Does Monopoly Undersupply Product Quality?*

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Abstract

I investigate the behaviour of a multiproduct monopolist supplying vertically differentiated varieties of the same good. The discrete model adopted here allows to obtain a continuous model when, in the limit, the number of varieties becomes infinitely large. The main finding establishes that the tendency on the part of the monopolist to undersupply all qualities but the top one can take two alternative forms, i.e., either qualities correspond to the socially optimal ones but the allocation of consumers across qualities is distorted by the price schedule, or qualities are indeed lower than those supplied under social planning. The first case arises when the monopolist finds it profitable to restrict output, while the second obtains when the market is rich enough to induce the monopolist to supply the same quantity a social planner would produce. Policy implications are discussed.

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1 Introduction

The behaviour of a vertically di®erentiated monopolist has received a considerable amount of attention in the literature. The main issue at stake is whether a monopolist has any incentive to supply the same quality that would be available under perfect competition (or social planning), or to distort it so as to induce self-selection on the part of consumers. The earliest contributions (Spence, 1975; Sheshinski, 1976) deal with a single-product monopolist whose cost function is convex in quality and linear in quantity. The main conclusions reached here (Spence, 1975) are that (i) for a given output level, quality is over or undersupplied by the monopolist as compared to social planning, depending on whether the marginal valuation of quality is above or below the average valuation of quality; if they coincide, the monopolist supplies the same quality as the social planner; and (ii) the monopolist undersupplies quality if his output is close to the socially optimal one.

Several other contributions investigate a continuous model where the monopolist supplies a range of qualities, with a technology analogous to that assumed in Spence (Mussa and Rosen, 1978; Itoh, 1983; Maskin and Riley, 1984; Besanko, Donnenfeld and White, 1987; Champsaur and Rochet, 1989).[1] All these authors emphasise that di®erentiation within her own product range enables the monopolist to discriminate among buyers with di®erent characteristics. In order to do so, the monopolist increases the slope of the price-quality gradient compared to the social optimum. This is achieved by o®ering a quality range broader than the one that would be available under social planning or perfect competition. This points to the adoption of Minimum Quality Standards to correct quality distortion (Besanko, Donnenfeld and White, 1987).

The latter statements apparently contrast with Spence's ¯ndings, according to which the di®erence (or coincidence) between monopoly qualities and their socially optimal levels depend upon the consumers' valuation of quality itself, and the choice of which distortion to operate, whether in the quality or the quantity dimension, is taken by the monopolist accordingly. In this paper, I use a discrete model of a multiproduct monopolist sharing the same basic features of the models employed in the above mentioned literature. I derive the continuous case as the limit of the discrete model when the number

[1] See also White (1977). The case where the cost function involves a ¯xed cost of quality improvement is analysed by Gabszewicz, Shaked, Sutton and Thisse (1986).

of varieties tends to infinity, and prove that (i) there exists at least one case, that of a uniform consumer distribution, where the continuous model follows the rules identified by Spence; (ii) the continuous model lacks the crucial information as to where the marginal consumer locates at equilibrium; this produces the relevant consequence that partial market coverage is treated as a case of full market coverage of a restricted population of consumers, yielding quality distortion; (iii) the monopolist always operates a distortion in the allocation of consumers across qualities, either by undersupplying qualities or by pricing above marginal cost (thereby restricting output) while supplying the socially optimal qualities. Then, since Spence's conclusions hold in a continuous setting as well, the opportunity of a regulation policy based on the adoption of an MQS must be reassessed, in that the considerations put forward in the existing literature may not hold true, and there can be cases where an MQS is completely ineffective.

The remainder of the paper is organised as follows. Section 2 contains a summary of the existing literature. The discrete model is presented in section 3. A discussion of the results and policy implications is in section 4. Section 5 contains concluding remarks.

2 Preliminaries: review of the literature

Consider a population of consumers distributed over the interval $[\underline{\theta}; \overline{\theta}]$ according to a continuously differentiable distribution function $F(\theta)$. The associated density function $f(\theta)$ is assumed positive everywhere over the support $[\underline{\theta}; \overline{\theta}]$. Parameter θ denotes consumer's marginal willingness to pay for quality $q \in [0; \infty)$, produced at constant unit cost $C(q)$, where $C(q)$ is twice continuously differentiable, with $C(0) = 0$, $C'(q) > 0$, and $C''(q) > 0$. Total production costs of variety q are $\Gamma = x \cdot C(q)$, where x is the output level. In the remainder of the analysis, it is assumed that the market is supplied by a single firm who is unable to observe the taste parameter θ.

