

# A DESIGN APPROACH FOR SUB-HARMONIC GENERATION OR SUPPRESSION IN NON-LINEAR CIRCUITS

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## ABSTRACT

*A design approach for the generation or suppression of sub-harmonic frequencies in non-linear circuits under large-signal drive is described. A standard Harmonic-Balance algorithm is first used for the large-signal analysis at fundamental frequency and higher-order harmonics; a conversion matrix is then computed for the calculation of potential frequency conversion between harmonic and sub-harmonic frequency components. Cross-frequency stability circles are drawn based on the conversion matrix, and the condition for the insurgence of sub-harmonic instabilities is set as the fulfilment of the Barkhausen oscillation criterion between different frequencies. The instability can be either imposed or avoided by suitably choosing the loads at sub-harmonic frequencies.*

## INTRODUCTION

The stability of non-linear microwave circuits is not a well-assessed issue yet. Only one general method is currently available for stability verification, based on the application of Nykvist's criterion to the determinant of the conversion matrix for a swept perturbation frequency [1], but the results are not always very stable and clear. No method is currently available for the direct design of instabilities, or for the stabilisation, of a general non-linear circuit. In this paper a method is proposed that allows the identification of potential instabilities in general non-linear circuits under large-signal operations, allowing either the generation of instabilities, in particular at sub-harmonic frequencies, or the stabilisation of the circuit, by a suitable choice of the loading impedances at the potentially unstable frequencies. The method is based on a standard Harmonic-Balance analysis, and on the calculation of the conversion matrix between the potentially unstable frequency components.

The method is based on the properties of the conversion matrix, that can be considered as the generalisation of the small-signal parameters to non-linear circuits in large-signal periodic regime, when a small-amplitude signal is superimposed to the non-linear circuit in periodic large-signal regime. An incremental small-signal circuit is derived, that includes a frequency-converting linear(ised) network representing the active device under large-signal drive, and the passive linear circuitry [4]. This small-signal circuit must be stable for any small-signal frequency; this can be verified by the use e.g. of the Barkhausen criterion. If this is not the case, the stability circles of the frequency-converting network can be used to identify the 'dangerous' loads, and to move them away from the unstable region of the Smith Chart. On the other hand, an instability may be desired to take place, e.g. for frequency division. In this case, if potential instabilities are detected by means of stability circles, the Barkhausen criterion can be imposed in such a way that an instability occurs at the desired frequency, by a suitable choice the loads at the potentially unstable frequencies.

## THE DESIGN-ORIENTED APPROACH

The non-linear circuit is first analysed by means of a standard Harmonic-Balance method, in order to identify the periodic large-signal regime due to a large input signal. Only the loads at fundamental frequency and higher harmonics (plus the DC) are relevant for the large-signal periodic regime. The convergence of the analysis however does not guarantee the stability of the circuit, since frequency components outside the harmonic spectrum of the input signal are *a priori*

not included in the analysis, and cannot be detected. A sub-harmonic oscillation, or an instability not frequency-correlated to the input signal, could actually be present in the circuit without being detected; the oscillation however, if present, is easily identified by a time-domain analysis. In order to illustrate the proposed method, we will design a stable and an unstable amplifier based on the same Harmonic Balance analysis with the same loads at harmonic frequencies.

The amplifier is based on a medium-power,  $0.5 \text{ } \mu\text{m}$  gate MESFET with  $1 \text{ mm}$  total periphery, from the AMS foundry. The device is modelled by means of a standard non-linear equivalent-circuit [2]; the simulations are performed by means of the commercial Agilent-ADS package. The source and load impedances that must be supplied by the input and output matching networks are specified only at  $DC$ , at fundamental frequency  $f_0 = 3 \text{ GHz}$  and higher harmonics up to the 5<sup>th</sup> ( $15 \text{ GHz}$ ).

A large input signal at  $f_0 = 3 \text{ GHz}$  is now applied, driving the amplifier into non-linear operations. The drain voltage is shown in fig.1; no instability appears from the Harmonic Balance analysis. We shall now synthesise the actual matching networks so that they present the specified loads at fundamental and harmonic frequencies; as a demonstration of the method, we will do this in such a way that a sub-harmonic at  $f_0/3$  ( $1 \text{ GHz}$ ) also appears.

