A Novel Closed-Form Approach for Comparing the Q-Factor Responses between the Asymmetric and Symmetric On-Chip Inductors

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***Abstract* — This paper proposes an equivalent transmission-line circuit for on-chip inductors and derives the quality (Q) factor response in terms of the transmission-line circuit parameters in closed form. The derived formulas are general and suitable for all kinds of on-chip inductors to account for their frequency dependences of Q factors. For demonstration, a series of asymmetric inductors and another series of same valued symmetric inductors have been fabricated on the same silicon substrate. The measured Q-factor responses for both kinds of inductors agree quite well with the formula predictions. The symmetric inductors have a higher equivalent characteristic impedance so as to correspond to a higher peak Q-factor frequency than the asymmetric inductors. The presented formulas can also uniquely distinguish the improvement in Q-factor responses due to the reduction of conductor loss or dielectric loss.**

I. INTRODUCTION

In today's radio-frequency integrated circuit (RFIC) design, the asymmetric and symmetric on-chip inductors [1],[2] are quite popularly used in the tank, matching and choke circuits. It is without controversy that the quality (Q) factor is always the most important parameter to evaluate the inductor performance. Many researches have been devoted to the high-Q asymmetric and symmetric inductor designs by employing the physical modeling [3],[4] and electromagnetic simulation [5],[6] techniques. However, so far no analytical approach has been reported to account for the distinction in the Q-factor responses between two kinds of inductors. Although the increase of conductor thickness and the removal of substrate material have been widely used to lower the conductor loss and the dielectric loss, respectively, in some special semiconductor processes for the purpose of enhancing the inductor Q factor, there is still no theory to accurately predict the improvement of Q factor due to the above loss reduction schemes.

This paper uses a simple equivalent transmission-line circuit to represent any kind of on-chip inductor. A closed-form formula is further found for the inductor Q-factor response when expressed in terms of the transmission-line circuit parameters. Such a closed-form Q-factor formula is concise, and its differentiation can be also derived in closed form. These closed-form formulas can help determining the peak Q-factor value and its corresponding frequency for an arbitrary kind of inductor. Therefore, one can apply the transmission-line theory to characterize an on-chip inductor and simultaneously make accurate prediction of its Q-factor response based on our presented closed-form formulas.

II. EQUIVALENT TRANSMISSION-LINE CIRCUIT

As shown in Fig. 1(a), an on-chip inductor when viewed as a two-port reciprocal component can have a generalized equivalent transmission-line circuit whose parameters can be derived as functions of the Z-network parameters in the following forms:

$$Z_0 = R_0 + jX_0 = \sqrt{Z_{11}^2 - Z_{12}^2} \tag{1}$$

$$\gamma_0 l = a_0 + j\theta_0 = \cosh^{-1}\left(\frac{Z_{11}}{Z_{12}}\right) \tag{2}$$

$$Z_L = Z_{22} - Z_{11} \tag{3}$$

where Z_0 in (1) is the characteristic impedance with the real part R_0 and the imaginary part X_0. In (2), a_0 and θ_0 denote the attenuation and the electrical length, respectively. Z_L in (3) is an additional impedance to account for the inductor asymmetry.

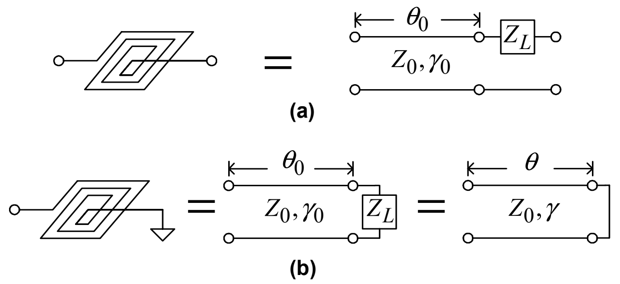

Fig. 1. (a) Two-port on-chip inductor and its equivalent transmission-line circuit. (b) One-port configuration of on-chip inductor with another port shorted and its equivalent transmission-line circuit.

