PRIVATE AND PUBLIC PENSION SCHEMES: A CASE OF VERTICAL SEGMENTATION

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No 163

Private and Public Pension Schemes:

A Case of Vertical Segmentation

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May 1993

Abstract:

This work develops an overlapping generations model in order to analyze a transition to a partial privatization of the social security system. We find that such a reform induces a vertical segmentation in the market of pension schemes. Moreover the redistributive aim of the government is limited by the possibility to switch from the private to the public pension scheme.

JEL classification number: H55

* I wish to thank Normand Ireland, Alberto Niccoli and Stefano Toso for their most frutiful comments. The usual disclaimer applies.

INTRODUCTION

The reform of the social security system represents a recurring argument in the economic policy debate and in the political agenda of governments of most industrialized countries.

As social security system is typically a mandatory, public system usually it is financed on a pay-as-you-go basis i.e. contributions on labour income of current working generations are used to finance pension benefits of current retired generations.

There are a number of possible explanations for the emergence of social security in the society. The economic literature stresses the existence of "individual equity" and "social adequacy" goals for the government (MUNNELL 77, THOMPSON 83, KOTLIKOFF 90).

According to the first goal social security's function is the provision of insurance. The risk to be covered would be that of disability and uncertain longevity. Benefit payments are provided to elderly in form of annuities which continue until the death of the recipient. With perfect markets the actuarial value of contributions paid to the pension system equals the expected actuarial value of benefits received.

The existence of a public, mandatory system is explained by the particular kind of risk which the system covers, to the adverse selection problem and to the myopic behaviour of individuals (DIAMOND 77, BARR 85, KOTLIKOFF 90). The pay-as-you-go method of financing the system is optimal for the society as long as the sum of the rates of population growth and productivity growth is bigger than the real interest rate (AARON 66).

The "social adequacy" goal is a welfare criterion according to which benefits are not measured against lifetime contributions but rather against a standard of living beneath which society feels no one should fall (MUNNELL 77).

This second goal necessarily implies the public provision of social security. Financing the system entails a redistribution of income both between generations and also among members of the same generation. The social security system becomes a part of the tax-transfer system: contributions and benefits are fixed in a way such to permit the government to reach its redistributive goal (THOMPSON 83).

The favourable demographic evolution of the population in the past decades permitted the state to pursue both objectives by financing the system with a pay-as-you-go method. Therefore the introduction and maintenance of a pay-as-you-go social security system represented a Pareto improving policy since it increased the welfare of all generations (SAMUELSON 75).

The reversal of the time pattern of the ratio between working and retired population created concerns about the financial viability of the public, mandatory social security systems[1]. Uncertainty has arisen about the capacity of the public pension system to provide an adequate retirement income for future generations i.e. to pursue both the "social adequacy" and the "individual equity" objectives.

Different proposal in order to reform the social security system are emerged. There are three kind of policies a government can adopt in order to face the crisis of social security:

i) the government can implement a reform of the pension system by changing its functional parameters i.e. the retirement age, the contribution rate or the substitution rate between wage and pension benefits. However the system would maintain the pay-as-you-go financing method;

ii) the government can borrow resources from the private market in order to finance the increasing number of old age pensions;

iii) the government can implement a policy of partial privatization of the pension system. Contracting out in the U.K. is an example of such a kind of policy[2].

Any of these three policies presents new problems which the government never met in the past in undertaking the management of the pension system. Principally contrasts will emerge between generations: any possible reform policy will make some generation better off at the expenses of some other generation.

Moreover the implementation of the third option competes with the government's aim to maintain the "social adequacy" goal (DIAMOND 77). Indeed private insurances

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[1] A detailed description of the evolution of the population in the industrialized countries is given in OECD (86).

[2] For a detailed description of the reform of social security in the United Kingdom see (CREEDY and DISNEY 85)

are not involved in any kind of redistribution between generations or among members of the same generation.

This paper will examine the third policy option in the theoretical framework of an overlapping generations model.

We find that the possibility of choose between a private and a public pension scheme generates a vertical segmentation in the market of pension schemes. In our model people with low income levels prefer a flat rate, redistributive pension benefit whereas people with high income levels prefer to pay a contribution to the government for the financing of the public scheme and opt for a private pension scheme offered in a competitive market.

