Long Run Growth and Income Distribution in an OLG Model with Strategic Job-Seeking and Credit Rationing

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Abstract

In the human capital literature, it is usually assumed that human capital is paid according to its marginal productivity. Nevertheless, in the real world labor compensation is linked to a fixed hierarchy due to the division and organization of labor. Access to privileged positions in the hierarchy depends on schooling credentials, which in turn are a function of individual learning abilities and of individual spending in education. People compete in education in order to achieve the best job positions: positional competition is like a rent-seeking activity, based on the different levels of credentials. In this paper, a simple OLG economy with two agents and two kinds of jobs is modeled, and the strategic solutions are analyzed. The model shows different outcomes depending on the hypotheses regarding the type of strategic interaction (sequential or simultaneous) and the characteristics of the capital market. In the sequential equilibrium, the presence of credit market imperfections and risk-aversion makes the asympthotic wealth distribution dependent on initial conditions (non ergodicity). In the simultaneous equilibrium, a non monotonic relationship between income inequality and long run growth is shown; in the long run, job allocation is mainly determined by the innate learning abilities and it is unrelated to the initial wealth distribution (ergodicity).

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1 Introduction

Recent literature on growth theory has renewed economists' interest in the decisions of investment in education and accumulation of human capital, underlining the relationships between distribution of income, human capital and economic growth.[1] From the point of view of a single agent, investing in education is mainly a way to signal his individual ability; that is each subject invests in education in order to get a better job.[2] If individual human capital is not transferable and the capital market is imperfect, the amount of inherited wealth is crucial in establishing the investment in education. Since education represents the main way to better one's economic condition[3], a complex relationship emerges between educational choices and the dynamics of income distribution. In this paper, we aim to investigate this relationship by means of a model of positional competition on the labour market[4]. According to Hirsch (1976), agents try to acquire credentials by investing in education, to distinguish themselves from others and to get a privileged role in the organization of production (and therefore a higher income or more social prestige[5]). If the organization of roles is fixed and, with the development of the economic system, the number of the privileged roles does not increase at the same pace as the number of agents investing in education, then, as time passes, an increasing level of investment in education is required to get the same socio-economic roles (the quantity of "intermediate" good increases without any change in the final output: an inefficient outcome). As we know, in the real world the number of high paying (high prestige) positions does not increase with the number of individuals with a high degree of education[6]. This yields to a competition among individuals in order to get such positions, competition which takes place through the investment in education. Such an investment, independently from the increase of productivity that it might involve, acts like a signal.

Although each agent competes against an indefinite number of subjects (all the potential aspirants for the same role), it is possible to assume that in a labour market characterized by a fixed number of high-paying positions, the behavior of agents should be strategic. It would be useless for a poor agent to invest money and time (which also involves an opportunity cost equal to the time spent without working as an unskilled worker) to compete with other agents whose level of education is far greater. In this respect, it is rational for the poor agents to immediately undertake an unskilled activity, choosing not to invest[7].

We assume that the economy is characterised by a particular institutional setting, where the available working positions are fixed over time (the types and number of available jobs and relative wages are exogenous[8]). In this institu-

[1] See [11], [10], [8], [7], [5].

[2] See [20], [4], [23].

[3] See [18].

[4] On positional competition, see [14]. Orsini (1997) presents a simulation of a model with positional competition by education among heterogeneous agents.

[5] See [9].

[6] Davis - Haltiwanger (1991) provide some interesting empirical evidence about this trend in the United States after 1975.

[7] In the model, we always assume that agents work only one period. Therefore, the opportunity cost of investing in education is represented only by the amount of forgone consumption.

[8] This assumption about the job structure is not at variance with empirical evidence: see

tional framework, agents compete for the better jobs by means of investment in education (credentials), likewise in *rent-seeking* models.[9] Another peculiarity of the model is that the signal role of school achievements does not come from the equilibrium of a game with incomplete information, as in signalling models, but it is taken for granted, as a starting assumption. There are different jobs with different pay, and their allocation is such that more educated agents have higher chances of getting better jobs.[10] We assume that the process of job assignment is uncertain: an agent can invest in education and, depending on his learning abilities, can reach a particular level of credentials. The latter influence the probability of getting a high paying job, but it is never conclusive. Credentials are obtained at school, several years before the application for the position: therefore the future structure of the labour market (the demand side) is unknown. Investing in education increases the chances of winning the positional competition in the future, but the process is never deterministic: the agent with better credentials cannot be sure to get the best position.[11]

Our main interest regards the consequence of the institutional setting (the schooling system and the labour and credit markets) on the distributive dynamics. If we consider an economy with risk-neutral agents and perfect capital markets, independently from individual wealth, the individual innate abilities determine the probabilities of getting the high-paying positions: in the long run the distribution of wealth among the different dynasties will be ergodic, that is, every dynasty will always have a positive probability of getting the highest pay. However, if agents are risk averse and credit is rationed (due to possible imperfections in the capital market), the asympthotic wealth distribution may depend on the initial wealth, which means that it may no longer be ergodic.

The educational system can be seen as an institution whose purpose is to provide people a signal for the job market that indicates their potential productivity. In the actual economy there are many differences among the educational systems of different countries (more or less private, more or less expensive[12]); the results of the model can help explain, at least partially, these differences. For example, in the American educational system there is a small set of elitarian scholastic institutions, which are very expensive and affordable only by individuals with high wealth, or with a notable grant (Harvard, Yale, Princeton, etc..). These institutions help effectively in the competition for a good job.[13] Rich

[13]. As time passes, the type of credentials necessary to get the top positions change. This fact leads to the first period uncertainty about the future assignment process.

[9] See [21].

[10] Spence (1973) is the first model in which education is only a signal. Education works as a signal related to the unobservable innate ability of workers; in this respect, more years of education can mean a higher perseverance (which means fewer absenteeisms and a smaller risk of abandoning the job) and other characteristics generally correlated with the scholastic curriculum; Weiss (1995) writes: "better-educated workers are not a random sample of workers: they have lower propensities to quit or to be absent, are less likely to smoke, drink or use illicit drugs, and are generally healthier. [...] we would expect employers to favor better-educated workers as a means of reducing their costs of sickness and job turnover". Employers hire on the basis of the credentials, but the latter are only *screening devices*: the studies carried out signal that the worker has a higher probability of belonging to the set of agents with greater productivity.

[11] We think that this approach is quite realistic: education increases the chances of success, but it cannot assure a better job with certainty.

[12] See [3] and [12].

[13] Talented individuals who study at Harvard are bound to find a much better job in comparison to talented individuals who do not. This is not to say that anybody can get a degree from

agents can be willing to pay a great amount of money in order to weaken the competition for high wage positions. Since this willingness to pay exists, there are the conditions for the rise of an institution capable of giving a particularly strong signal, that can exclude the poor by imposing high fees[14]. Of course, this kind of analysis of the factors that determine the degree of social mobility is only partial, because it neglects many other institutional aspects. Nevertheless, the relationship between the level of initial wealth, the educational choices, and the income dynamics, seems to be crucial.[15]

We will show that in a strategic environment, a low wealth agent might not be able to compete with high wealth agents for the high-paying positions, due to capital market imperfections and risk-aversion. Moreover, we will highlight how the educational institutions can somehow favor the competitors with high wealth, and how this could lead to a misallocation of resources[16]; as a result the society can be partitioned in different income classes, with a very low degree of social mobility.

The paper is organized as follows. Section 2 develops the structure of the model, paying attention to the specific assumptions about the rules of the positional competition in the labour market. In section 3 a case with risk aversion and credit market imperfections is analyzed. Section 4 analyzes the distributional dynamics. Section 5 concludes with some comments. In the Appendix, the results in the simple case with risk neutral agents are briefly illustrated.

2 The model

The economy is composed of N agents who live for two periods. In the first period agent j has to decide how to allocate his own inheritance $b_j(t)$ among the investment in education $a_j(t)$, the financial investment $s_j(t)$ and consumption $c_j(t)$, that is[17]:

$$b_j(t) = a_j(t) + s_j(t) + c_j(t). \tag{1}$$

In the second period the agent will have to decide how to allocate his income deriving from previous investments between consumption and inheritance. The

Harvard. In the model we propose, learning abilities are heterogeneous, therefore any agent can choose any amount of investment in education, but the credentials eventually achieved depend upon individual abilities.

[14]These institutions, having greater resources, are able to attract the best professors too. Rarely in the real world the selection of candidates is based only on their willingness to pay, since the access by means of grants of the most talented individuals, indipendently from their wealth, allows for a raise in the value of entering into the institution itself. From this point of view, attracting the most talented students raises the value of the signal awarded by the institution; full exclusion of the (most talented) poors by means of prohibitive fees would therefore not be coherent with the maximization of profit by the istitution (see [1]).

[15]See [5] for empirical evidence about Italy and the United States. A high quality private schooling system can be seen as a means of perpetuating inequality, while the observed degree of social mobility can be seen as an unavoidable concession which is necessary to legitimate the existing inequality. Lasch (1995) about this point says: "A high level of mobility is not contradictory with a system of stratification which concentrates power and privileges in a dominant élite. On the contrary, the circulation of the élite strengthens the hierarchical principle, because it always provides new talented élite and legitimates their ascent as a function of merit and not of birth". See also [22].

[16]See [17].

