Asymmetric Races of Research and Development

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ASYMMETRIC RACES OF RESEARCH AND DEVELOPMENT

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Abstract

In this paper we study a one-shot game of R & D between two price-setting firms that are asymmetrically placed as they produce at different cost levels. First we prove the existence and the properties of a noncooperative equilibrium. Then, we show that the higher (lower) the discount rate, the lower (higher) the probability of innovating of the current leader. In a specialised version of the model we establish the effect of the productivity of R & D expenditure, initial cost gap, and market size on the expected identity of the winner of the patent race.

1. Introduction

In this paper we analyse a game of Research and Development (R & D) for a cost reducing innovation between two technologically _asymmetric_ firms. A large part of the recent game theoretic literature on R & D has focused on symmetric games (see Reinganum 1984 for a survey of the early literature and our 1988 a,b papers). However, there are at least two strands of literature in which the problem of asymmetric R & D races arises.

The first one includes papers by Gilbert and Newbery (1982), Reinganum (1983), and the subsequent debate in the _American Economic Review_. This debate deals with the comparison of the incentives to obtain a patentable innovation of an incumbent firm and a potential entrant. The analysis aims at establishing which firm invests more in R & D.

Asymmetric games have also been investigated in models with a sequence of R & D races (e.g., Reinganum 1985, Vickers 1986, Beath _et al._ 1987). Since a race is a contest in which it is possible to distinguish sharply between the winner and the loser, after the first race firms will be in an asymmetric position even if they were symmetrically placed at the outset. In other words, the second race of a multistage model will necessarely be one in which there is a high cost and a low cost firm.

To the best of our knowledge, all the contributions dealing with asymmetric races of R & D have focused on two rather extreme cases. Either the race is modeled as a deterministic game, so that not only the type, but also the timing of the innovation is known at the outset, or the innovation is assumed to be drastic, that is so dramatic as to give the winner monopoly power. The first line of research has been pursued, e.g., by Gilbert and Newbery (1982), Dasgupta (1986), Vickers (1986) and Beath et al. (1987), while the second one has been explored, e.g., by Reinganum (1983) and (1985).

These two extreme hypotheses often lead to opposite results. In the incumbent vs. entrant debate, Gilbert and Newbery (1982) show that if the R & D race is deterministic, then the current monopolist will have a greater incentive to innovate than the challenger. On the other hand, Reinganum [1983] proves that in an uncertain environment, with a drastic innovation (so that the loser of the race makes zero profits) in equilibrium the challenger would invest more than the incumbent.

The reason why these hypotheses allow one to reach a definite conclusion may be explained in terms of what Fudenberg and Tirole (1986, p. 32) have christened efficiency effect and replacement effect. The former operates in favour of the firm whith the greatest difference in payoffs between winning the race and letting the rival win it. The latter operates against the firm which is currently making positive profits, when there is uncertainty on the timing of innovations: the existence of current positive profits induces the leading firm to reduce its effort so that the time of successful completion of the new technology is postponed. Since in Gilbert and Newbery's model the timing of innovations is fixed, there is no replacement effect; then, the asymmetric structure of firms' incentives gives a higher preemptive payoff to the incumbent than to the entrant. On the other hand, the drastic character of innovations implies that the efficiency effect in Reinganum's (1983) model is the same for the two firms, so that only the replacement effect operates.

The same argument explains the different results of Vickers (1986) and Reinganum (1985). In Vickers' model, there is no replacement effect because the timing of innovations is fixed. Since under Bertrand competition in the product market the efficiency effect always works in favour of the technologically leading firm, it follows that the leader will always win (Increasing Dominance). On the other hand, Reinganum's (1985) model can only accomodate the replacement effect, because she assumes drastic innovations; then, the efficiency effect is the same for both firms, and the replacement effect gives the challenger the greatest incentive to innovate.

In a more general setting, however, both effects must be taken into account. In this paper we shall show that under Bertrand competition in the product market the efficiency effect and the replacement effect go in opposite directions. As a consequence, generally speaking it is not possible to unambiguously identify the expected winner of the R & D race. Nevertheless, we are able to find out which economic variables are responsible for the prevalence of either effect.

Specifically, in a general setting we show in section 2 that the higher the discount rate, the more likely the replacement effect dominates. Hence, the higher the discount rate, the lower chance has the current leader to win the technological race. To proceed further, we specialise the model by assuming a specific well-behaved hazard function (section 3) and a linear demand function (section 4). We show that a higher productivity of R & D expenditure increases the probability that the current leader invests more than the challenger; the same conclusion follows from an increase in the size of the market and the initial cost gap. On the other hand, the effect of an increase in the size of the innovation (as measured by the difference between the current lowest cost and the post innovation cost) on the two competing firms' efforts in R & D is ambiguous. Section 5 contains some concluding remarks. Finally, some technical issues are relegated to three appendices.

2. The asymmetric R & D game under Bertrand competition: the general case

In this section we study a R & D race between two price setting firms which are asymmetrically placed as far as the technology is concerned. We assume that firms compete in prices in a homogeneous product market, so that either the market is monopolised or a Bertrand equilibrium is established.

Production takes place under constant returns to scale. Let B be the low cost firm and A be the high cost firm. Denote by c_A and c_B the constant marginal and average costs of the two firms, with $c_B < c_A$. Let π_B be B's current profit; obviously A's current profit is zero.