To date the most often used Q factors are defined as the ratio of imaginary to real parts of the input impedance in a one-port configuration with another port shorted. As depicted in Fig. 1(b), such a one-port configuration can be finally represented by a short-circuit terminated transmission line with the following parameters:

$$\gamma l = a + j\theta \tag{4}$$

$$a = a_0 + \frac{1}{2}\mathrm{Re}\left\{\ln\frac{Z_0 - Z_L}{Z_0 + Z_L}\right\} \tag{5}$$

$$\theta = \theta_0 + \frac{1}{2}\mathrm{Im}\left\{\ln\frac{Z_0 - Z_L}{Z_0 + Z_L}\right\} \tag{6}$$

where a and θ denote the modified attenuation and electrical length, respectively, after including the impedance Z_L.

It is well known to define the transmission-line Q factor as

$$Q_0 = \frac{\theta}{2a} = \left[\frac{1}{Q_{0c}} + \frac{1}{Q_{0d}}\right]^{-1} \tag{7}$$

where

$$Q_{0c} = \frac{\theta}{2a_c} \tag{8}$$

$$Q_{0c} = \frac{\theta}{2a_d}\,. \tag{9}$$

Note that $a = a_c + a_d$ where a_c and a_d represent the attenuation due to conductor loss and dielectric loss, respectively. Q_{oc} and Q_{od} can be therefore regarded as the conductor Q factor and the dielectric Q factor related to a_c and a_d, respectively.

III. General Formulas for the Quality-Factor Responses

Under low-loss condition, the inductor Q factor can be formulated in the electrical-length domain as

$$Q(\theta) = \frac{\mathrm{Im}\{Z_0 \tanh(a + j\theta)\}}{\mathrm{Re}\{Z_0 \tanh(a + j\theta)\}} \approx \left(\frac{2a}{\sin 2\theta} - \frac{X_0}{R_0}\right)^{-1}. \tag{10}$$

As illustrated in Fig. 2, the transmission-line formulation transforms the inductor Q-factor response from the frequency domain between 0 and self-resonant frequency (SRF) to the electrical-length domain between 0 and $\pi/2$. This gives a unique advantage to be able to compare the Q-factor responses of different valued inductors within a fixed range of electrical length from 0 to $\pi/2$.

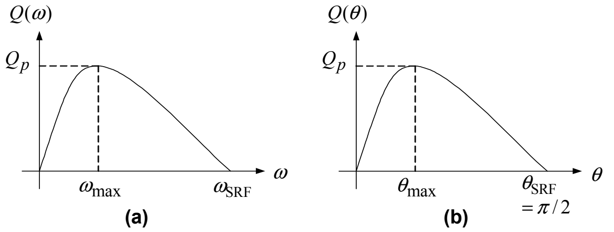

Fig. 2. Transformation of the Q-factor response from (a) the frequency domain into (b) the electrical-length domain.

In (10), the term X_0/R_0 under low-loss condition can be further derived as

$$\frac{X_0}{R_0} \approx \frac{a_d - a_c}{\theta}\,. \tag{11}$$

Substituting (11) into (10) gives

$$Q(\theta) \approx \left[\frac{a_c}{\theta}\left(\frac{2\theta}{\sin 2\theta} + 1\right) + \frac{a_d}{\theta}\left(\frac{2\theta}{\sin 2\theta} - 1\right)\right]^{-1}$$

$$= \left[\frac{1}{2Q_{0c}}\left(\frac{2\theta}{\sin 2\theta} + 1\right) + \frac{1}{2Q_{0d}}\left(\frac{2\theta}{\sin 2\theta} - 1\right)\right]^{-1}. \tag{12}$$

One can deduce from (12) that at low frequencies ($\theta \to 0$) the attenuation due to conductor loss (a_c) or the conductor Q factor (Q_{0c}) is the dominant factor to determine the inductor Q factor. As illustrated in Fig. 2(b), the peak Q factor occurs at $\theta_{\max}$, whose value can be determined by setting the derivative of (12) with respect to θ equal to zero. According to our observation, $Q_{0c} \propto \theta$ and $Q_{0d} \propto \theta^0$ in the vicinity of $\theta_{\max}$. After derivation, $\theta_{\max}$ can be determined from solving the following equation:

$$\left.\frac{Q_{0c}}{Q_{0d}}\right|_{\theta_{\max}} = J(\theta_{\max}) = \frac{\sin 2\theta_{\max} + 2\theta_{\max}}{2\theta_{\max}(1 - 2\theta_{\max}\cot 2\theta_{\max})} - 1\,. \tag{13}$$

Note that (13) is general and good for all kinds of on-chip inductors to predict their peak-Q factor positions in the electrical-length domain.