[17]Notice that a simple way to model credit rationing is to assume that $s_j(t)$ cannot be negative or, more generally, inferior to a negative threshold.

crucial hypothesis is that the return of the investment in education (the individual labour compensation) depends on the investments of the other agents, that is a positional competition in the job market exists. Let A_{-j} be the vector of investments in education of all the agents with the exclusion of agent j and $V = (v_1, ..., v_N)$ be the vector of individual innate learning abilities, so that the return of the investment in education for agent j can be represented as $w\left(a_j\left(t\right), A_{-j}\left(t\right), V\left(t\right)\right)$. Therefore in the second period the budget constraint is:

$$y_j\left(t+1\right) = R(t+1) \cdot s_j\left(t\right) + w\left(a_j\left(t\right), A_{-j}\left(t\right), V\left(t\right)\right) = b_j\left(t+1\right) + c_j\left(t+1\right), \tag{2}$$

where $y_j\left(t+1\right)$ is the agent j's income in period $t+1$, $b_j\left(t+1\right)$ is the bequest for his offspring, and $R(t+1)$ is the rate of return of the financial investment. The vector $V\left(t\right)$ is common knowledge.[18]

The simplest way to model the positional competition in education is to suppose that the return of the investment in education is determined by a lottery, whose probabilities are functions of the individual investments in education and the innate abilities, so that agent $j's$ problem becomes:

$$\max_{c_j(t), c_j(t+1), b_j(t+1), a_j(t)} E\left[U_j\right] = E\left[U\left(c_j\left(t\right), c_j\left(t+1\right), b_j\left(t+1\right)\right)\right],$$

subject to:

$$\begin{aligned} b_j\left(t\right) &= a_j\left(t\right) + s_j\left(t\right) + c_j\left(t\right) \\ R(t+1) \cdot s_j\left(t\right) + w\left(a_j\left(t\right), A_{-j}\left(t\right), V\left(t\right)\right) &= b_j\left(t+1\right) + c_j\left(t+1\right) \\ c_j\left(t\right) &\geq 0 \\ c_j\left(t+1\right) &\geq 0 \\ a_j\left(t\right) &\geq 0 \\ b_j\left(t+1\right) &\geq 0. \end{aligned}$$

After substituting $c_j\left(t\right)$ and $c_j\left(t+1\right)$, we have:

$$\begin{aligned} \max_{b_j(t+1), a_j(t)} E\left[U_j\right] = E\left[U\left(b_j\left(t\right) - a_j\left(t\right) - s_j\left(t\right), R(t+1) \cdot s_j\left(t\right) + w\left(a_j\left(t\right), A_{-j}\left(t\right), V\left(t\right)\right) + \right.\right. \\ \left.\left. -b_j\left(t+1\right), b_j\left(t+1\right)\right)\right], \end{aligned}$$

subject to:

$$\begin{aligned} b_j\left(t\right) - a_j\left(t\right) - s_j\left(t\right) &\geq 0 \\ R(t+1) \cdot s_j\left(t\right) + w\left(a_j\left(t\right), A_{-j}\left(t\right), V\left(t\right)\right) - b_j\left(t+1\right) &\geq 0 \\ a_j\left(t\right) &\geq 0 \\ b_j\left(t+1\right) &\geq 0. \end{aligned}$$

[18] Assuming instead that $V(t)$ is unknown, would have different implications according to the hypotheses about agents' attitude torwards risk. With risk neutral agents, and with homogeneous prior beliefs about the vector $V(t)$, nothing would change. With risk aversion, agents' behaviour would be different, because the investment in education would assume the role of a form of insurance against possible unfavourable realizations of the vector of learning abilities.

In general, the first order condition for $b_j(t+1)$ is:

$$\frac{U'_{b_j(t+1)}}{U'_{c_j(t+1)}} = 1,$$

where the index represents the variable with respect to which U has been derived. Notice that when there is no more uncertainty about the income of the second period, the choice about the amount of bequest is deterministic (even if still conditioned to the realization of the return of the investment in education). Under general hypotheses on the form of U (i.e. omothetic preferences), inheritance will be equal to a constant fraction β of second period income[19]: $c_j(t+1) = (1-\beta) \cdot y_j(t+1)$. Therefore we have:

$$b_j(t+1) = \max\left[\beta \cdot \left[R(t+1) \cdot s_j(t) + w(a_j(t), A_{-j}(t), V(t))\right], 0\right]. \tag{3}$$

Replacing (3) in the expected utility function, we have:

$$\max_{s_j(t), a_j(t)} E[U_j] = E\left[\hat{U}\left(b_j(t) - a_j(t) - s_j(t), R(t+1) \cdot s_j(t) + w(a_j(t), A_{-j}(t)), V(t)\right)\right],$$

where $\hat{U}$ is the indirect utility function modified in order to consider (3).

2.1 Imperfect capital market

The capital market can provide agent j with additional resources, so that it becomes possible to invest an amount of money in education that is greater than the bequest $b_j(t)$. However, if there are market imperfections, the investment in education $a_j(t)$, besides the inferior limit equal to 0, also shows a superior limit related to the level of inheritance $b_j(t)$ (initial wealth).[20]

A simple hypothesis about the form of credit market imperfections, is not to allow agents to borrow at all in the capital market, that is $s_j(t) \geq 0$. In such a way, investment in education is constrained in the following manner:

$$0 \leq a_j(t) \leq b_j(t). \tag{4}$$

This seems like a restrictive hypothesis, because in the second period agent j gets an income which is at least equal to the minimum wage, so that the maximum borrowing threshold might be higher; however this does not qualitatively affect the results.

2.2 Positional competition

The return of the investment in education does not follow the standard marginal productivity rule. We assume that the process of job assignment is uncertain: an

[19] However, this partition rule of income is very favorable to the fraction of income set aside for bequest; a more realistic formulation would consider that, below a certain income threshold, all the resources are devoted to consumption (a minimum consumption exists): in such a case the ratio between inheritance and consumption would be equal to zero for low levels of income and increasing with respect to it (i.e. almost-linear preferences).

[20] For example, these imperfections could be caused by the impossibility of making the child pay possible debts of the parents (indeed for hypothesis $b_j(t+1) \geq 0$), so that the lender results as being subject to the risk of default (second period consumption cannot clearly be negative).

agent can invest in education and, depending on his learning abilities, can reach a particular credential. The probability of getting a high paying job depends on the individual relative level of credentials, but this latter is never conclusive. Investing in education increases the chances of winning the positional competition in the future, but the process is not deterministic: the agent with better credentials can never be sure to get the best position.

Moreover, we suppose that there is a limited number of job positions relative to a different level of wages. Therefore the two crucial points are:

1. the specification of the wage scale
2. the determination of the probability of each agent of getting a certain position, given his credentials and those of his competitors.

We suppose that the wage scale is composed of K different possible wages w_k (where $k = 1, ...K$), relative to n_k positions for each level of wage.

Let $p_k(a_j(t), A_{-j}(t), V(t), n_k)$ be the probability that agent j gets a return equal to w_k, given his investment $a_j(t)$, the vector of the investments of the other agents $A_{-j}(t)$, and the vector of individual innate learning abilities $V(t)$. Therefore, agent j's problem can be expressed as:

$$\max_{a_j(t), s_j(t)} E[U_j] = \sum_{k=1}^{K} p_k(a_j(t), A_{-j}(t), V(t), n_k) \cdot \hat{U}(b_j(t) - a_j(t) - s_j(t), R(t+1) \cdot s_j(t) + w_k),$$

subject to:

$$\begin{aligned} b_j(t) - a_j(t) - s_j(t) &\geq 0 \\ R(t+1) \cdot s_j(t) + w_k &\geq 0 \qquad \forall k \\ a_j(t) &\geq 0 \\ s_j(t) &\geq 0. \end{aligned}$$

The form of the function p_k is determined by the lottery used to assign the available positions to the different agents. The assignment process, given the investments in education, takes on the form of a sampling without replacement, in which the positions relative to high wages are drawn out first[21]. Since after each sampling it is necessary to remove the investment of the drawn out agent from the total calculation, and therefore to recalculate the single probability, the calculus of the relative probabilities is quite complicated. Some simplification is desirable at this point.

3 Job seeking with risk aversion: a simple case with two agents

We choose to limit our attention to an economy with two agents, (agent i and agent j) and with two levels of wages ($w_H(t+1)$ and $w_L(t+1)$, where $w_H(t+1) > w_L(t+1)$).[22]

[21] See [2].

[22] As the number of agents (and of possible job positions) increases, the positional competition becomes more complex. We study only the simplest case (2×2). For the simulation of a similar model with a greater number of agents, see Orsini (1997).