Let us denote by $p_M(c)$ the monopoly price associated with a constant marginal and average cost c. If

$$p_M(c_B) \leq c_A \qquad (1)$$

then B is a monopolist, A is a potential entrant and the equilibrium price is $p_M(c_B)$. On the other hand, if

$$p_M(c_B) > c_A \qquad (2)$$

then B is a Bertrand leader, A is the inactive firm in the asymmetric Bertrand equilibrium, and the equilibrium price is c_A. Notice that in both cases A's profits are null, while B's profits are positive. Obviously, B's profits are larger in case (1) than in case (2).

Besides competing in the product market, firms compete for a single patentable innovation; the winner of the patent race will get the exclusive right to pruduce forever at a cost level $c^* < c_B$. We assume through the paper that

$$p_M(c^*) > c_B \qquad (3)$$

that is, the innovation is not drastic in the sense that if the initially less efficient firm wins the technological race, it will not monopolise the market. On the other hand, if B wins the race, three cases are possible, i.e.:

(i) (1) holds so that B <u>remains</u> a monopolist;

(ii) (2) holds, and $p_M(c^*) \leq c_A$, so that B <u>becomes</u> a monopolist;

(iii) (2) holds, and $p_M(c^*) > c_A$, so that B <u>remains</u> a Bertrand leader, while increasing its technological lead.

In the post-innovation equilibrium (monopoly or Bertrand equilibrium), if B has innovated its profit will be π_B^* per unit of time and A will get nothing forever; if A has innovated, its profit will be π_A^* per unit of time and B will receive nothing forever. In all the three cases above, $\pi_B^* > \pi_A^*$, provided that the marginal revenue curve is decreasing (see Appendix 1).

We assume that the timing of the innovation is uncertain, and that each firm's probability of innovating is an increasing exponential function of its R & D expenditure (see Reinganum 1984 for details). As far as the R & D process is concerned, the two firms share the same technology. For sake of simplicity, we assume that the hazard function $h(x_i)$, $i = A,B$, is strictly concave and satisfies the following conditions

$$h(0) = 0 \qquad (4.i)$$

$$\lim_{x_i \to \infty} h'(x_i) = 0 \qquad (4.ii)$$

$$\lim_{x_i \to 0} h'(x_i) = \infty \qquad (4.iii)$$

These conditions guarantee that the firms' maximisation problem will always yield an interior solution, and that the second order conditions are satisfied.

Let x and y be the R & D expenditure of firms A and B, respectively. We assume that R & D costs are non contractual as in Lee and Wilde [1980]. Firms noncooperatively choose the R & D expenditure in order to maximise the discounted value of expected profits net of R & D cost. Thus, A's payoff is

$$W_A = \frac{h(x)\pi_A^*/r - x}{r + h(x) + h(y)} \qquad (5)$$

and B's payoff is

$$W_B = \frac{h(y)\pi_B^*/r + \pi_B - y}{r + h(x) + h(y)} \qquad (6)$$

where r is the discount rate. Differentiating (5) and (6) with respect to x and y, respectively, we get

$$\frac{\delta W_A}{\delta x} = \frac{h'(x)\pi_A^* + h(y)h'(x)\pi_A^*/r - r - h(x) - h(y) + h'(x)x}{[r + h(x) + h(y)]^2} \qquad (7)$$

and

$$\frac{\delta W_B}{\delta y} = \frac{h'(y)(\pi_B^* - \pi_B) + h(x)h'(y)\pi_B^*/r - r - h(x) - h(y) + h'(y)y}{[r + h(x) + h(y)]^2} \qquad (8)$$

The equilibrium R & D expenditures x^* and y^* are therefore the solutions of the following system

$$h'(x)\pi_A^* + h(y)h'(x)\pi_A^*/r - r - h(x) - h(y) + h'(x)x = 0 \qquad (9)$$

$$h'(y)(\pi_B^* - \pi_B) + h(x)h'(y)\pi_B^*/r - r - h(x) - h(y) + h'(y)y = 0 \qquad (10)$$

It can be shown [1] that the hypothesis $h'' < 0$ implies that $\delta f/\delta x < 0$ and $\delta g/\delta y < 0$, where f and g denote the L.H.S. of (9) and (10), respectively.

As a preliminary result, we show that an equilibrium of the R & D game exists.

Proposition 1. An equilibrium x^*, y^* exists in which both $x^*, y^* > 0$.

Proof. See Appendix 2.

We are interested in analysing which firm makes the largest R & D effort. Diagrammatically, we want to establish under what conditions the intersection of the reaction curves lies above or below the 45° straight line.

A quick inspection to (9) and (10) reveals that two opposite forces are at work. Firstly, there is the efficiency effect, that is the difference in the payoff of a firm between winning the race and letting the rival win it. In the present model, this gives a greater incentive to the current low cost firm than to the currently high cost firm, since $\pi_B^* > \pi_A^*$. Secondly, current profits reduce B's incentive as π_B enters with a negative sign in expression (8) which implicitly defines B's incentive to innovate. This is the so-called replacement effect. As Fudenberg and Tirole [1986] argue, generally speaking either effect may dominate, that is, the efficiency effect, measured by $(\pi_B^* - \pi_A^*)$, may be greater or lower than π_B, which measures the replacement effect.

