Simulation of Large-Signal Cyclostationary Noise in Microwave Devices: from Physics-Based to Compact Modelling Approaches

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Abstract— **The paper presents a review of the available modelling techniques for cyclostationary noise analysis. Physics-based models are presented first, and exploited as a reference solution for the validation of the phenomenological compact modelling approaches currently used in the circuit simulation area, based on different modulation schemes. The results shows that a unique modulation approach is missing, and that a device and noise source dependent strategy is required.**

I. Introduction

The modelling of cyclostationary noise in large-signal periodic operation has become, during the last few years, an active research topic, with reference to emerging RF applications such as mixers and oscillators, in which noise conversion plays a fundamental role. We review here the available modelling techniques, starting from physics-based simulation, and further discussing the derivation of compact models based on the modulation of stationary (small-signal) noise expressions. Results are shown concerning the conversion of $1/f$ noise in a resistor, and the noise behaviour of pn junction diodes.

The main feature of large-signal operation is the (periodic or quasi-periodic) time-varying nature of the device working point. At least for the case of forced operation (the autonomous case, namely oscillators, requires more care, as discussed in [1]), the periodic modulation effect from the working point makes the stochastic processes representing noise *cyclostationary* [2], [3]. From a statistical standpoint, cyclostationarity implies that the frequency components of the resulting modulated processes are no longer uncorrelated, but rather a correlation takes place. The distinctive feature is that such correlation is not null only for those (angular) frequencies having the same distance ω from one of the harmonics ω_k of the noiseless working point[1]: this allows to partition the spectrum into upper and lower *sidebands* $\omega_k^{\pm} = \omega_k \pm \omega$, where ω is called *sideband frequency*, and to represent the second order statistical properties of the process through the *sideband correlation matrix* (SCM) $\mathbf{S}(\omega)$, whose (k, l)-th element $(\mathbf{S}(\omega))_{k,l}$ is the correlation spectrum between sidebands k and l.

II. Physics-based models

The estimation of cyclostationary noise through physical models can be carried out either by solving the Boltzmann transport equation (BTE), or making use of partial-differential equation (PDE) approaches, namely moment-based simulations. Concerning the BTE, the Monte Carlo approach is the most common tool for its solution: noise evaluation is, in this case, naturally embedded in the very numerical solution technique [4]. Despite these advantages, due to numerical burden Monte Carlo approaches are unfeasibile for the simulation of complex device structures, in particular for bipolar devices and for the analysis of generation-recombination (GR) noise. PDE-based models, on the other hand, are far more computationally efficient, in particular the standard drift-diffusion (DD) system [2], [5], [6]. Noise can be simulated by adding properly defined stochastic forcing terms to the relevant PDEs, called *microscopic noise sources* [7]. Due to local (space-dependent) modulation, such sources are themselves cyclostationary, and their expression can be derived from the stationary (small-signal) microscopic noise sources according to the procedure outlined in [2], [3], [8]. The microscopic noise sources are then propagated to the device terminals, evaluating the induced current or voltage fluctuations (the device noise generators), by means of a linearized analysis: the model equations are assumed to be linearly perturbed by the (small-amplitude) noise sources, thus allowing for a Green's function-based analysis for the propagation step [2]. The model equations are linearized around the instantaneous, large-signal time-periodic working point, therefore the resulting linear system is periodically time-varying (LPTV) [9]: this means that the Green's functions include frequency conversion capabilities, thus giving rise to noise conversion effects. For this reason, noise analysis is more naturally carried out in the frequency domain, where frequency conversion due to a LPTV system is easily represented through the *conversion matrix* formalism [10]: the Green's functions are actually (space dependent) conversion matrices, and therefore are termed *conversion Green's functions* (CGF).

Concerning applications, physics-based cyclostationary noise analysis has been implemented, in general-purpose PDE models, for the bipolar drift-diffusion transport description only [2], [6]. In this case, the diffusion (thermal) noise source is included in the carrier continuity equations, while the GR noise source must be added to all the charge conservation equations. This means that, for trap-assisted GR phenomena, a rate equation for each trap level must be added to the DD system [3]; since the rate equation also includes a microscopic noise source, a CGF must also be evaluated. In the current implementations, the noiseless working point is determined in the frequency domain through the harmonic balance (HB)

[1]For strictly periodic operation, $\omega_k = k\omega_0$, where k is an integer and ω_0 is the fundamental frequency of the applied signals.

approach [2], thus allowing for a direct extension of the efficient numerical technique developed in [11] for the stationary Green's functions evaluation.