We first compute the conversion matrix for an  $f_{in} = 1 \text{ GHz}$  test input signal; the conversion matrix will relate all the upper and lower sidebands generated within the transistor by interaction with the large  $3\text{-GHz}$  signal, i.e. all the spectral lines at frequencies [3]:

$$f_{\pm n} = n \cdot f_0 \pm f_{in} \quad n = 0, \dots, 5$$

A small-signal equivalent circuit is now defined for the frequency-converted components; a 24-port frequency-converting linear(ised) network represents the transistor under large-signal drive, where the electrical ports represent the two physical ports (gate and drain ports) at the 12 converted frequencies as in eq.(1), i.e. at the converted frequencies. The passive circuitry is represented as the loads at the 24 ports. In fact, only low-order conversion is usually important, and the total number of the electrical ports can be practically limited to 6 or 10 ( $n = 0, \dots, 1$ , or  $n = 0, \dots, 2$ , respectively).

In order to generate a signal at  $1 \text{ GHz}$  without any input at this frequency, the autonomous linearised network in fig.2 must oscillate at some port at  $1 \text{ GHz}$ . This can be accomplished only if the network is potentially unstable, and if the loads are chosen in such a way that the Barkhausen oscillation condition is fulfilled. In order to make sure that the instability is due only to non-linear frequency conversion, we will try to impose the oscillation condition between gate port at  $1 \text{ GHz}$  and drain port at  $2 \text{ GHz}$ . The loads at all other frequencies, so far unspecified, are chosen in such a way that the two-port network between gate port at  $1 \text{ GHz}$  and drain port at  $2 \text{ GHz}$  is potentially unstable. We now draw the stability circles corresponding to this two-port network, and select the gate and drain loads within the stability circles, and such that the Barkhausen oscillation condition is fulfilled both in amplitude and in phase (fig.3):

$$\mathbf{G}_{s,1\text{GHz}} \cdot \mathbf{G}_{in,1\text{GHz}} = 1 \quad \mathbf{G}_{out,2\text{GHz}} \cdot \mathbf{G}_{d,2\text{GHz}} = 1$$

The actual matching networks are then synthesised, such that they present the specified impedances. A time-domain analysis is now performed for verification of the design: sub-harmonic frequencies are present in the output spectrum (fig.4). For small amplitudes of the  $3\text{-GHz}$  input signal the sub-harmonic lines are not present, confirming the frequency-conversion origin of the instability. For larger input signal amplitudes the sub-harmonics grow larger, as intuition would suggest.

For further confirmation of the method, a stable  $3 \text{ GHz}$  non-linear amplifier is now designed. All steps are repeated until the stability circles are drawn. The gate load at  $1 \text{ GHz}$  and the drain load at  $2 \text{ GHz}$  are now chosen in such a way that they lie outside the stability circles. The matching networks are synthesised, and the output spectrum for several input power levels shows a stable large-signal output with a  $3\text{-GHz}$  fundamental frequency (fig.5).

## CONCLUSIONS

An effective design approach for sub-harmonic generation or suppression in non-linear circuits has been demonstrated, and the frequency-conversion origin of sub-harmonic generation is illustrated. To the authors' knowledge, this is the first attempt to define a systematic approach for the design of sub-harmonic instabilities.

## REFERENCES

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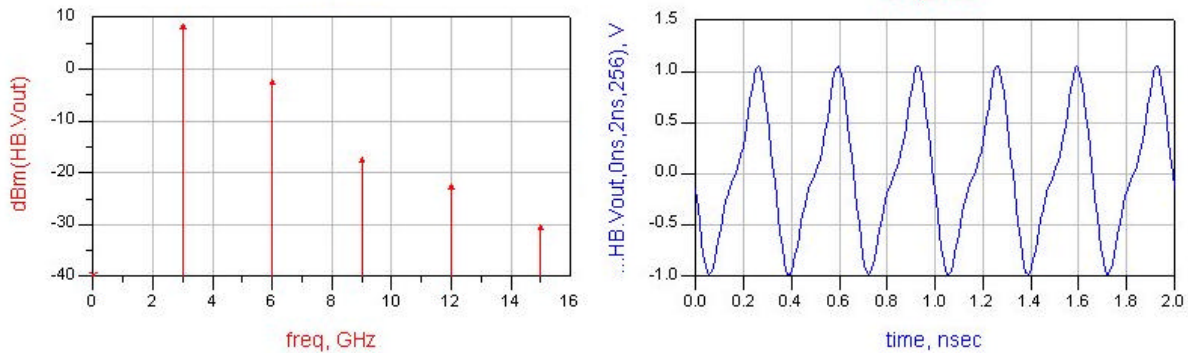