Another way to look at the incentives to innovate of the two firms is to consider the first two terms which appear on the L.H.S. of (9) and (10). In this way, one may distinguish two notions of incentives: the first one is the difference between the flow of profits accruing forever to the winner and those accruing to the losers; the second one is the difference between the prospective profits to the winner and his current profit. The former (i.e., $\pi_B^* - \pi_A^*$) reflects the presence of rivalry in the technological competition: B anticipates that, should it fail to innovate, A would succeed and gain a technological lead. Obviously, an analogous reasoning would apply to A. Thus, this effect characterises situations of strategic interaction. Secondly, the differences $(\pi_B^* - \pi_B)$ and $(\pi_A^* - \pi_A)$ measure the incentives to invest in R & D irrespective of the presence of rivals. This is the only incentive taken into account in the decision theoretic approach to R & D. Clearly, under Bertrand competition in the product market, our factorisation of the incentives coincides with that of Fudenberg and Tirole (1986).

First of all, we shall show that the equilibrium R & D expenditure of each firm is positively related to both its incentives.

Proposition 2. The equilibrium R & D investment y (resp., x) is an increasing function of π_B^* (resp., π_A^*) and $(\pi_B^* - \pi_B)$ (resp., $(\pi_A^* - \pi_A)$).

Proof. Implicitly differentiating the equilibrium condition (10) one gets:

$$\frac{\delta y^*}{\delta \pi_B^*} = - \frac{h(x)\ h'(y)}{r(\delta g/\delta y)} > 0 ,$$

and

$$\frac{\delta y^*}{\delta(\pi_B^* - \pi_B)} = - \frac{h'(y)}{\delta g/\delta y} > 0 ,$$

where we have used the inequality $\delta g/\delta y < 0$. Analogously,

$$\frac{\delta x^*}{\delta \pi_A^*} = - \frac{h(y)\ h'(x)}{r(\delta f/\delta x)} > 0 ,$$

by the inequality $\delta f/\delta x < 0$. (Notice that $\pi_A = 0$.) ∎

This result implies that in order to compare the effects on R & D investment of different initial costs one can simply focus on the comparison between the incentives. That is, one has to compare π_B* and π_A* on the one hand, and $(\pi_B* - \pi_B)$ and π_A on the other hand. We already know that $\pi_B* > \pi_A*$. We show in Appendix 3 that $\pi_A* > (\pi_B* - \pi_B)$, in all the three cases we have distinguished above. Thus, the two effects go in opposite directions. As a consequence, generally speaking one cannot unambiguously establish the identity of the firm which has the greatest incentive to innovate.

At this level of generality, the discount rate r is the only parameter of the model. We now show that r may be responsible for one effect to prevail over the other one. More precisely, the relative strength of the two effects is a strictly monotonic function of r.

Proposition 3. There exists a unique $\hat{r} > 0$ such that, if $r > \hat{r}$ then $x* > y*$, whereas if $r < \hat{r}$ then $y* > x*$.

Proof. Let us suppose that $x = y$. Inserting this condition into equation (10) yields a unique value of x and y, say z*. Diagrammatically z* is the abscissa of the intersection point of A's reaction curve and the 45° line (see figure 2). Clearly, if it turns out that $\delta W_B/\delta y$, evaluated at $x = y = z*$, is positive, then it follows that $y* > x*$; if it turns out that $\delta W_B/\delta y$, evaluated at $x = y = z*$, is negative, then it follows that $y* < x*$. Proceeding in this way we get

$$\left.\frac{\delta W_B}{\delta y}\right|_{\substack{x=y \\ \delta W_A/\delta x=0}} = (\pi_B^* - \pi_B - \pi_A^*) + h(x)(\pi_B^* - \pi_A^*)/r \qquad (11)$$

Clearly, the R.H.S. of (11) is a strictly decreasing function of r, which tends to $(\pi_B^* - \pi_B - \pi_A^*) < 0$ as r goes to infinite, and tends to infinite as r goes to zero. (Notice that x decreases with r, and h is an increasing function of x.) This suffices to prove the Proposition. ∎

The economic intuition behind Proposition 3 is that a large discount rate reduces the present value of post-innovation profits, and thus makes the innovation less attractive to both firms. As a consequence, the efficiency effect becomes less important and the replacement effect ends to prevail.

3. Parametrisation of the model I: the hazard function

In this section we specialise the model presented in section 2 by assuming the following well-behaved hazard function:

$$h(z) = 2\mu\sqrt{z} \qquad z = x, y \qquad (12)$$

where $\mu > 0$ is a parameter measuring the productivity of R & D expenditure.

It can be shown that the effects of μ on the firms' incentives to innovate are exactly opposite to those of the discount rate r. Actually, under this parametrisation of the hazard function, it turns out that a single parameter, i.e. the ratio $\theta = \mu/r$, captures the role of both the productivity of R & D expenditure and the discount rate.

Proposition 4. There exists a unique $\theta\hat{} > 0$ such that, if $\theta > \theta\hat{}$ then $x^* > y^*$, whereas if $\theta < \theta\hat{}$ then $y^* > x^*$.

