SELF-REINFORCING MECHANISMS, RECONTRACTING PROCESSES AND PUNCTUATED EQUILIBRIA: IS LOCK-IN TO A MARKET POSITION A PERMANENT OUTCOME?

by

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1. INTRODUCTION

Dynamical systems of self-reinforcing mechanisms tend to possess a multiplicity of asymptotic states or possible "emergent structures". The initial starting state combined with early random events or fluctuations acts to push the dynamics into the domain of one of these asymptotic states and thus to "select" the structure that the system eventually "locks into".

This lock-in by fluctuations to one pattern or structure out of several possible has parallels in thermodynamics, ferromagnetism, laser theory and chemical kinetics (Haken, 1978). Ferromagnetic materials, spin glasses, solid-state lasers and other physical systems that consist of mutually reinforcing elements show indeed the following properties: they "phase lock" into one of many possible configurations, small perturbations at critical times influence which outcome is selected, and the chosen outcome may have higher energy (that is, be less favourable) than other possible end states.

Economic examples have been recently analysed. Sequential-choice lock-in may occur in the presence of competing technologies with increasing returns to adoption. (Arthur, 1989). If one technology gets ahead by good fortune, it gains an advantage, with the result that the adoption market may "tip" in its favour and may end up dominated by it. Given other circumstances, a different technology might have been favoured early on, and it might have come to dominate the market. Thus in competition between technologies with increasing returns to adoption ordinarily there are multiple equilibria. As to which actual outcome is selected from these multiple candidate outcomes , it is argued that the prevailing outcome turns out to depend on the path which has been initially chosen. In particular, the resulting outcome can be inefficient; that is to say, the market can be locked-in to the "wrong" technology. Therefore, it turns out that equations governing self-organization are intrinsically non linear, market-share dynamics are non ergodic, and possible inefficiency may result.

An important question arising in this context is the following. If an economic system is locked-in to an inferior local equilibrium, is "exit" or escape into a superior one possible? Are there policies or spontaneous actions of the economic system for removing harmful lock-in?

An answer depends on the degree to which the advantages accrued by the inferior equilibrium are reversible or transferable to an alternative one. Where learning effects and specialized fixed costs are the source of the self-reinforcing mechanisms, usually advantages are not reversible and not transferable to an alternative equilibrium. Where coordination effects, which confer advantages to "going along" with other agents taking similar action, are the source of lock-in, often advantages are transferable (Schelling,1978).

For example, users of a particular technological standard may agree that an alternative would be superior, provided that everybody "switched". If the current standard is not embodied in specialized equipment and its advantage-in-use is mainly that of commonality of convention, then a changeover to a superior collective choice can provide exit into the new "equilibrium" at negligible costs. Farrell and Saloner (1985) show that in the presence of "network externalities" and as long as agents know other agents' preferences, each will decide independently to switch if a superior alternative is available. Put it another way, if a technology's advantage is mainly that most adopters are "going along" with it, a coordinated changeover to a superior collective choice can provide escape. But if there is uncertainty of others' preferences and intentions, there can be "excess inertia": this uncertainty can lead all the firms to remain with the status quo even when in fact they all favour switching, because they are unwilling to risk switching without being followed. As a result, if the economic system is locked-in to an inferior local equilibrium, escape into a superior outcome is not guaranteed. Permanent lock-in to one market position may result.

The question about technological switching in the presence of benefits arising from "network externalities" seems to be particularly relevant, since the issues of compatibility and standardization have become more important than ever. This is especially true within the computer and telecommunications industries, which are characterized by urgent demands for compatibility and rapid innovation of new products and services. The cost of switching is very low, almost negligible with respect to the benefits of compatibility (Farrell and Saloner, 1986).

In the following pages we tackle this question again, but examine this case taking advantage of phase transitions models in synergetics. To do this, we assume that transition between technologies occur probabilistically. The transition probabilities for each firm depend in general on the market shares of each technology, i.e. on their "network". This obviously makes self-reinforcement possible.