The SCM of the short circuit noise generators connected to device terminals i and j can be expressed as [2], [7]:

$$\mathbf{S}_{i_i,i_j}(\omega) = \sum_{\alpha,\beta=n,p,t} \int_\Omega \mathbf{G}_{\alpha,i}(\boldsymbol{r},\omega) \cdot \mathbf{K}_{\gamma_\alpha,\gamma_\beta}(\boldsymbol{r},\omega) \cdot \mathbf{G}^\dagger_{\beta,j}(\boldsymbol{r},\omega)\ \mathrm{d}\boldsymbol{r}, \quad (1)$$

where Ω is the device volume, † denotes the hermitian conjugate, $\mathbf{G}_{\alpha,i}$ is the CGF corresponding to injection in equation α and observation on the current fluctuation induced on terminal i, $\mathbf{K}$ is the SCM of the local noise source for the spatially uncorrelated microscopic fluctuation $\gamma_\alpha(\boldsymbol{r})$, and the sum spans all of the model equations including a microscopic noise source: in particular, t denotes, collectively, the trap level rate equations. Notice that in the case of diffusion noise, $\gamma_\alpha = \nabla \cdot \boldsymbol{\xi}_\alpha$ ($\alpha = n, p$ only), where $\boldsymbol{\xi}_\alpha$ is the (vector) current density fluctuation due to velocity fluctuations.

Despite the significant efficiency improvement with respect to Monte Carlo approaches, even PDE based physical models are by far too computational intensive to be used for circuit-oriented analysis. Nevertheless, apart from the very important application aimed at the design and optimization of the device structure, physical models can, and should, play a significant role as a tool to derive and validate effective, and thus predictive, compact models, as will be discussed in the following section. Besides suggesting the possible simplifications that can lead to the formulation of a physically consistent compact model, physics-based simulators can also provide the reference solution for the compact model validation.

III. Compact models

Since compact cyclostationary noise models are of course an indispensable tool for the design and optimization of noise performance in circuit CAD environment, their development is the subject of significant effort. One of the main difficulties in developing such models is the lack of direct measurements of the statistical properties of the cyclostationary noise processes: this makes the development of black-box approaches very difficult. For this reason, compact models have been derived either from properly approximated physical models, such as the pn diode model presented in [12], or from heuristic methodologies, basically the modulation schemes of stationary compact models based on the seminal work by Dragone [13]. The latter approach is the most commonly employed in the circuit simulation community.

Heuristic compact model derivation is discussed in [9]. The idea is to assume that the (DC) bias dependence of a stationary compact noise model still holds in cyclostationary conditions, but with periodic time-varying bias dependent parameters, thus leading to a modulation effect. The very process of modulation poses severe problems when non-white stationary spectra are involved, as discussed in [9], [8], [7]. The main assumption is to consider a stationary process $\gamma(t)$ with power spectrum $S_{\gamma,\gamma}(\omega_{\mathrm{SS}}) = f^2 \left|\tilde{h}(\omega_{\mathrm{SS}})\right|^2$, where f is a working point-dependent factor, and $\tilde{h}(\omega_{\mathrm{SS}})$ is the impulse response of a properly defined linear time-invariant system taking into account the frequency dependence of the spectrum. Process

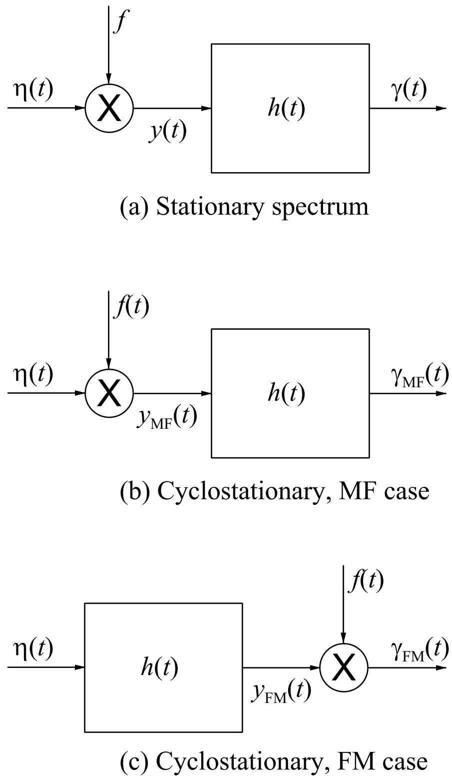

Fig. 1. System interpretation of modulation schemes for cyclostationary compact model derivation: (a) stationary (small-signal) spectrum; (b) MF modulation scheme for the cyclostationary case; (c) FM modulation scheme for the cyclostationary case (from [7]).