Proof. Proceeding as in the proof of Proposition 3, using (12), we get

$$\left.\frac{\delta W_B}{\delta y}\right|_{\substack{x=y \\ \delta W_A/\delta x=0}} = (\pi_B^* - \pi_B - \pi_A^*) + 2\theta\sqrt{z^*}(\pi_B^* - \pi_A^*) \qquad (13)$$

where z^* is the positive solution of the following equation (obtained by setting $x = y$ in equation (9))

$$\mu\pi_A^*/\sqrt{z} + 2\mu^2\pi_A^*/r - r - 3\mu\sqrt{z} = 0$$

or

$$3\theta z - (2\theta^2\pi_A^* - 1)\sqrt{z} - \theta\pi_A^* = 0 \qquad (14)$$

This equation has two roots of opposite sign. Clearly, only the positive solution is economically meaningful. Differentiating (14) with respect to θ we get

$$\frac{d\sqrt{z^*}}{d\theta} = -\frac{3z - 4\theta\pi_A^*\sqrt{z} - \pi_A^*}{6\theta\sqrt{z} - (2\theta^2\pi_A^* - 1)} \qquad (15)$$

The denominator of (15), evaluated at $z = z^*$, is positive. Using (14), the numerator of (15) reduces to $[-2\theta\pi_A^* - 1/\theta]\sqrt{z} < 0$. It follows that $d\sqrt{z^*}/d\theta > 0$. Clearly, then, $\theta\sqrt{z^*}$ is an increasing function of θ, which tends to 0 as θ goes to 0 and tends to infinite as θ goes to infinite. By (13), this suffices to prove Proposition 4. ∎

4. Parametrisation of the model II: the hazard function and the demand function

In this section we further specialise the model presented in section 2 by assuming (12) and the following linear market demand function

$$p = a - q \quad , \quad a > 0 \qquad (16)$$

Let us define the following strictly positive parametrs:

$s = a - c_A$ is the size of the market,

$m = c_A - c_B$ is initial cost gap,

$n = c_B - c^*$ is the cost improvement (strictly speaking n measures the cost improvement as of firm B, whereas for firm A the cost improvement would be measured by $m + n$).

We mantain that, even winning the patent race, A cannot gain a monopolistic position (see (3)). In the present setting this condition is equivalent to

$$s > m + n \qquad (17)$$

In what follows we shall focus only on case (iii) defined in section 2. Under the parametrisation of the demand function (16), A's profits in case he wins the race (so that he becomes a Bertrand leader) are

$$\pi_A* = n(s + m) \qquad (18)$$

On the other hand, B's profits (if B is a Bertrand leader both before and after the innovation) are

$$\pi_B = ms \qquad (19)$$

$$\pi_B* = s(m+n) \qquad (20)$$

Plugging (16), (17) and (18) into (11), we get the following expression

$$\left.\frac{\delta W_B}{\delta y}\right|_{\substack{x=y \\ \delta W_A/\delta x=0}} = [s(m+n) - ms - n(s+m)] + \frac{2\mu\sqrt{z*}}{r}[s(m+n) - n(s+m)]$$

which can be written as

$$\left.\frac{\delta W_B}{\delta y}\right|_{\substack{x=y \\ \delta W_A/\delta x=0}} = - mn + \frac{2\mu\sqrt{z*}}{r} m (s-n) \qquad (21)$$

where z* is the solution of

$$\frac{\mu}{\sqrt{z}} n(s+m) + \frac{2\mu^2 n(s+m)}{r} - r - 3\mu\sqrt{z} = 0 \qquad (22)$$

Let us define $\beta = n(s+m)$. Implicitly differentiating (22) with respect to β we get

$$\frac{dz*}{d\beta} = \frac{2(2\mu\sqrt{z}/r + 1)}{\beta/z + 3} > 0 \qquad (23)$$

From (23) it follows that z* is an increasing function of s, m, and n. Hence we have the following Proposition.

Proposition 5. For s and/or m large enough, the current leader has a greater incentive to innovate (hence, a greater probability to win the patent race) than the follower.

Proof. Clearly, the R.H.S. of (21) is an increasing function of s, both directly and via z*. It is apparent that the R.H.S. of (21) increases at a greater speed than s, and therefore will become positive for s large enough.

As far as m is concerned, let us divide the R.H.S. of (21) by m. The resulting expression, i.e. $[-n + 2\mu\sqrt{z*}(s-n)/r]$, depends on m via z*. We have shown that z* is an increasing function of m; furthermore, since (22) is quadratic in $\sqrt{z*}$, it is clear that $\sqrt{z*}$ goes to infinite as m goes to infinite. It then follows that for m large enough, the R.H.S. of (21) is positive. ▪

Notice that the effect of n (i.e., the cost improvement) is ambiguous, since an increase in n reduces the first term on the R.H.S of (21), and has two opposite effects on the second term: one is direct and negative, the other is positive via z^*.

The content of Proposition 5 is not obvious. Indeed, an increase in the size of the market s has two contrasting effect in that it affects both the pre-innovation and the post-innovation profits. More precisely, an increase in s increases the current profits of the leader, thus enhancing the replacement effect; on the other hand, it increases the post-innovation profits of the winner. From (18) and (20) it is clear that the effect on post-innovation profits is stronger for the leader than for the follower. Proposition 5 shows that the net effect of the variations in the replacement and the efficiency effect favours the leader. Similar arguments exlain the effects of an increase in m.