Each consumer buys at most one unit of the good of quality q per period of time. A generic consumer's utility function is defined as follows:

$$U = y + V(q; \theta); \tag{1}$$

where y represents consumption of all other goods. The consumer buys if net unit surplus is non-negative, i.e., if $u(\theta) = V(q; \theta) - p \geq 0$. I assume that

V is thrice continuously differentiable[2] for all q and θ; with (see Besanko, Donnenfeld and White, 1987, p. 745; Champsaur and Rochet, 1989, pp. 536-542):

Assumption 1 $V(0;\theta) = V(q;0) = 0$.
Assumption 2 $V_q(q;\theta) > 0$; $V_{qq}(q;\theta) \le 0$; $\partial[qV_{qq}/V_q]/\partial\theta \le 0$.
Assumption 3 $V_\theta(q;\theta) > 0$ $\forall q > 0$; $V_{\theta\theta}(q;\theta) \le 0$.
Assumption 4 $V_{\theta q}(q;\theta) \ge 0$; $V_{q\theta\theta}(q;\theta) \le 0$.

A further assumption is adopted concerning the distribution function:

Assumption 5 $(1 - F(\theta))/f(\theta)$ is nonincreasing in θ.

2.1 The single-product monopolist

Spence (1975) and Sheshinski (1976) investigate the behaviour of a single-product monopolist facing a continuum of consumers. The firm chooses the optimal quality q of the unique variety being supplied, and the price p (or output x). The demand function for the product is

$$x = \int_{\hat{\theta}}^{\bar{\theta}} f(\theta)d\theta, \tag{2}$$

where $\hat{\theta}$ denotes the marginal willingness to pay of the marginal consumer. If $p/q > \underline{\theta}$, then $\hat{\theta} > \underline{\theta}$, i.e., the price-quality ratio at equilibrium is such that the poorest consumer in the market is unable to buy, and partial market coverage obtains. Otherwise, if $p/q \le \underline{\theta}$, then $\hat{\theta} = \underline{\theta}$, and full market coverage obtains, with $x = F(\theta)$. The monopolist's profit function is defined as $\pi = (p - C(q))x$. Consumer surplus is

$$CS = \int_{\hat{\theta}}^{\bar{\theta}} u f(\theta)d\theta = \int_{\hat{\theta}}^{\bar{\theta}} (V(q;\theta) - p) f(\theta)d\theta, \tag{3}$$

and social welfare is

$$SW = \pi + CS. \tag{4}$$

The following holds (see Spence, 1975, p. 419):

[2] Mussa and Rosen (1978, p.303) assume that $V(q;\theta) = \theta q$.

Proposition 1 For a given output level x, the monopolist undersupplies quality compared to the social optimum if

$$\frac{1}{x}\int_{\theta}^{\bar{\theta}} \frac{\partial p}{\partial q} d\theta > \frac{\partial p}{\partial q};$$

i.e., if the average valuation of quality (at the margin) is larger than the marginal valuation of quality (at the margin), and conversely.

Moreover, the tendency of a monopolist to restrict the output level compared to social planning, for a given quality, must also be accounted for. Let x^S and x^M denote the output observed under social planning (or perfect competition) and monopoly, respectively. Spence (1975, p. 421) establishes that, if x^M is near x^S, then

$$\frac{1}{x^S}\int_{\theta}^{\bar{\theta}} \frac{\partial p}{\partial q} d\theta > \frac{\partial p}{\partial q}(x^M); \tag{5}$$

and consequently

$$\frac{\partial SW}{\partial q}\Big|_{q=q^M} > 0; \tag{6}$$

i.e., the derivative of social welfare in correspondence of the optimal monopoly quality is positive, entailing that the monopolist undersupplies quality compared to the social optimum. The reverse holds when x^M is small.[3]

Therefore, the behaviour of the monopolist is determined by the interplay between (i) his incentive to produce a suboptimal quality, according to the relationship between the marginal consumer's and the average consumer's valuation of quality, and (ii) his incentive to restrict the output level. On the one hand, the above Proposition implies that there exists a class of consumers' distributions producing the coincidence between the monopolist's optimal quality and the socially efficient one, i.e., those for which $(1/x)\int_{\theta}^{\bar{\theta}}(\partial p/\partial q)d\theta = \partial p/\partial q$ holds. This can be expected to be the case if the monopolist restricts output. On the other, the monopolist may find it profitable not to restrict output and discriminate among consumers by undersupplying quality.

