THE STATE AS A NON-COOPERATIVE EQUILIBRIUM

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August 1990

Preliminary draft

Abstract

We propose a theory of the origin of the State as an institution originating from the voluntary interaction among decentralized and fully informed agents. A given population in anarchy have to decide to enter into a State where they can perform an economic activity guaranteeing positive payoffs. The existence of the State is a necessary condition for markets to exist. In the State there are two sectors where agents may operate. By means of a free mobility condition, the distribution of agents across sectors is made endogenous. We interpret the equilibrium of the non-cooperative one-shot game as the State. In the State, agents simultaneously play an economic and political game under different rules governing their single maximization problem. We compare the effects of two political settings - a lobbying and a party system - on individual and aggregate indicators of performance. We show that: (i) a State may fail to exist even if the outcome of the game entails a Pareto improvement with respect to anarchy; (ii) a State may fail to exist if costly political activity cannot be paid back through profits in the economic stage; (iii) under the lobbying (party) technology small differences across sectors yield (do not yield) very uneven distributions of agents; (iv) under lobbying agents invest more in political activity than under the party system; (v) the performance of the party system is superior to the one of the lobbying system; (vi) differences in rules governing sectors are detrimental to individual and aggregate payoffs; (vii) if the lobbying and the party systems coexist in a State, lobbying is bound to be the political practice of the minority.

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We thank Vincenzo Denicolò for helpful comments on an earlier draft. The usual caveat applies.

1 Introduction

> "Yet, although the problem of an appropriate social order is today studied from the different angles of economics, jurisprudence, political science, sociology, and ethics, the problem is one which can be approached successfully only as a whole".
>
> Friederich A. von Hayek

This paper can be regarded as an attempt to present in a fully fledged economic model the problem of the origin of collective decision making (e.g., the origin of the State). A classic tenet in welfare economics, as well as in political economy at large, is that if the interaction between decentralized agents fails to produce Pareto-efficient allocations, then institutions arise such as to benefit all parties[1].

For instance, Inman (1987, p.650) argues that "there is a common problem which underlies all market failures, and that that common problem is uniquely handled by an institution which can enforce cooperative - that is collective behaviour - in a world where non-cooperative behaviour is the preferred individual strategy".

There is a clear *non sequitur* in Inman's argument, as in a fully non-cooperative world also the origin of such an enforcing institution should be based on non-cooperative behaviour. If one adheres to such a non-cooperative perspective, one coherent way

1 This position has been criticized as functionalist. See Rowe (1989) who also provides a rich bibliography on the issue.

to found the institution Inman refers to relies upon the idea of repeated games[2].

However, we shall not offer a theory of the State based on supergames for two reasons. Firstly, because of the well known endemic multeplicity of equilibria featuring repeated games. Secondly, because the theory of supergames does not allow a sufficiently detailed description of the constituent game (the ingredients of which are typically black-boxed in a simple normal form).

We propose a theory of the origin of the State as an institution originating from the voluntary interaction among decentralized and fully informed agents[3].

We shall study a context where a given population of agents in a situation of anarchy have to decide to enter into a State where they can perform an economic activity guaranteeing positive payoffs. The existence of the State is therefore a necessary condition for markets to exist because of the need for establishing rules (i.e., property rights, enforcement of the law, regulation of trades in and out of the State, taxes to finance the institution of markets)[4]. Therefore, the creation of a State

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2 As is well known, the Folk Theorem states the conditions under which any Pareto efficient equilibrium can be reached when the constituent game is repeated. Under complete information, enough patience ensures that players choose the strategies leading to the Pareto efficient outcomes. See Inman (1987) for a complete survey of the applications of supergames to the problem of the origin of the State.

3 In this respect we do not endorse Baumol (1967, pp. 180–1) classic analysis of the relationship between market failure and the origin of the State which is based on a coercitive conception of the State: "In those cases where the welfare of the various members of the economy is partly dependent on each others' activity, it is possible that persons in pursuit of their own immediate interests will be led to act in a manner contrary to the interests of others ... It may be that the disadvantage to each of them of such activity is so great and of such a sort that arrangements can be made on a purely voluntarily base whereby it can be avoided altogether. Where such arrangements cannot be relied on, it becomes advantageous to the members of the economy to have activities restricted by coercive measures ... The fundamental importance of this point to the theory of government should be noticed for in the very concept of government there is an element of coercion".

4 Of course, any of these examples can be seen as types of public goods whose provision entails Pareto-improvements with respect to the anarchy state and for which collective finances have to be raised. See Coase (1988) for an interesting historical evaluation of the costs of establishing markets.

implies a Pareto improvement with respect to anarchy.

In our State there are two sectors where agents may operate. By means of a free mobility condition, which requires equilibrium payoffs to be equal across sectors, the distribution of agents is made endogenous. We interpret the equilibrium of the one-shot non-cooperative game as the State. By this we mean a distribution of agents across sectors which guarantees a non-negative payoff level (equal in both sectors via the free mobility condition).

More precisely, we model a setting where agents to enter the State simultaneously play an economic and political game under different rules governing their single maximization problem. In particular, we are interested in comparing the effects of two political technologies - that we label *lobbying* and *party* - on individual and aggregate indicators of performance.

Among the main results, we show that:

(i) due to the free mobility condition, a State (i.e., an equilibrium of our game) may fail to exist even if the outcome of the game entails a Pareto improvement with respect to anarchy[5];

(ii) a State may fail to exist if costly political activity cannot be paid back through profits in the economic stage;

(iii) under the lobbying (party) technology small differences across sectors yield (do not yield) very uneven distributions of agents;

(iv) under lobbying agents invest more in political activity than under the party

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5 While our approach displays an explicit Wicksellian (1896) overtone, in that the State (as instance of collective decision making) involves Pareto improvements, this conclusion goes in opposite direction. This is due to the contractualistic conception of the State which permeates Wicksell's approach. In this respect we are nearer in spirit to the non-cooperative vision of the State typical of the Italian school of public finance and most brilliantly put forward by Montemartini (1900). See also Buchanan (1960) for a comparison between Italian and Anglo-saxon foundations of public economics.

system;

(v) the performance of the party system is superior to the one of the lobbying system;

(vi) differences in rules governing sectors are detrimental to individual and aggregate payoffs;

(vii) if lobbying and the party system coexist in a State, lobbying is bound to be the political practice of the minority.

This paper is organized as follows. In next section we discuss the set of issues within which we would like to locate our contribution and we outline some of the distinctive features of our paper as compared with the related literature[6]. In section 3 we present our basic model and the features of the political technologies. In section 4, 5 and 6 we focus on the working of the model when lobbying, the party system or a mixed technology are introduced together with a Nash-Cournot game in the economic stage. The overall efforts in both individual and aggregate political activity will be the object of section 7. Section 8 will deal with such political frameworks when Bertrand-Nash behaviour prevails. In all these sections we shall focus on: (i) the problem of the existence of the State; (ii) how the distribution of agents across sectors varies with the parameters of the model; (iii) the interplay between rules of the economic stage and rules of the political stage. In section 9 the levels of individual payoffs and of some indicators of aggregate welfare will be examined for each of the aforesaid settings. In section 10 we will discuss some implications of our approach for the analysis of public decision making.

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6 Paraphrasing Wittman (1989), this article is directed to those who believe in the "homo economicus" assumption, for they can perhaps accept that "homo politicus" is a rational maximizer too. For those unwilling to share such assumption, no benefits can arise from reading it.

2 The setting

2.1 Basic lines

In what follows, we model a community where individuals take economic and political decisions in order to maximize their payoffs. The effects of the two decisions on payoffs are intertwined through the distribution of agents across sectors which is determined by a free mobility condition.

Looking at the economic sphere of agents' decisions, we shall focuss on agents' strategies as producers. Due to our purposes this requires some justifications. we have in mind a given population where each agent produces his consumption bundle. Agents can undertake productive activities entering the State, which means to choose an output level and a level of political activity. If this twofold interaction among agents yields an equilibrium (i.e., a State), then the outcome of the production activities is not consumed inside our community, but it is entirely exported. Hence, the productive activity does not alter directly the consumption one since the latter continues through self-consumption plus the possibility to import from other communities. On the contrary, undertaking productive activity in the State affects directly agents' payoffs as it entails positive returns[7]. This notwithstanding, we understand that neglecting the analysis of agents' as consumers in this context is more than a simplifying assumption.

Except than in corner equilibria - on which we shall return below - in equilibrium (i.e., ex post) agents group into two types, different as for the output they produce. Ex ante (in anarchy) agents can be either indistinguishable or already labelled according to the output they can produce. Their payoffs under anarchy - arising from their self-produced consumption bundles - are generally different and we normalize the

7 In other words, our community is one which has market power vis-à-vis the rest of the world in the markets where the produced commodities are exchanged, whereas it is price-taker as for the commodities it consumes.

largest among them to zero. If agents are initially indistinguishable, the equilibrium distribution reveals ex post how many of them will be active in either sector. If agents are initially labelled, the equilibrium distribution (which generically does not coincide with the original one) reveals ex post how many of them have to change sector. In both cases, our model is silent about the specific mechanism by means of which the equilibrium distribution is reached. In this sense, we are proceeding as in oligopoly models with free entry in which only a subset of the potential entrants will be active in equilibrium and nothing hints at which specific subset will be operating.

As for the political sphere, if a State is formed, public decision making involves the production of social norms influencing agents' objective functions. Aggregate (i.e., sectorial) political activity is positively related to individual payoffs. Therefore, the amount of political activity produced in each sector can be regarded as the input in the social norms' production function. Such norms, which can also be seen as public inputs, reduce the set-up costs to enter the State[8].

The collective decision procedure is therefore an instance of legislative production with allocative effects. But there is also a distributive effect. Indeed, the individual political capacity of producing such norms depends on the number of agents in his sector and, for the total number of agents is given, free mobility entails distributive effects[9].

This type of social rules characterized by both allocative and distributional effects are not only general in spirit, but they capture the essence of most public intervention

8 Notice that here we do not tackle the more general issue of the equilibrium number of sub-communities which would arise if agents were free to form any number of sectors. For a work on this line of research see Guesnerie and Oddou (1981).

9 For models where such a choice between coalitions is modelled in an abstract setting see Aumann and Kurz (1977) Aumann, Kurz and Neyman (1987) and the interesting related literature. For a less technical treatment of this issue a classic references is Olson (1968).

mechanisms which are hardly if ever purely distributive or purely allocative. Moreover, in our framework it will be possible to compare the performances of different specifications (a lobbying or a party system) of this general technology of legislative production[10].

To handle such a framework, we need to impose simple rules on how the game is played. We assume that agents play according to Cournot-Nash or Bertrand-Nash rules in the economic stage of the game. As for the political stage, we shall consider three sets of rules that we label the party, the lobbying and the mixed system where with the latter we mean a setting where the former two coexist.

Needless to say, it is somehow hazardous to identify phenomena as complex as these political settings with a few stylized features which simply describe the way in which the political activity enters the individual payoffs. However, as we shall see below, our specification of such technologies entails radically different patterns of behaviour. In turn, such patterns can be understood as individual responses to either a party or a lobbying political system.