Our main result is that permanent lock-in to one market position is not possible when the source of lock-in advantages is network externalities. The model exhibits "punctuated equilibria" (Gould, Eldredge, 1972). The market lingers at prevalence of one type of technology with intermittent transitions to prevalence of the other. That is to say, "lock-in" is only a temporary outcome.

2. THE MODEL

Consider two technological standards, A and B, which are available for performing the same task and are competing passively for a market. There are n adopters in the market which have to decide whether to stick with one standard or to switch to the other. Each standard has positive "network" externalities, that is, there are benefits to "going along" with other adopters of a given standard.

Each potential adopter is characterized by a preference parameter ν. We assume that the firms preferring A have $\nu \geq 0$, and those preferring B have $\nu \leq 0$; $\nu = 0$ is the case of indifference.

We suppose that adopters do not know the others' preferences and intentions, that is, a firm does not know whether the others would like to switch or not. In deciding whether to switch to the other technological standard or to stick to the old one, a firm takes into account its own preference over standards, and the market shares of the standards, that is, the number of adopters, or the "network" of each standard. In particular, we assume that the transition probabilities for each firm depend in general on the "network" of adopters. This, obviously, makes self-reinforcement possible.

Let $f(n_A, n_B, t)$ denote the probability of finding, at time t, n_A firms with standard A and n_B firms with standard B. Obviously, $n_A + n_B = n$. Let us consider how the function $f(n_A, n_B, t)$ changes over time. For simplicity we assume that transitions occur one unit at a time [1]. We have the following :

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[1]The analysis can be generalized to the case where transitions do not occur one unit at a time only (see Stratonovich, 1967)

(1) $$n[f(n_A,n_B,t+\tau) - f(n_A,n_B,t)] =$$

$$= (n_A + 1)\, P^{AB}_{t\,,\,t+\tau}(n_A+1,\, n_B-1) f(n_A+1,\, n_B-1,\, t) + (n_B + 1)\, P^{BA}_{t\,,\,t+\tau}(n_A-1,\, n_B+1) \cdot$$

$$\cdot f(n_A-1,\, n_B+1,\, t) - [(n_A\, P^{AB}_{t\,,\,t+\tau}(n_A,n_B) + n_B\, P^{BA}_{t\,,\,t+\tau}(n_A,n_B)) f(n_A,\, n_B,\, t)]$$

where $P^{AB}_{t\,,\,t+\tau}(n_A,\, n_B)$ denotes the probability that a firm switches from A to B in the interval (t, t+τ), given that n_A firms have standard A and n_B firms have standard B, and $P^{BA}_{t\,,\,t+\tau}(n_A,\, n_B)$ denotes the probability that a firm switches from B to A in the interval (t, t+τ), given the numbers n_A and n_B of adopters of standards A and B respectively.

Let $\tilde{P}^{ij}_{t,t+\tau}(n_A,\, n_B) = P^{ij}_{t,t+\tau}(n_A,n_B)/\tau$. We require the following conditions:

(a) $\lim_{\tau\to 0} \tilde{P}^{ij}_{t,t+\tau}(n_A,n_B) = P_{ij}(n_A,n_B)$
(b) $\lim_{\tau\to 0} [f(n_A,n_B,t+\tau) - f(n_A,n_B,t)]/\tau = \partial f(n_A,n_B,t)/\partial t$

Normalizing to variable x in the continuous interval (−1/2, 1/2) by setting:

$$x = (n_A - n_B)/2n$$

$$\Delta x = 1/n$$

and writing f(x, t) for $f(n_A,\, n_B,\, t)$ and $P_{AB}(x)$, $P_{BA}(x)$ instead of $P_{AB}(n_A,\, n_B)$, $P_{BA}(n_A,\, n_B)$, we get:

(2) $$\partial f(x,t)/\partial t = \omega_{AB}(x+\Delta x) f(x+\Delta x,\, t) + \omega_{BA}(x-\Delta x) f(x-\Delta x,\, t) -$$

$$[\omega_{AB}(x) + \omega_{BA}(x)]f(x, t)$$

where $n\omega_{AB}(x) = n_A P_{AB}(n_A, n_B) = n(1/2 + x)P_{AB}(x)$ and $n\omega_{BA}(x) = n_B P_{BA}(n_A, n_B)$ $= n(1/2 - x)P_{BA}(x)$.