[3] The possibility that a monopolist oversupplies product quality is further investigated in Donnenfeld and White (1988) and De Meza (1997).

The incentive to distort the quality level depends on how the surplus that the firm may appropriate varies with quality. Define

$$\bar{\pi}(q) = \max_p \pi(p, q); \quad \overline{SW}(q) = \max_p SW(p, q) = SW(p = C'(q), q) \quad (7)$$

and

$$\beta(q) = \frac{\bar{\pi}(q)}{\overline{SW}(q)}, \quad (8)$$

the slope of $\beta(q)$ determines whether quality is over or undersupplied as compared to the socially efficient level. Taking logs and differentiating w.r.t. q, one obtains

$$\frac{\beta'(q)}{\beta(q)} = \frac{\bar{\pi}'(q)}{\bar{\pi}(q)} - \frac{\overline{SW}'(q)}{\overline{SW}(q)}, \quad (9)$$

implying that, when $\bar{\pi}'(q) = 0$, $\beta'(q)/\beta(q) = -\overline{SW}'(q)/\overline{SW}(q)$. This leads to the following Proposition (Spence, 1975, p. 421):

Proposition 2 The profit-maximising monopolist undersupplies quality compared to the social optimum if $\beta'(q) < 0$, and conversely.

A corollary to the above result is that, when $\beta'(q) = 0$, the profit-maximising quality coincides with the socially optimal one. This is the case when the inverse demand function is linear in the output level (Spence, 1975, p. 422, fn. 7).

2.2 The multiproduct monopolist with a continuum of qualities

Consider now the setting where the monopolist supplies a continuum of varieties $q_i \in [0, \infty)$. Alternatively to any q_i, a consumer may purchase an outside good whose quality is normalised to zero.[4] The rationale behind this assumption lies in the fact that otherwise the derivative of the profit function

[4] This low-end alternative is assumed, either implicitly or explicitly, by Mussa and Rosen (1978), Itoh (1983), Maskin and Riley (1984) and Besanko, Donnenfeld and White (1987). The alternative at the high-end of the product spectrum is considered only by Champsaur and Rochet (1989).

w.r.t. price would be positive everywhere, entailing seemingly an infinitely high price (see Champsaur and Rochet, 1989, p. 538).[5]

Being unable to observe each consumer type, the monopolist cannot perfectly discriminate. As a result, she sets price $p(\theta)$ so as to maximize profits taking into account consumer's reaction, defined as follows:

$$q(\theta) = \arg\max_{q \geq 0} V(q;\theta) - p(\theta). \tag{10}$$

Then, writing the price schedule as $p(\theta) = V(q(\theta);\theta) - u(\theta)$, the monopolist's problem translates into

$$\max_{q(\theta),u(\theta),\hat{\theta}} \int_{\hat{\theta}}^{\bar{\theta}} [V(q(\theta);\theta) - u(\theta) - C(q(\theta))] f(\theta)d\theta, \tag{11}$$

subject to

$$u'(\theta) = V_\theta(q(\theta);\theta) \; \forall\theta \in [\hat{\theta},\bar{\theta}], \tag{12}$$

$$q(\theta) \text{ is nondecreasing} \tag{13}$$

$u(\hat{\theta}) = 0$, and $\hat{\theta} \in [\underline{\theta},\bar{\theta}]$ is the marginal willingness to pay of the marginal consumer. This problem can be treated as an optimal control problem where $q(\theta)$ is the control variable and $u(\theta)$ is the state variable, the relevant Hamiltonian being

$$H = [V(q;\theta) - u - C(q)] f(\theta) + \mu V_\theta(q;\theta), \tag{14}$$

where μ is the co-state variable associated with the constraint (12).

