Compact Modelling of SiGe HBTs

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Abstract— **The main goal of this paper is to outline principle ideas and conceptual differences in modern compact modelling procedures for SiGe HBTs. Instead of going into description of particular model structures, the principle approaches to SiGe HBT compact modelling have been analyzed in terms of main transfer current and corresponding charges. Special emphasis is put on the modelling of the Early effect, quasi-neutral base recombination, high-injection and temperature effects.**

I. INTRODUCTION

The worldwide interest in silicon germanium (SiGe) heterojunction bipolar transistors (HBTs) as a commercial IC technology is growing rapidly [1]. The introduction of the Ge mole fraction in the bipolar transistor base permits the reutilization of transistor geometry and doping profiles that improves the transistor current gain and high-frequency performance. The corresponding circuit design activities require also compact modelling procedures that accurately describe SiGe HBTs in all operation modes.

The evolution of the compact bipolar transistor models is characterized by continuously growing complexity, mainly following the ideas of the famous Gummel-Poon (GP) model introduced in 1970 [2], [3]. In its various form, especially as a Berkeley SPICE GP (SPGP), GP model is still very often used for circuit design. However, it was early recognized that the GP model is not accurate enough for high-frequency and high-speed large-signal transient applications requiring high collector current densities. A new wave of BJT compact modelling developments has been initiated resulting in sophisticated modern bipolar transistor compact models as Mextram, Hicum and Vbic [4].

Most of the existing bipolar compact modelling procedures employ the idea of integral charge control relationship (ICCR) to express the transistor transfer current [4]. ICCR is essentially based on the assumption that the total hole base charge and the base Gummel number change proportionally with applied biases. Such an assumption could be adequate for the bipolar transistors having uniform intrinsic carrier concentration in the base. However, with a nonuniform doping and Ge distribution in the transistor base, the bias induced modulation of the base hole charge and the base Gummel number have quite different rates [5]. As a consequence, standard ICCR fails to correctly represent the Early effect in the transistor transfer current characteristics. A high mole fraction of Ge in the base produces a non-negligible neutral base recombination [6] which requires also to introduce a base current *Early effect* into the compact modelling procedure. Moreover, the presence of base-collector hetero-junction requires to treat the high-injection effects in a different manner. Finally, the temperature dependence of the SiGe HBT is different owing to the bandgap variations along the transistor structure.

The main goal of this paper is to outline the principle ideas and conceptual differences in the modern compact modelling procedures in treating the peculiar features of SiGe HBTs.

II. INTEGRAL CURRENT AND CHARGE CONTROL

Assuming one dimensional transistor structure, as shown in Fig. 1, the transfer current is generally defined in the integral form [4]

$$I_N = -qn_i^2 A_E \frac{\xi(x_E) - \xi(x_C)}{G\,(x_E, x_C)}$$
$$G\,(p; x_E, x_C) = \int\limits_{x_E}^{x_C} \frac{p(x)}{D_n(x)} \left(\frac{n_i}{n_{ie}(x)}\right)^2 \frac{1}{\eta(x)}\, dx \tag{1}$$

where $\xi = \exp(-\phi_n/V_T)$ and $\eta = \exp(\phi_p/V_T)$ are *Slotboom variables*, ϕ_n and ϕ_p are electron and hole quasi-Fermi levels, p is the hole concentration, n_i and n_{ie} are the intrinsic and position dependent effective intrinsic carrier concentrations, D_n is the electron diffusivity, A_E is the effective emitter area, V_T is the thermal voltage and q is the elementary charge. The choice of control interval (x_E, x_C) allows to trade the problem of evaluating the integral term G with the evaluation of boundary terms $\xi(x_E)$ and $\xi(x_C)$. The simplest way to control G is to restrict (1) to the quasi-neutral base (QNB)