Moreover the government has a limit in pursuing its "social adequacy" goal. People choosing the private option have to pay a contribution to the government in order to finance the public pensions. We find that the government can not raise this contribution over a certain maximum level. Indeed higher levels of the contribution imply lower values for the public pension benefits.

The work is organized as follow. Subsection 1. deals with the description of the model of the choice between public and private pension schemes. A pension system is mandatory. The government pursues a paternalistic aim inducing people to save for their old age. With the implementation of the pension system the government pursues also a redistributive aim (DIAMOND 77).

When the public option is chosen the individual has to pay contributions for the financing of pension benefits of the current old generation. He/she acquires the right to receive a pension in the second period of life.

When the private option is chosen the individual has to pay contributions to a private insurance. The private pension benefits are determined by the return of saving in a competitive capital market.

The private option does not excuse individuals from the payment of a tax for the financing of the redistributive component of the public pension scheme.

By introducing a simple hypothesis on income distribution and a linear tax rule for the financing of the public pension scheme we show that a vertical segmentation in the market for pension schemes emerges. We find a value in the distribution of income which divides the market. People with lower income levels opt for the public pension whereas people with higher income levels prefer the private option.

Subsection 2. analyzes the limit a government can encounter in pursuing its "social adequacy" goal when such a reform is implemented. The possibility for individuals to switch from the private pension scheme to the public one when the first implies a high level of the tax for the financing of the redistributive component of the public pension scheme, put a maximum level to the tax rate itself.

Successively an optimal policy for the government is found when the public pension is considered as a poverty line, i.e. a minimum level of income for old people fixed by the government.

The last section summarizes the results.

1. A MODEL OF THE CHOICE BETWEEN PUBLIC AND PRIVATE PENSION

1.1 Description of the model

Individuals have the possibility of choosing between a public and a private pension scheme. The public pension system is financed by the government with a pay-as-you-go method whereas private pensions are provided by private insurances which act in a competitive market. The individual has to choose one of the private and the public pension schemes.

The explanation for this limit to consumer sovereignty can be thought of as being related to government's aim of avoiding myopic behaviour of agents who would not save enough during the first period of life (DIAMOND 77). Moreover the government uses the pension system for its redistributive purposes.

The individual's life consists of two periods. Preferences are assumed to be represented by a utility function $U = u(C_t, C_{t+1})$ which is increasing and concave in C_t and C_{t+1} where C_t is consumption at time t and C_{t+1} is consumption at time t+1.

Only two generations are living during each period.

Each individual has a given income level in the first period of life and no income in the second period. In order to support consumption in the old age each individual accumulates a certain amount of savings and also decides whether to participate either in a public or a private pension system in the young age.

We also assume that individuals differ with respect to their gross income, Y. The distribution of income across individuals is assumed to have density $f(Y) = 1$ and distribution function $F(Y) = Y$. Then income is uniformly distributed with range $0 < Y$

ϵ

< 1.

We do not make any assumptions about the supply side of the model. Therefore we do not need to consider the existence and form of a production function and the possible effects of a pension system on capital formation. Indeed we assume a constant real rate of interest. Moreover the income level is assumed not to vary over time.

According to whether the individual opts for the public or for the private system he/she will face different budget constraints.

1.2 <u>The public option</u>

Equations (1) and (2) describe the budget constraints of an individual when he/she chooses the public pension scheme.

$$C_t = Y_t - \theta Y_t - l_t \qquad (1)$$

where $0 < \theta < 1$.

$$C_{t+1} = P_s + (1+r)\, l_t \qquad (2)$$

where $P_s > 0$, $r > 0$.

Equation (1) says that consumption for a person of generation "t" when young is equal to the person's initial income Y_t, minus the contribution to the state for the public pension θY_t, minus the amount of resources privately saved l_t. The contribution to the public pension scheme is a fraction of the first period income.

Consumption of a person of the same generation when old is equal to the public pension benefit P_s, plus the capitalized value of saving.

We do not take into account the possibility of bequests. Every individual uses all his/her income during his/her two periods' life.

