A CAD-oriented model for the ohmic losses of multiconductor coplanar lines in hybrid and monolithic MIC's

G. Ghione, M. Goano and C. U. Naldi,

Dipartimento di Elettronica, Politecnico di Torino,
Corso Duca degli Abruzzi 24, I-10129 Torino, ITALY

Abstract

The paper presents CAD-oriented closed-form expressions for the high-frequency resistance matrix of general multiconductor coplanar lines for analog or digital GaAs IC's. Together with the quasi-TEM characterization presented in [1] the present results enable the evaluation of the modal impedances and attenuations of the multiconductor line. The approach is based on an exact conformal mapping technique, and allows for for coupled lines and multiconductor buses of arbitrary geometry, *i.e.* having arbitrary strip widths and spacings. Results are presented to demonstrate the accuracy and efficiency of the approach.

Introduction

Multi-conductor coplanar lines are a basic building block in both analog components (directional couplers in coplanar IC's) and in digital circuits (high-speed interconnections, see [1]). During the past few years, several numerical techniques have been presented for the numerical analysis of the conductor losses of coplanar lines; nevertheless, CAD-oriented closed-form expressions were only developed for single lines. Conformal-mapping approximations for the high-frequency (skin-effect) conductor attenuation of symmetric coplanar waveguides (CPW) and striplines (CPS) in air were first proposed by Wu and Owyang [2]; the approach was later extended by Ghione [3] to asymmetric lines; other closed-form expressions, derived from the incremental inductance approach, were presented by Gupta, Garg and Bahl [4]. More recently, corrections were proposed to properly account for the low-frequency behaviour of lines partly or completely penetrated by current density [5] and investigations were carried out on edge-shape effects [6]. From the standpoint of lossless line characterization, a closed-form conformal mapping method for the analysis of multiconductor coplanar waveguides (MCPW, see Fig.1) was presented by Linner [7] in 1974 and later extended to the case of multiconductor coplanar striplines (MCPS, see Fig.1) by Ghione [1].

In the present paper, the approaches in [3] and [1] are combined so as to obtain a closed-form conformal mapping estimate of the high-frequency conductor losses of multiconductor coplanar lines. The outcome of the method is the skin-effect per-unit-length (*p.u.l.*) resistance matrix of the line. Arbitrary line geometries (strip widths and spacings) and coupling between distant lines are allowed for. For the sake of brevity, only the MCPW case will be discussed here; the case of the MCPS, which is quite similar, will be presented elsewhere. From the results in [1], which are briefly summarized in the paper, the *p.u.l.* inductance and capacitance matrices can be derived, thus allowing the characteristic impedance matrix and modal propagation constants to be evaluated through standard multiconductor line analysis in the quasi-TEM approximation. Dielectric losses, if significant, can be trivially introduced by using a complex value for the substrate permittivity.

The paper is structured as follows. After a short review of the conformal mapping approach for the evaluation of the characteristic parameters of the lossless MCPW, the computation of the *p.u.l.* resistance matrix is discussed and closed-form expressions for its elements are presented. Such elements are generally expressed in terms of hyperelliptic integrals, whose efficient numerical evaluation is carried out by means of Gauss-Chebychev quadrature formulae. Finally, some numerical results are presented to compare the present approach with already available analytical expressions.

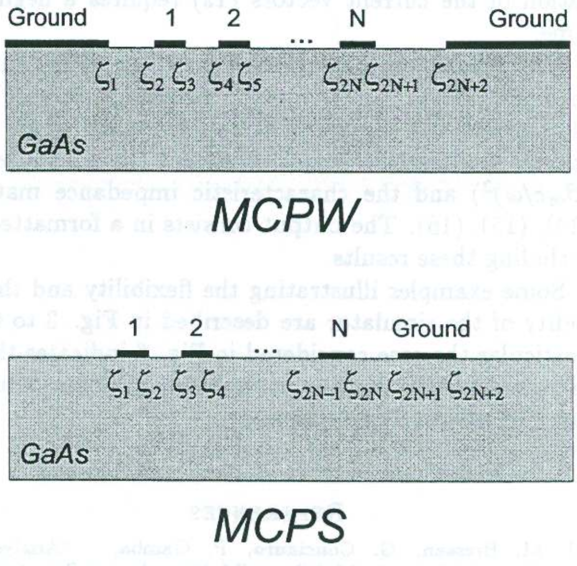

Figure 1: Multiconductor coplanar waveguide (above) and multiconductor coplanar stripline (below). The strip thickness is denoted by t; the parameters ζ_1 etc. define the line geometry.