Of course, both the behaviour of agents inside the sectors and their joining one of them depends on the rules imposed on the working of both the economic and the political stages. In this respect, we define political settings by means of the technology transforming political activity into social norms and then into the payoff functions of the members of the State.

Our previous references to the party and the lobbying systems were quite informal. As we turn to a formal illustration of our model, a closer description of these political settings is in order.

We define a *party system* as one where: 1) there can be *redistribution*, in the

10 See Posner (1986) for a broad introduction to the literature focussing on this issue.

sense that from the legislative production process the majority can obtain to transform the set-up costs in subsidies to its economic activity; 2) free riding occurs in the sector where *more* agents are grouped; 3) net individual political payoffs *decrease* with the number of agents in the sector if the latter groups their majority.

On the other hand a *lobbying* system is one where: 1) *no redistribution* - in the sense explained above - can occur; 2) free riding occurs in the sector where *less* agents are grouped; 3) net individual political payoffs always *decrease* with the number of agents in the sector.

Of course, as the distribution of agents across sectors is endogenous, features (1) - (3) in both cases have to be understood as equilibrium properties.

2.2 Methodological issues

In welfare economics most models which tackle normative issues regard as exogenous that stage of the social interaction where a political equilibrium is reached. In this respect, the standard - Bergson-Samuelson - social welfare function can be seen as a reduced form of an unspecified political model. However, such a specification does not rest on a fully-fledged theory of agents' behaviour in both stages of their interaction[11]. In our opinion this is due to a lack of interest towards the nature of processes of collective decision making in general and the emergence of the State in particular. Of course, the benefits from such a restriction are great.

However, if non-cooperative behaviour is taken seriously through a further stage

11 The need for an integration between the economic and the political spheres has been commonly felt, but scarcely pursued even in modern public economics. A signal of this state of affairs is summarized in the following quotation: "(T)he wider political structure must be taken into account in any realistic assessment of the prospects for (economic) reform. In this return to "political economy" a great deal remains to be done". Atkinson and Stiglitz (1980, p.576).

of the process of public decision making, one should investigate how both economic and political equilibria are explicitly determined in a game where decentralized agents act simultaneously in both stages[12]. This is not the same as studying how different weights in a social welfare function change the outcome of public interventions. In that case the focus is on the dependence of economic allocations on political weights. In our view, such a dependence cannot be fully founded on non-cooperative individual behaviour, unless both stages are solved simultaneously by self-seeking agents. Therefore, while this practice can be fruitful in many applications, there is a need to look at the political and economic spheres not merely as interacting, but as essentially *one* domain of agents' maximizing activity.

In fact, when decentralized agents act strategically, they try also to change the political equilibrium from which the mechanism is derived in the first place. Hence, in order to understand the features of public intervention one needs to study mechanisms which not only are incentive compatible (that is strategy-proof with respect to informational manipulations), but also equilibria in the game including the political stage (that is strategy-proof with respect to political manipulations). Only when mechanisms possess both features, they can be regarded as implementable in the game where economic and political strategies are intertwined[13].

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12 We are aware that our model too does not fully base collective rules on individual rationality, because the choice among different political settings is beyond the scope of the model itself. At any rate we try to integrate the analysis of some given rules of political behaviour into the political and economic game thought as a whole.

13 Pareto (1913) in a relatively neglected contribution (but see Bergson 1983, and the debate cited therein) already advanced the idea of modelling collective decision making by means of social welfare functions. According to Pareto, such functions should express a reduced form of the political equilibrium of the conflicts between different social groups. In this respect, the social welfare functions were not to be used in models of pure economic analysis, but instead tackling sociological issues. At the same time Pareto encouraged younger economists to investigate the socio-economic equilibria which sustain different modes of public intervention in the economy, even if in doing so they were forced to abandon more rigourous forms of analysis. (Notice the convergence with the concept expressed in the quotation of von Hayek at the start of the paper).

3 The basic model

We consider a population formed by a finite number (N) of agents. There are two productive sectors denoted by A and B which ex post group two different types of agents. We model a two-stage non-cooperative game. In each sector agents compete in the political arena by choosing simultaneously the level of political activity; moreover, agents in each sector compete in their (homogeneous) product market.

To solve their maximization problem agents proceed as follows. First, they choose simultaneously an optimal output (price) level and a Cournot-Nash (Bertrand-Nash) equilibrium is established in each sector. Second, they simultaneously solve for the optimal level of political activity. Clearly, both the equilibrium levels of output and political activity depend on the number of agents operating in the two sectors and the parameters of the model. Hence, agents equalize the resulting individual (i.e., representative) payoffs of the two sectors and solve for the equilibrium number of agents in each sector. This condition of free mobility across sectors allows agents to calculate the optimal level of the strategic variable chosen in each stage (but see below for homogeneous communities).

To summarize, there are three unknowns in the model - the output (price) level, the level of political activity, and the number of agents in each sector - that are simultaneously determined by using the first order conditions (FOCs) in the two stages and the free mobility condition (FMC, henceforth). We now turn to the formal description of the model.

3.1 In sector i (i=A,B) a homogeneous product is produced with the following technology:

$$c(q_{ij}) = c_i q_{ij} \qquad j = 1, 2, \ldots, n_i. \tag{1}$$

The demand of product i (i=A,B) is summarized by:

$$p_i = a_i - \sum_j q_{ij} \qquad j = 1,2,\ldots,n_i . \tag{2}$$

In what follows we shall normalize to one the term $(a_i - c_i)$, i=A,B.

3.2 We consider two different specifications of the political technology reflecting differences at the institutional level in the production of the political activity. Under lobbying, in sector i (i=A,B) the net payoff from political activity, to agent j is:

$$-\left[\frac{k_i}{\epsilon_i \sum_j s_j} + \frac{s_j^2}{2}\right] \qquad j = 1,2,\ldots,n_i \tag{3}$$

where k_i, which is non-negative and finite, is a set-up cost to enter sector i, s_j is agent j level of political activity, and ϵ_i, again non-negative and finite is a measure of the latter effectiveness.

Under the party system, the net payoff from political activity to agent j in sector i is:

$$-\left[k_i\left(1 - \epsilon_i \sum_j s_{ij}\right) + \frac{s_j^2}{2}\right] \qquad j = 1,2,\ldots,n_i \tag{4}$$

where symbols are as above.

Notice that in both (3) and (4) the reduction in the set-up cost depends on the *sectorial* level of political activity. More precisely, under lobbying, due to the non-linear way in which s_j enters agent j payoff, some political activity must be undertaken to keep payoffs non-negative. This is not the case with the linear technology (4). Another relevant difference - which will appear more clearly in section 5 - between (3) and (4) is that the latter but not the former allows for a subset of the population (the majority) to have the sign of (3) changed, that is, to transform costs into benefits.

In other words, the lobbying technology, though very powerful in altering individual payoffs, does not permit redistribution in the sense referred above through political activity. In this respect, formulation (3) can be thought of as a simplified representation of a political technology in a Wicksellian State where unanimous consensus is required[14].

On the other hand, the party system is not as powerful in altering individual payoffs as the lobbying one, but it allows for redistribution through political activity. Then the party system as expressed in formulation (4) can be a political technology operating in a Welfare (more interventionist) State[15].

The lobbying and the party political settings can be thought of as *meta-constitutional* rules, whereas our fundamentals $k_A, k_B, \epsilon_A, \epsilon_A$, can be thought of as *constitutional* rules. The choice of this terminology needs to be justified. By constitutional rules we mean the rules that determine the effects of political activity on individual payoffs. For instance, ϵ_i can be thought of as a measure of the critical majority needed to pass legislation in favour of agents in sector i: as ϵ_i grows, the equilibrium level of (costly) political activity needed to legislate declines. In the same vein, k_i can be regarded as an entry cost in the political arena[16].

In what follows we shall adopt the following definition of equilibrium for the game just presented.

Given the economic technologies (1) and (2), the rules governing the competition

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14 But see note 5 and recall that unanimity in the Wicksellian sense was thought of as the analogous of Pareto optimality.

15 The possibility of obtaining net subsidies out of the whole range of public interventions is a distinctive feature of political settings where well-organized political parties jockey for awarding their members. This is not the case under lobbying where redistribution is vetoed by either groups irrespective of their size. This relates to the aforementioned non-linear way in which the political activity affects payoffs under the lobbying technology.

16 One can imagine educational costs to operate in the political arena for agents of type i.

in the product market (Bertrand of Cournot), the political technology (3) or (4), and the values of $k_A, k_B, \epsilon_A, \epsilon_B$, an *equilibrium* of our game (n_A^*) is a value of n_A in the interval [0,N] which *equalizes* payoffs (denoted by V_i i=A,B) across agents and such that $V_i(n_A^*)$, is *maximum* and *non-negative.*

Our definition captures the individual rationality of agents: they enter the State if this entails a Pareto improvement upon anarchy and, among the feasible sectorial distributions, the one with larger payoffs represents an equilibrium.

Hence, agents solve the free mobility condition with respect to n_A. If there is a solution yielding positive payoffs they compare the latter with the payoffs in both corners. The equilibrium will be that value of n_A which gives larger payoffs. If the free mobility condition has no (internal) solution a corner solution is an equilibrium only if it guarantees positive payoffs.

Centering on this definition of equilibrium, it becomes clear why we mantain that to be a Pareto-improvement upon anarchy is only a necessary but not a sufficient condition for the State to arise. Actually it may happen that there exist values of n_A which make all agents better off with respect to anarchy but as they obtain different payoffs such values are not equilibria[17] In this respect, a necessary and sufficient condition for the State to emerge is that it ensures equality of final (non-negative) payoffs. Recall that we did not need to assume that payoffs in anarchy are the same across agents, but only that the payoff of the better off is zero.

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[17] Figure 1 illustrates this point. No equilibrium exists because: (i) corner solutions yield negative payoffs and (ii) the free mobility condition is solved only for a value of n_A yielding negative payoffs.

With reference to the related literature on the origin of the State, our definition of equilibrium differs because it accommodates the case of heterogeneous agents. This is generically true in anarchy, but not in the State. As we shall discuss in Section 10, the notion of mobility featuring our definition of equilibrium prevents equilibrium payoffs to be completely determined by the initial distribution of agents between types.

We shall analyze in next section the model under the lobbying technology and in section 5 the model under the party system. In both cases, the competition in the product market follows Cournot rules.

4 The lobbying system

4.1 We start considering the maximization problem faced by agent j in sector A. The problem in sector B is analogous. In the first stage our agent maximizes with respect to q_j the following payoff:

$$V_A = \left(a_A - \sum_j q_j\right) q_j - c_A q_j - \frac{k_A}{\epsilon_A \sum_j s_j} - \frac{s_j^2}{2}. \tag{5}$$

The FOC gives:

$$\left(a_A - \sum_j q_j\right) - q_j - c_A = 0. \tag{6}$$

Imposing symmetry and remembering that $(a_A - c_A) = 1$, we get:

$$q^* = \frac{1}{(1 + n_A)}. \tag{7}$$

Notice that q* does not depend on the choice made in the other stage[18]. In the second stage, each firm solves (5) with respect to s_i. The FOC is:

$$\frac{k_A}{\epsilon_A\left(\sum_i s_i\right)^2} - s_i = 0 \tag{8}$$

which yields, imposing symmetry:

$$s_A^* = \left(\frac{k_A}{\epsilon_A n_A^2}\right)^{1/3}. \tag{9}$$

From (6) and (9) it is easy to see that both second order conditions are met[19]. To save on notation, when no ambiguity arises, we write s_A^* as s^*.