From the necessary first order conditions, one obtains that the optimal quality assignment are given by the following expression (see Besanko, Donnenfeld and White, 1987, p. 748; Champsaur and Rochet, 1989, p. 540):

$$\Pi(q;\theta) = V(q;\theta) - C(q) - \frac{1 - F(\theta)}{f(\theta)} V_\theta(q;\theta), \tag{15}$$

leading these authors to state that monopoly deteriorates quality for all consumers but those located at $\bar{\theta}$, and to sell a larger spectrum of varieties

[5] In the next section, I am going to show that the derivative of the profit function w.r.t. price is indeed always positive under full market coverage, but this does not imply that the equilibrium price can be infinite.

compared to the social optimum. In particular, the profit-maximising monopolist enlarges the quality range downwards.[6] The generic socially optimal quality $q^S(\theta)$ is defined implicitly by the solution of the first order condition

$$V_q(q;\theta) - C'(q) = 0; \qquad (16)$$

under Assumption 4, $q^S(\theta)$ is indeed nondecreasing. Define the socially efficient range of product varieties as $[q^S_L(\tilde{\theta}); q^S_H(\bar{\theta})]$; where $\tilde{\theta}$ identifies the marginal consumer under social planning; and the optimal monopoly range as $[q^M_L(\hat{\theta}); q^M_H(\overset{1}{\theta})]$: The above discussion is summarised by the following

Remark 1 $q^S_H(\bar{\theta}) = q^M_H(\bar{\theta})$; $q^S_L(\tilde{\theta}) > q^M_L(\hat{\theta})$; $q^S(\theta) > q^M(\theta)$ for all $\theta \in (\hat{\theta}; \overset{1}{\theta})$; and $\hat{\theta} \geq \tilde{\theta} \geq \underline{\theta}$:

This is correct if the quality produced by the monopolist is nondecreasing in θ; which in turn holds if the cross partial derivative $\Phi_{q\theta}$ is positive. Otherwise, bunching consumers with different tastes onto the same variety becomes optimal (see also Lemma 1 in Besanko, Donnenfeld and White, 1987, p. 749). Finally, notice that the fourth inequality in Remark 1 establishes that the monopolist may restrict the output level compared to the social optimum.

3 The discrete model

The analysis of (i) the single-product case, and (ii) the multiproduct case, with a continuum of varieties leads to some contradictory conclusions. On the one hand, the discrete model where a unique good is produced reveals that the quality supplied by the monopolist can be lower, equal or higher than the socially optimal quality, depending on consumers' tastes. Moreover, we should expect the monopolist to distort quality downwards as output approaches the output observed under social planning. On the other hand, the continuous model yields that the optimal quality range of a monopolist is strictly larger than the efficient product lines, and contains additional qualities located between the lower bound of the socially optimal spectrum and

[6] Obviously, the opposite holds if the outside good is located at the high-end of the quality spectrum.

the outside good (Champsaur and Rochet, 1989, p. 540). Moreover, the solution of the optimal control problem does not convey a clearcut information as to the extent of market coverage.

Here, I am going to show that Spence's (1975) results hold in the multiproduct case as well. I will prove that distortion always obtains in that the allocation of consumers across product varieties is distorted compared to the social optimum, but this is not always the result of a distortion in qualities. It can be the consequence of pricing above marginal cost (or restricting output), while producing the same qualities a social planner would supply. In order to analyse the multiproduct case where quality is a discrete variable, I assume that

- $q_i \in [0; \infty); i = 1; 2; 3...n; q_k \geq q_{k-1}$ for all $k; k - 1 \in \{1; 2; 3...n\}$;
- $V(q; \theta) = \theta q$; consequently, $u(\theta) = \theta q - p$;
- $\Gamma_i = tq_i^2 x_i$; i.e., $C_i(q_i) = tq_i^2$; with $t > 0$;
- consumers are uniformly distributed with unit density over $[\underline{\theta}; \bar{\theta}]$; with $\underline{\theta} > 0$ and $\underline{\theta} = \bar{\theta} - 1$. Hence, $f(\theta) = 1$. Observe that the uniform distribution satisfies Assumption 5.