Following Haken (1978) and for $n >> 1$, we can treat x as a continuous variable and transform the difference equation in (2) into a partial differential equation by expanding the right-hand side up to the second order in Δx. We obtain:

(3) $$\partial f(x,t)/\partial t = -\partial[K_1(x)f(x,t)]/\partial x + \frac{1}{2}\partial^2[K_2(x)f(x,t)]/\partial x^2$$

where

$K_1(x) = \Delta x(-\omega_{AB}(x) + \omega_{BA}(x))$, $K_2(x) = (\Delta x)^2(\omega_{AB}(x) + \omega_{BA}(x))$.

Equation (3) is a Fokker-Planck equation for the probability distribution function f(x,t). Standard methods allows us to find the solutions to (3), and in particular the stationary time dependent solution of (3).

Until now we have not specified the transition probabilities $P_{AB}(x)$ and $P_{BA}(x)$. Let:

(4) $$P_{AB}(x) = \alpha e^{-(kx + \nu)}$$

(5) $$P_{BA}(x) = \alpha e^{kx + \nu}$$

as simplifying expressions for $P_{AB}(x)$ and $P_{BA}(x)$, which will enable us to capture the idea that there are benefits to doing what others do, and α, k, ν are constants.

The value ν measures the individual preference parameter; k is the network externalities parameter and measures the willingness of one firm to join the network. Expressions (4) and (5) capture the idea that the larger the network of standard A (B), that is, the more adopters choose A (B), the higher the net benefits from going along with adopters of A (B), and therefore the higher the probability of switching fron B (A) to A (B), the smaller from A (B) to B (A).

With the explicit forms (4) and (5) of the transition probability it is possible to obtain an explicit form for the stationary solution to (3).

The introduction of

$$g(x, t) = K_1(x)f(x, t) - \frac{1}{2}\,\partial[K_2(x)f(x, t)]/\partial x \tag{6}$$

allows us to write (3) in the form:

$$\partial f(x, t)/\partial t + \partial g(x, t)/\partial x = 0 \tag{7}$$

Expression (7) can be interpreted as the equation of conservation of probability. Expression (6) describes the amount of probability crossing the abscissa x in the positive direction per unit time. Consider the interval $-1/2 \leq x \leq 1/2$. Then $g(-1/2)\tau$ is the amount of probability entering this interval in time τ across the abscissa $x = -1/2$, and $g(1/2)\tau$ is the amount of probability leaving the interval in time τ across the abscissa $x = 1/2$. If probability never disappear inside this interval and if there are no sources of probability inside it, then the difference $g(-1/2)\tau - g(1/2)\tau$ is the increment of the total probability $\int f(x)dx$ in the interval $(-1/2,1/2)$, i.e., passing to the limit $\tau \to 0$, equation (7) is satisfied.

In what follows we assume that the boundary condition $g(x = \pm\frac{1}{2}, t) = 0$ holds. This condition implies that there is no "flow" of firms across the boundary. That is to say, no random trajectory can enter the interval $-1/2 \leq x \leq 1/2$ by crossing the boundary, and every random trajectory terminates when it arrives at the boundary. This condition is a natural one, given our assumption that there is noise which prevents the possibility of having absorbing barriers. With the boundary condition $g(x = \pm\frac{1}{2})$ we can derive the stationary time-dependent solution $f_{st}(x)$ of (3) or (7), that is $\partial f_{st}(x)/\partial t = 0$, from $g_{st}(x) = 0$, that is the first-order differential equation for the stationary case is:

$$0 = K_1(x)f_{st}(x) - \frac{1}{2}\,\partial[K_2(x)f_{st}(x)]/\partial x \tag{8}$$

which has the following solution:

$$f_{st}(x) = cK_2^{-1}(x)\exp[2\int_{-1/2}^{x} \frac{K_1(y)}{K_2(y)}\,dy] \tag{9}$$