The characteristic parameters of the MCPW

As well known, a N-conductor MCPW (N strips lying between two lateral ground planes) supports N quasi-TEM propagation modes whose phase velocities and characteristic impedances can be derived from the *p.u.l.* capacitance and inductance matrices via standard techniques from multiconductor line analysis. Since the *p.u.l.* inductance matrix $\mathbf{L}$ is proportional to the inverse of the capacitance matrix *in vacuo* $\mathbf{C}_0$, the analysis can be confined to finding the capacitance matrices of the line *in vacuo* and with dielectrics.

The results presented in this section are taken from [1] and are based on a Schwartz-Christoffel conformal mapping method similar to the one proposed by Linner [7] in connection with stripline couplers. The analysis of [7] is formulated here in a more compact, CAD-oriented matrix form.

The geometry under consideration is shown in Fig.1; the line is made of N strips of thickness t, $N+1$ slots, and two lateral ground planes. For the evaluation of the *p.u.l.* capacitance and inductance matrix, the strip thickness will be neglected. The *p.u.l.* capacitance matrix $\mathbf{C}$ can be expressed as:

$$\mathbf{C} = (1+\epsilon_r)\mathbf{C}_1 \tag{1}$$

where $\mathbf{C}_1$ is the *in vacuo* capacitance of the upper half-structure. The latter can be evaluated by introducing a set of N Schwarz-Christoffel mappings [1]; each mapping transforms the upper half plane into the interior of a rectangle, with one of the strips on the upper side, the ground planes on the lower side, and all other strips folded so as to have zero total charge [1]. The final result is expressed as:

$$\mathbf{C}_1 = \epsilon_0 \mathbf{U}^{-1}\mathbf{G}^{-1}\mathbf{F} \tag{2}$$

where $\mathbf{U}$ is defined as: $U_{ij} = 0$ for $i > j$, $U_{ij} = 1$ otherwise, while the matrices $\mathbf{G}$ and $\mathbf{F}$ have elements:

$$G_{ij} = \frac{1}{\sqrt{-1}} \int_{\zeta_{2j-1}}^{\zeta_{2j}} \frac{\zeta^{i-1}}{D(\zeta)} d\zeta, \tag{3}$$

$$F_{ij} = \int_{\zeta_{2j}}^{\zeta_{2j+1}} \frac{\zeta^{i-1}}{D(\zeta)} d\zeta \tag{4}$$

in which:

$$D(\zeta) = \prod_{i=1}^{2N+2} \sqrt{\zeta - \zeta_i} \tag{5}$$

where the ζ_i are defined in Fig.1.

The matrices $\mathbf{F}$ and $\mathbf{G}$ are defined in (4), (3) in terms of *hyperelliptic* integrals. For $N=1$ or for $N=2$ in presence of symmetries such integrals reduce to complete elliptic integrals of the first kind. In general, however, such simplifications do not occur.

Although hyperelliptic integrals can be analytically expressed in terms of generalized hypergeometric functions, this is of no advantage from a computational standpoint, since such functions have to be computed through multiple power series which do not perform particularly well even for the elliptic case. Thus, numerical quadrature formulae are best suited for the evaluation of the capacitance matrix. Since the integrand is singular on the limits of the integration interval, either the singularity is extracted and accounted for analytically (as proposed in [7]) or proper Gaussian quadrature formulae have to be used.