It is of some interest to compare the solution (9) and the individual level of political activity that would be chosen by n_A coordinated agents:

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18 As we shall see below, however, q* depends on the level of political activity through the number of agents.

19 To illustrate the equilibrium in the second stage of the game let us consider two agents in sector A, denoted by h and j. Their two FOCs are:

$$s_h(s_h + s_j)^2 = \theta$$

$$s_j(s_h + s_j)^2 = \theta$$

where $\theta = k_A/\epsilon_A$, whose unique solution is clearly:

$$s_h^* = s_j^* = \left(\frac{\theta}{4}\right)^{1/3}$$

Notice that s_h $(s_j) = \theta^{1/3}$ as $s_j(s_h) \rightarrow 0$, whereas $s_h(s_j)$ approaches zero as s_j (s_h) approaches infinity. (See Figure 2).

Proposition 1: In a Nash equilibrium, the lobbying technology involves underproduction in political activity with respect to the cooperative solution.

Proof. If agents minimize total cost of political activity, each of them solves the following problem:

$$\operatorname*{Min}_{s_j} \; n_A\left[\frac{k_A}{\epsilon_A \sum_j s_j} + \frac{s_j^2}{2}\right] \qquad j = 1, 2, \ldots, n_A$$

which gives:

$$s^{**} = \left(\frac{k_A}{\epsilon_A n_A}\right)^{1/3}$$

which is clearly greater than s* for $n_A > 1$.■

This finding is reminiscent of free riding in the political activity. We are aware that a more proper test of such behaviour would be to look into how individual and aggregate political activity vary with n_A. However, as n_A is endogenous in our model, we cannot perform standard comparative statics with respect to n_A. We postpone the discussion of this issue to next sub-section.

4.2 Notice that q* and s* in (7) and (9) are not equilibrium *levels*, but equilibrium *functions* of the proportion of agents across sectors, so that agents need a further condition to determine their levels. This condition is provided by equating the equilibrium payoffs of the two sectors, that is imposing a FMC across sectors[20]. From now on, without loss of generality, we set $N = n_A + n_B = 1$, so that $n_A (n_B)$ now means the proportion of agents in sector A (B). Hence, remembering that we have also normalized to one the difference between market size and marginal cost in the product market, we plug q* and s* into V for each sector and we obtain:

$$\left[\frac{1}{(1+n_A)^2} - \frac{1}{(2-n_A)^2}\right] + [\beta(1-n_A)^{-4/3}(3/2-n_A) - \alpha n_A^{-4/3}(1/2+n_A)] = 0 \tag{10}$$

where $\alpha = (k_A/\epsilon_A)^{(2/3)}$ and $\beta = (k_B/\epsilon_B)^{(2/3)}$. Our notion of equilibrium requires the non-negativity of the equilibrium payoffs. For sector A, by plugging (7) and (9) into (5), this means:

$$V_A = \frac{1}{(1+n_A)^2} - \alpha n_A^{-4/3}(1/2+n_A) \geq 0. \tag{11a}$$

Proceeding analogously for sector B, we get:

$$V_B = \frac{1}{(2-n_A)^2} - \beta(1-n_A)^{-4/3}(3/2-n_A) \geq 0 \tag{11b}$$

which restrict the acceptable ranges of α and β. A sufficient condition on α and β to guarantee the non-negativity of (11a) and (11b) is $\alpha < 0$ and $\beta < 0$. Of course, this is

20 This condition can be defended on twofold ground. On one hand, it is a common assumption in multisectorial equilibrium models to adopt the FMC as a long run equilibrium condition. (General economic equilibrium models, models of trade, and so on). On the other hand, the FMC has to be introduced when the problem of the origin of the State is investigated under the assumption of no exogenous barriers to mobility (See section 10).

a far too strong restriction for our purposes and hence we shall consider values of α and β which, although positive, ensure V_A and V_B to be non-negative. Of course, we focus on *real* values of n_A in the closed interval [0,1].

4.3 We now investigate the existence and the properties of equilibria under lobbying. Let us define $h(n_A)$ as the LHS of (10).

Lemma 1: If $n_A \geq 1/2$, then $\partial h(n_A)/\partial n_A > 0$.

Proof. Implicitly differentiating $h(n_A)$ we get:

$$\frac{\partial h(n_A)}{\partial n_A} = \left[\frac{2}{(2-n_A)^3} - \frac{2}{(1+n_A)^3}\right] + \left[\beta(1-n_A)^{-4/3}\left(\frac{3-n_A}{3(1-n_A)}\right) + \alpha n^{-4/3}\left(\frac{2+n_A}{3n_A}\right)\right].$$

The first term is non-negative for n_A in the interval [1/2,1] and the second term is always positive.■

Clearly Lemma 1 gives a sufficient condition. In other words, there exist values of α and β, yielding through the FMC a value of n_A in the interval [0,1/2[such that $\partial h(n_A)/\partial n_A$ is still positive. However, when n_A solving (10) is in a neighbourhood of zero, such derivative is certainly negative.

Proposition 2: $n_A = 0$ ($n_A = 1$) can be an equilibrium. A necessary condition for $n_A = 0$ ($n_A = 1$) to be an equilibrium is $\alpha >$ (<) β.

Proof. Suppose there is no n_A in the interval]0,1[solving (10). (See Proposition 6 below). When $n_A = 0$, $V_B = 1/4 - 3\beta/2$. When $n_A = 1$, $V_A = 1/4 - 3\alpha/2$. $n_A = 0$ is then an equilibrium if $1/6 \geq \alpha > \beta$. If $1/6 \geq \beta > \alpha$ then $n_A = 1$ is an equilibrium.■

From now onwards we shall focus only on equilibria which are interior solutions (i.e., $0 < n_A^* < 1$).

Proposition 3: $\alpha = \beta \Leftrightarrow n_A^* = 1/2$.

Proof. ⇒ If $\alpha = \beta$, for any $n_A < 1/2$, both terms in $h(n_A)$ are positive. If $\alpha = \beta$, for any $n_A > 1/2$, both terms are negative. $n_A = 1/2$ is the only acceptable solution of (10) when $\alpha = \beta$[21].

⇐ By straightforward substitution of $n_A = 1/2$ into (10).■

As we are ensured of the existence of some equilibria and we want to perform some comparative statics, we now investigate the stability of our equilibria. A necessary condition for local stability requires that in equilibrium:

$$\frac{dV_B}{dn_A} > \frac{dV_A}{dn_A}.$$

From (11a) and (11b), this becomes:

$$\frac{2}{(2-n_A)^3} + \frac{2}{(1+n_A)^3} > \frac{\alpha(3+n_A)}{3n_A^{7/3}} + \frac{\beta(3-n_A)}{6(1-n_A)^{7/3}}.$$

One can easily check that this condition is less likely to be met as $(\alpha - \beta)$ tends to zero. This is because as α equals β the LHS of the above inequality reaches its minimum. (See Proposition 3).

21 Of course there are multiple solutions for n_A, but n_A=1/2, is the only real one in the acceptable range.

Proposition 4:

(i) $\alpha > \beta > 0 \quad \Rightarrow \quad n_A^* > 1/2$

(ii) $\beta > \alpha > 0 \quad \Rightarrow \quad n_A^* < 1/2$.

Proof. We proceed by contradiction and suppose that when $\alpha > \beta > 0$, $n_A < 1/2$. We show that for $n_A < 1/2$, the FMC is violated as $h(n_A)$ is always positive. First notice that the first term of $h(n_A)$ is never negative for n_A in the interval [0,1/2]. The second term of $h(n_A)$ equals $3\beta/2$ when $n_A = 0$ and, by Lemma 1, is monotonically increasing in n_A for any pair of α, β. Hence, the second term of $h(n_A)$ is always strictly positive and the first term is never negative, so that (10) is never met as n_A varies in the interval [0,1/2]. Remembering Proposition 3, the contradiction establishes the Proposition. The same procedure establishes part (ii).■

Proposition 5: If $\alpha\beta = 0$ and $(\alpha + \beta) \neq 0$ and there exists a solution, then:

(i) $\alpha = 0 \Rightarrow n_A^* > 1/2$

(ii) $\beta = 0 \Rightarrow n_A^* < 1/2$.

Proof. Follows easily from (10).■

Proposition 6: If α is much greater than β, then there is no equilibrium.

Proof. If α is much greater (smaller) than β, then both terms in $h(n_A)$ are negative (positive).■

We now check how the equilibrium level of individual political activity varies with the number of agents[22].

22 As n_A is endogenous, by this we mean the consequences of small changes in the fundamentals which bring about small changes in n_A^*.

Proposition 7: If $\alpha > \beta > 0$ or $\beta > \alpha = 0$, then $\partial s^{*}/\partial n_A^{*} > 0$.

Proof. Implicitly differentiating we get:

$$\frac{ds^{*}}{dn_A} = -\frac{\partial h(n_A)/\partial n_A}{\partial h(n_A)/\partial s^{*}}.$$

We know from Lemma 1 that the sign of the numerator is positive for $n_A \geq 1/2$. Isolating s^{*} into $h(n_A)$, the sign of the denominator is always negative. Since, by Propositions 4 and 5, we know that $\alpha > \beta > 0$ or $\beta > \alpha = 0$ entail $n_A^{*} > 1/2$, the conclusion follows.■

Proposition 8: If $\alpha = \beta$, $s_A^{*} = s_B^{*}$.

Proof. Follows from (9) and Proposition 3.■

Notice that from Propositions 3 and 8 $n_A^{*} s_A^{*} = n_B^{*} s_B^{*}$.

The production of political activity allows agents to reduce the set-up costs to enter the State. It is therefore of some interest to investigate how such costs vary with s_j $(j = 1, \ldots, n_A)$ and consequently with n_A. Focussing on the political stage of the game, it is as if agent j minimizes with respect to s_j the following:

$$\frac{k_A}{\epsilon_A \sum_j s_j} + \frac{s_j^2}{2} \quad .$$

Plugging (9) into this expression we get:

$$\alpha n_A^{-4/3}(n_A + 1/2) \qquad (ISC)_L$$

$(ISC)_L$ is the individual net set-up cost, that is the set-up cost (k_A) reduced through the fruits of the political activity. By implicitly differentiating $h(n_A)$ we now check how n_A affects $(ISC)_L$.

Proposition 9: $d(ISC)_L/dn_A > 0$.

Proof. Proceed as in the proof of Proposition 7.∎

We have seen that (10) is a crucial expression because it allows us to analyze the equilibrium distributions of agents across sectors. Unfortunately, it turns out extremely difficult to solve (10) for n_A as a function of $k_A, k_B, \epsilon_A, \epsilon_B$, which are the fundamentals of our model. Hence, we resort to numerical simulations after restricting the acceptable domain of parameters on the basis of theoretical arguments (see comments on (11)).