$\gamma(t)$ can be interpreted as the output of the linear filtering of a unit white gaussian noise process $\eta(t)$ according to the scheme in Fig. 1 (a). In cyclostationary conditions, the factor $f = f(t)$ periodically modulates the noise process. Such a modulation can be performed at least in two different ways:

- $\eta(t)$ is first modulated by $f(t)$, and thus converted into a cyclostationary process, then filtered by $h(t)$ (see Fig. 1 (b)): this mechanism will be denoted as "MF," and leads to process $\gamma_{\mathrm{MF}}(t)$ with SCM [7]:
$$\left(\mathbf{S}_{\gamma_{\mathrm{MF}},\gamma_{\mathrm{MF}}}(\omega)\right)_{m,n} = \tilde{h}(\omega_m^+) G_{m-n} \tilde{h}^*(\omega_n^+), \quad (2)$$
where G_k is the k-th harmonic amplitude of the periodic function $g(t) = f^2(t)$;
- $\eta(t)$ is first filtered by $h(t)$, then modulated by $f(t)$ (see Fig. 1 (c)): this mechanism will be denoted as "FM," and leads to process $\gamma_{\mathrm{FM}}(t)$ with SCM [7]:
$$\left(\mathbf{S}_{\gamma_{\mathrm{FM}},\gamma_{\mathrm{FM}}}(\omega)\right)_{m,n} = \sum_k F_{m-k} F^*_{n-k} \left|\tilde{h}(\omega_k^+)\right|^2, \quad (3)$$
where F_k is the k-th harmonic amplitude of the periodic function $f(t)$.

Of course, the two approaches lead to same result for a white stationary spectrum ($\tilde{h}(\omega_{\mathrm{SS}}) = 1$). If, on the other hand, $\tilde{h}(\omega_{\mathrm{SS}})$ is a low-pass function, from (2) follows that the MF modulation scheme yields a SCM with null elements apart from $(0,0)$, i.e. the baseband diagonal sideband, while the FM scheme (3) results in a SCM with nonzero elements, provided that $f(t)$ has a large enough number of harmonics. This markedly different behaviour calls for a choice between the two possible approaches. According to the literature, the

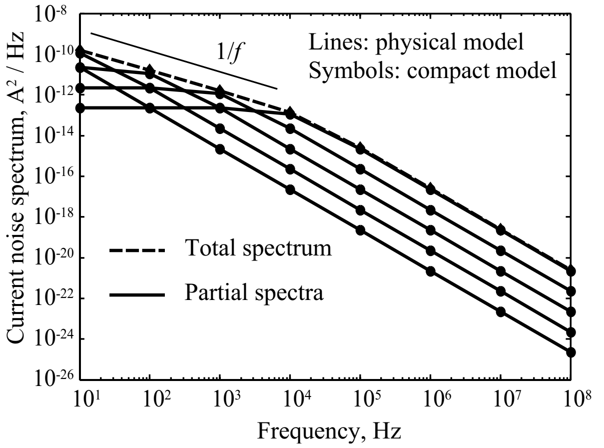

Fig. 2. Small-signal GR noise spectrum for a uniform sample with 5 noninteracting trap levels. Device cross section is normalized to 1 cm^2.