5. Concluding remarks

In this paper we studied an asymmetric race of R & D between two technologically asymmetric price-setting firms. In section 2 we proved a sufficiently general result about the role of the discount rate on the expected identity of the winner of the patent race. The intuition about that result (Proposition 2) relies upon the distinction between the replacement effect and the efficient effect and their relationship with the incentives to innovate. The effect of other economic parameters has been shown within a specialised version of the model in sections 3 and 4.

Among the extensions of our model, it is worth mentioning the case in which firms are quantity-setting oligopolists in the product market so that in the resulting equilibrium both rivals make positive (although different) profits.

Appendix 1

In this Appendix we show that $\pi_B* > \pi_A*$. This is obvious when B is a monopolist in the post-innovation equilibrium (cases (i) and (ii)). If B remains a Bertrand leader (case (iii)), its profits will be

$$\pi_B* = (c_A - c*)\ q(c_A)$$

whereas A's profits in case A wins the race will be

$$\pi_A* = (c_B - c*)\ q(c_B)$$

where q(.) denotes the demand function. Now, if $p_M(c*) > c_A$, then B's profits if B wins are given by the difference between the areas of the regions ABC and CDE, while A's profits in case A wins are given by the difference between the areas of the regions ABC and CFG (see figure 1).

If the marginal revenue curve is downward sloping, then the difference between the two areas is clearly positive. (Actually, it would suffice that the marginal revenue curve cuts the post-innovation marginal cost curve from above only once.)

(figure 1 here)

Appendix 2

In this Appendix we prove Proposition 1. Let us consider the two equilibrium conditions (9) and (10). They implicitly define the best responses of firms A and B, respectively. We now investigate the properties of these reaction curves. To begin with, let us consider firm A's reaction curve. When $y = 0$, (9) reduces to

$$h'(x)\pi_A^* - h(x) + h'(x)x = r \qquad (A.1)$$

Let us denote by $H(x)$ the L.H.S. of (A.1). By (4.i)-(4.iii) it follows that $H(0) = \infty$. By the concavity of h, $H'(x) = h''(x)(\pi_A^* + x) < 0$. By the concavity of h again, it follows that $h(x) - h'(x)x > 0$, which together with (4.ii) implies $H(\infty) < 0$. Hence we may conclude that equation (A.1) has a unique positive solution x^0.

Next, implicitly differentiating equation (9) we get

$$\frac{dx}{dy} = -\frac{\delta f/\delta y}{\delta f/\delta x} \quad \alpha \ \delta f/\delta y = h'(y)[h'(x)\pi_A^*/r - 1]$$

It follows that there exists a unique $x^\wedge > 0$ such that

$dx/dy > 0$ iff $x < x^\wedge$, and $dx/dy = 0$ iff $x = x^\wedge$

The critical value $x^\wedge$ is implicitly defined by $h'(x^\wedge) = r/\pi_A^*$. By the concavity of h, it follows easily that $x^\wedge > x^0$. Furthermore, since f is a strictly monotonic function of x, there is a unique best response of A to any strategy y chosen by B. Then, the reaction function must be a continuous, strictly increasing function which tends to infinite as x goes to $x^\wedge$.

Obviously, a similar reasoning applies to B's best response (see figure 2). Hence, an equilibrium exists in which both x and y are strictly positive [2].

(figure 2 here)

Appendix 3

In this Appendix we show that $\pi_A* > \pi_B* - \pi_B$. We distinguish between cases (i), (ii), and (iii).

Case (i). In this case, the difference $\pi_A* - (\pi_B* - \pi_B)$ is the shaded area in figure 3.

(figure 3 here)

Case (ii). In this case B is a Bertrand leader before the innovation, but becomes a monopolist after the innovation. Hence, the following inequalities hold:

$p_M(c_B) > c_A$, (B is not a monopolist before the innovation)

$p_M(c*) > c_B$, (the innovation is non-drastic)

$p_M(c*) \leq c_A$. (B becomes a monopolist winning the patent race)

If $p_M(c*) = c_A$, the result follows as in case (iii) below. Let us now suppose that c* falls. If the marginal revenue curve is downward sloping, then $p_M(c*) < c_A$. Clearly, π_B is independent of c*. Then, the result follows if π_A* grows more quickly than π_B* as c* decreases. Now,

$$\frac{d\pi_A*}{dc*} = - q(c_B)$$

because A becomes a Bertrand leader in case he wins the patent, so that his profits will be $(c_B - c*)\, q(c_B)$. Moreover,

$$\frac{d\pi_B*}{dc*} = - MR^{-1}(c*)$$

where MR^{-1} is the inverse marginal revenue function. Since by hypothesis $c_B < p_M(c*)$, we have $q(c_B) > MR^{-1}(c*)$. This completes the proof in case (ii).

Case (iii). In this case, the difference $\pi_A* - (\pi_B* - \pi_B)$ is the shaded area in figure 4.

(figure 4 here)

FOOTNOTES

(1) Indeed

$$\delta f/\delta x = h''(x)\pi_A^* + h''(x)h(y)\pi_A^*/r + h''(x)x$$

which is clearly negative if $h'' < 0$. By the same token,

$$\delta g/\delta y = h''(y)(\pi_B^* - \pi_B) + h''(y)h(x)\pi_B/r + h''(y) < 0.$$

(2) Further conditions on the third derivative of the hazard function h would be required to assure uniqueness of the equilibrium.