4.4 We have performed a large number of numerical simulations; Table 1 (see also Figure 3) below collects the values of n_A in correspondence of acceptable values of the parameters α and β.

On the basis of such findings it is now possible to make some remarks on the relations between economic and political variables in shaping the distribution of agents across sectors.

Claim 1. n_A is more sensitive to small changes in α the smaller $|\alpha-\beta|$. (See Table 1).

Claim 2. Keeping α/β constant, n_A^* is less (more) sensitive to changes in $(\alpha-\beta)$ the larger (smaller) is $(\alpha+\beta)$. (See Table 1).

Claim 3. When $\alpha > \beta$, $s_A^* > s_B^*$. (See Tables 14 and 15).

This finding comes from the following steps. From the equilibrium expressions for s^*_A and s^*_B , and Proposition 3, Claim 3 holds if n^*_A is very sensitive to changes in $(\alpha - \beta)$. This is indeed the case by Claim 1.

Claim 4. If $\alpha > \beta$. $n^*_B s^*_B < n^*_A s^*_A$. (See Tables 5, 6, 14 and 15).

We now turn our attention to the party system.

5 The party system

5.1 We consider the same problem as in section 4 when the political technology is summarized by (4). Then, agent j in sector A now maximizes:

$$V_A = \left(a_A - \sum_i q_i\right)q_j - c_A q_j - k_A\left(1 - \epsilon_A \sum_i s_i\right) - s_j^2/2 \qquad (13)$$

Clearly the FOC in the first stage is still given by (6), while the FOC in the second stage gives:

$$s^*_A = \epsilon_A k_A \qquad (14)$$

Notice that now s^* (we are still omitting the subscript) is independent of the number of agents in the sector (contrast (14) and and (9)). As a consequence, no comparison with respect to collusive actions in the political stage can be addressed in this context.

Building on the interpretation of ϵ and k suggested in section 3.2, (14) tells us that the equilibrium level of political activity is positively related to the cost of entering the political arena and inversely related to the majority needed to pass legislation. While the first relation does not deserve any comment, the second one can be rationalized in terms of incentives to undertake political activity: the larger the majority needed to legislate, the lower the incentive to produce political activity.

5.2 Under the same normalization as behind (10), the FMC under the party system becomes:

$$\left[\frac{1}{(1+n_A)^2}-\frac{1}{(2-n_A)^2}\right]+[\gamma+\delta(n_A-1/2)] = 0 \tag{15}$$

where $\gamma = k_B - k_A$ and $\delta = k_A^2\epsilon_A^2 + k_B^2\epsilon_B^2$. Equilibrium payoffs must be non-negative, i.e.:

$$V_A = \frac{1}{(1+n_A)^2} - k_A + \epsilon_A^2 k_A^2 (n_A - 1/2) \geq 0 \tag{16a}$$

$$V_B = \frac{1}{(2-n_A)^2} - k_B + \epsilon_B^2 k_B^2 (1/2 - n_A) \geq 0 \tag{16b}$$

These conditions require that δ varies in the interval]0,4[[23].

We now investigate the existence and the properties of equilibria under the party system. Let us define $f(n_A)$ as the LHS of (15).

Lemma 2: If $n_A^* \geq 1/2$, then $\partial f(n_A)/\partial n_A > 0$.

Proof. Proceed as in the proof of Lemma 1.∎

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23 The non-negativity of V_A requires:

$$k_A^2\epsilon_A^2 \leq 2[(2n_A-1)(1+n_A)^2]^{-1}$$

which, for all relevant values of n_A, implies that $k_A^2\epsilon_A^2 \leq 2$. Hence, the restriction follows.

Proposition 10:

(i) $n_A^* = 0$ can be an equilibrium;

(ii) $n_A^* = 1$ can be an equilibrium.

Proof. Suppose (15) has no solution. $n_A = 0$ is an equilibrium if $V_B(n_A = 0) > V_A(n_A = 0)$. This is the case when $\delta > 3/2 + 2\gamma$. Analogously, for $n_A = 1$ to be an equilibrium, it must be $\delta > 3/2 - 2\gamma$. Both conditions can be satisfied.∎

From now on we shall focus on internal solutions.

Proposition 11: $\gamma = 0 \Leftrightarrow n_A^* = 1/2$.

Proof. By simple inspection of (15).∎

Before proceeding, we want to ensure that our equilibria are stable. As for the internal solutions, proceeding as in section 4, a necessary condition for local stability is:

$$\frac{2}{(2-n_A^*)^3} + \frac{2}{(1+n_A^*)^3} > \delta .$$

As under lobbying, this condition is more stringent when fundamentals yield an equal distribution of agents across sectors. More precisely, this condition is certainly met as $\delta < 32/27$. which is the minimum value of the LHS. Hence, from now on, we adopt such a restriction on δ. On the other hand, equilibria which are corner solutions are locally stable[24].

Proposition 12: If $\gamma > 0$, then n_A^* increases (decreases) with γ.

Proof. Let us consider (15). Implicitly differentiating we get:

24 See section 7.1 below.

$$\frac{dn_A}{d\gamma} = -\frac{\partial f(n_A)}{\partial \gamma} / \frac{\partial f(n_A)}{\partial n_A}.$$

As the numerator is always positive, $dn_A/d\gamma$ has the opposite sign of $\partial f(n_A)/\partial n_A$. Now,

$$\frac{\partial f(n_A)}{\partial n_A} = \left[\frac{2}{(2-n_A)^3} - \frac{2}{(1+n_A)^3}\right] + \delta.$$

A sufficient condition for this expression to be positive is $\delta > 7/4$. However, this contrasts with the stability condition which imposes an upper bound on δ. If δ is sufficiently small $\partial f(n_A)/\partial n_A$ is negative and then $\partial n_A/\partial \gamma$ is positive.■

Proposition 13: If γ is large as compared to δ, then there is no equilibrium.

Proof. By inspection of (15).■

Now we look at how the individual level of political activity varies with n_A[25]. Isolating $s^* = \epsilon_A k_A$ into (15), and implicitly differentiating s^* with respect to n_A, we get:

Proposition 14: When $n_A^* \geq 1/2$, $ds^*/dn_A < 0$.

Proof. As $\partial f(n_A)/\partial s_A^* > 0$ when $n_A^* \geq 1/2$, by Lemma 2 the Proposition follows.■

As in section 4.2, we look at how n_A affects the (ISC), which now becomes:

$$k_A - n_A \epsilon_A^2 k_A^2 + \frac{1}{2}\epsilon_A^2 k_A^2. \qquad (ISC),$$

25 Remember footnote 20.

Proposition 15: When $n_A^* \geq 1/2$, then $d(ISC)_P/dn_A > 0$.

Proof. By implicit differentiation of (15).■

5.3 Now we perform numerical simulations to understand how n_A^* varies in the $\gamma - \delta$ space. The main findings are collected in Table 2 and Figure 4.

Claim 5. For $\delta < 3/2$, n_A^* is (is not) "very" sensitive to small changes in γ in a neighbourhood of $\gamma = 0$ for high (low) values of δ.

Claim 6. For $\delta > 3/2$, n_A^* is (is not) "very" sensitive to small changes in γ in a neighbourhood of $\gamma = 0$ for low (high) values of δ.

Claim 7. For $\epsilon_A = \epsilon_B$, $k_A <$ (>) k_B implies $n_A^* s_A^* <$ (>) $n_B^* s_B^*$.

6 Mixed political settings

So far we have assumed that both sectors employ the same political technology (either lobbying or the party system). In principle, however, different groups might interact in the process of legislative production by means of different technologies. Hence, we try to model the implications of such a circumstance by examining the working of our model where both political technologies coexist. When lobbying takes place in sector A, the FMC becomes:

$$\left[\frac{1}{(1+n_A)^2} - \frac{1}{(2-n_A)^2}\right][k_B + \epsilon^2 k_B^2 (n_A - 1/2) - \alpha n_A^{-4/3}(1/2 + n_A)] = 0 \qquad (17)$$

where, for the sake of comparison with the two pure technologies, we assumed that $\epsilon_A = \epsilon_B = \epsilon$.

Proposition 16: When A (B) is lobbying, if an equilibrium exists, $n_A^* < (>)\ 1/2$.

Proof. By simple inspection of (17).■

The sector where lobbying prevails sustains a smaller number of agents than the one with the party system. This is because, for any given configuration of the parameters, the same reduction in set-up costs requires a lower number of agents under the lobbying system than under the voting one. Notice also that $n_A = 1/2$ cannot be an equilibrium.

Proposition 17: When A (B) is lobbying, $n_A^* = 0$ $(n_A^* = 1)$ can be an equilibrium.

Proof. Suppose A is lobbying and (17) has no solution. $n_A = 0$ is an equilibrium if $V_B(n_A = 0) > V_A(n_A = 0)$, i.e., if $\epsilon_B^2 k_B^2 > 3/2 + 2k_B$. This condition can be satisfied. An analogous reasoning leads to show that $n_A = 1$ is an equilibrium when B is lobbying.■

This is reminiscent of what happens under both pure political technologies: see Propositions 2 and 10.

Proceeding as in sections 4 and 5, and focussing on (17), numerical simulations allow us to obtain Tables 5, 6, 17 and 18. In commenting these findings subscript L (P) refers to lobbying (party). Notice that, except for the first column (where $k_A = k_B$) in correspondence of any pair (k_B and ϵ) the values of n_A^* in Tables 5 and 6 do not add up to one. This is because, although the two political settings are switched, the asymmetry in set-up costs persists. Furthermore, the equilibrium fails to exist when the sector endowed with the lobbying technology experiences large set-up costs in correspondence of small values of ϵ[26].

Claim 8. When sector B is lobbying, n_A^* decreases as $(k_B - k_A)$ grows, for any given level of ϵ.

Claim 9. When $k_B = k_A$, the individual political activity is greater in the sector endowed with the lobbying technology.

We know from Proposition 16 that lobbying groups less agents than the party system for any equilibrium set of fundamentals. Although this entails larger profits in the lobbying sector, the distribution of agents is so uneven that the FMC requires the few agents in the lobbying sector to produce more political activity.

Claim 10. When $k_B > k_A$:

$s_{LA} > s_{PA}$;

$s_{LB} > s_{PB}$ for low values of ϵ.

As in a mixed political setting the lobbying sector groups a smaller number of agents for relatively low set-up costs, the individual political effort must be greater than under voting. For higher level of set-up costs and ϵ, this inequality is reversed.

26 Indeed the payoff in sector B becomes negative for large values of k_B and low values of ϵ. As the FMC requires equal payoffs in equilibrium, this situation is not an equilibrium.

7 Frameworks compared

The games we have analyzed so far differ both in the economic and political rules. Table 11 visualizes all combinations of settings we have considered.

Among the 16 scenarios collected in Table 11, we have mostly focussed on cases labelled CL-CL and CP-CP because they allow a more detailed analysis of both stages of our game. Coherently, in this section we compare some aspects of these two cases. As the performances of these two settings will be examined in section 8, the present comparison is organized around the distribution of agents across sectors and the production of political activity. For the sake of comparison next remarks are preceded by short headings referring to the relevant results obtained in previous sections.