The government finances the public pension scheme with a pay-as-you-go method. Pensions of the old generation are paid for by the contributions of the currently young generation. Therefore there is no accumulation of funds in the public pension system[3].

We can combine the two individuals' budget constraints and thereby look at the restrictions an individual faces when planning lifetime consumption possibilities.

We get:

$$C_t + \frac{C_{t+1}}{1+r} = Y_t(1-\theta) + \frac{P_s}{1+r} \qquad (3)$$

Equation (3) is the lifetime budget constraint and says that the lifetime consumption expressed in terms of goods at time (t) is equal to the lifetime income expressed in terms of goods at time (t).

1.3 The private option

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[3] In this model P_s the public pension benefit can be thought of as a choice variable for the government. In deciding its level the government is assumed to use contributions of the current young generation as the unique source of financing.

Equations (4) and (5) describe the budget constraints of an individual who decides to opt for the private pension system.

$$C_t = Y_t - \alpha\theta Y_t - \beta Y_t - I_t \qquad (4)$$

where $0 < \alpha < 1$; $0 < \theta < 1$; $0 < \beta < 1$;

$$C_{t+1} = (1+r)\,[\beta Y_t + I_t] \qquad (5)$$

The consumption in the first period of his/her life is equal to the initial income minus the compulsory contributions to a private insurance for the financing of the pension βY_t, minus the private saving, minus the contributions to the financing of the public pension $\alpha\theta Y_t$. The latter is a fraction of the contributions paid by individuals choosing the public pension scheme.

The parameter α is a policy variable. We will exclude from the analysis the two extreme situations $\alpha = 0$ and $\alpha = 1$[4]

If $\alpha \neq 0$ then the private option does not completely excuse the individual from the contribution to the public pension. This is the redistributive component of the model. An individual opting for the private pension scheme does not have any rights in receiving transfers from the state in form of pension benefits even though he/she has to contribute

[4] If $\alpha = 1$ then there is no remission at all and therefore nobody will choose the private pension. On the other hand if the value of α is zero the private pension plan variant will correspond exactly to private saving given the hypothesis of a unique constant interest rate.

for the financing of the public pension scheme.

The consumption at time (t+1) is equal to the capitalized value of the contributions paid to the private insurance at time (t) plus the capitalized value of saving[5].

The lifetime budget constraint will be:

$$C_t+\frac{C_{t+1}}{(1+r)}=Y_t(1-\alpha\theta) \qquad (6)$$

and it measures the present value of consumption and income when individuals choose a private pension scheme.

1.4 Maximization of utility and market segmentation

Any individual maximizes his/her lifetime utility. The utility maximization problem for the individual can be stated as:

$$\max U=u(C_t,C_{t+1}) \qquad (7)$$

$$s.t.\ C_t+\frac{C_{t+1}}{1+r}=\max\left(\ [Y_t(1-\alpha\theta)]\ ;\ [Y_t(1-\theta)+\frac{P_s}{1+r}]\ \right) \qquad (8)$$

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[5] We assume perfect markets. Therefore there is not difference between the return rate of the two forms of saving.

The solution of the maximization problem will give us the optimal consumption level at times (t) and (t+1).

Corresponding to each solution we will find a value for the indirect utility function $U^* = u(Y, r)$ where Y is the lifetime income level.

In this model individuals can change their lifetime income level only by choosing a different pension scheme.

Hence it is possible to measure the relative value in deciding to enter a public or a private pension system just by comparing the lifetime income levels which an individual would have in the two alternative situations.

Given the form of the utility function and in particular the hypothesis that utility is non-decreasing in Y, for any value of P_g,$\propto$ and θ a type Y individual will maximize his/her utility and will choose the private pension if the associated lifetime income is higher than the lifetime income assured to him/her by the public pension scheme, i.e. if:

$$Y_t(1-\alpha\theta) > Y_t(1-\theta) + \frac{P_g}{1+r} \qquad (9)$$

Since we assumed a uniform distribution function for Y with density f(y) = 1 then income can assume different values with constant probability.

Given particular values of the parameters ($\propto$, θ, P_s) there will be a level of income which divides the population in two groups with different preferences concerning the choice of the pension scheme.

We can find this income level by equating the left hand side and the right hand side of equation (9) and then solving for the value of Y.