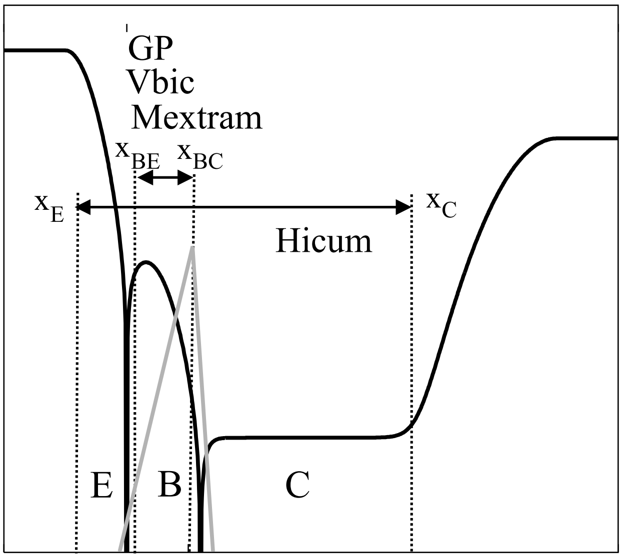

Fig. 1. The 1-D transistor structure and integral charge control intervals employed in the evaluation of the transistor transfer current.

interval (x_{BE}, x_{BC}), as it is done in GP, Vbic and Mextram. From the simplicity of evaluating $\xi(x_E)$ and $\xi(x_C)$, it is best to position x_E and x_C at emitter and collector contacts, or at least at points connected to the contacts via pure ohmic regions. Such an approach is followed by Hicum. The integral expression (1) implies the continuous quasi-Fermi potentials across the hetero-junction interfaces neglecting the effects of thermionic emission.

For the purpose of compact modelling it is convenient to split G into low-level injection component

$$G_L = G_0 + G_{jE} + G_{jC} \tag{2}$$

where

$$\begin{aligned} G_0 &= G\left(p_0; x^0_{BE}, x^0_{BC}\right) \\ G_{jE} &= G\left(p_0; x_{BE}, x^0_{BE}\right) \\ G_{jC} &= G\left(p_0; x^0_{BC}, x_{BC}\right) \end{aligned} \tag{3}$$

and high-injection component

$$G_H = G_B + G_E + G_C \tag{4}$$

where

$$\begin{aligned} G_B &= G\left(p - p_0; x_{BE}, x_{BC}\right) \\ G_E &= G\left(p; x_E, x_{BE}\right) \\ G_C &= G\left(p; x_{BC}, x_C\right) \end{aligned} \tag{5}$$

where x^0_{BE}, x^0_{BE}, p_0 and G_0 are zero bias values of x_{BE}, x_{BE}, p and G.

The ICCR is based on the assumption that

$$\frac{G(p; x_E, x_C)}{G_0} \rightarrow \frac{Q(p; x_E, x_C)}{Q_0} \tag{6}$$

where

$$Q(p; x_1, x_2) = qA_E \int\limits_{x_1}^{x_2} p(x)\, dx \tag{7}$$

represents the total hole charge in the region of integration and Q_0 is its zero-bias value. In a full analogy with the integral term G, the total hole charge Q can be split into Q_{jE} and Q_{jE}, representing base-emitter and base-collector depletion charges, and Q_B, Q_E and Q_E, being diffusion charges in the base, emitter and collector region. However, for the modelling of SiGe HBTs the concept of ICCR should be abandoned or at least appropriately modified.

III. EARLY EFFECT AND DEPLETION CHARGES

With nonuniform Ge (and doping) profiles in the transistor base the ICCR is not valid any more. The bias dependence of the low-level injection integral term G_L and the corresponding charge Q_L could have quite different rates. Introducing the zero bias forward and reverse Early voltages V^0_F and V^0_R as [7]

$$\frac{1}{V^0_{R(F)}} = \frac{1}{G_0}\left(\frac{\partial G_{jE(C)}}{\partial V_{BE(C)}}\right)_{V_{BE(C)}=0} \tag{8}$$

and zero-bias depletion capacitance

$$C^0_{jE(C)} = \frac{\partial Q_{jE(C)}}{\partial V_{BE(C)}} \tag{9}$$

the relative contribution of the integral terms $G_{jBE(C)}$ can be expressed as [7]

$$\frac{G_{jE(C)}}{G_0} = \frac{Q^{\mathrm{eff}}_{jE(C)}}{C^0_{jE(C)} V_{R(F)}} \tag{10}$$

where

$$Q^{\mathrm{eff}}_{jE(C)} = \int\limits_{x_{BE}(x^0_{BC})}^{x^0_{BE}(x_{BC})} p_0(x) \left(\frac{n_{ie}(x^0_{BE(C)})}{n_{ie}(x)}\right)^2 dx \tag{11}$$

represents the effective base-emitter and base-collector depletion charges. For piecewise linear Ge and exponential doping profiles, the effective depletion charges can be explicitly evaluated in terms of internal junction biases $V_{BE(C)}$. To this end, the base-emitter and base-collector depletion width modulation is related to the depletion charges $Q_{jE(C)}$ as