7.1 The distribution of agents

Propositions 6 and 13. As for the very existence of an equilibrium, both lobbying and party systems admit no equilibrium for equal values of ϵ when the difference in entry costs across sectors is sufficiently large. In this respect the two political frameworks seem to behave similarly. This does not happen when, for $k_A = k_B$, the difference in constitutional rules (ϵ_A and ϵ_B) gets larger. Indeed, while lobbying does not permit an equilibrium in such a case, the party system does.

Propositions 2 and 10. Complete specialization can be an equilibrium under both political technologies. Notice that such equilibria are stable in both cases. Under lobbying, the payoffs in the active sector are lower than the payoffs in the vanishing one. However, every agent anticipates that, if he moved, every other agent would follow, yielding (by definition) a lower individual payoff. Under party system the same situation may arise, but it could also be the case that the payoffs in the active sector are larger than those in the vanishing one. A fortiori the equilibrium in this case is stable.

Propositions 3 and 11. As for the role of fundamentals, the two technologies seem to display the same behaviour, that is, identical fundamentals yield $n_A^* = 1/2$. Actually, Proposition 3 is more restrictive than Proposition 11 for the latter imposes not just equality between values of k, but also a specific relation between values of k and values of ϵ.

Proposition 5. Under lobbying, when $\alpha = 0$, (9) tells us that there are no set-up costs to enter sector A. Therefore, for $n_A = 1/2$ we would get $V_A > V_B$, but this violates the FMC. In order to keep payoffs equalized, n_A has to increase. This yields an increase in V_B via the economic payoff, but a negative effect via the political payoff. Contrasting Propositions 4 and 5, one can see the relevance of the political element in determining the distribution of agents across sectors. For identical values of the fundamentals, such distribution is reversed according to whether or not set-up costs, and therefore political activity, is present in both sectors. Notice that such a "reversal" does not occur under the party system.

Propositions 4 and 12. Lobbying and the party system entail opposite relationships between the distribution of agents and differences in the fundamentals for large levels of political activity. This difference breaks down when the production of political activity is large, that is, when the economic component of individual payoffs is dominated by the political one in the party system. Remember, however, that equilibria in this case are not stable.

Proposition 4 sounds rather paradoxical from a purely economic viewpoint. Without political activity, it is well-known that larger set-up costs imply a lower number of active producers. In this sense it can be argued that, under lobbying, the role played by set-up costs in the political component of the payoff "dominates" over the role played by set-up costs in the economic one. This is because only grouping a larger number of agents in the sector with larger set-up costs allows the FMC to be met.

As for the party system, for low values of δ. it can be argued that the economic component of the payoff dominates over the political one. This is true also for $\delta = 0$. Such a case (for positive set-up costs) arises when the political stage disappears in both sectors. Then, a fortiori, the remark above applies. On the contrary, for high values of δ (but remember the stability condition), the FMC would push agents in sector B (where set-up costs are larger). This must be qualified because our δ may come about from a plethora of combinations of the four fundamentals. Our suggested interpretation seems plausible when none of such fundamentals approaches extreme values (e.g., ϵ_i approaching zero). Notice also that this result explains why the numerical relationship between n_A^* and γ changes dramatically in passing from $\delta = 1$ to $\delta = 3/2$. (See Table 2).

We now look at the sensitivity of the equilibrium distribution of agents with respect to fundamentals.

Claims 1 and 2. Under lobbying, an increase in k_A requires a smaller increase in n_A the larger is $(\alpha - \beta)$. This is because to meet the FMC different set-up costs are balanced by greater levels of aggregate political activities (greater n_A) the smaller $(\alpha - \beta)$. The distribution of agents across sectors does not only depend on the difference between fundamentals, but also on their absolute values. More precisely, the larger are the overall set-up costs, the less uneven is the distribution for given $(\alpha - \beta)$.

Claims 5 and 6. Under the party system, for low values of δ the political component is "dominated" by the economic one. By this we mean that the distribution of agents across sectors follows the logics of a purely economic model in that large set-up costs lead to a small number of producers. Starting from a situation where $\gamma = 0$, a small reduction in k_A requires a larger increase in n_A^* the higher the aggregate political activity in equilibrium (i.e., the higher δ). This is because when k_A decreases - ceteris paribus - s^* declines too (the more so the higher δ). Accordingly, in order to meet the FMC, agents group into sector A more "rapidly" the higher the level of politic[al] activity.

On the other hand, when δ is sufficiently large, the political component "dominate[s]" the economic one, so that the distribution of agents across sectors parallels the o[ne] emerging, under the same configuration of fundamentals, with lobbying. Starting fro[m] a situation where $\gamma = 0$, a small reduction in k_A requires a larger decrease in n_A^* th[e] lower the aggregate political activity in equilibrium. This is because when k_A decrease[s] s^* declines too, (the more so, the higher δ). In this case, however, due to the increase[d] effectiveness of the aggregate political activity, agents group in the sector with high[er] k - which is now B - the more "rapidly" the lower is the ex-ante level of politic[al] activity. This circumstance retards the arising of differences in the relative levels [of] such activity across sectors which in turn explains the asymmetric distribution [of] agents across sectors. However, this second case seems less relevant than the fir[st] one as we know that large δ are responsible for unstable equilibria.

7.2 The production of political activity

We start noticing that from (9) and (14) one may be tempted to compare the ro[le] of fundamentals in determining the equilibrium level of political effort in both setting[s]. While this is correct in the case of the party system (see (14)), it cannot be done [in] the case of lobbying because when fundamentals vary n_A^* varies too. However, fr[om] (9) and Propositions 3 and 11 we have:

Proposition 18: $n_A^* = 1/2$ is (is not) a sufficient condition for $s_A^* = s_B^*$ under the lobbyi[ng] (party) system.

The intuition behind Proposition 18 is that under lobbying $n_A^* = 1/2$ implies and is implied by the equality of parameters α and β embodying all fundamentals. On the contrary, under the party system, $n_A^* = 1/2$implies and is implied by the equality $k_A = k_B$ which leaves room for differences in the values of ϵ. (Remember (14)).

Propositions 7 and 14. Recalling the comments on Lemma 1, we can argue that under lobbying free riding in sector A occurs when fundamentals are such as to yield a value of n_A^* sufficiently small. Remember that in this case $\partial h(n_A)/\partial n_A < 0$ and then $ds^*/dn_A < 0$. However, there is a critical threshold of n_A^* in the interval [0,1/2[above which free riding disappears because of the larger payoffs achievable through the lobbying activity (compare this with Proposition 1). Needless to say, the conditions in Proposition 7 are also sufficient for the *aggregate* political activity in A to increase with n_A. On the other hand, when $\partial h(n_A)/\partial n_A < 0$, also the aggregate political activity decreases with n_A^*[27]. It is worth noticing that when incentives to lobbying are low ($\alpha < \beta$ and such that n_A^* is small), free riding is very pervasive, in that even the aggregate political activity declines with n_A^*. By these arguments, under lobbying, free riding occurs whenever fundamentals are sufficiently different. We shall see below that the minimum difference required for free riding to occur is actually very small.

Under the party system, the individual production of political activity decreases with n_A when $n_A \geq 1/2$. In other words, when fundamentals combine as to yield a simple majority for sector A ($n_A^* > 1/2$), agents in sector A start free riding. This is not the case when they are in minority. The above condition is not a sufficient one to guarantee that the aggregate political activity in A decreases with n_A. On the contrary, when fundamentals are such that $n_A^* < 1/2$ we cannot give similar conditions on the relationships between both the individual and aggregate levels of political

27 Isolating $n_A s^*$ in $h(n_A)$, one gets $dn_A s^*/dn_A < 0$.

activity and n_A.

Propositions 9 and 15. Under the lobbying technology the net set-up cost to joi the State in sector A increases with the proportion of agents entering in A. The ne individual cost to enter the political arena increases with n^*_A only if the majority c agents in the State is already in A. Propositions 7 and 14 allow us to give an intuitio for 9 and 15 in terms of different patterns of free-riding behaviour.

Claims 3 and 4. Under lobbying, when $\alpha > \beta > 0$ (and then $n^*_A > 1/2$) $s^*_A < s^*_B$, bu $n^*_A s^*_A > n^*_B s^*_B$. This can be explained as follows. For $\epsilon_A = \epsilon_B$, a small difference in set-u costs brings about, via the FMC, a highly uneven distribution of agents. Notice als that the sector with higher set-up costs experiences lower payoffs in the econom stage. For the FMC to be met, this sector needs a very large aggregate politic activity. Claims 3 and 4 show that this is obtained more via n_A than via individual (remember (9)).

Claim 7. Under the party system, agents in the sector with larger set-up cos produce a greater level of individual political activity (see (14)). By Tables 3, 12 an 13 this is also true at the sectorial level.

We now turn our attention to aggregate levels of political activity under tl various settings. We are still bound to resort to numerical simulations. Tables 12-2 collect individual and community levels of political activity in the relevant cases. Tl aggregate levels of such activity have been obtained summing individual level in eac sector multiplied by the number of agents in that sector.

Looking at Tables 12-21 one can compare the individual and aggregate levels political activity for different political settings, where subscript M indicates the mixe political setting. The main findings are the following:

Claim 11. When $k_A = k_B$, then $s^*_A = s^*_B$ under lobbying and the party system, but n under a mixed setting.

This is the consequence of the distribution of agents across sectors under a mixed political technology.

Claim 12. When $k_B = k_A$, then, for both sectors, $s^*_M > s^*_L > s^*_P$.

As the political activity is aimed at reducing the negative effects on payoffs of entering the community, a greater level of such activity has to be produced when the institutional setting is such as to entail an uneven distribution of agents in correspondence of $k_A = k_B$.

As for the community level of political activity ($S = s^*_A n^*_A + s^*_B n^*_B$) we can notice:

Claim 13. $S_L >$ (<) $S_M >$ (<) S_P for small (large) levels of ϵ .

Under the party system, the community level of political activity is lower than under lobbying or mixed settings when large majorities are needed to pass legislation while these inequalities are reversed when majorities are small.

8 Bertrand competition

So far we have modelled the economic stage of our game as one in which agents compete à la Cournot. As a consequence, their profits in the product markets depend on n_i (i=A,B). It is worth noticing that the only link between the economic and the political stage passed through n_i.

We now want to explore the consequences of a more intensive competition in the product market as is the one associated with Bertrand. This extension is not pursued merely for sake of completeness but, as we shall see, it will allow us to show that th sustainability of political settings depends on the rules prevailing in the econom stage.

Under our linear technology in the product market, price competition clear implies zero profits in the Nash equilibrium of the economic game. Then, the ultima payoff coincides with the net payoff from political activity. We now go to ascerta how this affects our two political settings.

7.1 Suppose there is lobbying technology in at least one sector (say A). Then:

Proposition 19: Bertrand competition and lobbying in at least one sector yields equilibrium, irrespective of the type of competition in the economic stage and t political technology prevailing in the other sector.

