

R&D for Quality Improvement and Network Externalities

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Abstract

We investigate the bearings of network externalities on product quality improvements requiring costly R&D investments. The model considers the dynamic behaviour of a monopolist alternatively maximising profits or social welfare. On the one hand, we confirm much of the acquired wisdom from the static literature on the same topic, about the arising of quality undersupply at the private optimum. On the other, we identify the initial conditions that must be met for R&D activity to be observed under profit-seeking behaviour. We also show that the presence of network externalities affects the optimal behaviour of the profit-seeking firm but not that of a benevolent planner, who serves all consumers and smooths the R&D costs leading to a steady state quality which is independent of network concerns.

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1 Introduction

The analysis of dynamic monopoly is a long standing issue, dating back to Evans (1924) and Tintner (1937), who investigated the pricing behaviour of a firm with convex costs. The analysis of intertemporal capital accumulation appeared later on (Eisner and Strotz, 1963). However, several other aspects of monopoly behaviour have never been looked upon with the tools of optimal control theory. One such aspect is the provision of product quality, which has been debated in static models to highlight the monopolist's incentive to undersupply quality as compared to the social optimum (Spence, 1975; Mussa and Rosen, 1978; Itoh, 1983; Gabszewicz *et al.* 1986; Besanko, Donnenfeld and White, 1987; Champsaur and Rochet, 1989).

We develop a monopoly model where the firm may invest to increase quality over time, and consumers enjoy both the utility attached to intrinsic quality and a network effect, whereby the satisfaction of a generic consumer is increasing in the number of individuals purchasing the same good or service (see Cabral, Salant and Wroch, 1999; Shy, 2000).¹ In a static model with the same ingredients, it is shown that the monopolist trades off quality for quantity as the network effect becomes more relevant (Lambertini and Orsini, 2001, 2003a). Here, the dynamic formulation of the problem permits to single out some additional features of such a market. There exist a parameter region where the monopolist does not find it convenient to improve product quality because the overall willingness to pay of consumers is too low. This must be contrasted with the behaviour of a benevolent social planner, who always improves quality irrespective of how affluent consumers are. As far as the

¹Our model is close in spirit to a stream of literature where product quality interacts with the formation of goodwill through advertising (see Feichtinger, Hartl and Sethi, 1994).

extent of market coverage is concerned, we show that (i) the optimal (private) monopoly output is always increasing in the amount of externalities; yet (ii) the profit-seeking firm never covers the entire market, whatever the network effect is, while the planner serves all consumers from the outset to the steady state.

The remainder of the paper is structured as follows. The basic model is in section 2. Section 3 contains the analysis of the profit-seeking monopoly equilibrium, while the comparison with the social planner's behaviour is investigated in section 4. Concluding remarks are in section 5.

2 The setup

Consider a monopoly market over an infinite (continuous) time horizon, $t \in [0, \infty)$. Consumers are indexed by their marginal willingness to pay for quality, measured by parameter θ , uniformly distributed with density 1 over $[0, \bar{\theta}]$.² Accordingly, the size of the market is $\bar{\theta}$. The generic consumer at $\theta \in [0, \bar{\theta}]$ buys one unit of the good iff:

$$U(t) = \theta q(t) + \alpha y(t) - p(t) \geq 0 \quad (1)$$

where $p(t)$ and $q(t)$ are the price and the quality of the good supplied by the monopolist at time t ; $\alpha y(t)$ is the network externality which is assumed to be linear in market demand $y(t)$. When inequality (1) is reversed, the consumer located at θ does not buy and his utility is $U = 0$. Under partial market coverage, there will be a marginal consumer at $\hat{\theta}(t)$, who is indifferent

²Parameter θ can be thought of as the reciprocal of the marginal utility of income, so that high-income consumers are indexed by high levels of θ , and conversely for low-income consumers (see Tirole, 1988, ch. 2).