$$\frac{w_{BE(C)}}{w_{B0}} = \frac{Q_{jE(C)}}{Q_0}. \tag{12}$$

Another approach to model the effects of the non-uniform SiGe base is to employ [8]

$$G_L \propto Q_0 + h_{jE} Q_{jE} + h_{jC} Q_{jC} \tag{13}$$

where the quantities h_{jE} and h_{jC} compensate for different bias dependence of $G_{jE(C)}$ and $Q_{jE(C)}$. Since the Early effects represent second order effect, the use of operation point independent parameters h_{jE} and h_{jC} is often sufficient.

The depletion charges are determined as

$$Q_{jE(C)}(V) = \int\limits_0^V C_{jE(C)}(V)\, dV \tag{14}$$

where $C_{jE(C)}$ are corresponding depletion capacitances.

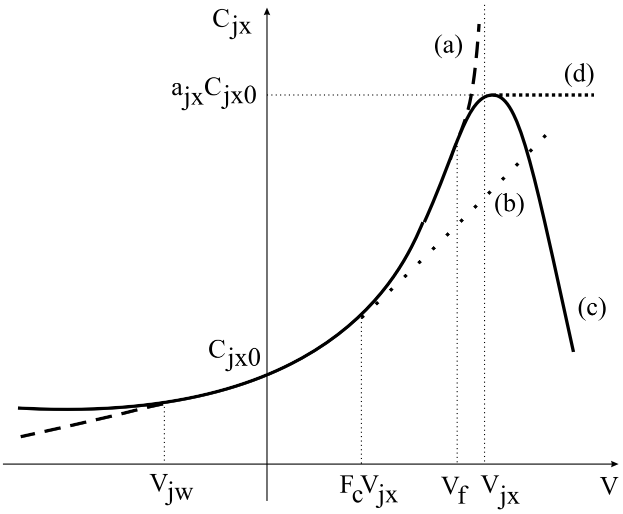

Fig. 2. The bias dependence of the depletion capacitances: (a) ideal theoretical function, (b) SPGP approach, (c) real physical behavior and (d) modern compact modelling approach (Mextram, Hicum).

The models of depletion capacitances are still based on the simple empirical expression inspired by the ideal abrupt and linear total depletion approximation as shown in Fig. 2. However, various modification have been introduced to account for the modulation of the depletion capacitances with the main transistor current, fully depleted epilayer (quantity V_{jw}), as well as to avoid singular capacitance behavior at the forward bias V_{jx} (quantities F_c and V_f).

IV. QUASI-NEUTRAL BASE RECOMBINATION

The carrier recombination in the quasi-neutral emitter and base-emitter space-charge region are considered as a dominant contribution to the bipolar transistor base current in the forward operation mode. However, in some SiGe HBT processes the base current component due to the recombination in QNB may be non-negligible having significant impact on the device performance [6].

The QNB recombination base current component could be in principle evaluated as

$$I_{BB} = qA_E \int\limits_{x_{BE}}^{x_{BC}} \frac{\Delta n}{\tau_n}\, dx \tag{15}$$

where Δn is the excess minority electron concentration and τ_n is the minority carrier lifetime in the QNB. Assuming that the excess electron concentration at the QNB boundaries is proportional to the injection currents [4], we have

$$\Delta n(x_{BE(C)}) \propto \left[\exp\left(\frac{V_{BE(C)}}{V_T}\right) - 1\right] \tag{16}$$

and the integral (15) can be expressed as [9]

$$I_{BB} = I_{BB}^{0}\left[\exp\left(\frac{V_{BE}}{V_T}\right) + \exp\left(\frac{V_{BC}}{V_T}\right) - 2\right]\frac{w_B}{w_{B0}} \tag{17}$$

where I_{BB}^{0} is the QNB recombination current prefactor and $w_B = w_0 + w_{BE} + w_{BC}$ is the QNB width. Notice that the width modulation of QNB, especially that due to the base-collector depletion capacitance, produces an Early-like effect in the forward base current.

V. HIGH-LEVEL-INJECTION EFFECTS

The modelling of the SiGe HBTs at high carrier injection require accurate description of the integral term G_H, as well as the integral charges Q_B, Q_E and Q_C.