Proof. The payoff for a generic agent in sector A becomes:

$$-\alpha n_A^{-4/3}(n_A + 1/2)$$

which is nothing but $(ISC)_L$ with the appropriate sign. This expression is paten negative. As producers can always guarantee themselves a null payoff, the conclus follows.∎

7.2 Under party system in both sectors we have:

Proposition 20: Bertrand competition and party system in both sectors yield equilibrium.

Proof. The payoff for a generic agent in sector A becomes:

$$-k_A + \epsilon_A^2 k_A^2 (n_A - 1/2) \tag{18}$$

which again is the $(ISC)_P$ with the sign changed. This expression cannot be non-negative for both sectors.■

Proposition 21: With Bertrand competition and party system in sector A (B), and Cournot coupled with lobbying or party system in sector B (A), an equilibrium exists provided that ϵ is sufficiently large. Moreover, $n_A^* > 1/2$.

Proof. Clearly, it suffices to show some examples. We shall give two that cover the cases in which there is Cournot competition and lobbying in sector B (case i) and in which there is Cournot competition and party system in sector B (case ii). In both cases there is Bertrand competition and party system in sector A.

(i) For $k_A = 1/10$ and $\epsilon = 14.83$, $n_A^* = 1$. (ii) For $k_A = 2, k_B = 1/10$ and $\epsilon = 14.14$, $n_A^* = 1$.■

The reason why ϵ must be large and, if an equilibrium exists, n_A^* must be greater than 1/2 is easily explained by looking at the payoff of a generic agent operating in sector A (see (18)). Only if ϵ is large, and parameters are such that $n_A^* > 1/2$, individual payoffs in sector A become positive and may equalize the positive payoffs arising under Cournot competition and lobbying political system in sector B.

From Propositions 20 and 21, we can argue that in our community, no equilibrium exists unless (costly) political activity is paid back through returns in the economic stage. This means that in at least one sector Cournot competition must prevail. Of course, this is not a sufficient condition.

9 Performance evaluations

In this section we compare the performances of all three political settings considered in the previous sections under Cournot competition in both sectors. By performance we mean the equilibrium levels of individual payoffs and an indicator overall welfare both at the sectorial and the community level.

Clearly, by the FMC, to evaluate the level of individual payoffs in the State suffices to look at one sector only. This is not the case with the other indicator performance. Unfortunately, the comparison between the three political regimes cann be handled analytically for the different ways in which parameters enter the releva expressions. Then we resort to numerical simulations. From now onwards, we s $k_A = 0.01$ and choose the range for k_B and ϵ in such a way that payoffs are non-negati in all settings compared and yield internal solutions (recall comments on (11) a condition (16)). Simulations have been undertaken keeping $\epsilon_A = \epsilon_B$ and varying set-costs across sectors. In order to better understand the outcome of numerical simulatio we present also Tables 3-6 which collect the equilibrium levels of n_A in correspondei of the same levels of parameters. Notice that the case of mixed setting is split i two sub-cases (see section 6). The levels of payoffs in the four cases are report in Tables 7-10.

Claim 14. In all political settings, payoffs are maximum when fundamentals yield $n_A^* = 1$.

In this respect, one could say that under the lobbying and the mixed technology, game is sub-additive with respect to the difference between the equilibrium level agents in the two sectors.

Claim 15. The party system gives greater payoffs than the lobbying and the mi ones.

Notice that the party system yields a sectorial distribution of agents which is unambiguously more even than the other technologies. However, even when $k_A = k_B$, and hence $n_A^* = 1/2$ in both party and lobbying technologies, payoffs in the latter are larger. Moreover, while under the party system payoffs fall "smoothly" as $(k_B - k_A)$ increases, under the lobbying technology such a fall is substantial.

As for the mixed political settings, Tables 5 and 6 confirm Proposition 16, that is that identical parameters across sectors is neither a necessary nor a sufficient condition for $n_A = 1/2$ to be an equilibrium. Accordingly, when $k_A = k_B$, payoffs are smaller than in the pure cases, but, contrary to the case of lobbying, they do not fall dramatically as $(k_B - k_A)$ increases.

We now look at the citizens' payoffs under different political settings[28]. Their sum is:

$$W = n_A V_A + (1 - n_A) V_B .$$

Building on Claim 14, we can state the following:

Claim 16. In all political settings, W is maximum when $n_A = 1/2$.

Claim 16 allows us to state:

Claim 17. The party system ensures the maximum W.

---

28 In our model agents are producers and no consumption activity is modelled explicitely. If one wants to assess the features of different equilibria in terms of consumer surplus, it can be shown that the latter is maximum when $n_A^* = 1/2$.

From Propositions 4 and 16 and Claim 14 we have:

Claim 18. The party system yields greater W than each mixed political setting w
set-up costs are identical across sectors. This can be reversed as the asymmetr
set-up costs gets larger.

Since the main methodological aim of this model amounts to stress the need collective decision making to be based on individual maximizing behaviour, no roo left for public intervention in the traditional vein. By this we mean interventions decision makers maximizing exogenously given objective functions (e.g., social wel functions).

As we mentioned in section 2, we are aware that our model too does not f base collective rules on individual rationality, but we try to integrate the analysis some given rules of political behaviour into the political and economic game thou as a whole[29].

Taking this perspective, we can pursue two levels of normative analysis. first one deals with the preferability of a particular meta-constitutional rule. second level deals with the preferability of a particular set of constitutional ru within a given meta-constitutional rule (recall the interpretation of these conce suggested in section 3.2)[30].

Let us begin from the first level. Suppose that an external observer, wish maximum overall welfare, chooses the political meta-constitutional rules after read this section. Then, he would go for the party system.

---

29 To this end one needs a consistent theory of the distribution of power in original state in order to model explicitly the origin of basic social rules.

30 One should notice here the analogies in spirit, if not in the methodological ai with Buchanan and Tullock (1962) and Brennan and Buchanan (1985).

As for the second level, for any given meta-constitution, he would choose sets of constitutional rules (i.e., parameters in our model) yielding equal distribution of agents across sectors. Of course, in the spirit of our approach, these normative remarks have a purely theoretical interest insofar this choice too should emerge out of a model dealing with "higher level" rules of political interaction. In this respect we do not aim at modelling a *regressio ad infinitum* of rules of higher order, but just at shifting the emphasis on a more integrated modelling of strategic behaviour.

10 Concluding remarks

The main features of our model that we wish to emphasise are the following.

An equilibrium of the community (i.e., the State) emerges from a purely strategic interaction between self-seeking agents.

We reject the functionalist justification for the origin of the State (based on market failure arguments). We instead stress a notion of State originated from the voluntary interaction among decentralized and fully informed agents.

Our setting is more general than the one in which Nozick elaborated his theory of the minimal State. Indeed, while we proceed as Nozick assuming an identity of payoffs in the State (due to the FMC), we allow for different payoffs under anarchy. Moreover, we allow for different economic activities in the State. Such a difference, which is modelled as a difference in constitutional rules across productive sectors allows for a richer set of outcomes than in the Nozickian theory.

In most economic theories of the origin of the State a necessary and sufficient condition for the State to be created is that it brings about a Pareto-improvement upon anarchy. This is necessary, but by no means sufficient in our model where the FMC imposes the more restrictive condition of equal payoffs.

We claim that the FMC is an imperative condition to be met in the long run

whenever unlabelled agents end up with occupying different positions in the Stat
Notice, however, that our model can also accommodate the case of labelled agents.

While the anarchy-state choice typically reduces to a comparison between tv boxes in a prisoner's dilemma matrix, we have developed a fully-fledged model political and economic interaction. In the economic stage we consider both Bertra and Cournot rules, whereas a political regime is identified by a meta-constitution rule (e.g., lobbying) and a set of constitutional rules (e.g., majority needed to pa legislation).

For the various economic and political rules listed in Table 11, we focus on ho changes in meta-constitutional and constitutional rules affect the distribution of agen across sectors, the levels of political activity, and some indicators of individual a aggregate performance.

In this paper we do not need to specify whether agents are differentiated not in the "original position". Though undifferentiated agents are usually assumed anarchy, in our approach they can be endowed with different attributes (e.g., talent that are relevant for the undertaking of their political and economic activities. In th framework the FMC could be thought of as a long-run equilibrium condition.

Allowing for different types innovates significantly with respect to the tradition literature where agents in anarchy are assumed to be all alike. This assumption, whi stems from the influential debate on the ethical foundations of social contracts, typica materializes in the highly artificial device known as "veil of ignorance". While reckon that this is appropriate either when purely ethical issues are at stake or wh agents are identical, we believe it unacceptable when modelling heterogeneous a self-seeking agents.

We are aware that the short run implications in studying the origin of the Sta that is when no FMC operates, should also be studied as mobility across types cou take very long to come about. When no mobility is assumed, the problem of the origin of the State reduces to determine the circumstances under which the levels of fundamentals and the initial distribution of agents between types bring about Pareto improvements upon anarchy. However, in such a case every features of the model in terms of individual political and economic strategies as well as of individual and aggregate indicators of performances would be determined by the exogenous distribution of types. To make such analysis more interesting one needs to give the opportunity to subsets of agents in anarchy to form different States so that the number of States becomes the crucial equilibrium variable. Such a number of course would vary with differences in the constitutional and meta-constitutional rules operating in each State. This will be the object of a companion paper.

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Table 1 Proportion of agents in A: lobbying system in general

β\α	0	.02	.04	.06	.08	.10	.12	.14	.16
0	.50	.05	.10	.17	-	-	-	-	-
.02	.95	.50	.96	.96	.96	.96	.97	.97	.97
.04	.90	.04	.50	.92	.93	.93	.93	.93	.94
.06	.83	.04	.08	.50	.88	.89	.89	.90	.90
.08	-	.04	.07	.12	.50	.84	.85	.87	.87
.10	-	.04	.07	.11	.16	.50	.80	.82	.83
.12	-	.03	.07	.11	.15	.20	.50	.73	.77
.14	-	.03	.07	.10	.13	.18	.27	.50	.70
.16	-	.03	.06	.10	.13	.17	.23	.30	.50

Table 2 Proportion of agents in A: party system in general

δ\γ	0	.1	.25	.5	.75	1
0	.50	.58	.74	.87	1	-
.25	.50	.61	.80	.94	-	-
.5	.50	.64	.82	1	-	-
1	.50	.83	-	-	-	-
1.5	0	-	-	-	-	-
2	.50	.27	0	-	-	-
2.5	.50	.39	30	0	-	-
3	.50	.43	.36	.21	0	-
3.5	.50	.48	.39	.28	.16	0

Table 3 Proportion of agents in A: party system in comparison

ε\k	.01	.02	.04	.06	.08	.10
1	.50	.501758	.503591	.504987	.507141	.508088
2	.50	.501759	.503597	.505123	.507144	.506356
4	.50	.501771	.503606	504878	.507190	.506401
6	.50	.501773	.503612	.505593	.507272	.506475
8	.50	.501774	.503641	.505894	.507503	.506582
10	.50	.500937	.503531	.505128	.504710	.506735