For future reference, observe that the interval of consumers' preferred varieties is $[\underline{\theta}/(2t); \bar{\theta}/(2t)]$ (see Cremer and Thisse, 1994; Lambertini, 1997). The objective of the monopolist is to maximise w.r.t. prices and qualities

$$\Pi = \sum_{i=1} \pi_i = \sum_{i=1} (p_i - tq_i^2)x_i \tag{17}$$

while that of the social planner consists in maximising

$$SW = \Pi + \sum_{i=1} \int_{\theta_i}^{\theta_{i+1}} u(\theta)d\theta = \sum_{i=1} \int_{\theta_i}^{\theta_{i+1}} (\theta q_i - tq_i^2)d\theta; \tag{18}$$

where $\theta_i = (p_i - p_{i-1})/(q_i - q_{i-1})$ defines the marginal willingness to pay for quality of the consumer indifferent between varieties i and $i-1$. All individuals for which $\theta \in (\theta_{i+1}; \theta_i)$ purchase variety i; all those for which $\theta \in (\theta_i; \theta_{i-1})$ purchase variety $i-1$; and so on. At the upper bound of the quality range, the demand for q_n is $x_n = \bar{\theta} - (p_n - p_{n-1})/(q_n - q_{n-1})$; at the lower bound, the demand for q_1 is

$$x_1 = \frac{p_2 - p_1}{q_2 - q_1} - \mathrm{Max}\left\{\theta_0 = \frac{p_1}{q_1}; \underline{\theta}\right\}; \tag{19}$$

i.e., either $x_1 = (p_2 - p_1)/(q_2 - q_1) - p_1/q_1 = (p_2 - p_1)/(q_2 - q_1) - \theta_0$; under partial market coverage, or $x_1 = (p_2 - p_1)/(q_2 - q_1) - \underline{\theta}$; under full market coverage.

Several of the results that can be derived in this setting are in Lambertini (1997). The reader interested in the details of the ensuing analysis is referred to that paper.

3.1 Partial market coverage

When $\theta_0 = p_1/q_1 > \underline{\theta} = \bar{\theta} - 1$; the poorest consumers are excluded from consumption of any variety. The following holds:

Proposition 3 For any number of varieties n, the profit-maximising monopolist produces the same qualities as the social planner, while supplying half the output as the social planner, both overall and for each variety. In the limit, as the number of varieties tends to infinity, the social planner serves all the market, while the monopolist serves only the upper half.

The complete proof is in Lambertini (1997, pp. 116-118). Here, I will resume some elements only. The intuition behind the result that, for any given n, equilibrium qualities are the same under both regimes lies in the fact that when the distribution is uniform and demands are linear, the average valuation of quality coincides with the marginal valuation for quality (Spence, 1975; see above),[7] and $p_i = q_i(\bar{\theta} + tq_i)/2$. On this basis, the distortion operated by the monopolist takes the usual form, i.e., an output restriction operated through the price mechanism. Given the monopoly price-output decision, it can be immediately verified that the first order condition relative to any quality q_i is the same under monopoly and social planning. Equilibrium qualities, quantities and prices are summarised as follows:

$$q_i^S = q_i^M = \frac{i\bar{\theta}}{t(2n+1)}; \quad i = 1; 2; 3...n; \tag{20}$$

[7] Straighforward calculations are needed to show that the same applies in the case of the triangular distribution described by $f(\theta) = 2(\bar{\theta} - \theta)$.

$$X^M = \sum_{i=1}^{n} x_i^M = \frac{n\bar{\theta}}{2n+1} = \frac{X^S}{2}; \tag{21}$$

$$x_i^M = \frac{X^M}{n} = \frac{x_i^S}{2}; \; i = 1, 2, 3, \ldots, n; \tag{22}$$

$$p_i^M = \frac{i\bar{\theta}^2(2n+i+1)}{2t(2n+1)^2}. \tag{23}$$

On the basis of (22), the following result can be established:

Corollary 1 In the monopoly regime, for any n, $x_n^M = x_n^S/2$ consumers are supplied with the same quality they would buy under social planning. All remaining consumers purchase a lower quality than under social planning. Since

$$\lim_{n \to \infty} x_n^M = 0,$$

as the number of varieties tends to infinity the consumer indexed by $\bar{\theta}$ is the only one in a position to buy the same quality as under social planning.

