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Figure 1

Figure 2


fosters collusion, as long as \( \theta \in ]0, (a - c)/4 [ \). Vickers (pp. 142-4) stresses the strategic advantage brought about by managerialization, which amounts to appointing the leader’s role to the firm that proceeds to the separation between ownership and control, provided the rivals do not imitate her. Since this is precisely what they would do in order to avoid playing the follower’s role, the result would be overproduction and lower profits respect to the case of competition between entrepreneurial firms. Our results re-establish a case in favour of managerialization, to the extent that such an organizational choice enhances firms’ ability to collude: owners can strategically use the tendency of managers to excessively expand production, in order to foster implicit collusion and gain higher profits.

5. Conclusions

Extending to the repeated game setting a framework originally introduced by Vickers (1985), we have tackled the issue of whether delegation can foster cartel stability in a duopoly where firms offer a homogeneous product under Cournot competition. The answer is twofold. If only one firm is managerial, then she has a stronger incentive to deviate from the collusive agreement as compared to what happens when both are entrepreneurial. The opposite holds for the rival that has not proceeded to the separation between ownership and control. Overall, cartel stability is reduced by the managerialization of only one firm. On the contrary, when both firms are run by managers interested in the level of sales, this enhances the stability of collusive agreements, since both firms expand production even if they play noncooperatively.

The emergence of implicit collusion in games of strategic delegation is not new: it has been shown by Fershtman, Judd and Kalai (1991) and Polo and Tedeschi (1992) specifying the compensation schemes for managers with and without relative performance evaluation, respectively. Here we have shown that analogous results hold in a repeated game model with no reference at all to the features of the contract between owners and managers. The inclusion of sales in the objective function in sufficient per se to create incentives towards cooperation through the overproduction due to managers’ behaviour and to stabilize such collusion, provided that all firms are managerial.
\( q_i^N = q_j^N = \frac{a - c + \theta}{3b}; \) 

the corresponding equilibrium profits are:

\( \pi_i^N = \pi_j^N = \frac{(a - c - 2t)(a - c + t)}{9b}. \) 

It is known (Vickers, 1985, p.142) that the optimal value of \( \theta \) in the strictly noncooperative game is \( \theta^* = (a - c)/5 \), which entails a larger production and lower profits as compared to the setting where both firms are strictly profit-maximizers. Both elements suggest that firms have a stronger incentive towards collusion than in the case in which at least one is entrepreneurial. Cartel profits are defined by (8). It is quickly shown that deviation quantities and profit are the same as in (18-19), so that the critical discount factor, for both firms, is

\( \tilde{\alpha}_i = \frac{9(a - c - 4\theta)}{17a - 17c - 4\theta}, \quad i = 1, 2. \) 

The critical discount factor is everywhere decreasing in \( \theta \), with a discontinuity at \( \theta = 17(a - c)/4 \). The behaviour of \( \tilde{\alpha}_i \) is illustrated in figure 2.

INSERT FIGURE 2 HERE

For all \( \theta > 17(a - c)/4 \), \( \tilde{\alpha}_i > 9 \), implying that the cartel would be less stable. Though, since firms would never choose \( \theta \) in this interval, we can conclude that managerialization of both firms
It is quickly verified that

\[ \tilde{\alpha}_2 < \frac{9}{17} \quad \forall \theta \in ]0, 2(a - c) [. \] (24)

In the intervals specified above for \( \theta \), the incentive to cheat for firm 2 is reduced by the managerialization operated by firm 1. Nevertheless, except for \( \theta \in ](7 + \sqrt{97})/(a - c)/8, 17(a - c)/8[ \), \( \tilde{\alpha}_1 > \tilde{\alpha}_2 \), and the optimal value of \( \theta \) is \( \theta^* = (a - c)/4 \), the cartel is indeed less stable as far as only one firm 2 is managerial.