Table 4 Proportion of agents in A: lobbying system in comparison

ε\k	.01	.02	.04	.06	.08	.10
1	.50	.089121	.080950	.076539	.071422	.050943
2	.50	.060018	.056810	.054356	.052546	.050940
4	.50	.040876	.039589	.038321	.037747	.037014
6	.50	.032801	.032038	.031495	.030916	.030459
8	.50	.028108	.027579	.271833	.026790	.026465
10	.50	.024957	.024559	.242782	.023957	.023707

Table 5 Proportion of agents in A: mixed political setting, lobbying in B

$\epsilon \backslash k$	.01	.02	.04	.06	.08	.10
1	.891126	.891050	-	-	-	-
2	.934208	.891140	-	-	-	-
4	.949112	.934152	.890976	.936694	-	-
6	.966145	.949019	.919793	.890766	.857314	.806134
8	.971177	.957069	.933929	.912497	.890392	.866023
10	.974485	.962257	.942653	.925121	.907938	.890147

Table 6 Proportion of agents in A: mixed political setting, lobbying in A

$\epsilon \backslash k$	.01	.02	.04	.06	.08	.10
1	.108874	.106491	.102223	.0984991	.0952074	.0922690
2	.0657922	.0648242	.063097	.0616113	.0603330	.0592354
4	.0508881	.0502664	.0492406	.0484736	.0479372	.0476115
6	.0338547	.0336464	.0336524	.0342275	.0354396	.0374509
8	.0288232	.0287901	.0293644	.0309155	.0338380	.0391194
10	.0255150	.0255150	.0268311	.0296854	.0356858	.0507205

Table 7 Individual payoffs under the party system

$\epsilon \backslash k$	.01	.02	.04	.06	.08	.10
1	.434444	.433405	.432326	.431872	.430249	.429690
2	.434444	.433404	.432326	.431795	.430246	430764
4	.434444	.433404	.432324	.431189	.430225	.430711
6	.434444	.433405	.432323	.431446	.430167	.430655
8	.434444	.433403	.432295	.431189	.430004	.430611
10	.434444	.433397	.432109	.430942	.429782	.430553

Table 8 Individual payoffs under the lobbying system

$\epsilon \backslash k$	.01	.02	.04	.06	.08	.10
1	.327483	.156133	.085786	.020768	−.0239222	−.448701
2	.370763	.193117	.149869	.194730	.0817421	.052369
4	.398028	.215435	.188662	.165844	.146299	.127980
6	.409022	.224177	.203897	.184381	.171772	.157873
8	.415204	.229013	.212350	.200783	.185931	.174495
10	419246	.233137	.217819	.207681	.195112	.185280

Table 9 Individual Payoffs: mixed political setting, lobbying in B

ε\k	.01	.02	.04	.06	.08	.10
1	.269924	.269675	-	-	-	-
2	.257530	.269766	-	-	-	-
4	.251785	.258006	.270284	.286971	-	-
6	.250361	.254866	.262837	.271127	.281173	.297651
8	.250380	.254013	.260151	.266039	.272329	.279530
10	.251248	.254332	.259404	.264077	.268787	.273805

Table 10 Individual Payoffs: mixed political setting, lobbying in A

ε\k	.01	.02	.04	.06	.08	.10
1	.269924	.259070	.238293	.218016	.198206	.178843
2	.257530	.247723	.229349	.212457	.197044	.183119
4	.251785	.245935	.234319	.228581	.228720	.234723
6	.250361	.245344	.245490	.259146	.286135	.483764
8	.250380	.249418	.265759	.305985	.369618	.480537
10	.251248	.255080	.292549	.366900	.476326	.612458

Table 11 The existence of equilibria

sector B\A	BL	BP	CL	CP
BL	NO(8)	NO(8)	NO(8)	NO(8)
BP	NO(8)	NO(8)	YES high ε (8)	YES high ε (8)
CL	NO(8)	YES high ε (8)	YES(4)	YES(6)
CP	NO(8)	YES high ε (8)	YES(6)	YES(5)

B,C,L,P read, respectively, Bertrand, Cournot, Lobbying and Party. Numbers in brackets indicate relevant Sections.

Table 12 Political activity in A under the party system

ε\k	.01	.02	.04	.06	.08	.10
1	.01	.01	.01	.01	.01	.01
2	.02	.02	.02	.02	.02	.02
4	.04	.04	.04	.04	.04	.04
6	.06	.06	.06	.06	.06	.06
8	.08	.08	.08	.08	.08	.08
10	.10	.10	.10	.10	.10	.10

Table 13 Political activity in B under the party system

ε\k	.01	.02	.04	.06	.08	.10
1	.01	.02	.04	.06	.08	.10
2	.02	.04	.08	.12	.16	.20
4	.04	.08	.16	.24	.32	.40
6	.06	.12	.24	.36	.48	.60
8	.08	.16	.32	.48	.64	.80
10	.10	.20	.40	.60	.80	1.00

Table 14 Political activity in the community under the party system

ε\k	.01	.02	.04	.06	.08	.10
1	.02	.03	.05	.07	.09	.11
2	.04	.06	.10	.14	.18	.22
4	.08	.12	.20	.28	.36	.44
6	.12	.18	30	.42	.54	.66
8	.16	.24	.40	.56	.72	.88
10	.20	.30	.50	.70	.90	1.10

Table 15 Political activity in A under the lobbying system

$\epsilon \backslash k$	.01	.02	.04	.06	.08	.10
1	.341995	1.07980	1.15129	1.19847	1.25153	1.26431
2	.271441	1.11549	1.15710	1.17334	1.21888	1.22199
4	.215443	1.14375	1.16839	1.83882	1.20612	1.21562
6	.188207	1.15706	1.17534	1.19003	1.20362	1.21298
8	.170997	1.16523	1.18067	1.19232	1.20315	1.21171
10	.158740	1.17092	1.18356	1.19158	1.20328	1.21023

Table 16 Political activity in B under the lobbying system

$\epsilon \backslash k$	.01	.02	.04	.06	.08	.10
1	.341995	.430886	.542883	.601533	.683990	.736806
2	.271441	.341995	.430886	.493242	.542830	.584803
4	.215443	.221521	.221632	.243150	.278494	.397939
6	.188207	.152738	.192337	.210765	.242143	.325430
8	.170997	.138772	.174215	.198643	.219379	.236266
10	.158740	.161437	.161393	.183482	.203259	.258036

Table 17 Political activity in the community under the lobbying system

$\epsilon \backslash k$	.01	.02	.04	.06	.08	.10
1	.341995	.487037	.588830	.668332	.720470	.766501
2	.271441	.386814	.471464	.543083	.575711	.612576
4	.215443	.257605	.267525	.285219	.315059	.428432
6	.188207	.183457	.221360	.241375	.269664	.352373
8	.170997	.168841	.201356	.223080	.246165	.262966
10	.158740	.174243	.185040	.214582	.239511	.25966

Table 18 Political activity in A: mixed political setting, lobbying in A

$\epsilon \backslash k$	.01	.02	.04	.06	.08	.10
1	.944895	.958939	.985449	1.01013	1.03328	1.05510
2	1.04923	1.05966	1.07890	1.09618	1.11164	1.12537
4	.988331	.996460	.893100	1.02088	1.02848	1.03317
6	1.13293	1.13746	1.24690	1.13760	1.09889	1.05919
8	1.14588	1.14676	1.13176	1.10796	1.13293	.934778
10	1.15381	1.14995	.955510	1.04304	.922576	.729811

Table 19 Political activity in B: mixed political setting, lobbying in B

$\epsilon \backslash k$	.01	.02	.04	.06	.08	.10
1	.939626	1.18994	-	-	-	-
2	1.04692	.944976	-	-	-	-
4	1.10916	1.04864	.944028	.825459	-	-
6	1.13290	1.08647	1.01195	.942818	.868411	.762573
8	1.14581	1.10742	1.04628	.993127	.940672	.866023
10	1.15378	1.11974	1.06744	1.02375	.980941	.939273

Table 20 Political activity in the community: mixed setting, lobbying in A

$\epsilon \backslash k$	.01	.02	.04	.06	.08	.10
1	.111785	.119988	.136646	.153586	.170759	.188126
2	.087698	.077407	.143027	.180132	.217412	.254496
4	.088259	.126067	.195945	.277849	.353912	.429995
6	.096322	.154233	.27365	.386609	.501929	.617227
8	.110722	.188408	.343785	.494660	.656663	.805269
10	.126890	.224360	.414904	.613148	.806217	.986540

Table 21 Political activity in the community: mixed setting, lobbying in B

$\epsilon \backslash k$	.01	.02	.04	.06	.08	.10
1	.112105	.147460	-	-	-	-
2	.087785	.138515	-	-	-	-
4	.085763	.143830	.245534	.335601	-	-
6	.096225	.169273	.301983	.423686	.535484	.631600
8	.110728	.200065	.367967	.524938	.672845	.808842
10	.126886	.234723	.438275	.631457	.816661	.993328