We find that:

$$Y_t^* = \frac{P_E}{\theta(1-\alpha)(1+r)} \qquad (10)$$

We define Y^* as the income level which divides the market of pension schemes. Therefore we will call it the separating income level.

When the model reaches its equilibrium, every individual is maximizing his/her utility. Individuals whose income is below the separating income level find the public option more profitable whereas individuals whose income level is above the separating income level will prefer the private option. A vertical segmentation of the market emerges.

More insights can be given by considering the lifetime budget constraint of people choosing the public pension scheme.

In the case where the introduction of the pension system does not modify the lifetime income level of an individual we can write:

$$Y_t(1-\theta) + \frac{P_B}{(1+r)} = Y_t \qquad (11)$$

$$Y_t = \frac{P_B}{\theta(1+r)} \qquad (12)$$

Therefore for income level $0 < Y_t < P_B / [\theta(1+r)]$, the introduction of a public pension scheme represents a net increase in the lifetime income level. For income level

$P_s / [\theta (1 + r)] < Y_t < P_s / [\theta (1 - \alpha) (1 + r)]$ the introduction of a public pension scheme represents a net loss in the lifetime income level but still the public option guarantees a higher lifetime income in comparison with the private option.

We now define π as the fraction of the population choosing the public pension scheme. We will have $0 < \pi < 1$. The value of depends on how many consumers decide to opt for the private pension scheme, and is thus also endogenous. Given that the hypothesis on the distribution function of income is $Y = F(Y)$ and income has range $0 < Y < 1$, the proportion of the population choosing the public pension is defined as: $\pi = F(Y^*) = Y^*$.

Therefore:

$$\pi = F\left[\frac{P_B}{\theta(1-\alpha)(1+r)}\right] = Y^* \qquad (13)$$

The following restrictions on parameters are required:

$P_s < \theta (1 - \alpha)(1 + r)$;

$P_s > 0$.

1.5.1 The government budget constraint

Government expenditures and revenues are uniquely related to the implementation of the public pension system. The government does not borrow resources from the private market but only uses the revenues obtained from the taxation of the young generation to finance the pension benefits of the current old generation. Therefore in every period revenues raised with taxation of income must equal expenditures for the financing of public pensions.

Through the implementation of the public pension system the government pursues its redistributive goal. Every individual contributes to financing the public pension scheme.

Since contributions are a constant fraction of income, people with high income levels are in fact contributing to a large extent to financing the public pension scheme. In the special case in which every generation is composed by the same number of individuals and income level is constant, the redistribution is in fact among members of the same generation even though the transfer of resources is between a young and an old generation. Variations in the ratio between the number of old and young people introduce also an intergenerational redistribution of income.

The introduction of the budget constraint permits us to stress the limits the government can encounter in pursuing its redistributive aim in a situation where the economic dependency ratio increases over time.

Government revenues are defined as:

$$R_t = \alpha\theta\int_0^1 Y_t dY_t + (1-\alpha)\theta\int_0^{\pi} Y_t dY_t \qquad (14)$$

where $\pi = P_s/[\theta(1-\alpha)(1+r)] = Y^*$ is the proportion of people choosing the public pension scheme as showed in equation (11).

Total revenues for the government are given by: i) the sum of the contributions which each individual has to make to finance the public pension whether they have opted to receive a public pension or not; ii) the sum of the remaining contributions of people choosing the public option for their pension.

$$E_t = d_t\int_0^{\pi} (P_g)\, dY_t \qquad (15)$$

Equation (15) specifies the expenditure of the government for the public pension scheme. It defines the government requirement to finance the pension system. The term "d_t" measures the number of old people every individual of the current young generation has to finance with his/her contributions.

By evaluating the integral terms in equations (14) and (15) we can calculate the value of revenues and expenditures of the government[6].

By equating the revenues and expenditures of the government we can get the equality:

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[6] Having assumed $0 < Y < 1$ and $f(Y) = 1$ i.e. a uniform distribution of income, the term $f(Y)$ has not been included in the integral terms.

$$\frac{1}{2}\alpha\theta+\frac{1}{2}\theta(1-\alpha)\left(\frac{P_B}{\theta(1-\alpha)(1+r)}\right)^2 = d_t * P_B\left(\frac{P_B}{\theta(1-\alpha)(1+r)}\right) \quad (16)$$

which gives a measure of revenues and expenditures for the financing of the public pension system in terms of the parameters of the model.