The strength of the signal in the job market (the credential) is proportional to the investment in education and to the individual innate abilities[23]. In particular, we assume that, if one agent invests $a(t)$ in education, and he has an ability to learn equal to v, he will obtain a schooling credential equal to $v \cdot a(t)$. Normalizing agent i's innate ability ($v_i = 1$), the parameter v_j becomes an index of agent j's relative learning ability. Agent j's probability of achieving the high wage $w_H\,(t+1)$ is given by:

$$p_j\,(t) = \frac{v_j \cdot a_j\,(t)}{v_j \cdot a_j\,(t) + v_i \cdot a_i\,(t)} = \frac{v_j \cdot a_j\,(t)}{v_j \cdot a_j\,(t) + a_i\,(t)} \tag{5}$$

The probability of getting the high-paying job is directly proportional to the investment in education[24]. The analogy with rent-seeking models is quite apparent. Agent j's problem therefore becomes:

$$\begin{aligned}\max_{a_j(t),s_j(t)} E\left[U_j\right] = {} & \frac{v_j \cdot a_j\,(t)}{v_j \cdot a_j\,(t) + a_i\,(t)} \cdot \hat{U}\left(b_j\,(t) - a_j\,(t) - s_j\,(t)\,, R(t+1) \cdot s_j\,(t) + w_H\,(t+1)\right) + \\ & + \frac{a_i\,(t)}{v_j \cdot a_j\,(t) + a_i\,(t)} \cdot \hat{U}\left(b_j\,(t) - a_j\,(t) - s_j\,(t)\,, R(t+1) \cdot s_j\,(t) + w_L\,(t+1)\right)\end{aligned}$$

subject to[25]:

$$b_j\,(t) - a_j\,(t) - s_j\,(t) \geq 0 \tag{6}$$
$$R(t+1) \cdot s_j\,(t) + w_k\,(t+1) \geq 0 \qquad k = H, L \tag{7}$$
$$a_j\,(t) \geq 0 \tag{8}$$
$$s_j\,(t) \geq 0. \tag{9}$$

Now we have to specify which kind of equilibrium we are looking for. Two types of strategic equilibrium are possible: the Cournot-Nash equilibrium (with simultaneous moves), and the Stackelberg equilibrium (with sequential moves). In our model, the hypothesis of sequential moves means that the richest agent can act as a leader, investing in education and so setting the standard of the expenses which are necessary to acquire useful schooling credentials. Therefore, when moves are sequential, we assume that the leader is always the wealthy agent[26]. We expect the outcome with one of the competitors that chooses not

[23] By "innate abilities" we mean the learning abilities that agents have after compulsory (and free) education. They represent the efficiency of the process which transforms higher education into credentials.

[24] A more general formulation is the following:

$$p_j\,(t) = \frac{(v_j \cdot a_j\,(t))^\alpha}{(v_j \cdot a_j\,(t))^\alpha + (a_i\,(t))^\alpha}$$

where α represents the degree of meritocracy of the process of positions' assignment. Indeed, as α increases, agents investing more resources in education will be more favoured in the competition. As an extreme case, when $\alpha = 0$, the level of investment does not affect probabilities at all.

[25] Notice that the second constraint is redundant taking the fourth constraint on $s_j\,(t)$ into account. However, the second constraint is not eliminated since it will become useful when, analyzing the results with risk neutral agents in the Appendix , we compare the two cases, with and without credit rationing.

[26] The rich dinasties always act as leaders in setting the standards of consumption and investment. Our hypothesis of a rich leader is then justified on the basis of the observation of consuetudinary behaviour.

to invest at all in education to be more probable in the case of sequential moves. In the case of simultaneous moves, on the contrary, it is not likely that one agent will not invest in education at all. Under uncertainty, agents' attitude towards risk is crucial. We focus on the behaviour of risk averse agents, confining the risk neutral case in the Appendix.

Risk aversion can be modeled by means of any concave utility function; we consider this simple form:

$$\hat{U}(\cdot) = \log\left(b_j(t) - a_j(t) - s_j(t)\right) + \hat{\theta} \cdot \log\left(R(t+1) \cdot s_j(t) + w_k(t+1)\right), \quad k = H, L$$

where $\hat{\theta} = \frac{\theta}{1-\beta}$ and $\theta < 1$ is the intertemporal preference factor[27].

To simplify the analysis, we consider only the extreme case with maximum borrowing constraints: agents are not allowed to borrow and agents always prefer consumption to the investment in capital market, which means that the variable $s_j(t)$ can be eliminated from the menu of choice (as if capital market were absent).

Therefore, agent j has to solve the following problem:

$$\begin{aligned} \max_{a_j(t)} E[U_j] = & \log\left(b_j(t) - a_j(t)\right) + \\ & + \hat{\theta} \cdot \left\{ \frac{v_j \cdot a_j(t)}{v_j \cdot a_j(t) + a_i(t)} \log\left[w_H(t+1)\right] + \frac{a_i(t)}{v_j \cdot a_j(t) + a_i(t)} \log\left[w_L(t+1)\right] \right\} \end{aligned} \tag{10}$$

subject to:

$$b_j(t) - a_j(t) \geq 0 \tag{11}$$

$$a_j(t) \geq 0 \tag{12}$$

We analyze first the Cournot, and then the Stackelberg equilibrium.

3.1 Cournot equilibrium

In this section we derive the Cournot equilibrium, corresponding to the intersection of the reaction curves of the two agents. The reaction curves can be derived from the FOC. For agent j, the FOC is:

$$\frac{\partial E[U_j]}{\partial a_j} = -\frac{1}{b_j - a_j} + \hat{\theta} \cdot \left[\frac{v_j \cdot a_i \cdot \log\left(\frac{w_H}{w_L}\right)}{\left(v_j \cdot a_j + a_i\right)^2} \right] = 0. \tag{13}$$

With the logarithmic utility function, the FOC automatically implies the constraint on first period consumption. If there were a minimum consumption threshold, b_j would have to be replaced with $\tilde{b}_j = \max[b_j - \tilde{c}, 0]$. An analogous condition with inverse index holds for agent i.

[27] With a logarithmic function, in order to have $b_j(t+1) = \beta \cdot y_j(t+1)$, the utility function must be:

$$U = \log c_j(t) + \theta \cdot \left[\log c_j(t+1) + \left(\frac{\beta}{1-\beta}\right) \log b_j(t+1)\right] + D,$$

where D is a constant which depends on the values of θ and β, so that $\hat{\theta} = \frac{\theta}{1-\beta}$.

From condition (13) we get agent j's reaction curve:

$$\frac{\partial E\left[U_j\right]}{\partial a_j} = 0 \Leftrightarrow \frac{\left(v_j \cdot a_j + a_i\right)^2}{v_j \cdot a_i \cdot \left(b_j - a_j\right)} = \hat{\theta} \cdot \log\left(\frac{w_H}{w_L}\right). \tag{14}$$

One solution is $a_j = 0$ and $a_i = \hat{a}_i = v_j \cdot \hat{\theta} \cdot \log\left(\frac{w_H}{w_L}\right) \cdot b_j$; this means that if agent i invests more than $\hat{a}_i$, the optimal choice of agent j is to abandon the competition (the optimal value of a_j is 0 whenever $a_i \geq \hat{a}_i$)[28]. Notice that $\hat{a}_i$ is a positive function of agent j's learning ability, which means that the investment in education is made to counterbalance the opponent's ability. Each agents values his own investment in education according to the level of his own learning ability, which is the degree of efficiency of his "credentials production function".

In the Cournot equilibrium, given by the intersection of the reaction curves of the two agents[29], we have:

$$a_j = \left(\frac{b_j}{b_i}\right) \cdot a_i, \tag{15}$$

which, substituted into (14) yields:

$$\bar{a}_j = b_j \cdot \left(\frac{v_j \cdot \hat{\theta} \cdot \log\left(\frac{w_H}{w_L}\right) \cdot b_i \cdot b_j}{v_j \cdot \hat{\theta} \cdot \log\left(\frac{w_H}{w_L}\right) \cdot b_i \cdot b_j + \left(v_j \cdot b_j + b_i\right)^2}\right). \tag{16}$$

Equations (16) and (15) highlight that, if the capital market is absent, each agent invests a quantity of resources in education that is a positive function of his wealth.[30] Notice that the optimal choice of agent i (likewise for j) belongs to the interval $[0, \hat{a}_i)$, as we can easily draw from the calculation of the maximum value of $\bar{a}_i$, that is

$$\lim_{b_i \to \infty} \bar{a}_i = v_j \cdot \hat{\theta} \cdot \log\left(\frac{w_H}{w_L}\right) \cdot b_j$$

which assures that (14) represents the part of the reaction curve where the solution lies.

The following Proposition describes the Cournot equilibrium with risk aversion.

Without capital market, the Cournot equilibrium of an economy with risk averse agents is given by:
$a_j^* = b_j \cdot \left(\frac{v_j \cdot \hat{\theta} \cdot \log\left(\frac{w_H}{w_L}\right) \cdot b_i \cdot b_j}{v_j \cdot \hat{\theta} \cdot \log\left(\frac{w_H}{w_L}\right) \cdot b_i \cdot b_j + \left(v_j \cdot b_j + b_i\right)^2}\right)$ and $a_i^* = b_i \cdot \left(\frac{v_j \cdot \hat{\theta} \cdot \log\left(\frac{w_H}{w_L}\right) \cdot b_i \cdot b_j}{v_j \cdot \hat{\theta} \cdot \log\left(\frac{w_H}{w_L}\right) \cdot b_i \cdot b_j + \left(v_j \cdot b_j + b_i\right)^2}\right)$.

[28] Analytically, this result is due to the negative sign of the first derivative of agent j for $a_j = 0$ and to the negative sign of the second derivative $\frac{\partial^2 E\left[U_j\right]}{\partial a_j^2}$ in the interval $[0, b_j]$.

[29] Agent i's reaction curve is:

$$\frac{\partial E\left[U_i\right]}{\partial a_i} = 0 \Leftrightarrow \frac{\left(v_j \cdot a_j + a_i\right)^2}{v_j \cdot a_j \cdot \left(b_i - a_i\right)} = \hat{\theta} \cdot \log\left(\frac{w_H}{w_L}\right).$$

[30] In equation (16) the expression in round brackets is the same for both agents.

If there were a minimum consumption $\tilde{c}$, that is $\frac{\partial E[U_q]}{\partial c_q(t)} = +\infty \ \forall c_q(t) \leq \tilde{c}$, b_q would have to be replaced with $\tilde{b}_q = \max[b_q - \tilde{c}, 0]$, for $q = i, j$.