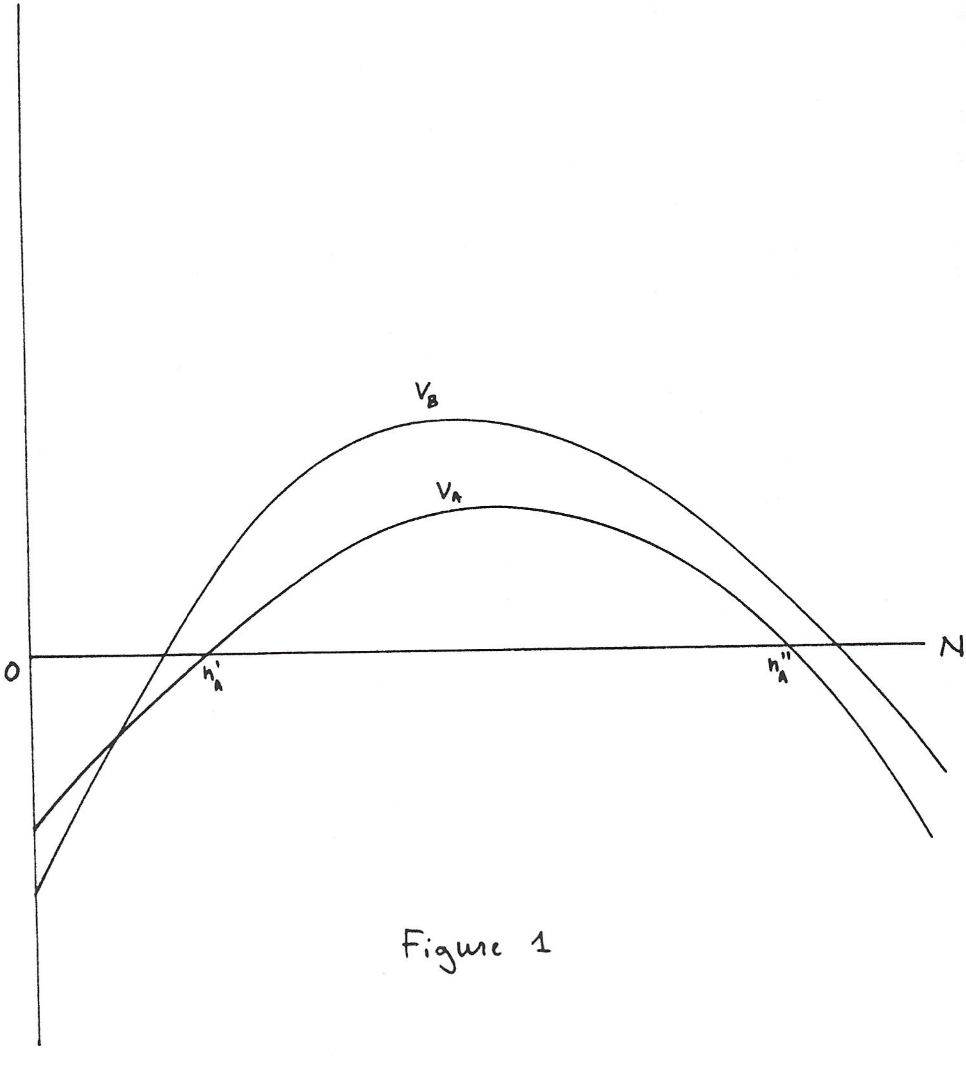
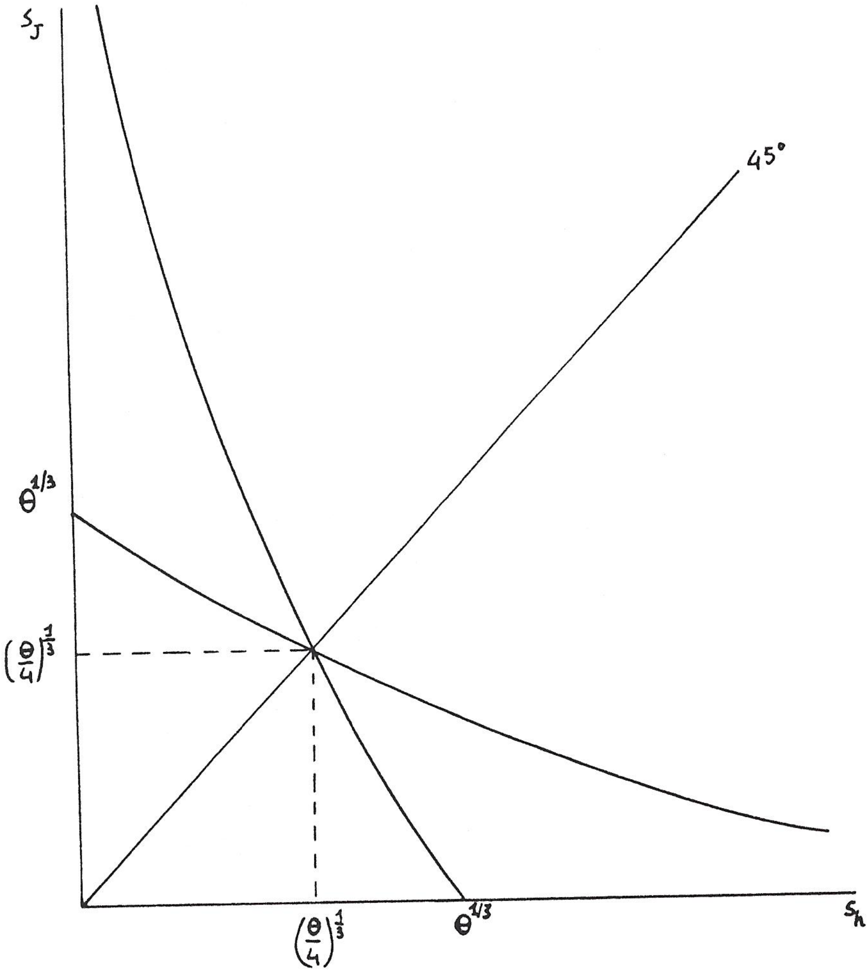


Figure 1



s_J
45°
$\theta^{1/3}$
$\left(\frac{\theta}{4}\right)^{\frac{1}{3}}$
$\left(\frac{\theta}{4}\right)^{\frac{1}{3}}$
$\theta^{1/3}$
s_h

Figure 2

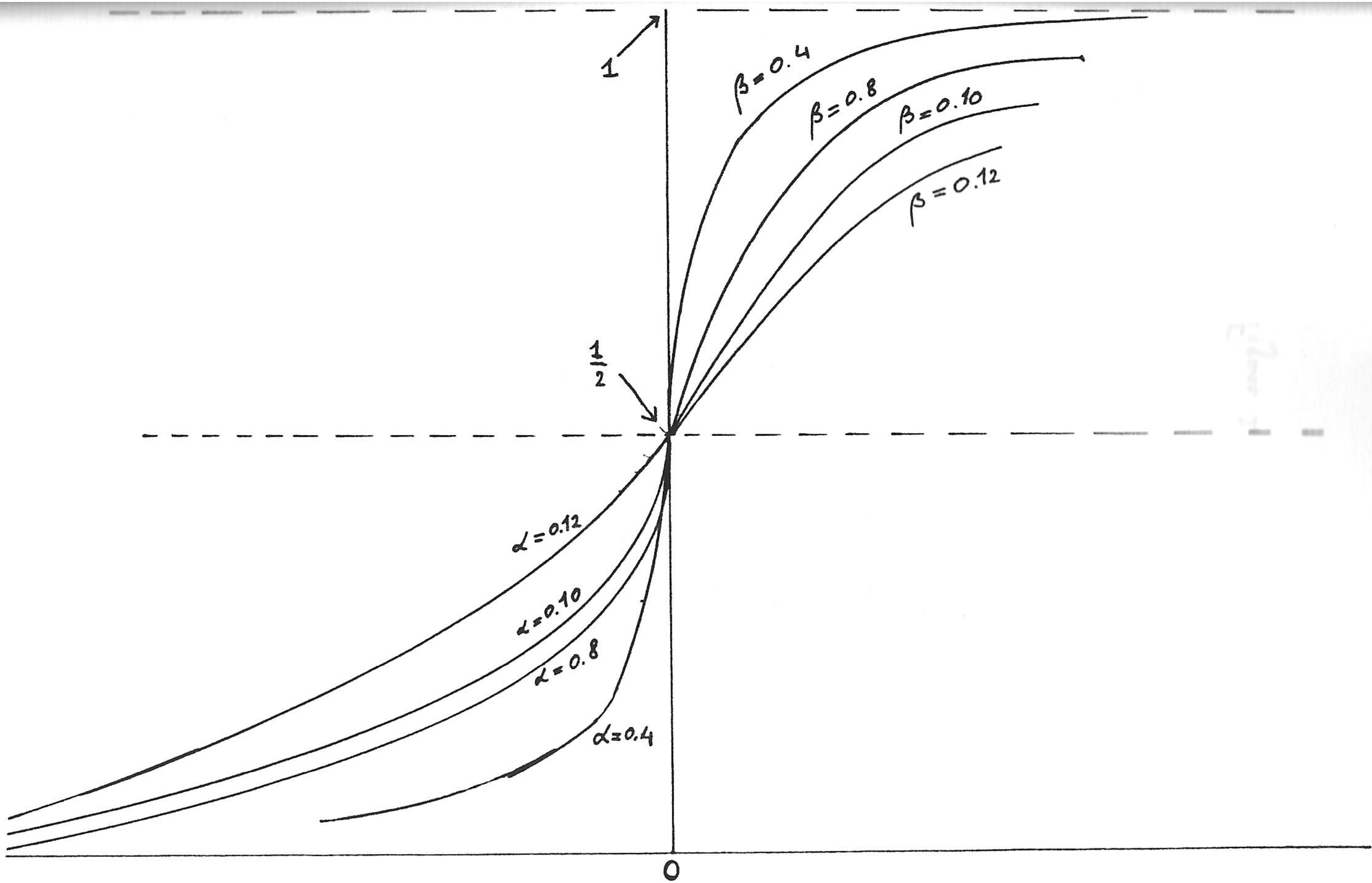
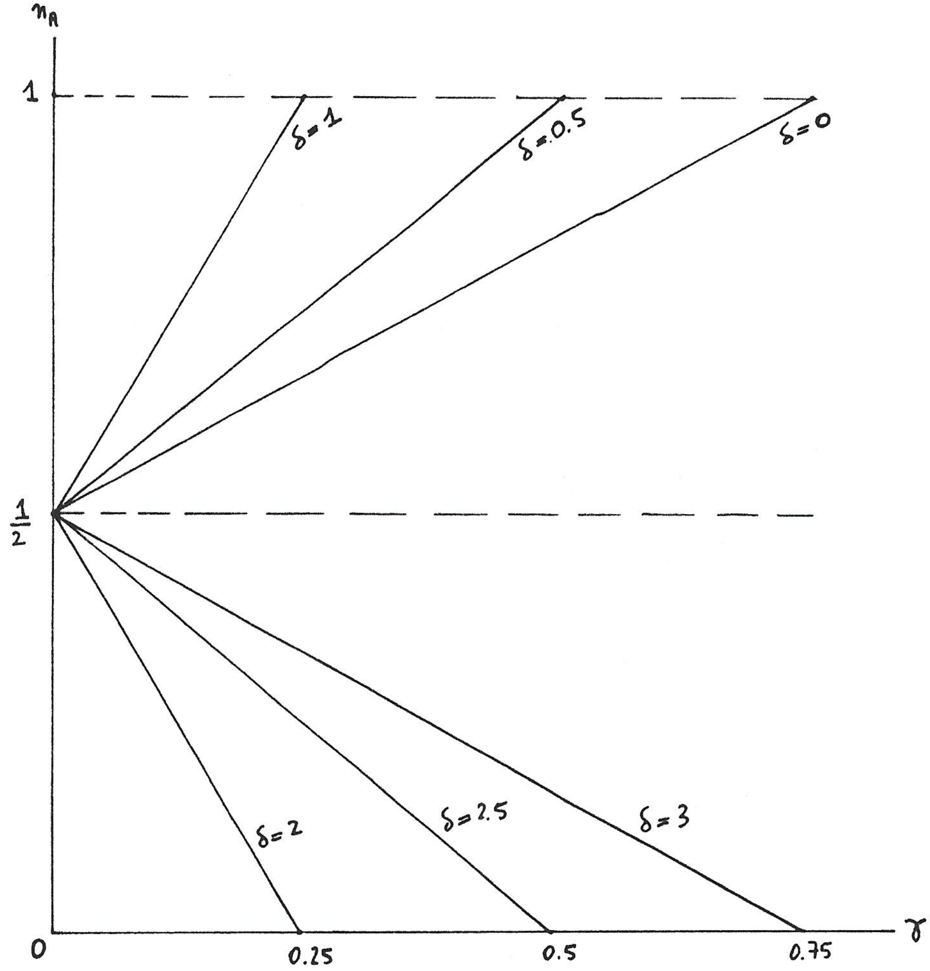


Figure 3



n_A
1
$\frac{1}{2}$
0
$\delta = 1$
$\delta = 0.5$
$\delta = 0$
$\delta = 2$
$\delta = 2.5$
$\delta = 3$
0.25
0.5
0.75
γ

Figure 4