1.5.2 Analysis of the separating income level

The amount of the public pension benefit paid to each individual is not related to previous contributions but only to the contributions of the current young generation. This allows a better focus on the major issues to be addressed. The pension system is considered as a tax transfer mechanism which the government uses to pursue its redistributive goal.

When the model is at its steady state equilibrium the government budget constraint is in balance and individuals maximize their utility function. Then equation (16) holds and Y^* in equation (11) defines individual behaviour.

Reconsider now equation (16): multiplying each term by 2 and dividing by $\theta(1 - \alpha)(1 + r)$ we get:

$$\frac{\alpha}{(1-\alpha)(1+r)}+\frac{1}{(1+r)}*\left(\frac{P_B}{\theta(1-\alpha)(1+r)}\right)^2=2d_t*\left(\frac{P_B}{\theta(1-\alpha)(1+r)}\right)^2 \quad (17)$$

and

$$\left(\frac{P_s}{\theta(1-\alpha)(1+r)}\right) = \left(\frac{\alpha}{(1-\alpha)[2d_t(1+r)-1]}\right)^{\frac{1}{2}} \qquad (18)$$

Notice also that $[P_s/\theta(1-\propto)(1+r)] = Y^*$ from equation (10). We can therefore find an equation for the separating income level which also implies the balance in the government budget:

$$Y_t^* = \frac{P_s}{\theta(1-\alpha)(1+r)} = \left(\frac{\alpha}{(1-\alpha)[2d(1+r)-1]}\right)^{\frac{1}{2}} \qquad (19)$$

Equation (19) expresses the value of the separating income level as a function of two parameters: the contribution rate $\propto$ and the economic dependency ratio , d_t.

As d_t increases the separating income level decreases. Intuitively, if the same number of young people has to finance the pension of an increasing number of old people then the per-capita value of the public pension has to be lower in order to keep the government budget constraint in balance. Since all the other parameters are constant the negative effect on Y^* is explained as well as the fall in the proportion of people choosing the public pension scheme.

As $\propto$ varies from 0 to 1 the cost of choosing the private option increases and therefore the proportion of the population opting for the private pension scheme diminishes. However changes on $\propto$ also affects the value of the public pension benefit P_s in a way which limits the government's redistributive aim.

This will be the topic of the next subsection.

2.1 Limit to the redistribution in a mixed pension system

By considering that $Y^* = P_s/[\theta(1-\alpha)(1+r)]$ in equation (20), we can find an expression which yields the public pension P_s as a function of α.

We get:

$$P_s = \left(\frac{\alpha(1-\alpha)}{[2d(1+r)-1]}\right)^{\frac{1}{2}} * [\theta(1+r)] \qquad (20)$$

If d, θ and r are fixed we find a functional relationship between P_s and α. This is shown in fig.1 below.

INSERT FIG. 1

The figure emphasizes the limits the government can encounter in its redistributive policy. The value of P_s reaches its maximum when $\alpha = 1/2$. Therefore $\alpha = 1/2$ is the level which guarantees the maximum effect of the tax parameter α on the level of the public pension benefit.

Every further increase of α from 1/2 makes the private pension scheme less profitable inducing people with high income levels to switch from the private option to the public one. The level of the public pension benefit will start to decrease because: i) more people will choose the public option; ii) less people will contribute to finance it without getting anything in exchange.

Therefore in the model here presented it is not worthwhile for the government to push the value of the contribution rate α over one half. Increasing it further implies a negative effect on the public pension as it is less targeted.

A lower value implies lost government income as contributions become smaller.

We here find two important results: i) the value $\alpha = 1/2$ represents an optimal choice for the government in order to maximize the level of the public pension benefit; ii) further increments of α over one half will have an opposite effect with respect to the government's aim to raise the level of the public pension benefit.

2.2 The public pension benefit as a poverty line

In our model the level of P_s can be considered as a poverty line i.e. the level of income the government considers the minimum that has to be reached in the old age.