Therefore, the borrowing constraints affect the quantity of investment in education but, if there is no minimum consumption, both agents will take part in the competition. Intuitively, if there were a minimum consumption, it would be possible for the rationed agent to not have enough resources to consume and invest so that, even with a positive inheritance, he would not make any investment in education. We observe that a lower inheritance can be counterbalanced by an higher learning ability: from (16) we find that $p_j = \frac{v_j \cdot b_j}{v_j \cdot b_j + b_i} > \frac{1}{2} \Leftrightarrow v_j \cdot b_j > b_i$. Being affected by both investment and learning abilities, the job assignment process is such that the most talented agent has the greatest probability of getting the best job if and only if his initial wealth is not too lower than that of his opponent.

3.2 Stackelberg equilibrium

In the present section we will show that, if agents are risk averse, the Stackelberg equilibrium differs from the Cournot equilibrium. In models of rent-seeking, the peculiar aspect of the Stackelberg equilibrium is the possibility that the follower does not invest at all. Linster (1993) shows that if the prize is equally valued by the two players, the Stackelberg equilibrium coincides with the Cournot equilibrium, but if the leader makes a higher evaluation of the prize, the follower could refrain from taking part in the competition. Analogously, in our model this means that with risk aversion, the poor agent can abandon the positional competition since intuitively he "evaluates" the additional income he could earn investing in education less than the leader. In particular, we will show that the leader can be able to exclude the follower from the competition by making an investment that is greater than a certain threshold, which is a function of follower's wealth. In some cases, learning ability heterogeneity can only mitigate this effect and, as we will show in section 4, it might not affect the distributive dynamics.

In the following we will always assume that agent i is the leader and agent j the follower, analyzing the case with $b_i > b_j$.[31] The first step is to find the follower's reaction curve, which is given by the FOC (14). Considering also that for $a_i \geq \hat{a}_i = v_j \cdot \hat{\theta} \cdot \log\left(\frac{w_H}{w_L}\right) \cdot b_j$ the curve (14) lies in the negative quadrant, the reaction curve of agent j for $a_i \geq \hat{a}_i$ is $a_j = 0$ (when leader's investment

[31] See note 26.

exceeds $\hat{a}_i$, the optimal choice of the follower is to not invest)[32]. Therefore, the follower's reaction curve is:

$$a_j(a_i) = \begin{cases} \frac{-a_i \cdot (2+\mu) + \sqrt{\mu \cdot a_i \cdot (\mu \cdot a_i + 4 \cdot a_i + 4 \cdot v_j \cdot b_j)}}{2 \cdot v_j} & \text{if } a_i < v_j \cdot b_j \cdot \mu \\ 0 & \text{if } a_i \geq v_j \cdot b_j \cdot \mu \end{cases} \tag{17}$$

where $\mu = \hat{\theta} \cdot \log\left(\frac{w_H}{w_L}\right)$.

The leader maximizes utility, taking into account that his own decision affects the choice of the follower:

$$\max_{a_i, s_i} E[U_i] = \log(b_i - a_i) + \hat{\theta} \cdot \left[\frac{a_i}{a_i + v_j \cdot a_j(a_i)} \cdot \log(w_H) + \frac{v_j \cdot a_j(a_i)}{a_i + v_j \cdot a_j(a_i)} \cdot \log(w_L) \right].$$

The FOC for the leader is then:

$$\frac{\partial U_i}{\partial a_i} = -\frac{1}{b_i - a_i} + v_j \cdot \hat{\theta} \cdot \log\left(\frac{w_H}{w_L}\right) \left[\frac{a_j(a_i) - a_i \cdot \frac{da_j}{da_i}}{(v_j \cdot a_j(a_i) + a_i)^2} \right] = 0.$$

From (14) we can calculate $a_j(a_i)$ and $\frac{da_j}{da_i}$, which substituted into the previous equation yield[33]:

$$\frac{\partial U_i}{\partial a_i} = -\frac{1}{b_i - a_i} + \frac{4 \cdot \mu^2 \cdot a_i \cdot b_j \cdot v_j}{M(a_i, b_j, v_j, \mu) \cdot [M(a_i, b_j, v_j, \mu) - \mu \cdot a_i]^2}, \tag{18}$$

where $M(a_i, b_j, v_j, \mu) = \sqrt{\mu \cdot a_i \cdot (\mu \cdot a_i + 4 \cdot a_i + 4 \cdot v_j \cdot b_j)}$.

Let $\bar{a}_i$ be the value of a_i which makes $\frac{\partial U_i}{\partial a_i}$ equal to zero:

$$\left. \frac{\partial U_i}{\partial a_i} \right|_{a_i = \bar{a}_i} = 0$$

Notice that $\bar{a}_i$ is never greater than $\hat{a}_i$, since $\hat{a}_i$ is enough to exclude agent j. Moreover it can be shown that $\frac{\partial^2 U_i}{\partial a_i^2} < 0$, $\lim\limits_{a_i \to 0} \frac{\partial U_i}{\partial a_i} = +\infty$ and $\lim\limits_{a_i \to b_i} \frac{\partial U_i}{\partial a_i} = -\infty$,

[32] Another way to see this point is by defining the difference between the utility values in the two cases: with and without a positive investment in education:

$$\Delta E[U_j] = E[U_j(a_j, a_i)] - U_j(0, a_i),$$

which is equal to:

$$\Delta E[U_j] = \log\left(\frac{b_j - a_j}{b_j}\right) + \theta \cdot \left[\frac{v_j \cdot a_j \cdot \log\left(\frac{w_H}{w_L}\right)}{v_j \cdot a_j + a_i} \right].$$

By calculating $\Delta E[U_j]$ with $a_j = 0$ we get:

$$\Delta E[U_j] \geq 0 \Leftrightarrow a_i \leq \hat{a}_i = v_j \cdot \theta \cdot \log\left(\frac{w_H}{w_L}\right),$$

which means that, if $a_i \geq \hat{a}_i$, then it is not convenient for agent j to invest.

[33] In particular:

$$\frac{da_j}{da_i} = -\left(\frac{2+\mu}{2 \cdot v_j}\right) + \frac{\mu^2 \cdot a_i + 4 \cdot \mu \cdot a_i + 2 \cdot \mu \cdot v_j \cdot b_j}{2 \cdot v_j \cdot \sqrt{\mu \cdot a_i \cdot (\mu \cdot a_i + 4 \cdot a_i + 4 \cdot v_j \cdot b_j)}}.$$

so that the sign of $\frac{\partial U_i}{\partial a_i}$ in $a_i = \hat{a}_i$ allows us to distinguish if both agents invest in education or if only the leader does:

$$\left.\frac{\partial U_i}{\partial a_i}\right|_{a_i=\hat{a}_i} \geq 0 \Leftrightarrow \bar{a}_i = \hat{a}_i \Longrightarrow \bar{a}_j = 0 \tag{19}$$

In Figure 1 an example of the follower's reaction curve (in light grey) and of the leader's utility U_i (in dark grey) is shown:

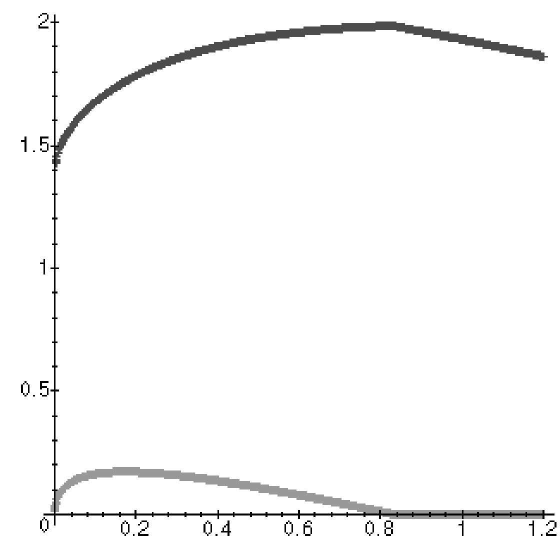

Figure 1 - The graph is drawn assuming b_j=1, v_j=1, b_i=4, w_H=4, w_L=1 and $\hat{\theta}$=0.6, so that $\hat{a}_i$=0.8318 and $\left.\frac{\partial U_i}{\partial a_i}\right|_{a_i=\hat{a}_i}$=0.0375. On the orizontal axis the variable a_i is measured. It can be noted that U_i reaches its maximum when a_i=$\hat{a}_i$, as we know from (19).

Replacing the value of $\hat{a}_i$ into (18), it is possible to demonstrate that

$$\left.\frac{\partial U_i}{\partial a_i}\right|_{a_i=\hat{a}_i} \geq 0 \Leftrightarrow b_i \geq 2 \cdot \left[1 + \hat{\theta} \cdot \log\left(\frac{w_H}{w_L}\right)\right] \cdot v_j \cdot b_j,$$

that is to say, in order to have $a_i = \hat{a}_i$ and $a_j = 0$ (see (19)), b_i has to be much greater than b_j (if $v_j > 1$).[34]

The following Proposition describes the Stackelberg equilibrium with risk aversion.

