A REMARK ON UNIT ROOT TESTS AND MEASURES OF PERSISTENCE

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In this paper we simulate a series which is a segmented trend plus noise. Despite the imposed data generating process, usual tests for unit roots and estimates of persistence fail to reject the random walk hypothesis.

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1. Introduction

Since the study of Nelson and Plosser in 1982 substantial empirical work has been done in the literature to test for *unit roots* in macroeconomic time series. Campbell and Mankiw (1989) suggest for example fluctuations in U.S. GNP series are quite persistent, while Cochrane (1988) presents much weaker evidence for GNP data spanning pre- and post-World War II. Perron (1989) doubts the presence of a unit root, once he allows for one change both in the intercept and in the slope of the trend of the series.

In this paper, we build up a framework that is very close in nature to that of Perron (1989). We consider a simulated series whose observations have been generated by different regimes over different periods of time. We reinforce Perron's result that standard tests cannot discriminate between stochastic and deterministic trends if the alternative hypothesis is modelled as a segmented trend representation. However, we shortly suggest why the segmented trend hypothesis should be preferred to the random walk hypothesis.

2. Unit root tests and estimates of persistence

Let us denote by y_t a series taken in natural logarithms. We wish to test empirically whether the series is better characterized by a stochastic trend in the form of a random walk (with drift) representation or by a deterministic linear time trend. A common procedure to test for a unit root is the augmented Dickey-Fuller (ADF) test: Here the regression equation is given by

$$\Delta y_t = \alpha_1 + \alpha_2 t + \alpha_3 y_{t-1} + \sum_{j=1}^{p} \gamma_j \Delta y_{t-j} + \varepsilon_t, \qquad (1)$$

and the null hypothesis is modelled as H_0: $\alpha_2 = \alpha_3 = 0$. The t statistics on the coefficients of interest are required not to be statistically different from zero based on the critical values tabulated in Fuller (1976, Table 8.5.2).

A more accurate test about the random walk hypothesis involves the estimation of some 'measures of persistence' introduced by Campbell and Mankiw (1987a, 1987b, 1989) and Cochrane (1988). Indeed, Cochrane (1991) has recently provided several convincing arguments that standard Dickey-Fuller tests for unit roots or trend stationarity "*have low power in finite samples against the local alternative of a root close to but below unity*" (p. 276).

As far as persistence is measured about how much of the current innovation gets passed into the levels of the series, the basic idea is that for a random walk the innovation gets entirely passed: Thus, it should be the case that running the regression $\Delta y_t=\mu+u_t$ no moving average component would be found in u_t. Of course, if a moving average component is contained in the error, shocks can have either bigger impact or no impact at all on the future values of the series, depending on the values of the coefficients in the moving average itself. The impact of a shock occurred in period t on the level of the series in period $t+k$ is $A_k(1) = 1+A_1+A_2+\ldots+A_k$, and a natural measure of persistence when $k \to \infty$ is given by

$$A(1) \equiv \lim_{k \to \infty} A_k(1) = \sum_{i=0}^{\infty} A_i. \tag{2}$$

It is the case that for a random walk representation $A(1)=1$, while for any stationary series around a deterministic trend $A(1)=0$. Another common measure of persistence is also Cochrane's (1988) *variance ratio*, defined as

$$V_k \equiv \frac{1}{k} \cdot \frac{\mathrm{Var}(y_t - y_{t-k})}{\mathrm{Var}(y_t - y_{t-1})} = 1 + 2 \sum_{i=1}^{k} \left(1 - \frac{i}{k+1}\right) \rho_i, \tag{3}$$

where ρ_i is the i-th autocorrelation of Δy_t, and k is the number of autocorrelation included to investigate the degree of trend reversion - if any - in the series. If a series affected by any exogenous shock has a tendency to revert to the original trend path, it must be the case that positive (negative) correlations are compensated somewhere in the future by negative (positive) correlations: Thus, V_k has to approach zero for large k. On the other hand, for a random walk representation, V_k will approach the unity for large k.[1] The limiting variance ratio is typically considered

$$V \equiv \underset{k \to \infty}{V_k} = 1 + 2 \sum_{i=1}^{\infty} \rho_i, \tag{4}$$

[1] Because values of k 'too small' relatively to the sample size may obscure the trend reversion manifested in higher autocorrelations, while 'too large' values of k may tend to find an excessive trend reversion, the measure is computed for different values of k.

which is related to $A(1)$ by the equation:

$$A(1) = \sqrt{\frac{V}{1 - R^2}}, \qquad (5)$$

where R^2=1-Var(ε)/Var(Δy_t).It is worth noting the two measure exactly coincide in the case the 'true' data generating process for y_t is a random walk.