between buying or not and identifies the lower bound of demand: $y(t) \equiv \bar{\theta} - \hat{\theta}(t)$. By definition, the indifference condition writes:

$$\hat{\theta}(t) q(t) + \alpha (\bar{\theta} - \hat{\theta}(t)) - p(t) = 0 \Rightarrow \quad (2)$$

$$\hat{\theta}(t) = \frac{\alpha \bar{\theta} - p(t)}{\alpha - q(t)}.$$

Then, plugging $\hat{\theta}(t)$ into $y(t) - \bar{\theta} + \hat{\theta}(t) = 0$ and solving w.r.t. the price, we obtain the inverse demand function:

$$p(t) = \bar{\theta} q(t) + (\alpha - q(t)) y(t). \quad (3)$$

Quality improvement involves an R&D investment process summarised by the following differential equation:

$$\dot{q} = bk(t) - \delta q(t), \quad b > 0 \quad (4)$$

where $k(t)$ is the instantaneous investment and $\delta \in [0, 1]$ is a constant depreciation rate. The instantaneous cost involved by investing $k(t)$ is $C(k(t)) = c[k(t)]^2$. For simplicity, we normalise the marginal production cost of output to zero. Hence, instantaneous monopoly profits are:

$$\pi(t) \equiv p(t) y(t) - c[k(t)]^2 \quad (5)$$

and, given a constant discount rate ρ , the monopolist must choose $y(t)$ and $k(t)$ so as to maximise:

$$\begin{aligned} \Pi &\equiv \int_0^\infty \{p(t) y(t) - c[k(t)]^2\} e^{-\rho t} dt \\ &s.t. : \dot{q} = bk(t) - \delta q(t). \end{aligned} \quad (6)$$

If instead the firm is run by a benevolent social planner, the scale of production and the intensity of R&D efforts are chosen to maximise the discounted

flow of social welfare, defined as the sum of profits and consumer surplus. The latter, at any time t , corresponds to:

$$cs(t) \equiv \int_{\hat{\theta}}^{\bar{\theta}} U(\theta) d\theta. \quad (7)$$

Therefore, the discounted stream of consumer surplus is:

$$CS \equiv \int_0^{\infty} cs(t) e^{-\rho t} dt. \quad (8)$$

Accordingly, the planner's problem is

$$\max_{y(t), k(t)} SW \equiv \Pi + CS \quad (9)$$

under (4).

3 Monopoly optimum

The Hamiltonian of the firm is:

$$\begin{aligned} \mathcal{H}_M = e^{-\rho t} \{ & [\bar{\theta}q(t) + (\alpha - q(t))y(t)]y(t) \\ & - c[k(t)]^2 + \lambda(t)[bk(t) - \delta q(t)] \} \end{aligned} \quad (10)$$

where $\lambda(t) = \mu(t)e^{\rho t}$, $\mu(t)$ being the co-state variable associated to quality. The initial and transversality conditions are $q(0) = q_0$ and

$$\lim_{t \rightarrow \infty} \mu(t)q(t) = 0. \quad (11)$$

The FOCs are (henceforth we omit the indication of time and discounting):³

$$\frac{\partial \mathcal{H}_M}{\partial k} = -2ck + b\lambda = 0 \quad (12)$$

$$\frac{\partial \mathcal{H}_M}{\partial y} = \bar{\theta}q + 2y(\alpha - q) = 0 \quad (13)$$

$$-\frac{\partial \mathcal{H}_M}{\partial q} = \dot{\lambda} - \rho\lambda \Rightarrow$$

$$\dot{\lambda} = \lambda(\rho + \delta) - y(\bar{\theta} - y). \quad (14)$$

FOC (12) yields:

$$\lambda = \frac{2ck}{b}; \dot{k} = \frac{b\lambda}{2c}. \quad (15)$$

From (13), we have $y_M^* = \bar{\theta}q / [2(q - \alpha)] > 0$ for all $q > \alpha$, which entails $\partial y_M^* / \partial q \leq 0$ for all $\alpha \geq 0$.⁴ On this basis, we can claim:

Lemma 1 *The monopolist trades off quantity and quality along the equilibrium path, provided any positive network effect operates.*

The above Lemma illustrates what is by now a well known result in the static models on the interplay between network effects and product quality, according to which the presence of the externality, while inducing the monopolist to expand output, brings also about an otherwise undesirable reduction of the quality level (see, e.g., Lambertini and Orsini (2001, 2003a)). Here, we extend this conclusion to a dynamic setting.