As anticipated above, (20-22) hold if and only if the inequality $\theta_0 > \bar{\theta} - 1$ is satisfied. Consider the monopoly setting where a single variety is produced. In this case, $\theta_0^M(1) = 2\bar{\theta}/3$. When two varieties are produced, $\theta_0^M(2) = 3\bar{\theta}/5$; when three varieties are produced, $\theta_0^M(3) = 4\bar{\theta}/7$; and so on. In general,

Proposition 4 In the monopoly regime, the marginal willingness to pay for quality of the consumer who is indifferent between the lowest quality and the outside good is $\theta_0^M(n) = (n+1)\bar{\theta}/(2n+1)$. Under social planning, $\theta_0^S(n) = \theta_0^M(n)/2$.

Proof. In order to prove the first statement in the above Proposition, two alternative routes can be taken. The first consists in a simple argument by induction. Note first that $\theta_0^M(n) = \theta_0^M(1)$ if a single good is supplied. Then, observe that, in the case of $n+1$ varieties, one obtains $\theta_0^M(n+1) = ((n+1)+1)\bar{\theta}/(2(n+1)+1)$; defining $\nu = n+1$, the former expression becomes $\theta_0^M(\nu) = (\nu+1)\bar{\theta}/(2\nu+1)$, which differs from $\theta_0^M(n)$ only for the presence of ν in place of n. The second consists in deriving the location of the marginal consumer from (20-23) when $i = 1$. Then, the proof that $\theta_0^S(n) = \theta_0^M(n)/2$ follows from the straightforward comparison between total outputs in (21). ■

Therefore, I can also state

Corollary 2 Under monopoly (respectively, social planning), a necessary and sufficient condition for partial market coverage to obtain is $\mu_0^M(n) > \underline{\mu}$; (respectively, $\mu_0^S(n) > \underline{\mu}$) i.e., $\overline{\mu} < (2n+1)/n$ (respectively, $\overline{\mu} < (2n+1)/(2n)$).

Consider the monopoly regime. The sufficiency relates, obviously, to the condition that the profit associated with partial market coverage be larger than the profit associated with full market coverage. A simple argument suffices to prove this. Consider that profit maximisation requires the choice by the monopolist of the price-quality ratio $\mu_0^M(n) = p_1/q_1$ defining the location of the marginal consumer over the interval $[\underline{\mu}; \overline{\mu}]$. If profit maximisation w.r.t. prices and quantities yields $\mu_0^M(n) > \underline{\mu}$, this implies that the price-quality schedule chosen by the monopolist is indeed optimal if and only if $\overline{\mu} < (2n + 1)/n$. If the latter inequality is not satisfied, the monopolist must take into account that the market is so rich that no consumer can be priced out, i.e., full market coverage is to be expected from the outset. As to the behaviour of the social planner, notice that the critical threshold of $\overline{\mu}$ below which the planner prices out some consumers is half the monopolist's critical threshold. The policy implications of this result are discussed in section 4.

3.2 Full market coverage

Suppose all consumers are in a position to buy, so that $X^S = X^M = F(\mu) = 1$. Demands are defined as in the previous subsections (see 17-18), with $x_1 = (p_2 - p_1)/(q_2 - q_1) - \underline{\mu}$. Moreover, $p_1 = \underline{\mu} q_1$. The following holds:

Proposition 5 As long as the number of varieties is finite, the monopolist undersupplies all qualities compared to the social optimum. As the number of varieties tends to infinity, the highest quality coincides with the socially optimal one, while the difference between the lowest monopoly quality and the socially optimal one is increasing in the number of varieties and, in the limit, is equal to the range of consumers' preferred qualities.