3. A simulated example

In section 2 we reviewed some procedures commonly used to test for a unit root and to estimate the size of the unit root itself. In this section, we simulate a series which is the sum of a deterministic component plus a random noise. The deterministic component we model as a segmented trend, and then we add a white noise error about each local linear trend. The goal of the simulation is to show that a series generated in such a way still passes all usual tests for the random walk hypothesis, that is standard tests cannot discriminate in fact between stochastic or deterministic trends under the alternative hypothesis of a *segmented* trend. The series we simulate accords to the model:

$$y_t = Z_i(t) + \eta_t, \qquad (6)$$

where $Z_i(t)=\alpha_i+\beta_i t$ and η_t~N(0,0.01). The values imposed on the α's and the β's are reported in table 1, and the outcome of the simulation is plotted in fig. 1.[2]

Table 1: Parameters underlying the simulated series.

α_i	11.9514	12.6544	12.6969	12.8719	12.9089
β_i	0.052	-0.025	0.0425	0.005	0.030

Note: for i=1,2,...,5 the α_i are imposed at time 1960, 1975, 1976, 1981, and 1984, while the β_i are constant growth rates in the intervals 1960-74, 1975, 1976-80, 1980-83, and 1984-88.

[2] We wish to remark the simulated series, although artificially generated, is not faraway from real observed output series. Indeed, the series was build up to resemble the real GDP series for Italy over the period 1960-1988 (see the Appendix for a comparison).

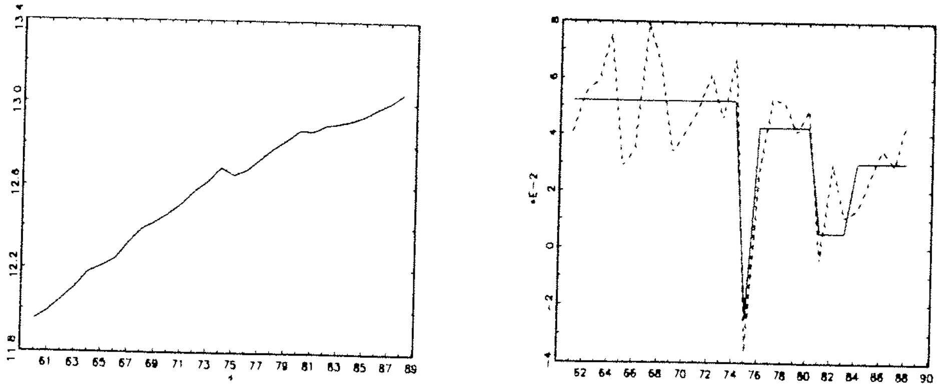

Fig. 1. Left: Simulated series in levels. Right: growth rates of the simulated series (dotted line) and underlying imposed structure (dashed line).

We consider now the simulated series and (i) test for a unit root and (ii) estimate the measures of persistence.[3] The main results reported in table 2 suggest the random walk hypothesis is not rejected by the data, despite the series was generated as a segmented trend plus noise, and not as a random walk process (even estimates of persistence indicate values much closer to one than to zero). The simulation signals therefore a result of observational equivalence between random walks and segmented trends which reinforces Perron's (1989) conclusion that "*choosing one view over the other is a matter of convenience for interpreting the data*" (p. 1389). Nonetheless, at least two main problems should be recognized with the random walk hypothesis.

On the one hand, the random walk model implies shocks have permanent effects at each point of the time: As it has been recently remarked by Rappoport and Reichlin (1989), and Balke and Fomby (1991), this has little economic appealing as far as economists are accustomed to attribute movements in actual series to shocks which occur infrequently. On the statistical front, on

[3] In the former case we run the regression equation (1), compute the ADF 't' statistic, and compare it with the critical values tabulated in Fuller (1976). In the latter case we get the estimate for V_k by equation (3), and derive $A_k(1)$ through equation (5), by replacing the R^2 with the square of the first autocorrelation (although this would understate a little bit the value of $A(1)$).

the other hand, fig. 2 demonstrates how imprecisely a random walk representation would fit actual data in presence of non-linearities: The lack of flexibility in the functional form doesn't allow the random walk model to capture the non linear structure possibly contained in the growth rates series.[4]

Table 2: Unit root test and measures of persistence.