Now we are in a position to characterise the steady state equilibrium. Using y_M^* , we may write the dynamics of the R&D investment as follows:

$$\dot{k} = \frac{8c(\rho + \delta)(\alpha - q)^2 k - \bar{\theta}^2 b q (q - 2\alpha)}{8c(q - \alpha)} \quad (16)$$

³Throughout the paper, we also omit the analysis of second order (concavity) condition, which are always satisfied at saddle point equilibria.

⁴Throughout the paper, we use stars to indicate optimal controls and states along the path to the steady state, and superscript *ss* to identify steady state levels.

and imposing $\dot{k} = 0$, we get

$$k_M^* = \frac{\bar{\theta}^2 b q (q - 2\alpha)}{8c(\rho + \delta)(\alpha - q)^2} > 0 \forall q > 2\alpha. \quad (17)$$

Note that the positivity of k_M^* also involves a requirement on the initial condition, i.e., $q_0 > 2\alpha$. Should this condition not be met, the monopolist would not start R&D activities for quality improvement.

From (4), $\dot{q} = 0$ in $q_M^* = bk/\delta$. Plugging it into (17), we have three steady state levels of the R&D effort: $k_{M1}^{ss} = 0$, which is economically meaningless, and

$$k_{M2,3}^{ss} = \frac{16\alpha c \delta (\delta + \rho) + b^2 \bar{\theta}^2 \mp b \bar{\theta} \sqrt{\Psi}}{16bc(\delta + \rho)} \quad (18)$$

where

$$\Psi \equiv b^2 \bar{\theta}^2 - 32\alpha c \delta (\delta + \rho) \geq 0 \quad (19)$$

for all $\bar{\theta} \geq \sqrt{32\alpha c \delta (\delta + \rho)}/b$. On the basis of (4) and (16), we can write the Jacobian matrix:

$$J_M \equiv \begin{bmatrix} \frac{\partial \dot{q}}{\partial q} & \frac{\partial \dot{q}}{\partial k} \\ \frac{\partial \dot{k}}{\partial q} & \frac{\partial \dot{k}}{\partial k} \end{bmatrix} \quad (20)$$

where:

$$\frac{\partial \dot{q}}{\partial q} = -\delta; \quad \frac{\partial \dot{q}}{\partial k} = b \quad (21)$$

$$\frac{\partial \dot{k}}{\partial q} = -\frac{\alpha^2 \bar{\theta}^2 b}{4c(q - \alpha)^3}; \quad \frac{\partial \dot{k}}{\partial k} = \rho + \delta. \quad (22)$$

Hence, the trace and determinant of the Jacobian matrix J_M are:

$$T(J_M) = \rho > 0 \quad (23)$$

$$\Delta(J_M) = \frac{\alpha^2 \bar{\theta}^2 b^2}{4c(q - \alpha)^3} - \delta(\rho + \delta) < 0.$$

Using $\Delta(J_M)$, one finds that $(q_M^*(k_{M3}^{ss}), k_{M3}^{ss})$ is a saddle point, while the other solution is an unstable focus.⁵

The discussion carried so far establishes:

Proposition 2 *Provided $q_0 > 2\alpha$ and $\Psi \geq 0$, the monopolist reaches a unique saddle point equilibrium at*

$$\begin{aligned} k_M^{ss} &= \frac{16\alpha c\delta(\delta + \rho) + b^2\bar{\theta}^2 + b\bar{\theta}\sqrt{\Psi}}{16bc(\delta + \rho)} \\ q_M^{ss} &= \frac{b}{\delta}k_M^{ss}. \end{aligned}$$

The associated price and output are:

$$y_M^{ss} = \frac{3b\bar{\theta} - \sqrt{\Psi}}{4b}; p_M^{ss} = \frac{b\bar{\theta}}{2\delta}k_M^{ss}. \quad (24)$$

On the basis of (24), without further proof, we can state:

Corollary 3 *The steady state output of the profit-seeking monopolist is smaller than $\bar{\theta}$ in the whole admissible range of parameters.*

In other words, the monopolist always prices some consumers in the lower part of the income distribution out of consumption.

Now we consider the issue of introductory price offers, which has been largely discussed in the existing literature on network externalities.⁶ The price dynamics obtains by differentiating the inverse demand function w.r.t. time:

$$\dot{p} = \frac{\dot{q} [2q(\bar{\theta} - y) - \alpha(\bar{\theta} - 2y)]}{2(q - \alpha)} \quad (25)$$

⁵The details are omitted for brevity.