Without capital market, the Stackelberg equilibrium of an economy with risk averse agents (where agent i is the leader and agent j is the follower), is

[34] This result is quite intuitive: given agent j's investment and learning ability, the higher is agent i's wealth, the lower is the risk that agent i perceives investing in education. The logarithmic utility function is *D.A.R.A.* (decreasing absolute risk aversion).

the following:

1) if $b_i < 2 \cdot \left[1 + \hat{\theta} \cdot \log\left(\frac{w_H}{w_L}\right)\right] \cdot v_j \cdot b_j$ then $a_i^* = \bar{a}_i$ and $a_j^* = \frac{-\bar{a}_i \cdot (2+\mu) + \sqrt{\mu \cdot \bar{a}_i \cdot (\mu \cdot \bar{a}_i + 4 \cdot \bar{a}_i + 4 \cdot v_j \cdot b_j)}}{2 \cdot v_j}$, where $\bar{a}_i$ solves $\left.\frac{\partial U_i}{\partial a_i}\right|_{a_i = \bar{a}_i} = 0$ and $\mu = \hat{\theta} \cdot \log\left(\frac{w_H}{w_L}\right)$;

2) if $b_i \geq 2 \cdot \left[1 + \hat{\theta} \cdot \log\left(\frac{w_H}{w_L}\right)\right] \cdot v_j \cdot b_j$ then $a_i^* = \hat{\theta} \cdot \log\left(\frac{w_H}{w_L}\right) \cdot v_j \cdot b_j$ and $a_j^* = 0$.

If there were a minimum consumption $\tilde{c}$, that is $\frac{\partial E[U_q]}{\partial c_q(t)} = +\infty \ \forall c_q(t) \leq \tilde{c}$, then b_q would have to be replaced with $\tilde{b}_q = \max[b_q - \tilde{c}, 0]$, for $q = i, j$.

From Proposition 3.2 we can conclude that a great distributive inequality involves an inefficient level of investment in education; indeed we notice that if the richer agent has an amount of wealth b_i greater than $2 \cdot \left[1 + \hat{\theta} \cdot \log\left(\frac{w_H}{w_L}\right)\right] \cdot v_j$ times that of the poorer (agent j), he will always invest $\hat{\theta} \cdot \log\left(\frac{w_H}{w_L}\right) \cdot v_j \cdot b_j$. By increasing agent j's resources, we would get, besides a decrease of inequality, an increase of the investment in education (case 1 of Proposition 3.2). Also notice that a higher learning ability of the follower (that is $v_j > 1$) is not a sufficient condition for a positive investment in education by both agents if the follower is much poorer than the leader.

4 Distributive dynamics

To analyze the dynamics of the wealth distribution between the two dynasties, we need to specify the temporal evolution of the two exogenous variables: $\frac{w_H(t)}{w_L(t)}$ and $v_j(t)$. We assume that the wage ratio remains constant over time ($\frac{w_H(t)}{w_L(t)} = \frac{w_H}{w_L} \forall t$) and that the relative learning ability $v_j(t)$ is uniformly distributed on the interval $[v_{\min}, v_{\max}]$, with $v_{\min} > 0$ and $E[v_j(t)] = 1$ (the two agents have the same learning ability on average).

From (3) we know that the inheritance is equal to a quota β of second period income: it is then equal to $\beta \cdot w_H$ or to $\beta \cdot w_L$.

The dynamics relative to the two types of strategic equilibria is different[35]. We first analyze the Cournot case.

4.1 Cournot equilibrium

From Proposition 3.1 we calculate the transition probabilities for agent j, reported in the following Table[36]:

	$w_H(t+1)$	$w_L(t+1)$
$w_H(t)$	$\frac{v_j \cdot w_H}{v_j \cdot w_H + w_L}$	$\frac{w_L}{v_j \cdot w_H + w_L}$
$w_L(t)$	$\frac{v_j \cdot w_L}{v_j \cdot w_L + w_H}$	$\frac{w_H}{v_j \cdot w_L + w_H}$

[35] For the risk neutral case, see the Appendix.

[36] If there were a minimum consumption threshold $\tilde{c}$ and $\beta \cdot w_L < \tilde{c}$, then the transition probabilities for agent j would become:

	$w_H(t+1)$	$w_L(t+1)$
$w_H(t)$	1	0
$w_L(t)$	0	1

that is, the distributive dynamics would be non ergodic.

Being $\frac{w_H(t)}{w_L(t)}$ a constant and $v_j(t) > 0 \ \forall t$, every transition probability is positive and depends on the values of $\frac{w_H}{w_L}$ and v_j. This means that the dynamics is ergodic because every agent always has a positive probability of getting any kind of job.

From Proposition 3.1 we can calculate the aggregate investment in education $A(t) = a_j^*(t) + a_i^*(t)$ and the aggregate credentials $C(t) = v_j(t) \cdot a_j^*(t) + a_i^*(t)$, which can be interpreted as a proxy of the aggregate human capital. It is possible to show that $\frac{\partial A}{\partial v_j} > 0 \Leftrightarrow v_j < \frac{b_i}{b_j}$ and that $\frac{\partial C}{\partial v_j} > 0 \ \forall v_j$. The first result comes from the strategic nature of the model: the less talented agent invests more in education as v_j increases, but only up to a point: if the distance in the learning abilities becomes greater than the wealth ratio, he decides to invest less resources in positional competition, leading to an aggregate decrease of investment. The most talented agent always invests more resources as his relative learning efficiency increases, leading to greater aggregate credentials (but not always to greater aggregate investment) as the distance between the individual abilities increases.

It is worth making a more in depth analysis of dynamics, focusing on the possible effect of the wage scale on the growth rate. If we suppose that aggregate credentials C are a proxy of the level of human capital and that this latter is the only productive factor, then the output of period t can be expressed as $Y(t) = \phi \cdot C(t-1)$ where $\phi > 0$; moreover letting $\alpha > \frac{1}{2}$ be the quota of output attributed to the high wage position, we get $w_H = \alpha \cdot Y(t)$ and $w_L = (1-\alpha) \cdot Y(t)$. To analyze the long run dynamics we set v_j equal to its expected value ($v_j = 1$) and, without being less general, $b_j = \beta \cdot w_L$ and $b_i = \beta \cdot w_H$. Then we obtain:

$$Y(t+1) = \phi \cdot C(t) = \phi \cdot Y(t) \cdot \beta \cdot \left[\frac{\hat{\theta} \cdot (1-\alpha) \cdot \alpha \log\left(\frac{\alpha}{1-\alpha}\right)}{\hat{\theta} \cdot (1-\alpha) \cdot \alpha \log\left(\frac{\alpha}{1-\alpha}\right) + 1} \right]$$

from which the growth rate is obtained:

$$g_Y = \frac{Y(t+1)}{Y(t)} - 1 = \phi \cdot \beta \cdot \left[\frac{\hat{\theta} \cdot (1-\alpha) \cdot \alpha \log\left(\frac{\alpha}{1-\alpha}\right)}{\hat{\theta} \cdot (1-\alpha) \cdot \alpha \log\left(\frac{\alpha}{1-\alpha}\right) + 1} \right] - 1$$

We notice that g_Y is positively affected by ϕ , $\hat{\theta}$ and β and in a non linear way by α. Deriving g_Y with respect to α we obtain

$$\frac{dg_Y}{d\alpha} = \phi \cdot \beta \cdot \hat{\theta} \cdot \left[\frac{1 - (2 \cdot \alpha - 1) \cdot \log\left(\frac{\alpha}{1-\alpha}\right)}{\left[1 + \alpha \cdot (1-\alpha) \cdot \hat{\theta} \cdot \log\left(\frac{\alpha}{1-\alpha}\right)\right]^2} \right]$$

that is,

$$\frac{dg_Y}{d\alpha} \geq 0 \Leftrightarrow \alpha \leq \hat{\alpha} \tag{20}$$

where $\hat{\alpha} \approx 0.823959$ solves $\log\left(\frac{\alpha}{1-\alpha}\right) = \frac{1}{2\alpha - 1}$[37]. Therefore, for $\alpha = \hat{\alpha}$ the economy performs the maximum growth rate. In other words, to maximize

[37]It is easy to demonstrate that $\hat{\alpha}$ is the only solution in the interval $\left[\frac{1}{2}, 1\right]$.

the growth rate a certain wage difference is necessary. Intuitively, too small a difference between high and low wages discourages agents from investing in education to get the best job and, if education also determines the level of output, this underinvestment causes a lower growth rate.

The following Proposition describes the distributive dynamics with risk aversion.

In an economy where agents are risk averse and equilibrium is Cournot, the distributive dynamics is ergodic, except in the case with a minimum consumption threshold $\tilde{c}$ and $\beta \cdot w_L < \tilde{c}$.
Aggregate investment in education can be both a positive and a negative function of learning ability v_j, while aggregate credentials are always a positive function of v_j.
Finally, if aggregate credentials are a proxy of the level of human capital, then the growth rate is maximized when the high wage receives a quota of output equal to 0.823959.

The relationship between wage inequality and growth is not monotonic. It is necessary to have a clear distinction between high-paying positions, which are mainly allocated to educated workers, and low-paying positions. This difference gives agents the incentives to invest in education. If the school is financed by means of progressive income taxes, these latter should not be too progressive, in order to mantain the right degree of (net) wage differentiation. If the wage difference is too high, the aggregate investment in education can decrease, due to the fact that the poor dynasties have not enough resources to compete[38].