According to this hypothesis the aim of the government is to guarantee a sort of minimum income for the poorer side of the population and to leave other individuals free to choose more profitable private pension schemes.

Thus the objective of the government policy would be that of ensuring nobody to fall below the poverty line while minimizing the cost for the society as measured by the contribution rate θ.

Formally we have:

$$\min \theta = \left(\frac{2d(1+r)-1}{\alpha(1-\alpha)}\right)^{\frac{1}{2}} * \frac{P_s}{(1+r)} \qquad (21)$$

$$s.t. \quad P_g = \overline{P}_g \qquad (22)$$

From the equation (22) we can immediately notice that independently from every value of the dependency ratio, d_t, the value of $\propto = 1/2$ will make θ minimum.

Therefore the optimal policy for the government in order to keep the poverty line constant is to fix $\propto$ at 1/2 and θ at

$$\theta = \frac{2\overline{P}_g}{(1+r)}\left[d(1+r)-1\right]^{\frac{1}{2}} \qquad (23)$$

We here find a confirmation of the result that the level of α, the redistributive component of the model, can not be raised over 1/2 whatever the value of the other parameters are.

Therefore the value of the second contribution rate, θ depends: i) on the level of P_s which is decided by the government; ii) on the ratio between old and young, d_t.

Therefore in the framework of this model the government uses two tax parameters in order to minimize the cost of the redistribution for the society.

The first parameter, $\propto$ which measures the contribution of the proportion of the population choosing the private option to finance the public pension system, is fixed as to maximize the value of the public redistributive pension benefit whatever the value of the other parameters are.

The second tax parameter, θ will vary with the economic dependency ratio in order to maintain constant the redistributive public pension benefit at the desired level.

A numerical example of this policy is given in table 1. The decision of the government to keep the poverty line constant when the economic dependency ratio increases has a cost measured by the increasing value of the tax rates θ.

Notice also that both the proportion of the population choosing the public option and the proportion of the population of "net gainers" fall.

INSERT TABLE 1.

3. CONCLUSION

In this concluding section we want to reconsider the main problems raised during this study in order to summarize the results obtained.

A two period overlapping generations model has been used to represent some of the problems the government has to face in the reform of the social security system.

Individuals are allowed to choose between a public and a private pension scheme. The public pension system give raise to a redistribution of income between generations and among members of the same generation. The private pension scheme is provided by private insurance companies in a competitive market. The private option does not excuse individuals from the payment of a tax for the finance of the redistributive public pension scheme.

A simple hypothesis of income distribution permits to determine a situation of vertical segmentation in the market of pension schemes. A level of income is found in the market of pension schemes: given an hypothesis of linear taxation, higher income level individuals prefer to choose the private pension scheme whereas lower income level individuals find the public option more profitable.

There is a limit in the level of the tax rate on income used to finance the redistributive public pension scheme. Higher values of this tax rate have an opposite effect respect to the government's aim of increasing the level of the public pension.

A feasible policy for the government in this model is then to pursue the "social adequacy" objective by considering the public pension as a poverty line. In this case tax rates used to finance the pension system have to be fixed such to minimize the cost for the society of maintaining a given level for the poverty line.

Appendix: Uniqueness of the crossing point

In this appendix we want to show that there is only one level of income Y^* which divides the market of pension schemes. In order to demonstrate this proposition we can imagine to have found a value for the separating income level Y^*. For any higher value of Y in the distribution of income it must be true that the increase in the marginal cost of the tax given by $[\beta - \theta\ (1 - \alpha)]Y_t$ is lower than the marginal increasing return derived from the shift to the private pension, $[(\beta Y_t - P_s/(1+r)]$.

Then we define a function $T = t(Y)$ as:

$$T = t(Y) = [\beta Y_t - \frac{P_s}{(1+r)}] - [\beta - \theta(1-\alpha)]\, Y_t \qquad (14)$$

$$T = \theta(1-\alpha)\, Y - \frac{P_s}{1+r}$$

The function T sums up the difference between the relative return and the relative tax cost of the two pension systems and assumes value zero at y^* where the two pension schemes give the same lifetime income level. Therefore we will have $T = t\ (Y^*) = 0$. f the value of the derivative of the function $T = t\ (Y)$ with respect to Y is always positive we are sure that for any $Y > Y^*$ the function $T = t(Y^*)$ is positive implying therefore the private choice.