The explicit approximations of the matrix elements F_{ij} and G_{ij} through Gauss-Chebyshev formulae of order M read:

$$F_{ij} \approx \frac{(-1)^{N+1-j}\pi}{M} \sum_{k=1}^{M} \frac{\theta_{jk}^{i-1}}{\prod_{m=1}^{2j-1}\sqrt{\theta_{jk}-\zeta_m}\prod_{m=2j+2}^{2N+2}\sqrt{\zeta_m-\theta_{jk}}} \tag{6}$$

$$G_{ij} \approx \frac{(-1)^{N+2-j}\pi}{M} \sum_{k=1}^{M} \frac{\sigma_{jk}^{i-1}}{\prod_{m=1}^{2j-2}\sqrt{\sigma_{jk}-\zeta_m}\prod_{m=2j+1}^{2N+2}\sqrt{\zeta_m-\sigma_{jk}}}, \tag{7}$$

where:

$$\theta_{jk} = \frac{\zeta_{2j+1}+\zeta_{2j}}{2} + \frac{\zeta_{2j+1}-\zeta_{2j}}{2}\cos\left(\frac{(2k-1)\pi}{2M}\right) \tag{8}$$

$$\sigma_{jk} = \frac{\zeta_{2j}+\zeta_{2j-1}}{2} + \frac{\zeta_{2j}-\zeta_{2j-1}}{2}\cos\left(\frac{(2k-1)\pi}{2M}\right). \tag{9}$$

The array element ζ_i define the position of strip and ground plane edges as shown in Fig.1.

Since all quasi-TEM modes supported by a MCPW or MCPS have the same effective permittivity $\epsilon_{\text{eff}} = (1+\epsilon_r)/2$, the characteristic impedance matrix $\mathbf{Z}$ of the line is proportional to the inductance matrix. Taking into account that $\mathbf{L} = \mathbf{C}_0^{-1}/c_0^2$, where c_0 is the light velocity *in vacuo*, and $\mathbf{C}_0$ the *in vacuo* capacitance matrix, one has:

$$\mathbf{Z} = -Z\mathbf{F}^{-1}\mathbf{G}\mathbf{U}/2 \tag{10}$$

where $Z = 120\pi/\sqrt{\epsilon_{\text{eff}}}$ is the wave impedance in an equivalent homogeneous medium of equivalent permittivity ϵ_{eff}. The phase velocity is the same for all propagation modes and can be approximated in the quasi-TEM limit as:

$$v_f = c_0/\sqrt{\epsilon_{\text{eff}}}. \tag{11}$$

Evaluation of the ohmic losses

Let us consider a MCPW with finite-thickness lines. In the same way as shown in [3] the finite-thickness structure can be transformed into a zero-thickness one by means of an intermediate Schwarz-Christoffel mapping. Then, the zero-thickness line undergoes N mappings into a rectangle, as for the computation of the characteristic parameters. The current density distribution for arbitrary line excitation can be derived from the superposition of the current distributions pertaining to each of the afore-mentioned mappings, on the basis of the approximation according to which the high-frequency current density distribution follows the static charge distribution. Thus, the power dissipated in each of the strips and in the ground plane can be recovered by properly integrating the square of the current density. With the same approximations exploited in [3] (strip thickness suitably smaller than the strip and slot widths) the dissipated power can be evaluated analytically, and the resistance matrix therefrom. In fact, the *p.u.l.* resistance matrix is the quadratic form associated to the *p.u.l.* dissipated power. The resulting resistance matrix is symmetric by definition and accounts for line-to-line coupling, although, for thin and uncoupled lines, it is diagonally dominant.

The *p.u.l.* resistance matrix of the MCPW can be finally expressed as:

$$R_{kh} = \sum_{n=1}^{N} R_{kh,n} + R_{Gkh} \tag{12}$$

where:

$$\begin{aligned} R_{kh,n} = {} & \frac{1}{w_k w_h}\left\{\pi + \log\left[\frac{4\pi}{t}(\zeta_{2n+1} - \zeta_{2n})\right]\right\} \\ & \cdot \left[\frac{\Phi_{kh}(\zeta_{2n})}{\Psi(2n-1, 2n+1, 2n)} + \frac{\Phi_{kh}(\zeta_{2n+1})}{\Psi(2n, 2n+2, 2n+1)}\right] \\ & + \sum_{i=1}^{2n-1} \frac{\Phi_{kh}(\zeta_i)}{\Psi(2n, 2n+1, i)} \log\frac{\zeta_{2n+1} - \zeta_i}{\zeta_{2n} - \zeta_i} \\ & + \sum_{i=2n+2}^{2N+2} \frac{\Phi_{kh}(\zeta_i)}{\Psi(2n, 2n+1, i)} \log\frac{\zeta_i - \zeta_{2n}}{\zeta_i - \zeta_{2n+1}} \end{aligned} \tag{13}$$