For demonstration, a series of asymmetric inductors and another series of same valued symmetric inductors have been fabricated on the same silicon substrate [7]. Fig. 3 illustrates their geometries. Fig. 4(a) shows the calculated ratios of Q_{0c} to Q_{0d} at $\theta_{\max}$ of these asymmetric and symmetric inductors for verifying the formula predictions from $J(\theta_{\max})$. Good agreement has been observed. (13) also reveals that a larger ratio of Q_{0c} to Q_{0d} generally moves the peak Q-factor position to a smaller electrical length.

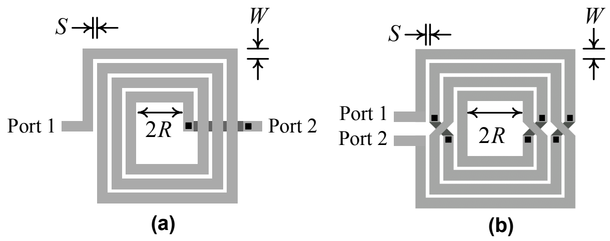

Fig. 3. The illustrated geometry of (a) asymmetric inductor and (b) symmetric inductor.

After substituting (13) into (12), the value of peak Q factor can be expressed as

$$\left.\frac{Q_p}{Q_0}\right|_{\theta_{\max}} = M(\theta_{\max})$$

$$= \left[\frac{2\theta_{\max}(1 - 2\theta_{\max}\cot 2\theta_{\max})}{\sin 2\theta_{\max} + 2\theta_{\max}} - \frac{\sin 2\theta_{\max} - 2\theta_{\max}}{2\sin 2\theta_{\max}}\right]^{-1}. \tag{14}$$

Fig. 4(b) plots the function of $M(\theta_{\max})$ and also marks the calculated ratios of Q_p to Q_0 at $\theta_{\max}$ for all six inductors. The agreement is still good. (14) implies that a smaller $\theta_{\max}$ results in a larger value of $M(\theta_{\max})$, which is beneficial to the increase of peak Q factor under the condition of the same Q_0.

To determine the peak Q-factor frequency ($\omega_{\max}$), we derive the relation between $\omega_{\max}$ and $\theta_{\max}$ as follows.

$$\frac{\theta_{\max}}{\omega_{\max}} = \tau \approx \frac{L(\omega \to 0)}{R_0(\theta \to 0)} \tag{15}$$

where τ denotes the propagation time delay and can be approximated as the inductance at the low frequency divided by the characteristic resistance at the small electrical length. Fig. 4(c) plots the linear relation between $\omega_{\max}$ and $\theta_{\max}$ according to (15) and simultaneously marks the evaluated results of the six inductors for comparison. Again, the comparison shows good agreement. As a matter of fact, the $\theta_{\max}$ determined by the ratio of Q_{0c} to Q_{0d} according to (13) is usually within a narrow range. Therefore, for the same valued inductor designs, a higher equivalent characteristic impedance can lead to a higher peak Q-factor frequency.

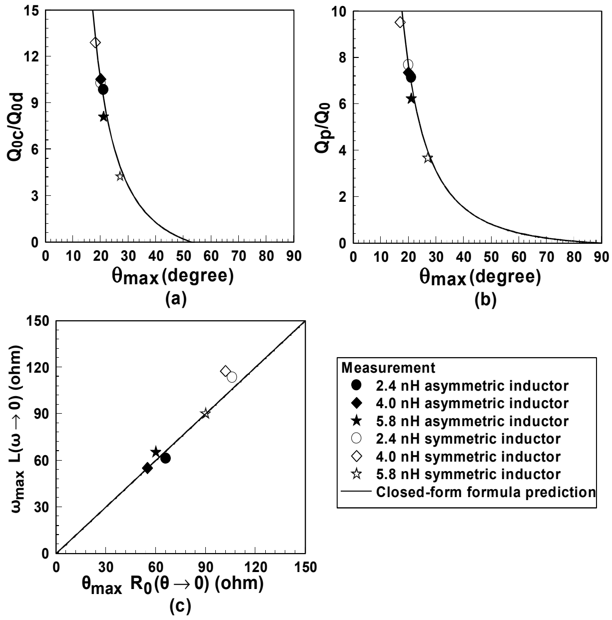

Fig. 4. Comparison of the closed-form formula predictions with the measured results in determining the inductor peak Q factor. (a) The ratio of the conductor Q factor to the dielectric Q factor versus $\theta_{\max}$. (b) The ratio of the peak Q factor to the transmission-line Q factor versus $\theta_{\max}$. (c) The product of the angular peak Q-factor frequency and inductance versus the product of the characteristic impedance and $\theta_{\max}$.