Fig.1 – Drain voltage spectrum and waveform from Harmonic Balance analysis for the amplifier

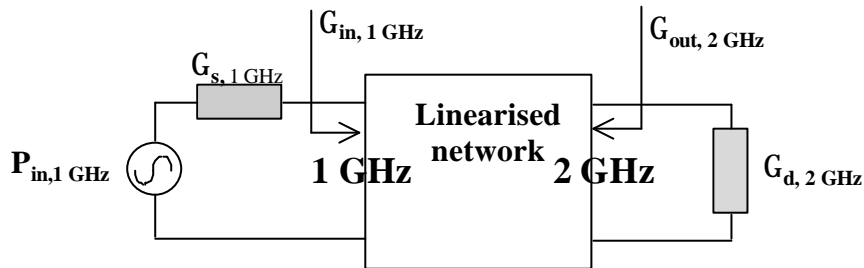


Fig.2 – The linearised two-port 1-to-2 GHz frequency-converting reduced network

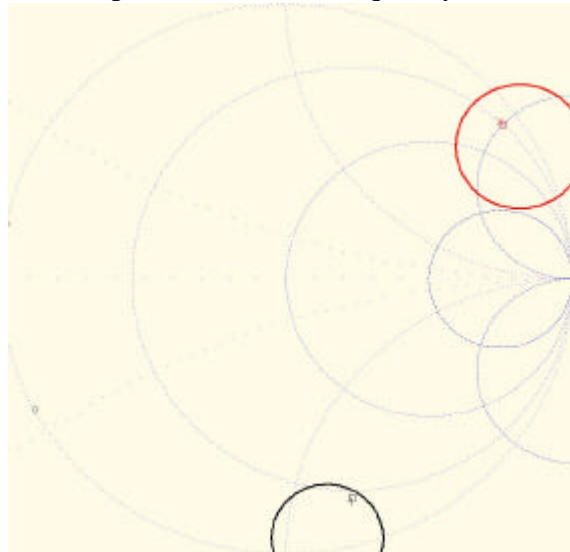


Fig.3 – The stability circles for the 1-to-2 GHz frequency-converting two-port network with the synthesised loads for frequency division (instability)