Obviously, $f_{st}(x)$ has to be normalized by $\int_{-1/2}^{+1/2} f_{st}(x)dx = 1$. With expressions (4) and (5) the functions $K_1(x)$ and $K_2(x)$ take the following form:

$$K_1(x) = \alpha[\sinh(kx+\nu) - 2x\cosh(kx + \nu)] \tag{10}$$

$$K_2(x) = (\alpha/n)[\cosh(kx+\nu) - 2x\sinh(kx+\nu)] \tag{11}$$

where sinh and cosh are the hyperbolic sine and cosine, respectively. Now the explicit form of the stationary solution $f_{st}(x)$ can be computed numerically. Figure 1 represents the stationary distribution in the trivial case $k = 0$ and $\nu = 0$, that is, in the case of no network externalities and no preference attitude.

[FIGURE 1]

If we let increase the parameter k continuously from the value 0 to values about 2.5 we observe that a typical phase transition of second order occurs in this range of k. In particular, for $k < 2$ there is only one maximum, that is, the stationary distribution is unimodal; for $k > 2$ the distribution becomes bimodal with maxima corresponding to the relative prevalence of one technology over the other. Figure 2 represents the case $k = 2.5$.

[FIGURE 2]

In this case, the market lingers at prevalence of one type of standard with intermittent transitions to prevalence of the other. Observe that this "recontracting process"[2] shows convergence in distribution rather than strong convergence to a point. Instead, it exhibits "punctuated equilibria"[3] in the shape of sojourns in the neighborhood of local maxima and transitions between them. Permanent lock-in to one market position is therefore not possible.

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[2]"Recontracting processes" models are models which allow recontracting within the market once it has formed. They differ from "allocation processes"models, which are appropriate to study how an allocative pattern form.

[3]The notion of punctuated equilibrium has been introduced by Eldredge and Gould (1972) withim models in paleobiology. Such a notion originates from the observation that <<long periods of morphologic stability are "punctuated" here and there by rapid events of speciation in isolated subpopulations>> (ibidem, p.110).