ADF regression: $\Delta y_t = 0.46 - 0.033\, y_{t-1} + \varepsilon_t$
(0.16) (0.01)

Diagnostics: R^2=0.23, DW=2.08, LM(3)=1.12, ARCH(3)=0.07

Unit root '*t*' statistic: -2.59.
90% and 95% critical values: -2.63, -3.00.

Measures of persistence: V_k = 1.23, 1.36, 1.59; k=3,5,7.
A_k = 1.12, 1.18, 1.27; k=3,5,7.

Note: Standard errors in parenthesis. The LM statistic for the absence of autocorrelation of order 3 is F(3,23) distributed. The ARCH statistic for the absence of heteroschedasticity of order 3 is F(3,20) distributed. Source for the critical values: Fuller (1976, table 8.5.2). Standard errors for V_k are 0.44, 0.61, 0.82, for k=3,5,7.

[4] This limitation of the random walk model has been pointed out by Hamilton (1989), who has proposed an approach to modeling changes in regime based on the hypothesis the mean growth rate of a non stationary series is subject to occasional shifts which "*follow a nonlinear stationary process rather than a linear stationary process*" (p. 357).

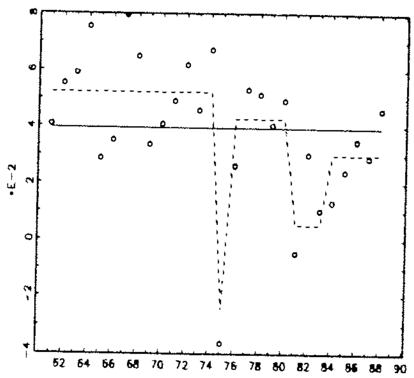

Fig. 2. Scatter plot of Δy_t against time (circles), OLS estimate of the drift in the model Δy_t=μ+error for the simulated series (dashed line), and real underlying structure (dotted line). Estimated μ is 0.039.

4. Conclusions

In this paper we considered a simulated series generated by a segmented trend plus noise. By running unit-root tests and estimates of persistence we found the null hypothesis of a random walk was not rejected, despite the true underlaying data generating process was given by a segmented trend plus noise. We concluded standard tests for stochastic trends have low power against the alternative of a deterministic but segmented trend. Despite this was not a surprising result to the extent conventional tests were originally designed to cope with the alternative hypothesis of a *global* (and not local) linear time trend, we presume any series characterized as random walk process according to the usual diagnostics might hidden in fact a segmented trend representation.

APPENDIX

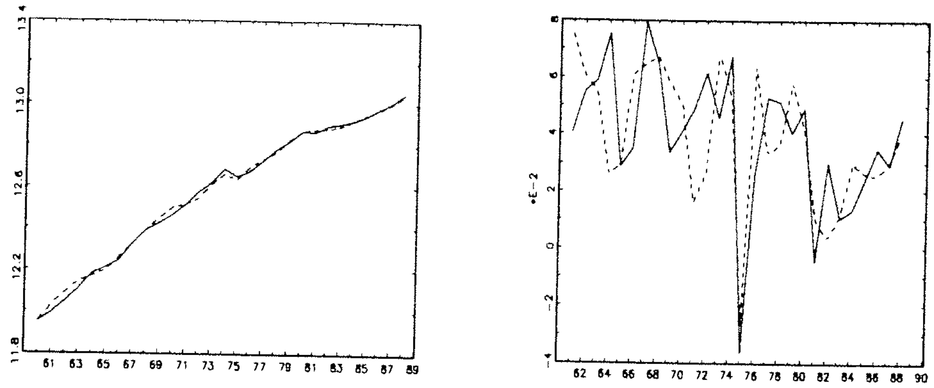

Fig. 3. Left: Simulated series in levels against actual GDP series observed for Italy (dashed and dotted line respectively). Right: the same series in growth rates.

Comparison of persistence estimates.			
Lags	V_k	SE	A_k
k=3	1.23	(0.44)	1.12
	1.29	(0.46)	1.15
k=5	1.36	(0.61)	1.18
	1.39	(0.63)	1.20
k=7	1.59	(0.82)	1.27
	1.61	(0.83)	1.29

Note: results are given for the simulated and the Italian GDP series respectively.

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