The value of the derivative of the function T with respect to Y is given by:

$$\frac{\delta T}{\delta Y}=\theta(1-\alpha)>0 \qquad (13)$$

and is always greater than zero if:

$\theta > 0$; $\alpha < 1$ and

$\theta(1 - \alpha) > 0$.

The first two requirements are consistent with the hypotheses of the model (pg.2 and pg. 4). The last always holds respected if the first two hold.

We can represent the function T in a diagram.

INSERT FIG. 2

For any given level of parameters we will observe a linear, monotonic increasing function in Y, the income level. There is only one point where the function T as sumes value zero. When the income level is at zero the function T assumes value $T = - a/(1+r)$.

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ec. dep. ratio	tax rates θ	poverty line P_s	separating income	net gainers
0,8	0,418	0,265	0,845	0,422
0,9	0,460	0,265	0,767	0,383
1	0,500	0,265	0,707	0,340
1,1	0,533	0,265	0,666	0,333
1,2	0,564	0,265	0,632	0,315
1,3	0,601	0,265	0,587	0,293
1,4	0,632	0,265	0,559	0,279
1,5	0,661	0,265	0,537	0,267

Table 1.

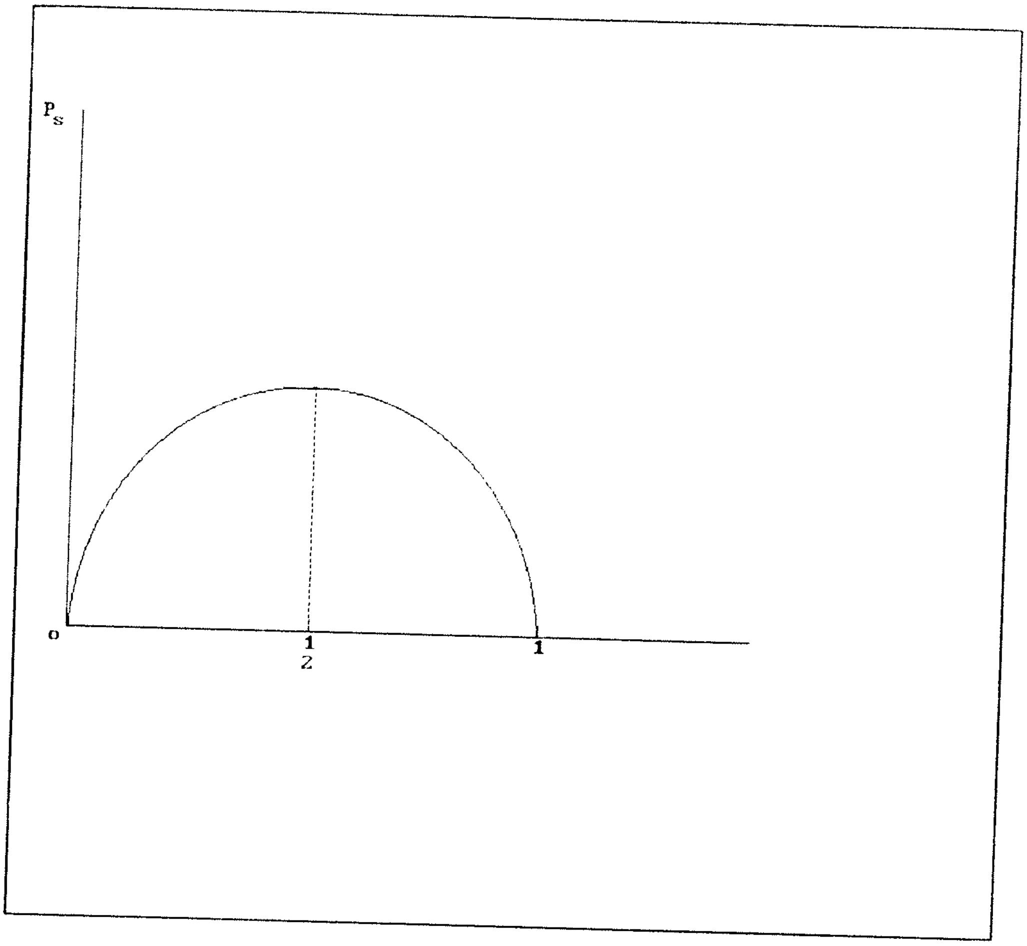

Figure 1.