4. Collusion between managerial firms

Assume now both firms have operated a separation between ownership and control, and both managers assign a positive weight \( \theta \) to sales. Given the symmetry of the problem, this weight needs no indexation (verifica). The generic objective function looks now as follows:

\[ M_i = \pi_i + \theta q_i; \] (25)

the outcome of the present framework can be summarized in

**PROPOSITION 2**: when both firms are managerial, cartel stability is enhanced in the relevant range of parameters.

**PROOF.** Differentiating (25) w.r.t. \( q_i \) yields the FOC:

\[ \frac{\partial M_i}{\partial q_i} = a - c - 2bq_i - bq_J + \theta = 0; \] (26)

from (26) we can derive the optimal quantities:
it is easy to verify that $\tilde{\alpha}_1 > 9/17$ $\forall \theta > 0$, so that the managerialization of firm 1 lowers cartel stability from the point of view of the same firm, i.e., it does not decrease her incentive to cheat. If $\theta = 0$, the two discount factors coincide, so that if $\theta$ is properly chosen after deviation the observed behaviour of the deviating firm is the same independently of her structure.

Let us turn to firm 2. Assuming firm 1 is loyal, the deviation output for firm 2 can be derived from condition (16):

$$q_2^p = \frac{3(a - c)}{8b},$$

(21)

and the profit accruing to the entrepreneurial firm from deviation is

$$\pi_2^p = \frac{9(a - c)^2}{64b}.$$

(22)

The critical discount factor in this case is the following:

$$\tilde{\alpha}_2 = \frac{9(a - c)^2}{(17a - 17c - 8\theta)(a - c + 8\theta)} > 0 \quad \forall \theta < \frac{17}{8}(a - c).$$

(23)

The behaviour of $\tilde{\alpha}_1$ and $\tilde{\alpha}_2$ is represented in figure 1. The behaviour of $\tilde{\alpha}_2$ beyond $\theta = 17(a - c)/8$ is reported only for completeness.

INSERT FIGURE 1 HERE
(13-14) yields:

\[
\pi_i = \frac{(a - c - \theta)(a - c + 2\theta)}{9b}, \quad \pi_2 = \frac{(a - c - \theta)^2}{9b}.
\]  

(17)

As pointed out by Vickers (1985, p.142), the value of \( \theta \) maximizing \( \pi_i \) is \( \theta^* = (a - c) / 4 \). The managerial firm earns a profit which is twice as much as that accruing to the rival, and is virtually appointed the Stackelberg leader’s role.

If firms collude in quantities, they obviously obtain the same profits \( \pi_i^c \) as defined in (8). What distinguishes this setting from the previous is that deviation gives different profits and consequently different critical discount factors for the two firms. Consider first the deviation by the managerial firm, assuming firm 2 sticks to the collusive output. The deviation quantity for firm 1 is obtained from condition (15):

\[
q_i^p = \frac{3(a - c) + 4\theta}{8b},
\]  

(18)
yielding

\[
\pi_i^p = \frac{(3a - 3c + 4\theta)(3a - 3c - 4\theta)}{64b}
\]  

(19)
as the deviation profit. Notice that, since \( \pi_i^p \) is decreasing in \( \theta \), its optimal value in case of deviation from the agreement is nil. We can now proceed to compute the critical discount factor relative to the managerial firm,

\[
\tilde{\alpha}_i = \frac{9(a - c + 4\theta)}{17a - 17c + 4\theta};
\]  

(20)
so that the punishment strategy is credible. The critical value of the discount factor sustaining collusion is then

$$\tilde{\alpha}_i = \tilde{\alpha}_j = \frac{\pi^D - \pi^C}{\pi^D - \pi^N} = \frac{9}{17},$$

(12)

so that, if both firms have a discount factor greater than 9/17 neither finds it profitable deviating from the cartel.