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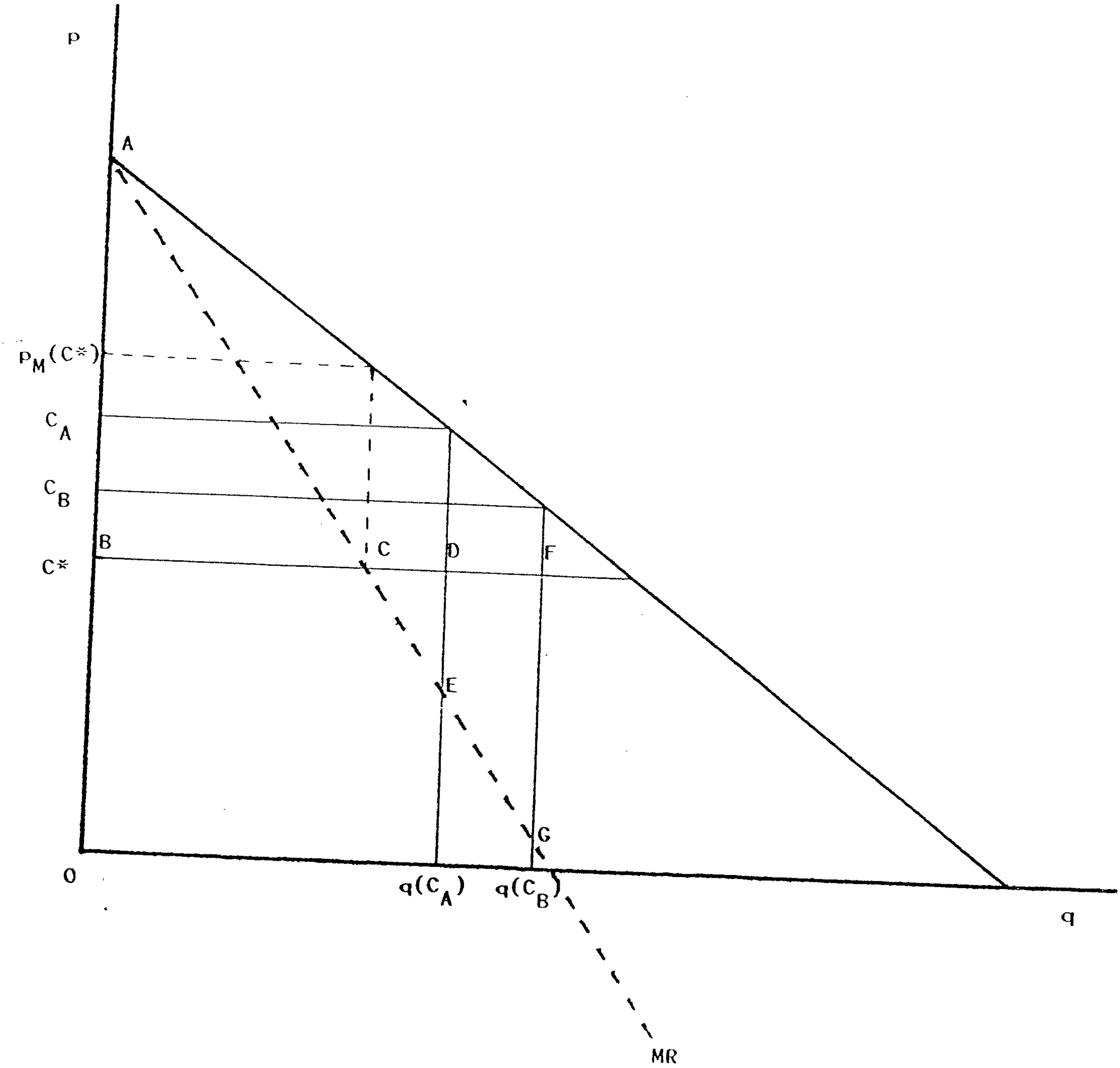