is related to the power dissipated in the strips, while:

$$\begin{aligned} R_{Gkh} = {} & \frac{1}{w_k w_h}\left\{\pi + \log\left[\frac{4\pi}{t}(\zeta_{2N+2} - \zeta_1)\right]\right\} \\ & \left[\frac{\Phi_{kh}(\zeta_1)}{\Psi(1, 2, 1)} + \frac{\Phi_{kh}(\zeta_{2N+2})}{\Psi(2N+1, 2N+2, 2N+2)}\right] \\ & + \sum_{i=2}^{2N+1} \frac{\Phi_{kh}(\zeta_i)}{\Psi(0, 1, i)} \log\frac{\zeta_{2N+2} - \zeta_i}{\zeta_i - \zeta_1} \end{aligned} \tag{14}$$

is related to the power dissipated in the ground plane. In the previous formulae one has:

$$\Psi(n_1, n_2, i) = \prod_{\substack{j=1 \\ j\neq i}}^{n_1} (\zeta_i - \zeta_j) \prod_{\substack{j=n_2 \\ j\neq i}}^{2N+2} (\zeta_j - \zeta_i) \tag{15}$$

and:

$$\Phi_{kh}(\zeta_i) = \left(\sum_{j=1}^{N} a_{ki}\, \zeta_i^{j-1}\right)\left(\sum_{j=1}^{N} a_{hi}\, \zeta_i^{j-1}\right). \tag{16}$$

The coefficients a_{ij} are the elements of the matrix $\mathbf{A}$ which is obtained as a by-product of the conformal mapping method [1] as:

$$\mathbf{A} = -\mathbf{W}\mathbf{F}^{-1} \tag{17}$$

where $\mathbf{W}$ is a diagonal matrix of elements w_i; these are the (arbitrary) widths of the rectangle in the transformed plane, see [1]. Since the final result does not depend on the choice of the w_i, one can choose $w_i = 1$ for any i. The resistance matrix thus defined is symmetric by construction; from a numerical standpoint, its evaluation requires a small overhead with respect to the computation of the capacitance matrix.

Once the resistance matrix has been evaluated, the *p.u.l.* impedance $\mathcal{Z}$ and admittance $\mathcal{Y}$ can be estimated as:

$$\mathcal{Z} = \mathbf{R} + j\omega\mathbf{L} \tag{18}$$
$$\mathcal{Y} = \mathbf{G} + j\omega\mathbf{C} \tag{19}$$

where the *p.u.l.* conductance matrix $\mathbf{G}$ is given as $\mathbf{G} = (\sigma/\epsilon_0)\mathbf{C}_1$, σ being the substrate conductivity. From $\mathcal{Z}$ and $\mathcal{Y}$ the modal propagation constants can be derived as the square roots of the eigenvalues of the product $\mathcal{Z}\mathcal{Y}$.

Results

In the present section, some results will be shown to assess the numerical accuracy and efficiency of the present approach. From a physical standpoint, the accuracy of the expressions provided is subject to the same limitations of the theory in [2] and [3], *i.e.* the approach is accurate in the presence of fully developed skin effect, while at low frequency the line resistance matrix must be evaluated accounting for the increasingly uniform current distributions. Such corrections can be implemented within the present approach, at least empirically, as it will be discussed elsewhere. At any rate, recent comparisons [6] suggest that the surface resistance approach, with edge corrections if required, can be exploited to obtain practically meaningful estimates of the line attenuation.

The purpose of Fig.2 is to estimate the numerical accuracy of the present approach when compared to the analytical expressions of [2] in the single conductor CPW. The relative error for the characteristic impedance and high-frequency attenuation is plotted as a function of the line impedance and of the number of integration samples exploited in the computations, which are performed in double precision. With 32 samples double precision accuracy is obtained for almost any line shape ratio, but also with a lower number of samples ($M = 8$) the accuracy is practically excellent.