For SiGe HBTs, the relative contribution of integral terms G_E and G_C to G_H is much higher then for the silicon BJTs. It has been proposed to generalized ICCR approach [8] using

$$G_H \propto Q_B + h_E Q_E + h_C Q_C \tag{18}$$

where the weighting factors h_E and h_C serve to compensate for the different relative contribution of the minority charges in the emitter and collector regions. One should be aware that, like h_{jE} and h_{jC}, the quantities h_E and h_C are operation point dependent. Moreover, it quite difficult to experimentally separate emitter and collector charges.

Restricting the integral current control to QNB region eliminates above problem but require an accurate description for the electron quasi-Fermi potential drop across the epilayer. It is essential for the correct evaluation of the transfer current I_N in the quasi-saturation mode of operation. To this end, the epilayer, with constant doping concentration N_D, is split into quasi-neutral injection region $0 < x < x_i$ and drift region $x_i < x < w_{\text{epi}}$ as shown in Figure 3. The injection region could be treated in the similar way as the quasi-neutral base while the drift region has resistive behavior with linear electric field distribution due to the electron velocity saturation.

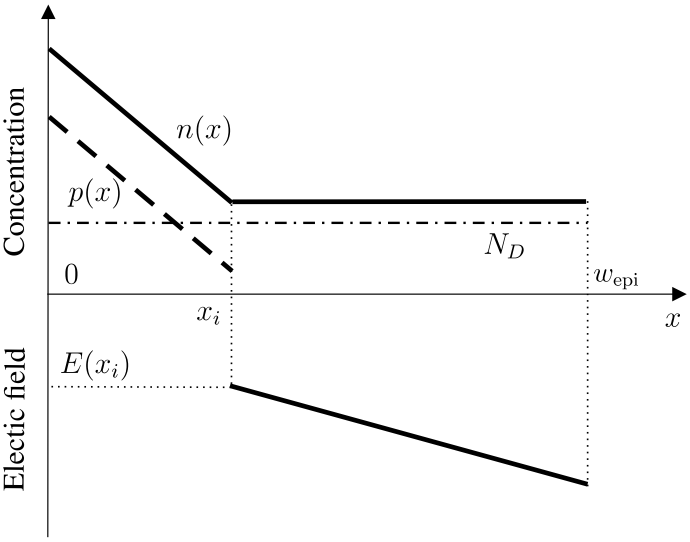

Fig. 3. Distribution of the carrier concentration and electric field in the epilayer.

In a straightforward generalization of the Gummel's charge control approach, the diffusion charges Q_B, Q_E and Q_C may be defined as

$$Q_* = \int\limits_{0}^{I_T} \tau_*\left(V_{BE}, V_{BC}, I\right) dI \tag{19}$$

where τ denotes the operation point dependent transit time while $* = B, E, C$ stands for base, emitter and collector regions, respectively. In principle, transit times have much smoother dependence on bias and temperature then charges, that also include the nonlinear current bias dependence, and may be easier to model. However, only behavioral models of the transit time are currently available.

Alternatively, the charges may be calculated assuming the electron distribution in quasi-neutral base, emitter and collector regions. Mextram was the first model that has abandoned the classical Gummel-Poon transit time charge control and evaluated charges directly from the junction low at quasi-neutral base boundaries. For example the charge in the injection epilayer region in Fig. 3 can be related to the transfer current ICCR

$$Q_C = -q^2 n_i^2 A_E^2 \left\langle D_n \right\rangle \frac{\xi(0) - \xi(x_i)}{I_T} \tag{20}$$

where $\langle D_n \rangle$ is the average electron diffusivity in the injection epilayer region. The advantage is the physics based modelling instead of behavioral. However, due to various assumptions the adaptivity of such physical models to generic measured data may be limited.

It should be emphasized that for SiGe HBTs the carrier injection into epilayer region may be reduced due to the formation of potential barrier at the base-collector hetero-junction. In the transit time based modelling this effects can be incorporated as an additional base transit time term. On the other hand, the physical epilayer model require modification of the interface conditions at the base-collector hetero-junction.

VI. TEMPERATURE EFFECTS

The electrical characteristics of bipolar transistors are particularly susceptible to temperature variations due to the self-heating or thermal interaction with other devices. The electro-thermal effects are typically implemented as a combination of the electrical model (current and charge branches), with temperature dependent parameters, and thermal model, that links the device average temperature to the dissipated electrical power. The temperature scaling of the parameters are based on strong physical background of the electrical parameters.