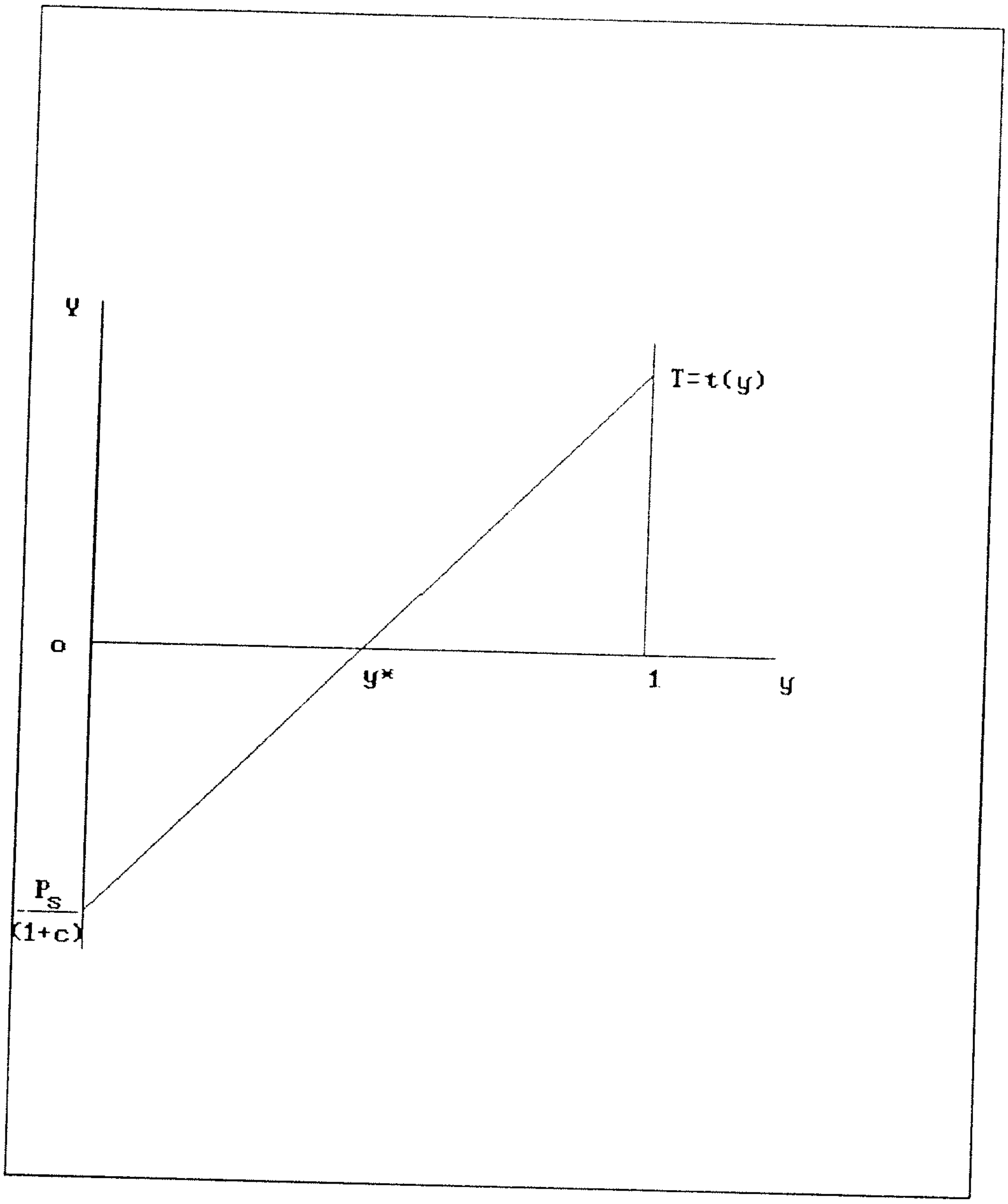

Figure 2.