4.2 Stackelberg equilibrium

In the Stackelberg equilibrium, assuming that agent j is the follower and agent i is the leader ($b_i \geq b_j$), we find that the follower will not invest in education if $b_i \geq 2 \cdot \left[1 + \hat{\theta} \cdot \log\left(\frac{w_H}{w_L}\right)\right] \cdot v_j(t) \cdot b_j$ (see Proposition 3.2), where $b_i = \beta \cdot w_H$ and $b_j = \beta \cdot w_L$. To have a stable dichotomy in the wealth distribution it is necessary that:

$$\beta \cdot w_H \geq 2 \cdot \left[1 + \hat{\theta} \cdot \log\left(\frac{w_H}{w_L}\right)\right] \cdot v_j(t) \cdot \beta \cdot w_L,$$

that is,

$$\frac{w_H}{w_L} \geq 2 \cdot v_j(t) \cdot \left[1 + \hat{\theta} \cdot \log\left(\frac{w_H}{w_L}\right)\right].$$

Define $\left(\frac{w_H}{w_L}\right)^*$ such that $\left(\frac{w_H}{w_L}\right)^* = 2 \cdot v_j(t) \cdot \left[1 + \hat{\theta} \cdot \log\left(\frac{w_H}{w_L}\right)^*\right]$, so that for $\left(\frac{w_H}{w_L}\right) \geq \left(\frac{w_H}{w_L}\right)^*$ the follower never invests in education. As one could expect, the wage scale, together with the learning abilities, is crucial in establishing the investment in education[39]. We note that if

[38] Comparing Italy and the U.S.A., we see that the italian wage differentials are much lower, but the human capital growth rate is higher (see [5]). This evidence could be rationalised on the basis of a value of α which is too high in the U.S.A.

[39] Note that the value of $\left(\frac{w_H}{w_L}\right)^*$ depends on the realization of $v_j(t)$.

$$v_{\max} < \frac{\frac{w_H}{w_L}}{2 \cdot \left[1 + \hat{\theta} \cdot \log\left(\frac{w_H}{w_L}\right)\right]}$$

then the distributive dynamics is non ergodic,[40] that is

	$w_H\ (t+1)$	$w_L\ (t+1)$
$w_H\ (t)$	1	0
$w_L\ (t)$	0	1

The following Proposition describes the distributive dynamics with risk aversion.

In an economy where agents are risk averse and equilibrium is Stackelberg, the distributive dynamics can be non ergodic, in particular:
1) if $\left(\frac{w_H}{w_L}\right) < \left(\frac{w_H}{w_L}\right)^*$ the distributive dynamics is ergodic;
2) if $\left(\frac{w_H}{w_L}\right) \geq \left(\frac{w_H}{w_L}\right)^*$ the distributive dynamics is not ergodic because the poorer agent never invests, where $\left(\frac{w_H}{w_L}\right)^*$ solves $\left(\frac{w_H}{w_L}\right)^* = 2 \cdot v_{\max} \cdot \left[1 + \hat{\theta} \cdot \log\left(\frac{w_H}{w_L}\right)^*\right]$.
Likewise in the Cournot equilibrium, when there is a minimum consumption threshold $\tilde{c}$, the distributive dynamics is non ergodic if $\beta \cdot w_L < \tilde{c}$.

This result can have interesting economic implications. The possibility that the distributive dynamics is non ergodic involves both a static and a dynamic inefficiency. In particular, if the richest agent moves first (competition is sequential), the growth rate can be sub-optimal. By financing schools with public funds, the poor agent can become able to invest in education, and the dicothomy can be overcome. However, the amount of public financing is constrained by the condition on the optimal value of α, which must not be less than $\hat{\alpha}$.

5 Conclusions

In this paper we analyzed the effects of credit constraints and risk aversion in a model of positional competition in the labour market. The investment in education has been modeled as a rent-seeking activity, which determines the probabilities of getting a high-paying job.

With risk averse agents, the borrowing constraints cause a static inefficiency in both the Cournot equilibrium and the Stackelberg equilibrium, but only in this latter a non ergodic distributive dynamics can emerge.[41] Indeed, if the wage difference is rather high, it can be optimum for the leader to exclude the follower from the competition, making an investment in education which is directly proportional to the follower's ability and wealth.[42] This result does not depend on the absolute levels of the two possible wages, but only on the ratio between them; therefore it remains valid if we assume that the wages evolve

[40] With the parameters choosen in the example of Figure 1, the condition for ergodicity is $v_{\max} \geq 1,092$. It is sufficient that the poor dynasty give birth to a descendant with a learning ability which is 10% greater than that of the richer dynasty.

[41] In the Appendix we show that with risk neutral agents, the distributive dynamics for both equilibria is always ergodic (except for the case with a minimum consumption threshold).

[42] With heterogeneous abilities, the possibility that the poor agent chooses not to invest arises in the case with risk neutrality too, but it is a case of little interest, because it stems only from the low level of learning abilities of the follower (see the Appendix).

in time, preserving the value of the ratio.[43] The temporal evolution of the dynasties' wealth also depends upon the (exogenous) realization of the innate abilities of the descendants; in the sequential game (Stackelberg equilibrium), the distributive dynamics can become ergodic if the range of possible values for the innate learning ability is wide enough. Finally, we show that in the Cournot equilibrium, in order to maximize the growth rate, a certain wage difference is necessary to give agents incentives to invest in education.

An interesting extension would come from the introduction of a time constraint, allowing for a first-period unskilled job. In this case, the opportunity-cost of investing in education would be increased by the amount of resources forgone by the agent if he was not working in the first period. The wage scale would then be enriched, introducing at least three wage levels: for the unskilled, for the skilled and for the top positions. A single agent would then face three different perspectives: a) not to invest, and work for two periods as unskilled[44]; b) to invest, losing the positional competition; c) to invest, winning the positional competition. The income stratification and the distributive dynamics would be different, and certainly more complex.

[43] Note that a constant wage ratio is compatible with an increasing wage difference.

[44] Even a fourth level of wage could be introduced, assuming that the first and the second period wages, for the unskilled worker, might differ (i.e., for learning by doing considerations). Second period wage could also be considered as an expected value, if unemployment exists.

A The case with risk neutrality

The simplest form of risk neutral preferences is the linear one:

$$\hat{U}_j(\cdot) = b_j(t) - a_j(t) - s_j(t) + \theta \cdot [R(t+1) \cdot s_j(t) + w_k(t+1)], \quad k = H, L \tag{21}$$

where, in terms of the preceding notation, under the hypothesis of a linear utility function, $\theta = \hat{\theta} \cdot (1-\beta)$.

Therefore, with risk neutral agents the problem becomes:

$$\max_{a_j(t), s_j(t)} E[U_j] = b_j(t) - a_j(t) - s_j(t) + \tag{22}$$

$$+ \theta \cdot \left[R(t+1) \cdot s_j(t) + \frac{v_j \cdot a_j(t) \cdot w_H(t+1) + a_i(t) \cdot w_L(t+1)}{v_j \cdot a_j(t) + a_i(t)} \right]$$

A.1 Cournot equilibrium

In order to find the Cournot equilibrium, it is necessary to calculate the agents' reaction curves by means of the two first order conditions (FOC) of problem (22), which are the following:

$$\frac{\partial E[U_j]}{\partial s_j} = -1 + R(t+1) \cdot \theta = 0 \tag{23}$$

$$\frac{\partial E[U_j]}{\partial a_j} = -1 + \theta \cdot \left[\frac{v_j \cdot a_i \cdot (w_H - w_L)}{(v_j \cdot a_j + a_i)^2} \right] = 0 \tag{24}$$

where the time index is omitted whenever this does not give rise to confusion in the notation.

On the other hand, notice that, if borrowing were possible and $b_j < \bar{a}_j(a_i)$, where $\bar{a}_j(a_i)$ indicates the optimum value of a_j, then agent j would borrow $\bar{s}_j = \bar{a}_j(a_i) - b_j$ (first period consumption has to be non negative, see (6)). Moreover, if there were a minimum level of consumption $\tilde{c}$, then the loan would be equal to $\bar{s}_j = \bar{a}_j(a_i) - b_j + \tilde{c}$. (ignoring the constraint on the consumption of the second period).

Condition (24) states that the optimum value of a_j has to satisfy the following:

$$\frac{v_j \cdot a_i \cdot (w_H - w_L)}{(v_j \cdot a_j + a_i)^2} = \frac{1}{\theta}. \tag{25}$$

This only represents the implicit reaction curve of agent j, ignoring possible borrowing constraints.

Since $\frac{\partial^2 E[U_j]}{\partial a_j^2}$ is negative, equation (25) admits only one solution, that we indicate with $\bar{a}_j(a_i)$. If $\bar{a}_j(a_i) > b_j$, for the constraints (6) and (9) (non negativity of consumption in the first period and of s_j), the admissible choice will be constrained and equal to b_j. Moreover, if there were a minimum level of consumption $\tilde{c}$ (so that if inheritance is inferior to this threshold, all the resources must be devoted to consumption, which in turn means that the marginal utility, when consumption is under the threshold $\tilde{c}$, approaches $+\infty$), the investment in education would be constrained to $\max[b_j - \tilde{c}, 0]$.