IV. Applications and Discussions

In Fig. 5(a) and 5(b), we transform the theoretical Q-factor responses in the electrical-length domain back to the frequency domain for the series of asymmetric inductors and symmetric inductors, respectively. Excellent agreements can be seen when compared to the measured frequency responses. It is interesting to find that for each pair of same valued inductors, the asymmetric inductor has a higher peak Q-factor value while the symmetric inductor has a higher peak Q-factor frequency. This is because the former inductor has a higher conductor Q factor while the latter one has a higher characteristic impedance.

The derived closed-form formula in (12) can be applied to distinguish the improvement of Q-factor response due to the reduction of conductor loss or dielectric loss. Fig. 6(a) and 6(b) show the predicted Q-factor responses in the frequency domain after increasing either the conductor Q factor (Q_{oc}) or the dielectric Q factor (Q_{od}) for the 4-nH asymmetric inductor and symmetric inductor, respectively. For both inductors, the peak Q-factor value increases as either Q_{oc} or Q_{od} increases. However, the peak Q-factor frequency behaves quite differently. When Q_{oc} increases alone, the peak Q-factor frequency moves to a lower frequency. On the contrary, when Q_{od} increases alone, the peak Q-factor moves to a higher frequency. This is because according to (13) the ratio of Q_{oc} to Q_{od} actually determines the value of $\theta_{\max}$ which is proportional to the peak Q-factor frequency. One can also find that the increase of Q_{oc} can raise the peak Q-factor value more effectively than the increase of Q_{od} . This can be also explained from (14). The increase of Q_{oc} alone causes a smaller $\theta_{\max}$ that corresponds to a larger value of $M(\theta_{\max})$. It is reminded from (14) that $M(\theta_{\max})$ is proportional to the peak Q-factor value.

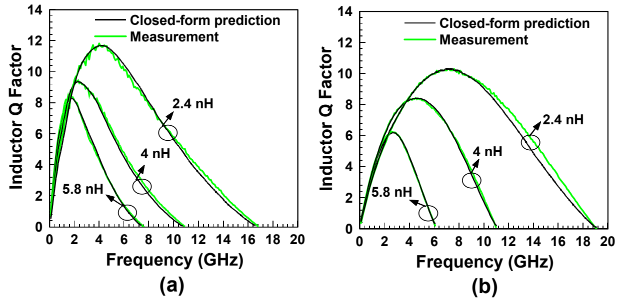

Fig. 5. Comparison of the Q-factor responses between closed-form results and measured results for the different valued (a) asymmetric inductors and (b) symmetric inductors.

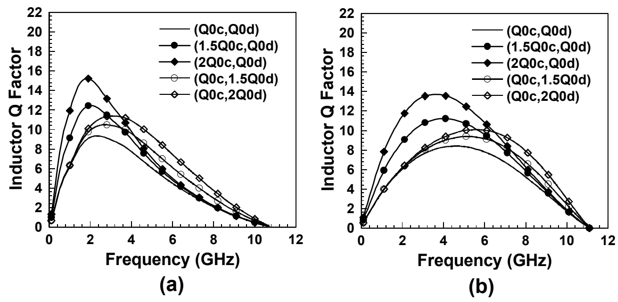

Fig. 6. The predicted Q-factor responses from closed-form results under the increase of either conductor Q factor or dielectric Q factor for the (a) 4 nH asymmetric inductor and (b) 4 nH symmetric inductor.

V. Conclusion

A novel approach for comparing the Q-factor responses between the asymmetric and symmetric inductors has been developed based on the equivalent transmission-line circuits. The derived closed-form expressions can account for the dependences of Q factor on the frequency as well as the attenuation due to conductor and dielectric losses. For the same valued asymmetric and symmetric inductor on silicon, the former has a higher peak Q-factor value due to a higher conductor Q factor while the latter has a higher peak Q-factor frequency due to a higher characteristic impedance.

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