The model parameters depend implicitly on temperature via intrinsic carrier concentration and carrier mobility temperature dependence. The temperature dependence of the intrinsic carrier concentration is [4]

$$\frac{n_{i*}^2}{n_{i*}^2(T_{\rm ref})} = \left(\frac{T}{T_{\rm ref}}\right)^3 \exp\left(-\frac{V_{g*}}{V_{\Delta T}}\right) \tag{21}$$

where $T_{\rm ref}$ is the reference temperature,

$$\frac{1}{V_{\Delta T}} = \frac{q}{k}\left(\frac{1}{T} - \frac{1}{T_{\rm ref}}\right), \tag{22}$$

k is the Boltzmann constant and $*$ may be `B`, `C`, `S` and `J` for the band-gap in base, collector, substrate and BE depletion region, respectively. This approach is particularly suitable for HBTs with varying band-gap across the device.

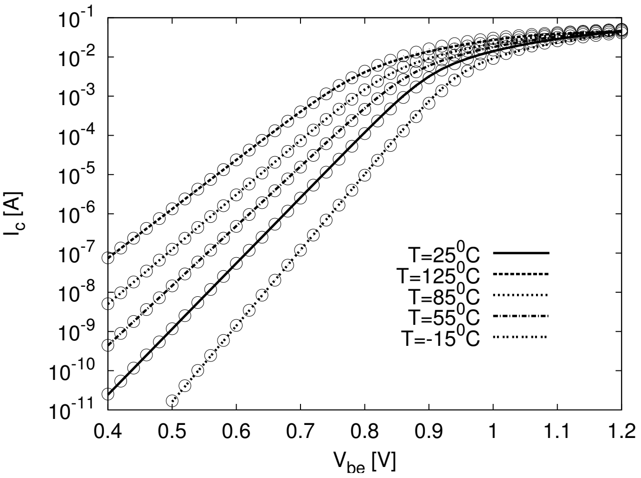

Fig. 4. Gummel plot of the collector current at different temperatures.

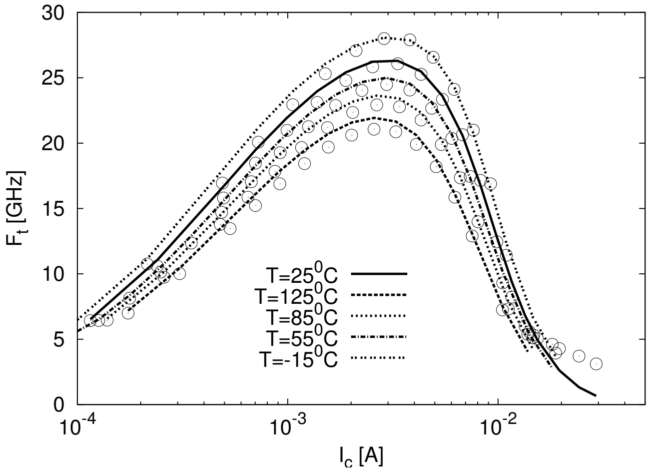

Fig. 5. Cut-off frequency vs. collector current at different temperatures.

For SiGe HBT applications the forward and reverse current gain depend on the difference of band-gaps at base-emitter and base-collector hetero-junctions. It is well known that the compensated charge in the base-emitter space-charge region could have significant effect on the dynamic performance of SiGe HBTs [4]. This charge also has different temperature dependence then the base diffusion charge.

As a practical example of temperature scaling with Mextram (Level 504) model, we present here results for the IBM self-aligned high-breakdown $0.32\mu m$ BiCMOS SiGe graded-base test technology. It has been offered by IBM as a non-production test vehicle for free data exchange in compact model evaluation and standardization effort. The measurement data have been provided for 5 different temperatures. The comparison between measured data and Mextram collector current and cut-off frequency for a single device, having emitter length of $16.7\mu m$, are shown in Fig. 4 and Fig. 5.

VII. CONCLUSIONS

The compact model based on ICCR cannot be directly applied to SiGe HBTs with nonuniform band-gap distribution across the transistor structure. The main modifications are required in the description of forward and reverse Early effects, introduction of QNB recombination terms and its bias dependence, accurate treatment of the high-level injection, as well as temperature related phenomena. Moreover, hetero-junction related effects like thermionic emission current and charge barrier effects have to be also incorporated in the future compact modelling procedure for SiGe HBTs. Finally, the geometry scaling od the model parameters has to be carefully revisited [10].

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