⁶For an overview, see Shy (2000). For static and dynamic analyses of this aspect in spatial monopoly models, see Rohlfs (1974) and Lambertini and Orsini (2003b), respectively.

which, using y_M^* , rewrites as $\dot{p} = \bar{\theta}\dot{q}/2 > 0$ as long as $\dot{q} > 0$. This entails the following corollary to Proposition 2:

Corollary 4 *As long as the monopolist invests in R&D to increase quality, he also monotonically increases the price over time. That is, the firm makes an introductory price offer.*

Note that the initial offer also involves a relatively low quality, both price and quality being bound to increase over time up to the steady state. Moreover, we can investigate the bearings of network effects on the steady state levels of controls, state and price:

$$\frac{\partial k_M^{ss}}{\partial \alpha} < 0; \frac{\partial y_M^{ss}}{\partial \alpha} > 0; \frac{\partial q_M^{ss}}{\partial \alpha} < 0; \frac{\partial p_M^{ss}}{\partial \alpha} < 0. \quad (26)$$

As the weight of the network externality increases, the steady state levels of R&D effort and quality shrink, since expanding the output is more convenient than increasing quality. To allow for a larger output, the price must be lower. In balance, the effects of a change in α on equilibrium price, output and quality entail that social welfare increases as the weight attached to network effects becomes more relevant.

4 Social optimum

Suppose first the planner may only partially cover the market. If so, the demand function is (3) and the Hamiltonian of the social planner is:

$$\begin{aligned} \mathcal{H}_{SP} = e^{-\rho t} \{ & [\bar{\theta}q + (\alpha - q)y]y + qy^2/2 \\ & - ck^2 + \beta(bk - \delta q) \}, \end{aligned} \quad (27)$$

where $\beta = \gamma e^{\rho t}$, $\beta(t)$ being the co-state variable associated to quality. Initial and transversality conditions are as in section 3. The FOCs of the planner are:

$$\frac{\partial \mathcal{H}_{SP}}{\partial k} = -2ck + b\beta = 0 \quad (28)$$

$$\frac{\partial \mathcal{H}_{SP}}{\partial y} = \bar{\theta}q + y(2\alpha - q) = 0 \quad (29)$$

$$-\frac{\partial \mathcal{H}_{SP}}{\partial y} = \dot{\beta} - \rho\beta \Rightarrow$$

$$\dot{\beta} = \beta(\rho + \delta) - y(2\bar{\theta} - y)/2. \quad (30)$$

From (28), one obtains:

$$\dot{k} = \frac{b\beta}{2c}; \beta = \frac{2ck}{b}. \quad (31)$$

From (29), $y_{SP}^* = \bar{\theta}q / (2\alpha - q)$, with

$$y_{SP}^* \in (0, \bar{\theta}) \quad \forall q \in (0, \alpha). \quad (32)$$

Using (30-31) and y_{SP}^* , one can impose $\dot{k} = 0$ to obtain:

$$k_{SP}^* = \frac{\bar{\theta}^2 bq(q - 4\alpha)}{4c(\rho + \delta)(2\alpha - q)^2} > 0 \quad \forall q > 4\alpha. \quad (33)$$

The condition for $k_{SP}^* > 0$ contrasts with the requirement for $y_{SP}^* \in (0, \bar{\theta})$. That is, (i) if $q \in (0, \alpha)$, then market demand is well defined but optimal R&D investment is negative; (ii) if $q \in (\alpha, 4\alpha)$, then $y_{SP}^* > \bar{\theta}$ and $k_{SP}^* < 0$; finally, if $q > 4\alpha$, then $y_{SP}^* > \bar{\theta}$ and $k_{SP}^* > 0$. This amounts to saying that partial market coverage is incompatible with social planning:

Lemma 5 *For all $\alpha \geq 0$, the social planner serves all existing consumers from the initial date to the steady state.*

