma + E_{\{i: w_0 \leq w_i \leq w_0\}} (\Psi' \gamma) + E_{\{i: w_0 \leq w_i < w_0\}} (\Psi' \gamma) - \lambda \sum_{i: w_0 \leq w_i} w_i \partial L / \partial b f(w_i) = 0
$$

(8)

where $E$ indicates the expected value. Calling $\phi$ the net marginal benefit, for each group of agents, normalizing it by $\lambda$ and using (8), we get the following optimality condition:

$$
\Upsilon + E_{\{i: w_i \leq w_i < w_0\}} (\phi_i) + E_{\{i: w_0 \leq w_i \leq w_0\}} (\phi_i) = 1
$$

(OC1)

where $\Upsilon = \frac{\eta \Psi_0 \gamma}{\lambda}$, $\phi_i = \frac{\Psi_0 \gamma}{\lambda}$ for working poors and $\phi_i = \frac{\Psi_0 \gamma + tw_i \partial L / \partial b}{\lambda}$ for any non-poor worker; that is exactly what stated in the proposition.

As it is straightforward to notice looking at $\phi_i$ for non-poor agents, in presence of wealth effects (i.e. $\partial L / \partial b < 0$) the introduction of a PI induces a severe reduction of their labor supply and consequently a smaller amount of yield available for financing the PI. Moreover, the expected net marginal benefit for non-poors may be negative whereas above effects are sufficiently strong. In this case, optimality imposes higher expected net marginal benefits for the rest of the population with a smaller participation income. From convexity of $v$, this will be more likely the relatively richer working poors and non-workers are.

With no wealth effects (i.e. $\partial L / \partial b = 0$), condition (OC1) can be written as:

$$
\eta \Psi' \gamma + 2E_{\{i: w_i \leq w_i \leq w_0\}} (\Psi' \gamma) = \lambda
$$

(9)

Following (10), the aggregate expected net marginal benefit for workers (poor and non-poors) must increase whereas the marginal cost of public funds increases. As above, if introducing a PI is particularly expensive optimality requires a low aggregate poverty in our economy. Symmetrically, an optimal PI may be sustained also in presence of deep income poverty if the marginal cost of public funds is relatively low.

Finally a more evident observation. Whether the portion of non-workers increases, the expected net marginal benefit for workers has *ceteris paribus*, to be reduced. Hence, the optimal PI in an economy with a large fraction of disable, sick or unemployed persons has to be small given the small tax base available. In economies where this coincide with generally high costs of public funds, this could involve an almost zero optimal PI.
Alternatively, let us imagine that a policy maker might introduce a negative income tax (NIT) for supporting working poor’s incomes. In its simplest formulation, a NIT may be seen as a flat-rate subsidy equal to \( s \) for any agent with a positive poverty gap. In the case of a flat rate tax levied on non-poors’ labor incomes, the government budget constraint is given by:

\[
 s \sum_{i: w_i \leq w_0} (z - w_i l_i) p(w_i) = t \sum_{i: w_0 \leq w_i \leq w} w_i L f(w_i) \tag{10}
\]

since non-workers do not receive any additional transfer. The available income is consequently \( \bar{M} \). On the contrary, any working poor receives an amount of non-labor income equal to \( s g(w_i) \) where, as defined above, \( g \) is the individual poverty gap.

Then, we can show the following:

**Proposition 2** A negative income tax is optimal if the working poor’s expected marginal benefit, weighted by their individual poverty gaps, minus the expected marginal loss of yield due to wealth effects on non-poors labor supply is proportional to the aggregate poverty gap of the economy.

**Proof.** Maximizing wrt \( s \) the following Lagrangian:

\[
 L = \eta \Psi \left( v \left( \bar{M} \right) \right) + \sum_{i: w_i \leq w_0} \Psi \left( v(w_i, s g(w_i)) \right) p(w_i) + \\
 + \sum_{i: w_0 \leq w_i \leq w} \Psi \left( v((1 - t) w_i; 0) \right) f(w_i) + \\
 - \lambda \left[ s \sum_{i: w_i \leq w_0} (z - w_i l_i) p(w_i) - t \sum_{i: w_0 \leq w_i \leq w} w_i L f(w_i) \right] \tag{11}
\]

we obtain the following foc:

\[
 E\{i: w_i \leq w_0\} \left( \phi_i g(w_i) \right) - \sum_{i: w_i \leq w_0} g(w_i)p(w_i) + t \sum_{i: w_0 \leq w_i \leq w} w_i \frac{\partial L_i}{\partial s} f(w_i) = 0 \tag{12}
\]

where \( \phi_i \{i: w_i \leq w_0\} = \frac{\psi_i \sigma}{\lambda}, \sigma = \frac{\partial \psi}{\partial w} > 0 \) and \( \frac{\partial L_i}{\partial s} < 0 \) if some relevant wealth effects exist. Equivalently, condition (13) may be written as:

\[
 E\{i: w_i \leq w_0\} \left( \phi_i g(w_i) \right) - E\{i: w_0 \leq w_i \leq w\} \left( tw_i \left| \frac{\partial L_i}{\partial s} \right| \right) = \tilde{G} \tag{OC2}
\]
Expression (OC2) implicitly features the optimal negative income tax. All other things equal, the expected marginal benefit of a negative income tax must augment if the working poors’ cumulative poverty gap increases. Given the convexity of \( v \) and working poors now relatively less rich, it involves an higher value for the optimal NIT. In opposition, the optimal NIT can be reduce in correspondence of a small \( G \).

Additionally, an high expected loss of yield due to strong wealth effects means less resources for financing the poverty alleviation program and hence a smaller optimal NIT. In this case for attaining optimality an higher increase in working poors’ expected marginal benefit with a reduced negative income tax is needed. This will be possible only if the latter are relatively richer.

Finally, without wealth effects (i.e. \( \partial L / \partial \sigma = 0 \)) it must be true that:

\[
E_{\{i:w_0 \leq w_i < w_0\}}(\phi_i g(w_i)) = \hat{G}
\]

where the discussed relation is even more straightforward to notice. Finally, maximizing the two Lagrangian with respect to \( t \) we get the same foc:

\[
E_{\{i:w_0 \leq w_i \leq \sigma\}}(\Psi \frac{\partial v}{\partial t}) + \lambda \sum_{i:w_0 \leq w_i \leq \sigma} w_i L f(w_i) + \lambda t \sum_{i:w_0 \leq w_i \leq \sigma} w_i \frac{\partial L}{\partial t} f(w_i) = 0
\]

(14)

With some manipulations and using the Roy’s Identity, condition (14) leads to the following optimality condition for the anchored flat rate tax:

\[
\frac{\hat{t}}{1 - \hat{t}} = \frac{E_{\{i:w_0 \leq w_i \leq \sigma\}}(w_i L (1 - \phi_i))}{E_{\{i:w_0 \leq w_i \leq \sigma\}}(w_i L \varepsilon)}
\]

(15)

where \( \varepsilon \) is the elasticity of substitution between labor supply and wage. Expression (15) is a slight modification of the traditional Ramsey Rule as noticed and explained in Atkinson (1995) (cfr.pp.36-37).
4 A necessary condition for Poverty Reduction using PI or NIT

Optimality conditions may not coincide with conditions for effectiveness of a PI and a NIT in alleviating aggregate poverty. The following proposition adds to conditions (OC1) and (OC2) a necessary conditions for effectiveness. As we will see, even here wealth effects play a crucial role.

Proposition 3 Independently by the composition of the set of poors, the Hagenaars’ poverty measure will surely decrease in $b$ and $s$ only if there are no relevant wealth effect on working poors’ labor supply.

Proof. Since a PI and NIT a progressive income transfers, these may modify the composition of the set of poors persons. Thus, there are four possible cases:

(i) **PI and no agents are crossing the poverty line.** In this case the Hagenaars’ Index is equal to

$$H = \left(1 - \frac{\ln \left(\frac{M + b}{\ln z}\right)}{\ln z}\right) + \frac{1}{\mu} \sum_{i: w_i \leq w_0 < w_i} \left(1 - \frac{\ln (w_i L + b)}{\ln z}\right)$$

with $\mu$ equal to the portion of working poors. It is straightforward to see that $\frac{dH}{db} < 0$ only if $\frac{dL}{db} = 0$

(ii) **PI and some agents are crossing the poverty line.** Now, the Hagenaars’ Index is equal to

$$H = \left(1 - \frac{\ln \left(\frac{M + b}{\ln z}\right)}{\ln z}\right) + \frac{1}{\mu} \sum_{i} \left(1 - \frac{\ln (w_i L + b)}{\ln z}\right) + \frac{1}{k} \sum_{j} \left(1 - \frac{\ln \left((1 - t)w_j L + b\right)}{\ln z}\right)$$

with $\mu \neq \mu'$ and $k$ equal to the portion of non-poors who have crossed from above the poverty line because of excessive fiscal pressure. Again, the above conclusion holds.