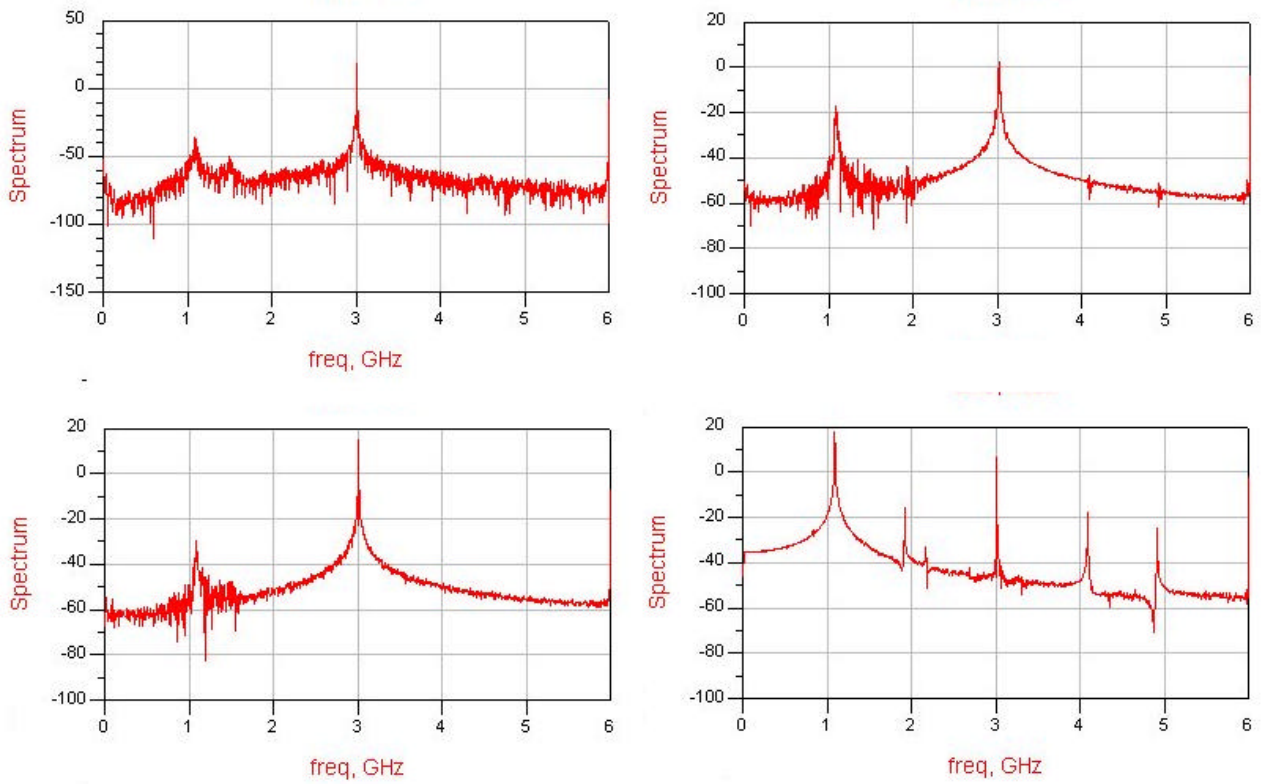


Fig.4 – The output spectrum of the frequency-dividing circuit for increasing input power levels, from top left to bottom right

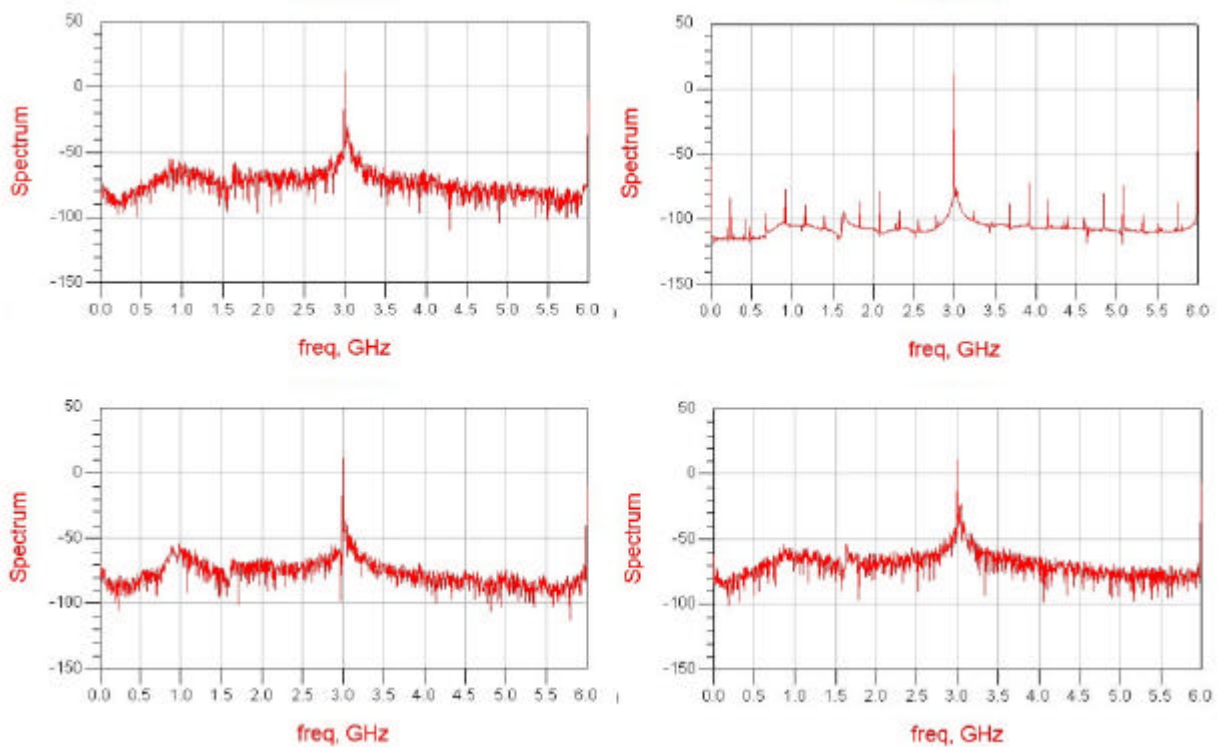


Fig.5 – The output spectrum of the stable amplifier for increasing input power levels