FIGURE 1

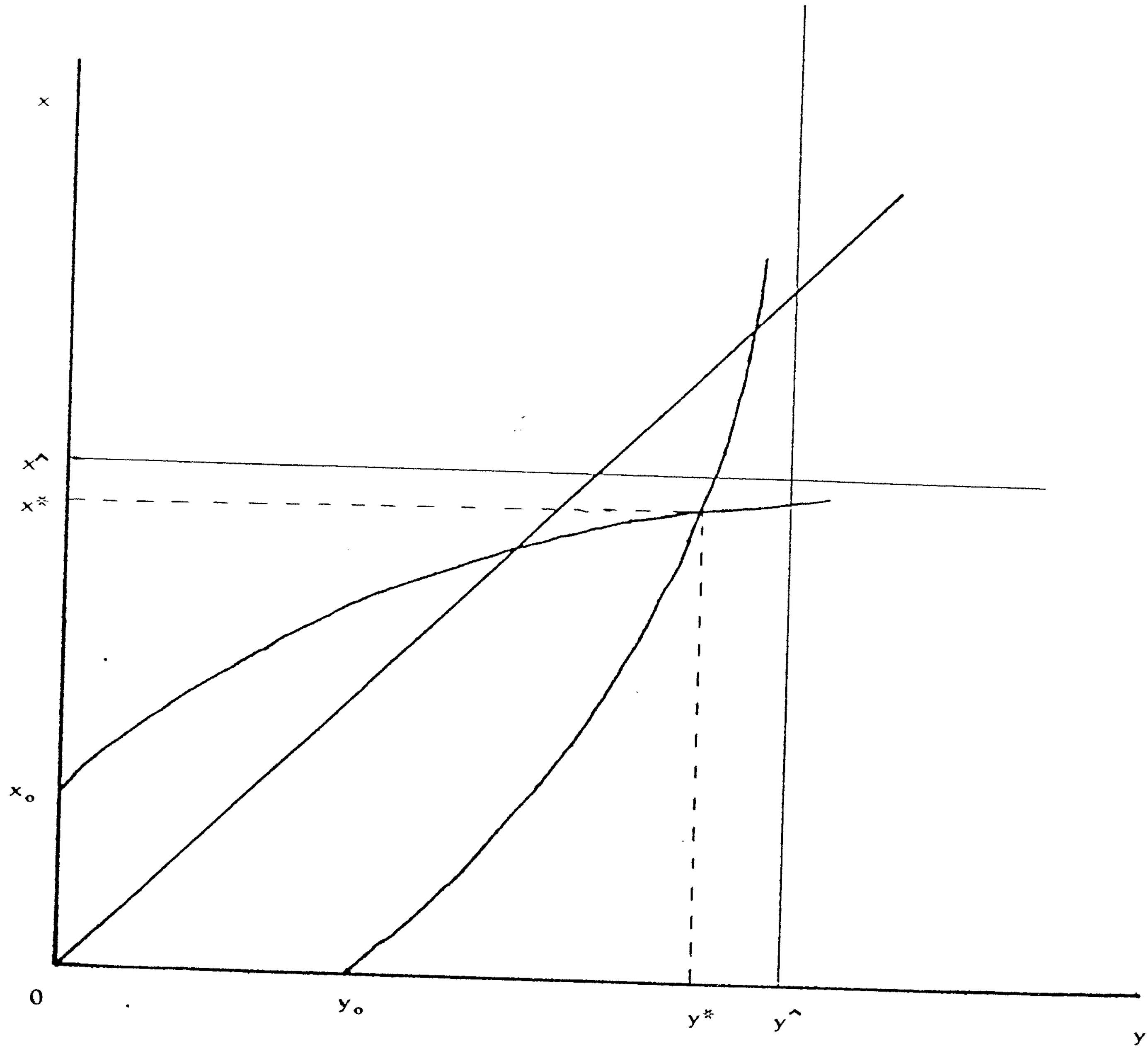

FIGURE 2