Again, the complete proof is in Lambertini (1997, pp. 112-116). The lower and upper bounds of the profit-maximising quality spectrum are

$$q_1^M = \frac{n(\overline{\mu} - 2) + 1}{2tn}; \quad q_n^M = \frac{n\overline{\mu} - 1}{2tn}; \tag{24}$$

while the socially optimal bounds are

$$q_1^S = \frac{2n(\overline{\mu} - 1) + 1}{4tn}; \quad q_n^S = \frac{2n\overline{\mu} - 1}{4tn}. \tag{25}$$

As a result, the degrees of differentiation in the two regimes are

$$q_n^M - q_1^M = \Delta q^M = \frac{n-1}{tn}; \quad q_n^S - q_1^S = \Delta q^S = \frac{n-1}{2tn}; \tag{26}$$

with $\Delta q^M = 2\Delta q^S$; and

$$\lim_{n \to \infty} \Delta q^M = \frac{1}{t}; \quad \lim_{n \to \infty} \Delta q^S = \frac{1}{2t}. \tag{27}$$

Hence, for a given output level, the distortion observed in the equilibrium quality levels under monopoly increases as one moves downwards along the quality spectrum. It is easily shown that

$$x_i^M = x_i^S = \frac{1}{n}. \tag{28}$$

As a result, given that the output level of any variety is the same in both regimes, undersupplying quality allows the monopolist to extract more surplus from rich consumers. In the limit, the result of "no distortion at the top" obtains:

$$\lim_{n \to \infty} (q_n^S - q_n^M) = 0; \quad \lim_{n \to \infty} (q_1^S - q_1^M) = \frac{1}{2t}; \tag{29}$$

while all qualities lower than q_n^M becomes more distorted as n increases.

Notice that the solution of the monopolist's problem when full market coverage is assumed from the outset implies solving $n-1$ first order conditions w.r.t. prices, since the price of the lowest quality is $p_1 = \underline{\theta} q_1$. Although known from the outset, this piece of information must be used after writing the first order conditions concerning the $n-1$ products above q_1. Moreover, $p_1 = \underline{\theta} q_1$ coincides with the price charged on the lowest quality under partial market coverage if and only if the location of the marginal consumer under partial coverage coincides with the lower bound of the support of consumers' distribution, i.e., $\theta_0 = \underline{\theta} = \overline{\theta} - 1$. Finally, observe that, in the papers analysing the continuous model, it is used from the outset the information that the surplus enjoyed by the marginal consumer in equilibrium must be nil, which *ex post* is indeed correct independently from the extent of market coverage, but is appropriately used *ex ante* only if full market coverage is expected to arise at equilibrium. Its use *ex ante* under partial market coverage eliminates one degree of freedom and modifies first order conditions, in that it transforms the optimisation problem under partial coverage into an optimisation problem under full coverage of a subset $[\hat{\theta}; \bar{\theta}]$ of the population of consumers. This procedure yields that the monopolist always undersupplies all qualities, and prevents from explicitly locating the marginal consumer along the support, thereby inducing to state that the monopolist may or may not restrict the extent of market coverage as compared to social planning or perfect competition.

4 Discussion and policy implications

I am now in a position to discuss the implications of the results derived in the previous section. The comparison between monopoly and social planning yields the following:

Proposition 6 For all $\bar{\theta} \geq (2n+1)/n$; full market coverage is observed irrespectively of the market regime. For all $\bar{\theta} < (2n+1)/(2n)$; partial market coverage is observed irrespectively of the market regime. For all $\bar{\theta} \in [(2n+1)/(2n); (2n+1)/n)$; the market is fully covered by the social planner while it is only partially covered by the monopolist.

When partial market coverage emerges at the monopoly equilibrium, qualities are the same as under social planning, i.e., they are undistorted. However, a distortion observed in the allocation of consumer across qualities is observed, due to the fact that price-quality gradient increases in the quality level and output is reduced as compared to social planning. Conversely, under full market coverage, the output level is not restricted, both overall and for any single variety, and the allocation of consumers across qualities is distorted by undersupplying each quality, but the top one in the limit when the number of varieties tends to infinity and consequently the product spectrum becomes continuous. Therefore, as the quality range becomes continuous, the result of "no distortion at the top" is common to both settings. As long as quality is discrete, i.e., for any finite value of n; there exists a group of consumers (those identified by $\theta \in (\theta_n; \bar{\theta}]$) that, under partial market coverage, are able to buy the same quality irrespectively of the firm's objective function, although they obviously pay different prices in the two cases. This can never happen under full market coverage, if quality is a discrete variable. To sum up, I can state

Proposition 7 Consider any finite n. If $\bar{\theta} < (2n+1)/n$; partial market coverage obtains and the monopolist supplies the socially optimal qualities and distorts the allocation of consumers across qualities through the price vector, for all values of μ except $(\mu_n; \bar{\mu}]$. If $\bar{\mu} \geq (2n+1)/n$, full market coverage obtains and the monopolist undersupplies all qualities, while producing the same output as the social planner for any variety.