3. Collusion with a managerial firm

Assume now firm 1 proceeds to separate ownership and control, and her manager attaches a positive weight $\theta$ to sales. Thus, the objective functions look as follows:

$$M_1 = \pi_1 + \theta q_1; \quad \pi_2 = (p - c)q_2.$$  

(13)  

(14)

In such a case, we can state the following

**PROPOSITION 1**: if only one firm is managerial, cartel stability is weakened as compared to the case in which both firms are profit-maximizers.

**PROOF**. The FOCs obtained by differentiating (13) and (14) w.r.t. $q_1$ and $q_2$, respectively, are:

$$\frac{\partial M_1}{\partial q_1} = a - c - 2bq_1 - bq_2 + \theta = 0,$$

(15)

$$\frac{\partial \pi_2}{\partial q_2} = a - c - 2bq_2 - bq_1 = 0; \quad \pi_2 = (p - c)q_2.$$  

(16)

solving the system (15-16) w.r.t. to quantities and substituting the latter into objective functions
Differentiating (6) w.r.t. $q_1$ and $q_2$ and solving, yields

$$q_1^c = q_2^c = \frac{(a - c)}{4b},$$

and equilibrium profits are

$$\Pi^C = \frac{(a - c)^2}{4b}; \quad \pi_1^C = \pi_2^C = \frac{(a - c)^2}{8b}.\quad (8)$$

Let us now turn to deviation profits. Given the symmetry of the model, we assume that firm $i$ sticks to the collusive output, while firm $j$ deviates. From (3), we compute the deviation output for firm $j$:

$$q_j^D = \frac{3(a - c)}{8b},$$

yielding as deviation profits

$$\pi_j^D = \frac{9(a - c)^2}{64b},$$

while the profit of the loyal firm is

$$\pi_i(q_j^D) = \frac{3(a - c)^2}{32b} < \pi_i^C,$$
where \( r_i \) is firm \( i \)'s discount rate, and \( \tilde{\alpha}_i \) is the critical level of the discount factor above which cartel stability is ensured.

Consider first the noncooperative setting. Two firms compete in quantities in a market for a homogeneous good. The inverse demand function is given by \( p = a - b(q_1 + q_2) \). Firms produce at constant marginal cost, \( C_i = cq_i, \quad i=1,2, \) with \( a>c \). The generic profit function is then

\[
\pi_i = (a - bq_i - bq_j - c)q_i; \quad i, j = 1, 2, \quad i \neq j; \quad (2)
\]
differentiating (2) respect to \( q_i \), we obtain the first order condition (FOC) for firm \( i \):

\[
\frac{\partial \pi_i}{\partial q_i} = a - 2bq_i - bq_j - c = 0; \quad (3)
\]
solving the FOCs w.r.t. quantities, we get

\[
q_1^N = q_2^N = \frac{(a - c)}{3b}. \quad (4)
\]
Substituting equilibrium quantities (4) into the above profit functions we obtain equilibrium profits,

\[
\pi_1^N = \pi_2^N = \frac{(a - c)^2}{9b}. \quad (5)
\]
Assume now that firms collude, setting quantities so as to maximize joint profits:
contracts are public information. This implies that the collusive outcome can be attained in noncooperative single-period games, without resorting to repeated ones.

Relying on a setting introduced by Vickers (1985), we analyse cartel stability under Cournot competition in a duopoly where at least one firm delegates control over her assets to a manager who is interested in the volume of sales. We show that, if only one firm is managerial, she has a stronger incentive to deviate from the collusive agreement as compared to the case in which both firms maximize only profits; on the contrary, the rival firm has a weaker incentive towards deviation, if the weight attached to sales is properly set by the managerial firm. If both firms operate a separation between ownership and control, then there exists an interval in which any positive weight attached to sales enhance cartel stability, and this interval contains the optimal value of the weight, i.e., the value that maximizes each firm’s profit. Thus, in the latter setting, the sustainability of collusion is enhanced, providing simply that the contract between each principal and her manager sets the weight of sales into the objective function at the level that agents would autonomously choose in the strictly noncooperative game.