At this point, we have to reformulate the optimum problem of the social planner under the assumption that $y_{SP} = \bar{\theta}$ from the very outset. This entails that instantaneous profits and consumer surplus write as follows:

$$\pi(t) \equiv p(t)\bar{\theta} - c[k(t)]^2 \quad (34)$$

$$\begin{aligned} cs(t) &\equiv \int_0^{\bar{\theta}} [\theta q(t) - p(t) + \alpha\bar{\theta}] d\theta \\ &= \frac{\bar{\theta}}{2} [\bar{\theta}(q(t) + 2\alpha) - 2p(t)] \end{aligned} \quad (35)$$

so that instantaneous welfare corresponds to:

$$sw(t) = \frac{\bar{\theta}^2}{2} [q(t) + 2\alpha] - c[k(t)]^2, \quad (36)$$

which is independent of the price. Therefore, we may suppose that the planner sets the lowest admissible price that allows to make up for R&D costs, in order to keep profits non-negative.

Accordingly, the planner's Hamiltonian function now becomes:⁷

$$\mathcal{H}_{SP} = e^{-\rho t} \left\{ \frac{\bar{\theta}^2}{2} [q + 2\alpha] - ck^2 + \beta(bk - \delta q) \right\} \quad (37)$$

The maximum problem now involves only one control and one state variable. The FOCs are:

$$\frac{\partial \mathcal{H}_{SP}}{\partial k} = -2ck + b\beta = 0 \quad (38)$$

$$\begin{aligned} -\frac{\partial \mathcal{H}_{SP}}{\partial y} &= \dot{\beta} - \rho\beta \Rightarrow \\ \dot{\beta} &= \beta(\rho + \delta) - \frac{\bar{\theta}^2}{2}. \end{aligned} \quad (39)$$

⁷Again, henceforth the indication of time is omitted for brevity.

Equation (38) yields the same value of β as well as the same dynamics of k as in (31). Then, using $\beta = 2ck/b$ and (39), we obtain:

$$\dot{k} = k(\rho + \delta) - \frac{b\bar{\theta}^2}{4c}. \quad (40)$$

Equation (40) shows that the planner's instantaneous investment along the equilibrium path is independent of quality. This stems from the fact that, all consumers being served at all times, the planner finds it convenient to fully smooth investment costs over time.

Solving the system $\left\{ \dot{q} = 0, \dot{k} = 0 \right\}$, one finds the steady state levels of quality and R&D effort:

$$q_{SP}^{ss} = \frac{b^2\bar{\theta}^2}{4c\delta(\rho + \delta)}, \quad k_{SP}^{ss} = \frac{b\bar{\theta}^2}{4c(\rho + \delta)}. \quad (41)$$

Using the Jacobian matrix of the dynamic system, which is defined as in (20), we can calculate the trace and determinant:

$$\begin{aligned} T(J_{SP}) &= \rho > 0 \\ \Delta(J_{SP}) &= -\delta(\rho + \delta) < 0 \end{aligned} \quad (42)$$

for all $\delta \in (0, 1]$. Therefore, the steady state (41) is stable in the saddle point sense. The foregoing discussion leads to

Proposition 6 *The pair $(q_{SP}^{ss}, k_{SP}^{ss})$ is a saddle point, unaffected by network externalities.*

Given that the price level is not univocally defined, and quality improvements hinges upon fixed costs only without interacting with the output level, the equilibrium R&D effort and quality are exactly the same that the planner would have chosen without network externalities.

5 Conclusions

We have assessed the bearings of network effects on the incentive to improve product quality through costly R&D efforts in a monopoly market where consumers have different marginal willingness to pay for quality and the firm may alternatively maximise profits or social welfare.

The analysis has been carried out in a dynamic model where the firm chooses the extent of market coverage together with the quality-improving investment.

Our results can be summarised in the following terms. While confirming much of the existing wisdom from the static analysis of network externalities, our model has singled out the initial conditions that must be satisfied for the R&D activity to start under the monopoly regime. Contrary to the result obtained in the static model (Lambertini and Orsini, 2003a), the monopolist never finds it profitable to cover the entire market, no matter how high the network externality can be. Provided that the profit-seeking firm does invest, in doing so she takes into account both the current quality level and the extent of the externality, while it would be socially optimal to smooth the R&D costs perfectly, by investing a constant amount of resources at every instant. Moreover, the planner always serves all consumers in the market, with a loose pricing rule, whose only requirement is to allow for the firm to cover R&D costs.

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