(iii) **NIT and no agents are crossing the poverty line.** Now the index is equal to

$$H = \left(1 - \frac{\ln \left(\frac{M}{\ln z}\right)}{\ln z}\right) + \frac{1}{\mu} \sum_{i} \left(1 - \frac{\ln (w_i L + sg(w_i))}{\ln z}\right)$$

and the above conclusion may easily be obtained. The same it is true for the last case. ■
Working poors’ labor supply responsiveness is the key element in ensuring a poverty alleviation effect of our redistributive measures. A PI involves perverse effects on poverty if working poors suddenly reduces their labor supply, thus receiving lower labor income. This will be more likely, the lower is the individual poverty gap. In other words, some well-known poverty trap effects may lower the impact of a participation income in alleviating poverty 9. The same effect may be recognized for a NIT 10. However in this case, since s decreases in correspondence to less pervasive cumulative deprivation, it is reasonable that poverty trap effects will be remarkably mitigated.

5 Concluding Remarks

The purpose of this paper is to provide some conditions for the utilization of a PI or a NIT inside poverty alleviation programs. We show that the optimal design of both measures crucially depends on the composition of the population and the magnitude of wealth effects on labor supply. Whereas strong wealth effects reduce non-poors’ labor supply, aggregate poverty has to be more pervasive in order to achieve optimality in the implementation of both redistributive measures. In this case, the impossibility of levying high labor income taxes on non-poor workers forces the government to introduce a relatively low PI or NIT. Thus, they will be able to assure sufficiently high net marginal benefits to poor persons only if poverty is sufficiently strong in the economy. If this is not the case, PI and NIT may optimally introduced whereas the level of aggregate poverty is not too high.

We also evaluate a necessary condition for effectiveness of both tools, again related to wealth effects on working poors’ labor supply. Since poverty trap effects may strongly reduce the impact of a PI or a NIT on aggregate poverty, it may be preferable to apply these redistributive policies in economies with a sufficiently weak preference for leisure time among poors in order to avoid unpleasant substitutions between labor income and public transfers.

The centrality of wealth effects seems here to reflect, as noticed by Van Parijs (1996), that a basic income "to which workers are unconditionally entitled [...] enables them to filter out jobs that are not sufficiently attractive". If the size of this effect is large a basic income, but the same may be said for a NIT related transfer, likely becomes a small sabbatical payment and a less effective tool in fighting poverty.

9 For a design of the basic income proposal specifically aimed to deal with poverty trap effects see Groot (1997) or Creedy and Dawkins (2002).
10 Ashenfelter and Plant (1990) provide an empirical estimation of labor supply responsiveness to the introduction of a NIT. Unfortunately their results present several ambiguities. Differently, De Jager et al. (1996) simulate the introduction of a NIT in a small welfare state, showing that, under some circumstances, the fall in labor supply may not coincide with an higher unemployment rate. This result is explained, in their framework, by the increasing attractiveness after a NIT is introduced of low-paid part-time jobs.
Obviously, any change in the poverty line used by the government to identify poors and non-poors alters the composition of the population and hence the optimal anchored flat rate tax. Moreover, it modifies the optimal NIT, through a different aggregate poverty gap. Hence, a NIT seems more sensitive than a PI to different thresholds set in the space of income.

Finally, we hope that our analysis has shown a possible application of the concepts of basic income\textsuperscript{11} and negative income tax in poverty studies. Further researches may introduce in our set up non-linear taxation schemes or differentiated public transfers.

\textsuperscript{11}This set up may be easily extended to different notions of basic income suggested by social scientists. For instance, if we adopt the notion of citizen income proposed by Robertson (1996) (i.e. a basic income paid to any citizen partecipative or not), we may introduce a fraction $\beta$ of non-partecipative agents, hence defining the portion of workers as $1 - \beta - \eta$. 


References