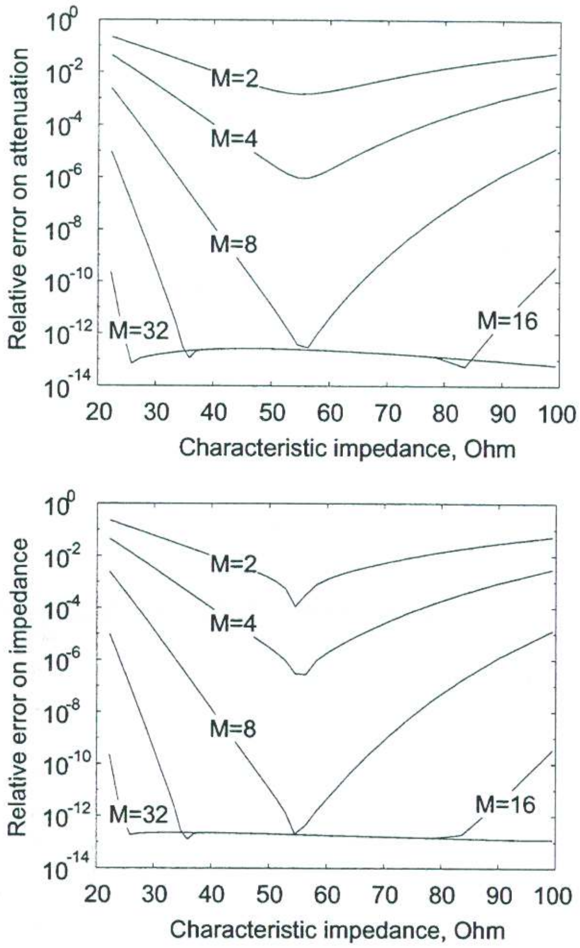

Figure 2: Relative error for the attenuation (above) and for the characteristic impedance (below) of a symmetric CPW between the exact (analytical) and the present approach as a function of the line impedance and the number of integration samples M. The line thickness is $t = 0.01b$ where b is the half-spacing between the ground planes.

Fig.3 tries a comparison between a CPW with finite-extent ground planes and the same structure treated as a 3-conductor MCPW with lateral ground planes removed far enough away to make their effect negligible (defining as $2d$ the ground plane spacing, this parameter was assigned as $d = 50b$). Clearly, the full MCPW has 3 propagation modes, a low-loss even mode, an odd mode with higher loss, and a high-loss second even mode. The MCPW numerical solution was compared, for two values of the ratio c/b, with analytical expressions for the CPW attenuation with finite ground planes [8]. Despite the slight differences mainly due to the fact that the exact and approximating structures do not exactly coincide, the agreement between the CPW attenuation and the attenuation of the second even mode of the MCPW is excellent. It can be noticed that the finite extent of ground planes slightly increases losses, as expected, and also changes the optimum impedance for minimum ohmic loss. With extremely thin lateral ground planes, a dramatic increase of losses can be detected.

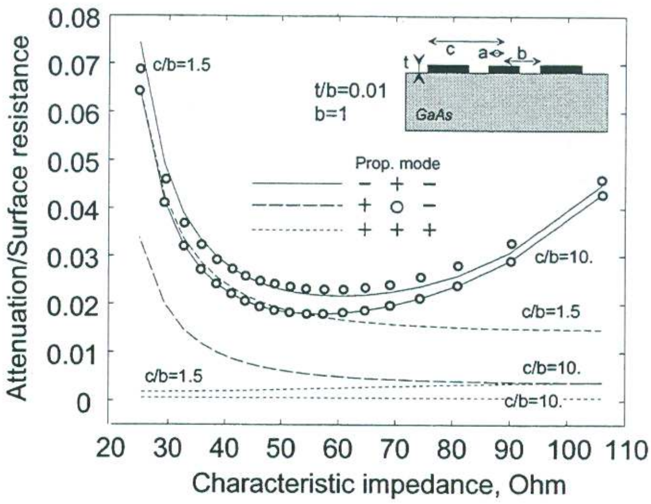

Figure 3: Normalized modal attenuation for a 3-conductor MCPW with lateral ground planes remobed so as to approximate a finite lateral ground plane CPW. The second even model attenuation is compared to the analytical expression for the CPW.

Conclusions

A CAD-oriented closed-form model for the high-frequency conductor losses of multiconductor coplanar waveguides has been presented. The comparatively low computational effort of the model, which can be complemented with corrections so as to account for the low-frequency (non skin-effect) regime makes it a candidate for inclusion into CAD tools for the analysis and design of analog and digital (M)MIC's.

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