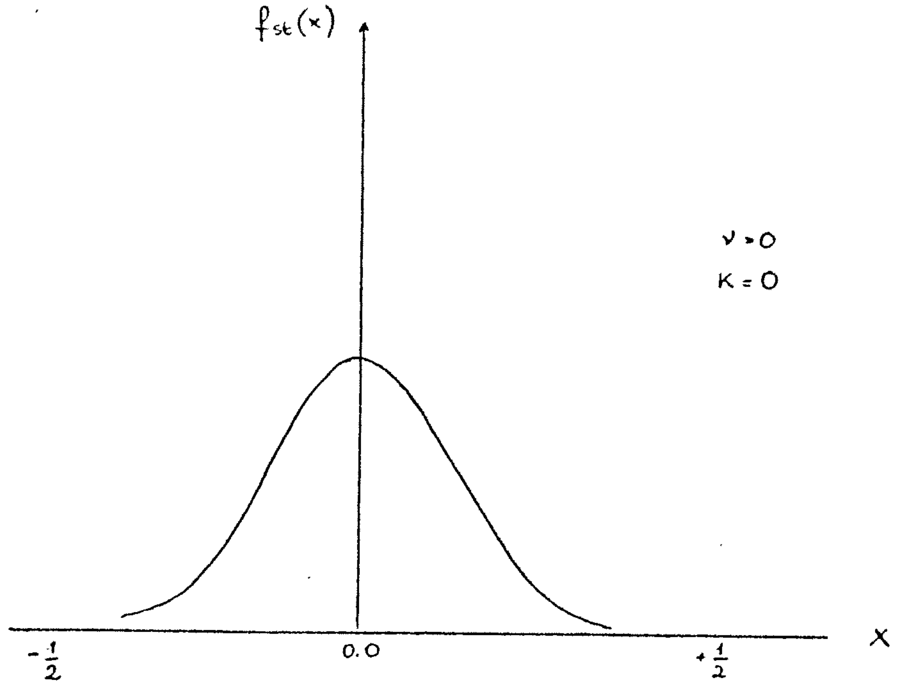

FIGURE 1

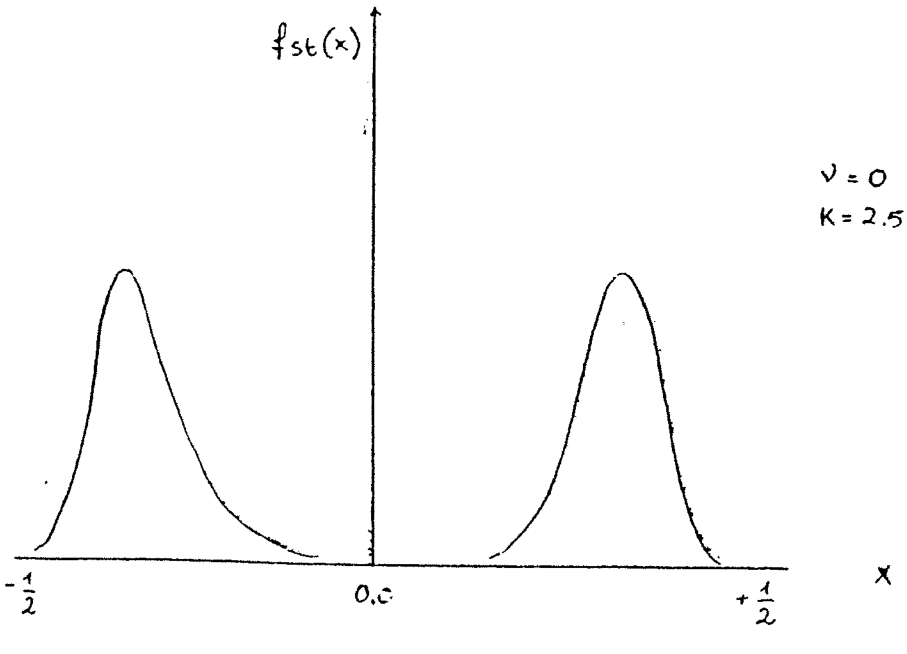

FIGURE 2

REFERENCES

ARTHUR, B. (1989), Competing Technologies, Increasing Returns, and Lock-in by Historical Events, *Economic Journal*, 116–31

ELDREDGE, N and S. J. GOULD (1972), Punctuated Equilibria: an Alternative to Phyletic Gradualism, in *Models in Paleobiology*, (ed by T. Schopf), San Francisco, Freeman Cooper & Company

FARRELL, J. and G. SALONER (1985), Standardization, Compatibility and Innovation, *Rand Journal of Economics*,70–83

FARRELL, J. and G. SALONER (1986), Competition, Compatibility and Standards: the Economics of Horses, Penguins and Lemmings, Working Paper E–86–73, The Hoover Institution, Stanford University

HAKEN, H. (1978), *Synergetics*, Berlin, Springer Verlag

KATZ, M. L. and C. SHAPIRO (1986), Technology Adoption in the Presence of Natwork Externalities, *Journal of Political Economy*, 145–67

SCHELLING, T (1978), *Micromotives and Macrobehaviour*, London, Norton & Company

STRATONOVICH, R. L. (1967), *Topics in the Theory of Random Noise*, Volume 1, Gordon and Breach, London

WEIDLICH, W (1971), The Use of Statistical Models in Sociology, *Collective Phenomena*,