In order to find the value of $\bar{a}_j(a_i)$, suppose that both agents are not constrained, so that the intersection of the two reaction curves[45] determines the optimal amount of investment in education, which is the same for both agents:

$$a_j^* = a_i^* = \frac{v_j \cdot \theta \cdot [w_H - w_L]}{(1+v_j)^2}. \tag{26}$$

If agent j is constrained, that is $a_j^* = b_j$, agent i modifies his own choice; in particular:

$$a_i^* = \sqrt{v_j \cdot \theta \cdot [w_H - w_L] \cdot b_j} - v_j \cdot b_j.$$

The following Proposition describes the Cournot equilibrium for an economy with risk neutral agents:

In the Cournot equilibrium with risk neutral agents, investment in education depends on the characteristics of the capital market:
1) if the capital market is imperfect (there are borrowing constraints: $s_j \geq 0$), then
$a_j^* = \min\left[b_j, \sqrt{v_j \cdot \theta \cdot [w_H - w_L] \cdot b_j} - v_j \cdot b_j, \frac{v_j \cdot \theta \cdot [w_H - w_L]}{(1+v_j)^2}\right]$ and
$a_i^* = \min\left[b_i, \sqrt{v_j \cdot \theta \cdot [w_H - w_L] \cdot b_j} - v_j \cdot b_j, \frac{v_j \cdot \theta \cdot [w_H - w_L]}{(1+v_j)^2}\right]$,
2) if the capital market is perfect, then $a_j^* = a_i^* = \frac{v_j \cdot \theta \cdot [w_H - w_L]}{(1+v_j)^2}$.
If there were a minimum consumption $\tilde{c}$, that is, $\frac{\partial E[U_q]}{\partial c_q(t)} = +\infty \; \forall c_q(t) \leq \tilde{c}$, b_q would have to be replaced with $\tilde{b}_q = \max[b_q - \tilde{c}, 0]$, for $q = i, j$.

We notice that the borrowing constraints could induce agents to underinvest in education: it is possible that in the Cournot equilibrium one agent does not invest at all, having insufficient resources in the first period (for instance $b_j(t) = 0$), while the other invests a quantity $\varepsilon > 0$, that is as small as he wishes, getting the high wage w_H. Finally, if there is a minimum level of consumption for each period, it is possible that the rationed agent may not set aside any resources to invest, so that, even though he received a positive inheritance, he does not make any investment in education. Ceteris paribus, heterogeneous learning abilities favour the most talented individuals in the positional competition. Without borrowing constraints, from (5) and (26) we get that $p_j = \frac{v_j}{1+v_j}$, which means that $p_j > \frac{1}{2} \Leftrightarrow v_j > 1$

A.2 Stackelberg equilibrium

The Stackelberg equilibrium is found assuming that one agent (the follower) considers the choice of the other agent (the leader) as given and maximizes accordingly.

Assume that agent i is the leader while agent j is the follower. In the first step we find the optimum choice of the follower depending on the choice of the leader; this is given by the FOC (25) if agent j is not rationed, and by b_j in the other case[46].

[45] Agent i's implicit reaction curve is specular to (25):

$$\frac{v_j \cdot a_j \cdot (w_H - w_L)}{(v_j \cdot a_j + a_i)^2} = \frac{1}{\theta}.$$

[46] If there were a minimum consumption, it would be given by $\max[b_j - \tilde{c}, 0]$.

The leader maximizes utility, taking the influence of his own action on the decision of the follower into account:

$$\max_{a_i,s_i} U_i = b_i - s_i - a_i + \theta \cdot \left[\frac{a_i \cdot w_H + v_j \cdot \bar{a}_j\left(a_i\right) \cdot w_L}{v_j \cdot \bar{a}_j\left(a_i\right) + a_i} + R(t+1) \cdot s_i \right].$$

The FOC with regard to s_i is the same as it is in the Cournot case (see (23)), while the FOC with regard to a_i is:

$$\frac{\partial U_i}{\partial a_i} = -1 + \theta \cdot \left[\frac{v_j \cdot (w_H - w_L) \cdot \left(\bar{a}_j\left(a_i\right) - a_i \cdot \frac{d\bar{a}_j}{da_i} \right)}{\left(v_j \cdot \bar{a}_j\left(a_i\right) + a_i\right)^2} \right] = 0.$$

From (25) we can calculate $\bar{a}_j\left(a_i\right)$ and $\frac{d\bar{a}_j}{da_i}$, which substituted into the previous equation yield:

$$\frac{\partial U_i}{\partial a_i} = 0 \Leftrightarrow \bar{a}_i = \frac{\theta \cdot (w_H - w_L)}{4 \cdot v_j} \tag{27}$$

Replacing $\bar{a}_i$ in (25), we find the optimal investment in education of agent j:

$$\bar{a}_j = \frac{\theta \cdot (2 \cdot v_j - 1) \cdot (w_H - w_L)}{4 \cdot v_j} \tag{28}$$

Notice that, if $v_j < \frac{1}{2}$, then the follower does not invest at all: if the poor agent also has less learning ability, he abandons the competition for the high-paying job. Moreover, in the Stackelberg equilibrium without borrowing constraints, we know from (27) and (28) that, if v_j were equal to 1, the two agents would invest the same amount of resources in education. If the follower is rationed, $\bar{a}_j\left(a_i\right) = b_j < \frac{\theta \cdot (2 \cdot v_j - 1) \cdot (w_H - w_L)}{4 \cdot v_j}$ and $\frac{da_j}{da_i} = 0$, so that

$$\frac{\partial U_i}{\partial a_i} = 0 \Leftrightarrow a_i^* = \sqrt{v_j \cdot \theta \cdot (w_H - w_L) \cdot b_j} - v_j \cdot b_j.$$

Finally, if there is a minimum consumption $\tilde{c}$, then b_q has to be replaced with $\tilde{b}_q = \max\left[b_q - \tilde{c}, 0\right]$, for $q = i, j$.

Therefore, we can conclude with the following:

The Stackelberg equilibrium with risk neutral agents is equivalent to the Cournot equilibrium only in the case of homogeneous learning abilities, or in the case of binding borrowing constraints. With an imperfect capital market, the Stackelberg optimal choices differ from the Cournot case:
$a_j^* = \min\left[b_j\left(t\right), \sqrt{\theta \cdot [w_H - w_L] \cdot b_i} - b_i, \frac{\theta \cdot (2 \cdot v_j - 1) \cdot (w_H - w_L)}{4 \cdot v_j}\right]$ and
$a_i^* = \min\left[b_i\left(t\right), \sqrt{\theta \cdot [w_H - w_L] \cdot v_j \cdot b_j} - v_j \cdot b_j, \frac{\theta \cdot (w_H - w_L)}{4 \cdot v_j}\right]$.
If there were a minimum consumption $\tilde{c}$, then b_q would have to be replaced with $\tilde{b}_q = \max\left[b_q - \tilde{c}, 0\right]$, for $q = i, j$.
If $v_j \leq \frac{1}{2}$, then $a_j^* = 0$ (the follower does not invest).

A.3 Distributive dynamics

In the Stackelberg case, we assume that $v_{\min} = \frac{1}{2}$, allowing us to ignore the case in which the follower has much less ability in addition to less wealth in comparison with the leader.[47]

If agents are risk neutral, then the Cournot and Stackelberg equilibria are qualitatively similar as far as the distributive dynamics is concerned (see Propositions A.1 and A.2), so that we analyze only the Cournot case.

From (3) we know that the inheritance is equal to a quota β of second period income: it is then equal to $\beta \cdot w_H$ or to $\beta \cdot w_L$. Defining $\Delta_H = \beta \cdot w_H - \frac{v_j \cdot \theta \cdot (w_H - w_L)}{(1+v_j)^2}$ and $\Delta_L = \beta \cdot w_L - \frac{v_j \cdot \theta \cdot (w_H - w_L)}{(1+v_j)^2}$, we have three possible cases:

1. $\Delta_H < 0$, where both agents are constrained;
2. $\Delta_L > 0$, where no agent is constrained;
3. $\Delta_H > 0$ and $\Delta_L < 0$, where only one agent is constrained, while the other is not.

In the first case the two agents always invest all their inheritance (b_j and b_i respectively). The transition probabilities for agent j are reported in the following Table[48]:

	$w_H\,(t+1)$	$w_L\,(t+1)$
$w_H\,(t)$	$\frac{v_j \cdot w_H}{v_j \cdot w_H + w_L}$	$\frac{w_L}{v_j \cdot w_H + w_L}$
$w_L\,(t)$	$\frac{v_j \cdot w_L}{v_j \cdot w_L + w_H}$	$\frac{w_H}{v_j \cdot w_L + w_H}$

while in the second case both invest $\frac{v_j \cdot \theta \cdot (w_H - w_L)}{(1+v_j)^2}$, so that the the transition probabilities for agent j are:

	$w_H\,(t+1)$	$w_L\,(t+1)$
$w_H\,(t)$	$\frac{v_j}{1+v_j}$	$\frac{1}{1+v_j}$
$w_L\,(t)$	$\frac{v_j}{1+v_j}$	$\frac{1}{1+v_j}$

.

The third case is the most interesting one, because we have one agent who is rationed in every period; if agent j gets w_L, then his offspring will receive $b_j\,(t+1) = \beta \cdot w_L$, and as a consequence he will invest $a_j^*\,(t+1) = \beta \cdot w_L$, while $a_i^*\,(t+1) = \sqrt{\beta \cdot v_j \cdot w_L \cdot \theta \cdot (w_H - w_L)} - v_j \cdot \beta \cdot w_L$, so that $p_j\,(w_H\,(t+1)\,/w_L\,(t)) = \frac{v_j \cdot \beta \cdot w_L}{\sqrt{\beta \cdot v_j \cdot w_L \cdot \theta \cdot (w_H - w_L)}}$. Therefore the transition probabilities for agent j will be

	$w_H\,(t+1)$	$w_L\,(t+1)$
$w_H\,(t)$	$\frac{v_j \cdot \left(\sqrt{\beta \cdot v_j \cdot w_L \cdot \theta \cdot (w_H - w_L)} - v_j \cdot \beta \cdot w_L\right)}{v_j \cdot \sqrt{\beta \cdot v_j \cdot w_L \cdot \theta \cdot (w_H - w_L)} + \beta \cdot w_L \cdot \left(1 - v_j^2\right)}$	$\frac{\beta \cdot w_L}{v_j \cdot \sqrt{\beta \cdot v_j \cdot w_L \cdot \theta \cdot (w_H - w_L)} + \beta \cdot w_L \cdot \left(1 - v_j^2\right)}$
$w_L\,(t)$	$\frac{v_j \cdot \beta \cdot w_L}{\sqrt{\beta \cdot v_j \cdot w_L \cdot \theta \cdot (w_H - w_L)}}$	$1 - \frac{v_j \cdot \beta \cdot w_L}{\sqrt{\beta \cdot v_j \cdot w_L \cdot \theta \cdot (w_H - w_L)}}$

.