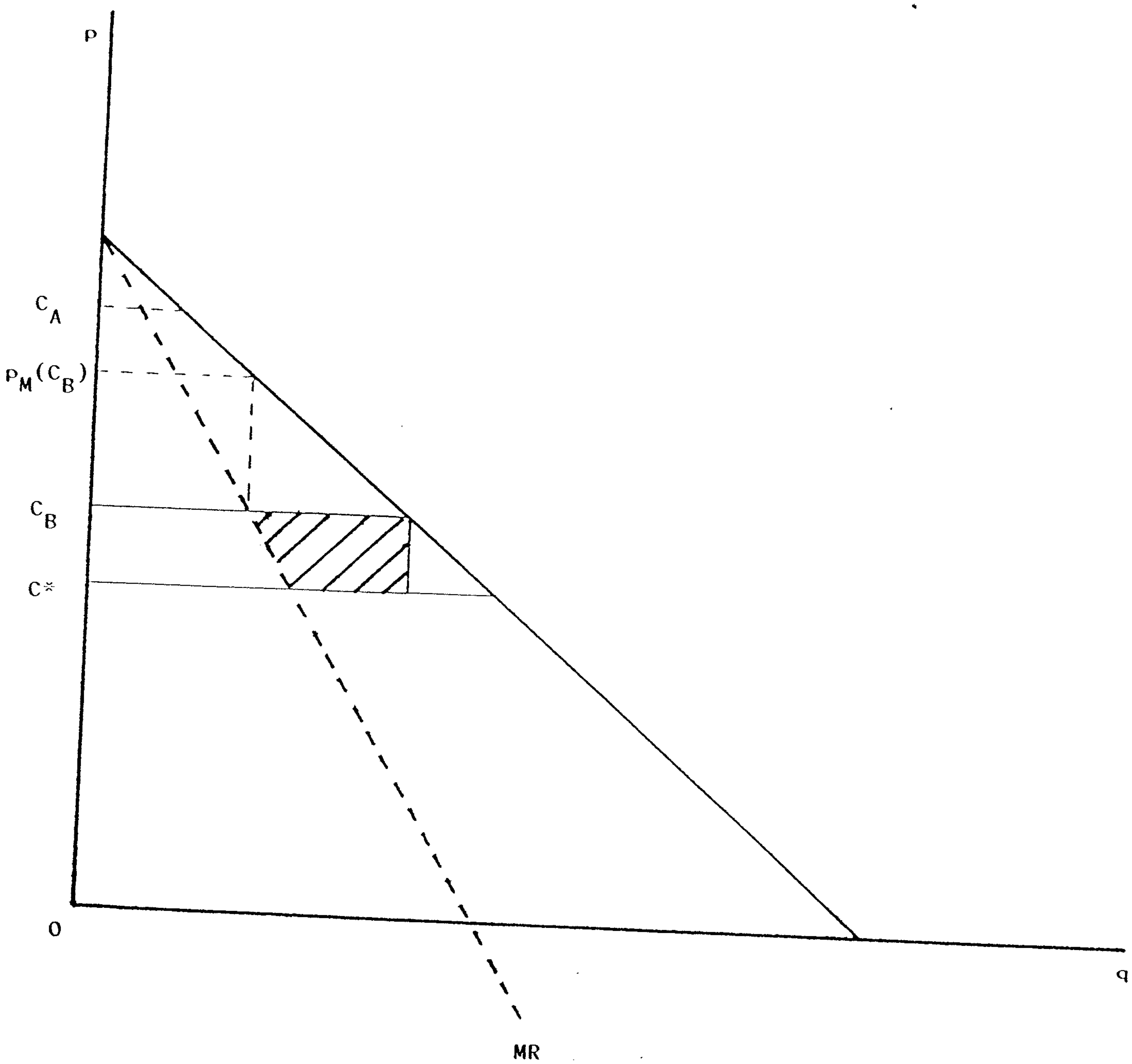

FIGURE 3

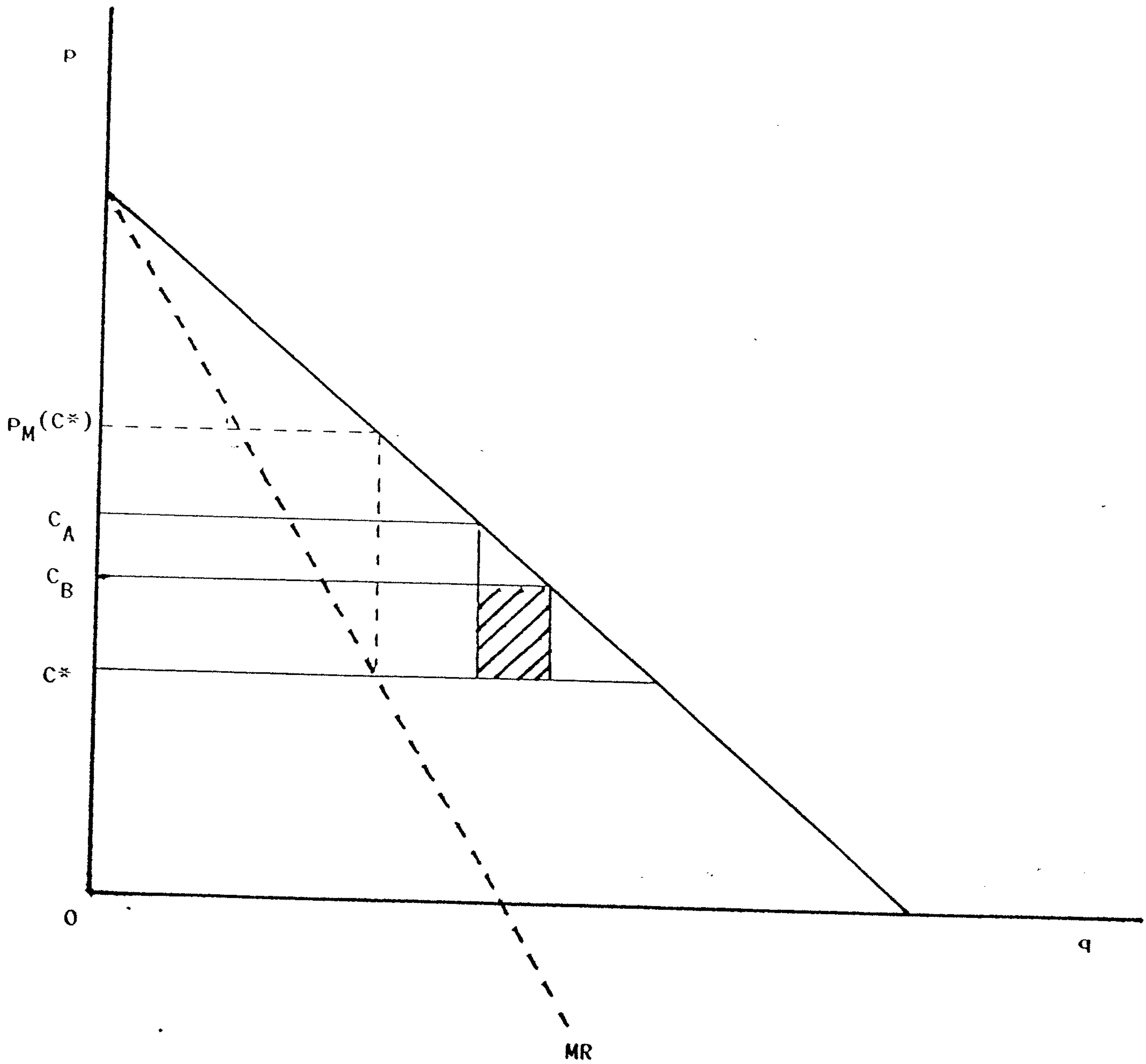

FIGURE 4