Proposition 8 As n tends to infinity and the quality range becomes continuous, partial market coverage obtains if $\bar{\mu} < 2$. Otherwise, the market is fully covered. In both cases, there exists a unique consumer, located at $\bar{\mu}$, purchasing the same quality as under social planning.

A few additional remarks are in order. First, if the market is relatively poor, the monopolist finds it optimal to restrict the output level, while the quality spectrum coincides with the socially efficient one. This implies that the misallocation of consumers is operated by the usual tendency for the monopoly to price above marginal cost. The distortion in qualities emerges only if the market is so affluent that no consumer can be profitably priced out. Second, since we can imagine that in real world situations the presence of fixed costs in production prevents the quality range from becoming continuous, we may reasonably expect to observe the former case rather than the second, when the marginal valuation of quality coincides with the average valuation of quality. Third, the above analysis has relevant implications concerning the possibility of regulating the monopolist's behaviour through the adoption of a Minimum Quality Standard (MQS). This policy is investigated in the continuous setting by Besanko, Donnenfeld and White (1987). In the light of the above discussion, the MQS is ineffective under partial market coverage, when qualities are not distorted. It can be used to raise the average quality available in the market either under full market coverage, or under partial market coverage, provided the marginal valuation of quality is lower than the average. Under the assumption that the lower bound of the monopolist's product range is lower than the MQS, Besanko, Donnenfeld and White (1987, Proposition 1, p. 750), find that consumers for whom the MQS is not binding purchase the same quality as in the unregulated equilibrium, while those for whom the MQS is binding receive a higher quality; moreover, some consumers may be excluded from the market after the adoption of an MQS. The first part of the statement is not generally true, in that the MQS induces a change in the prices of all varieties and a consequent modification in the assignment of consumers across qualities;[8] the second part of the state-

[8] In Besanko, Donnenfeld and White (1988), the population of consumers consists in two disjoint groups. In that case, it is indeed true that the introduction of the MQS affects

ment can be true, but it implies that the MQS policy might be discontinued if partial market coverage obtains.

Finally, a relevant policy implication of Proposition 6 is the following:

Corollary 3 If $\overline{\theta} \in [(2n+1)/(2n), (2n+1)/n)$; an MQS can be adopted only in combination with a price regulation such that the monopolist covers the entire market, i.e., $p_1 = \theta_0 q_1$.

This highlights that, used in isolation, an MQS policy may not be viable, in that whenever the distortion operated by the monopolist takes the usual form of a price increase (or output restriction) rather than a reduction in quality, authorities should rather adopt policy measures explicitly tailored on output or price, such as price caps or, perhaps, rate of return regulations.

5 Conclusions

Using a discrete model, I have shown that we should expect a monopolist offering a vertically differentiated range of varieties of the same good to behave according to the rules found by Spence (1975). I have illustrated the case where the distribution of consumers is uniform over a given support representing marginal willingness to pay for quality. The continuous case is obtained in the limit, as the number of varieties tends to infinity. The main conclusion emerging from the continuous model, namely, that no distortion at the top should be observed in equilibrium, i.e., all consumers but that (or those) characterised by the highest valuation of quality should buy a lower quality than under social planning, is qualified by establishing whether the market is fully or only partially served. Under partial market coverage, which obtains whenever the market is sufficiently poor to induce the monopolist to price so as to exclude some individuals from consumption, qualities coincide with the socially optimal one and the misallocation of consumers across qualities is entirely due to the price mechanism distorting output. Otherwise, under full market coverage, which obtains when the monopolist cannot profit from pricing any consumer out because the market is too rich to allow it, then no distortion in the output level can be operated and the misallocation of consumers across varieties is due to the monopolist stretching the quality range below the lower bound of the socially efficient spectrum.

only the quality supplied to the low-income group of consumers.

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