[47]¿From the last Proposition, we know that with $v_j \leq \frac{1}{2}$ the follower does not invest. We can exclude this case, which is of little interest.

[48]If there were a minimum consumption threshold $\tilde{c}$ and $\beta \cdot w_L < \tilde{c}$, then the transition probabilities for agent j would become:

	$w_H\,(t+1)$	$w_L\,(t+1)$
$w_H\,(t)$	1	0
$w_L\,(t)$	0	1

that is the distributive dynamics would be non ergodic.

Since $\Delta_L < 0$, we have $\beta \cdot w_L < \frac{v_j \cdot \theta \cdot (w_H - w_L)}{(1+v_j)^2}$, which means that

$$p_j \left(w_H \left(t+1 \right) / w_L \left(t \right) \right) < \frac{1}{2}$$

The richest agent has a greater probability of getting the high-paying job, but the distributive dynamics is ergodic because $p_j > 0 \; \forall t$.

We can observe that if there were not informational problems and credit markets were competitive, $R(t+1)$ would be a function of the wage scale. Consider the third case; in the first period inheritance is equal to $\beta \cdot w_L$, so that the level of the necessary loan is equal to $\left| s_j^* \right| = \frac{v_j \cdot \theta \cdot (w_H - w_L)}{(1+v_j)^2} - \beta \cdot w_L$. The probability of getting the high wage is equal to $p_j = \frac{v_j}{1+v_j}$, so that the expected income is equal to $\frac{1}{1+v_j} \cdot (v_j \cdot w_H + w_L)$. This latter, in turn, must be enough to repay the debt (the amount of loan plus the interests), that is $R(t+1) \cdot \left| s_j^* \right|$. Therefore the loan is granted only if the condition $\frac{v_j \cdot (1+v_j) \cdot (w_H + w_L)}{v_j \cdot \theta \cdot (w_H - w_L) - (1+v_j)^2 \cdot w_L} \geq R(t+1)$ is satisfied, from which the competition among banks would involve that $R(t+1) = \frac{v_j \cdot (1+v_j) \cdot (w_H + w_L)}{v_j \cdot \theta \cdot (w_H - w_L) - (1+v_j)^2 \cdot w_L}$.[49]

Likewise in the risk averse case, it is worth analysing the dynamics focusing on the possible effect of the wage scale on the growth rate. If we suppose that aggregate credentials C are a proxy for the level of human capital and that this latter is the only productive factor, then the output of period t can be expressed as $Y(t) = \phi \cdot C(t-1)$ where $\phi > 0$. Moreover let $\alpha > \frac{1}{2}$ be the quota of output attributed to the high wage position, so that $w_H = \alpha \cdot Y(t)$ and $w_L = (1-\alpha) \cdot Y(t)$. To analyze the long run dynamics we set v_j equal to its expected value ($v_j = 1$) and, without being less general, $b_j = \beta \cdot w_L$ and $b_i = \beta \cdot w_H$. The third case, (distributive dynamics which is always ergodic, while an agent is always rationed), which occurs when $\alpha > \max \left[\frac{\theta}{2 \cdot (\theta - 2\beta)}, \frac{4 \cdot \beta + \theta}{2 \cdot (2 \cdot \beta + \theta)} \right]$, seems to be the most interesting one. By simple algebra, we can calculate the growth rate of output

$$g_Y = \left[\phi \cdot \theta \cdot \beta \cdot (2 \cdot \alpha - 1) \cdot (1 - \alpha) \right]^2 - 1$$

from which

$$\frac{\partial g_Y}{\partial \alpha} \geq 0 \Leftrightarrow \alpha \leq \frac{3}{4}$$

and therefore $\alpha = \hat{\alpha} = \frac{3}{4}$ is the division of output that maximizes the growth rate. This result is analogous to what we found for the risk averse case.

The following Proposition describes the distributive dynamics with risk neutrality:

In an economy with risk neutral agents, the distributive dynamics is not affected by the type of equilibrium we consider.
In the Cournot equilibrium three cases are possible:
1) if $\beta \cdot w_H < \frac{v_j \cdot \theta \cdot (w_H - w_L)}{(1+v_j)^2}$, then all agents are rationed and the distributive dynamics is ergodic;
2) if $\beta \cdot w_L > \frac{v_j \cdot \theta \cdot (w_H - w_L)}{(1+v_j)^2}$, then nobody is rationed and the distributive dynamics is ergodic;

[49] Notice that banks can know v_j given s_j^*.

3) if $\beta \cdot w_H > \frac{v_j \cdot \theta \cdot (w_H - w_L)}{(1+v_j)^2}$ but $\beta \cdot w_L < \frac{v_j \cdot \theta \cdot (w_H - w_L)}{(1+v_j)^2}$, then only one agent is rationed. The rationed agent has a lower probability of getting the high-paying position, but the distributive dynamics is still ergodic.
In this case, considering the aggregate credentials as a proxy of the aggregate level of human capital, the growth rate is maximized when the high wage receives a quota of output equal to $\frac{3}{4}$.
If there were a minimum consumption $\tilde{c}$, then the distributive dynamics could be non ergodic if $\beta \cdot w_L < \tilde{c}$.

References

[1] Basu K. (1989), "A Theory of Association: Social Status, Prices and Markets", *Oxford Economic Papers* 41, pp. 653-671.

[2] Berry S.K. (1993), "Rent-Seeking with Multiple Winners", *Public Choice* vol. 77, pp. 437-443.

[3] Bertocchi G. and Spagat M. (1996), "The Evolution of Modern Educational Systems", mimeo, Brown University..

[4] Bertola G. and Coen Pirani D. (1996), "Market Failures, Education, and Macroeconomics", Working paper n.25.96 Fondazione Eni Enrico Mattei.

[5] Checchi D., Ichino A. and Rustichini A. (1996), "More Equal but Less Mobile? Education Financing and Intergenerational Mobility in Italy and in the United States", CEPR Discussion Paper n. 1496.

[6] Davis S.J. and Haltiwanger J. (1991), "Wage Dispersion between and within U.S. Manufacturing Plants, 1963 - 1986", *Brookings Paper on Economic Activity* vol. 3, pp. 115-180.

[7] De Gregorio J. (1996), "Borrowing Costraints, Human Capital Accumulation, and Growth", *Journal of Monetary Economics* vol. 37, pp. 49-71.

[8] De Gregorio J. and Se-Jik K. (1995), "Credit Markets with Differences in Abilities: Education, Distribution, and Growth", Mimeo, IMF, Washington DC.

[9] Fershtman C., Murphy K.M. and Weiss Y. (1996), "Social Status, Education, and Growth", *Journal of Political Economy* vol. 104, n. 1, pp. 108-132.

[10] Galor O. and Tsiddon D. (1996), "Technological Progress, Mobility and Economic Growth", CEPR Discussion Paper n.1431.

[11] Galor O. and Zeira J. (1993), "Income Distribution and Macroeconomics", *Review of Economic Studies* vol. 60, pp. 35-52.

[12] Glomm G. and Ravikumar B. (1992) "Public versus Private Investment in Human Capital: Endogenous Growth and Income Inequality", *Journal of Political Economy* 100, pp. 818-34.

[13] Goldin C. (1994), "Labour Markets in the Twentieth Century", *NBER Historical Working Paper* n.58.

[14] Hirsch F. (1976), *Social Limits to Growth*, London, Routledge and Kegan Paul.

[15] Lasch C. (1995), *The Revolt of the Elites and the Betrayal of Democracy*, New York, Norton & C.

[16] Linster B. (1993), "Stackelberg Rent Seeking", Public Choice vol. 77, pp. 307-315.

[17] Murphy K.M., Shleifer A. and Vishny R.H. (1991), “The Allocation of Talents: Implications for Growth”, *Quarterly Journal of Economics* vol. 106, pp. 503-530.

[18] Neelsen J.P. (1975), "Education and Social Mobility", *Comparative Education Review* 19, pp. 129-43.

[19] Orsini R. (1997), "The Labour Market as a Job-Seeking Contest: Human Capital, Social Mobility, and Growth", paper presented at SIEC Conference on Economics with Heterogeneous Interacting Agents, Ancona.

[20] Spence A.M. (1973), “Job Market Signalling”, *Quarterly Journal of Economics* vol. 87, pp. 355-374.

[21] Tullock G. (1980), “Efficient Rent-Seeking”, in J.M. Buchanan - R.D. Tollison - G. Tullock (eds.), *Toward a theory of the rent-seeking society*, College Station, Texas, A&M University Press.

[22] Verdier T. and Ades A. (1993), "The Rise and Fall of Elites", mimeo, Harvard University.

[23] Weiss A. (1995), “Human Capital vs. Signalling Explanation of Wages”, *Journal of Economic Perspectives* vol. 9, n. 4, pp. 133-154.