Collusive behaviour in the absence of delegation is analysed in section 2. Section 3 is devoted to the asymmetric case in which only one firm is managerial. The symmetric setting in which both firms are managerial is described in section 4. Section 5 contains final comments.

2. Collusion between profit-seeking firms

By now, the conditions underlying collusion in an infinitely repeated game are familiar, so we can sketch them very briefly. As first shown by Friedman (1971), in a repeated setting firms can sustain collusion, which would not be possible should they interact once and for all. The implicit collusion thus arising can be thought of as a contract that is not legally enforceable, so that to be sustainable it must be defined as a subgame perfect equilibrium of the repeated game. Collusion can be an equilibrium if the discount factor is high enough and the one-shot Nash equilibrium is adopted as a punishment mechanism. Thus, let $\pi_i^N$, $\pi_i^C$ and $\pi_i^D$ be firm $i$’s profits from the noncooperative one-shot game, collusion and deviation, respectively, with $\pi_i^D > \pi_i^C > \pi_i^N$. Then collusion is sustainable if
1. Introduction

The issue of cartel stability has received wide attention in the literature. The question how variations in cartel size affects the fortunes of those inside and outside the cartel has been explored by D’Aspremont et al. (1983), Donsimoni (1985) and Donsimoni et al. (1986), reaching the conclusion that stable cartels exist whenever the number of firms is finite. The relative efficiency of Bertrand and Cournot competition in stabilizing cartels composed of firms whose products are imperfect substitutes has been analysed by Deneckere (1983), showing that when substitutability between products is high, collusion is better supported in price-setting games than in quantity-setting games, while the reverse is true in case of low substitutability. Majerus (1988) has proved that this result is not confirmed as the number of firms increases, and Rothschild (1992), in contrast with Deneckere’s findings, has shown that in a price-setting duopoly, the greater the degree of substitutability, the greater is the incentive to deviate and therefore the less stable the cartel is, while in quantity-setting games cartel stability is monotonically increasing in the degree of substitutability between products. Finally, the issue of the influence of product differentiation on the stability of collusion has been tackled by Chang (1991), Ross (1992) and Häckner (1994). The main finding reached by these contributions is that, under vertical differentiation, collusion is more easily sustained, the more similar the products are, while the opposite applies under horizontal differentiation.

The early literature on strategic delegation (Fershtman, 1985; Vickers, 1985; Fershtman and Judd, 1987; Sklivas, 1987), in which the agents’ game strategies are not conditional upon compensation schemes, leads to the conclusion that delegation may yield more competitive equilibria. A few recent contributions suggest though that separation between ownership and management may more easily give rise to collusive behaviour by firms, i.e., owners can induce a more collusive behaviour in the market game between managers (see Fershtman, Judd and Kalai, 1991; Polo and Tedeschi, 1992), provided that each principal is fully committed to the contract signed with her agent and all contracts are fully observed, and thus can be conditioned upon in the agents’ game. This extra commitment is strictly needed to implement the collusive outcome by delegation. It is not necessary to resort to settings where contracts between the main players can be enforced, what is only needed is the possibility of hiring agents providing that
Delegation and Cartel Stability

Luca Lambertini*

Dipartimento di Scienze Economiche
Università degli Studi di Bologna
Strada Maggiore 45
40125 Bologna
Italy
tel 39-51-6402600
fax 39-51-6402664
e-mail lamberti@boph01.cineca.it

and

Linacre College
Oxford OX1 3JA
United Kingdom
e-mail econlla@vax.ox.ac.uk

JEL classification numbers: L13, L22

Abstract

The effect of delegation on cartel stability is addressed in a duopoly for a homogeneous product, under Cournot competition. The main findings are that if only one firm is managerial, the critical discount factor is increased by the presence of a weight attached to sales, so that cartel stability is decreased, while if both are managerial the opposite holds. As a consequence, the inclusion of sales in both firms’ objective function represents an incentive towards collusion.

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# Please send correspondence to the Italian address.