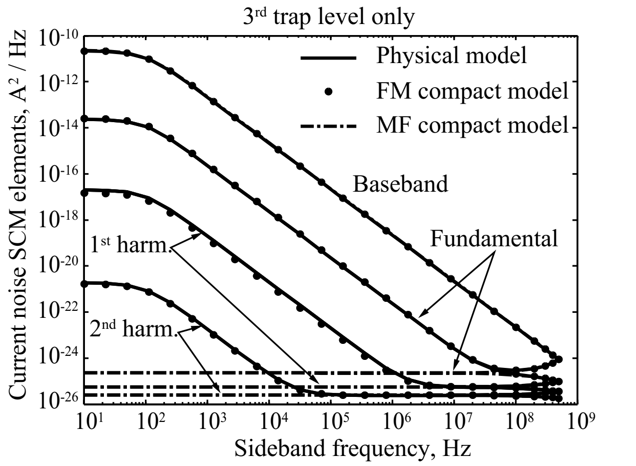

Fig. 3. Sideband frequency dependence of the diagonal elements of the GR noise SCM for the third trap level in the uniform sample with 5 noninteracting traps. Modulated compact models are compared to the physics-based simulation.

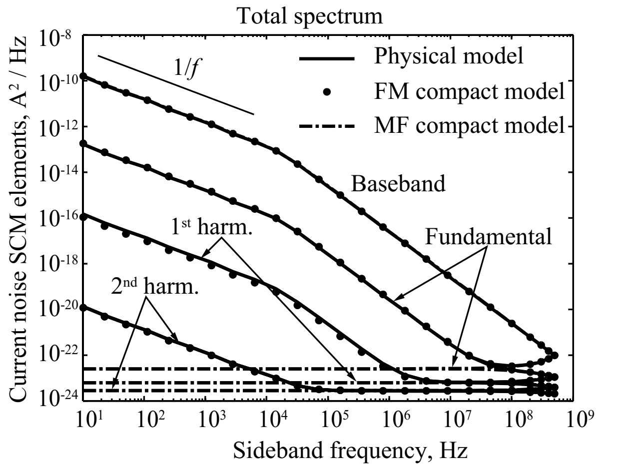

Fig. 4. Sideband frequency dependence of the diagonal elements of the GR noise SCM for the uniform sample with 5 noninteracting traps. Modulated compact models are compared to the physics-based simulation.

FM approach is probably the most commonly applied in circuit simulators, but the MF approach has been exploited as well (see the discussion in [7]).

Physics-based simulation can be used as a reference solution for a comparison between the two approaches, thus allowing for a physically sound indication on the better solution. Notice that, a priori, there is no reason to assume that either modulation scheme will provide exact results, both because of the complex distributed nature of terminal noise generation (see Sec. II), and of the strong assumption on the full factorization of the stationary spectrum model, which in many cases requires further approximations to be fulfilled.

The first case study is the simulation of GR noise due to several noninteracting traps in a uniform Si sample (doping $N_{\mathrm{D}} = 10^{16}$ cm^{-3}, length 2 μm), made nonlinear through the velocity saturation effect for electrons (low-field $\mu_n = 1390$ cm^2 V^{-1} s^{-1}, saturation velocity 10^7 cm/s), while holes have constant mobility ($\mu_p = 470$ cm^2 V^{-1} s^{-1}). The bias was set by a 1 V DC voltage, while in large-signal we superimposed a 0.2 V tone at $f_0 = 1$ GHz. We included in the simulation 5 noninteracting traps (total trap density: $N_{\mathrm{t}} = 10^{13}$ cm^{-3} for each level), all treated according to the Shockley Read Hall model with energy level 0.26 eV below the conduction band. The trap $c_{n,p}$ parameters (the trap electron and hole cross section times the carrier thermal velocity) were chosen logarithmically spaced so as to yield, for a homogeneous sample, a $1/f$ spectrum over a prescribed frequency range [14]: $c_n = c_p = 5.7 \times 10^{-12}$, 5.7×10^{-13}, 5.7×10^{-14}, 5.7×10^{-15}, 5.7×10^{-16} cm^3/s. Fig. 2 shows the current noise spectrum due to GR noise: the total spectrum actually exhibits the expected $1/f$ behaviour on the frequency tange 1 Hz – 10 kHz. Also shown (symbols) are the results of an analytical compact model, assuming that each trap yields a lorentzian spectrum with amplitude proportional to the (squared) DC current I flowing in the device [15]:

$$S_{i,i,k}(\omega_{\mathrm{ss}}) = C_k \frac{I^2}{1+\omega_{\mathrm{ss}}^2 \tau_k^2} \qquad k = 1, \ldots, 5. \tag{4}$$

The agreement is excellent by using the fitted parameters $C_1 = 2,7 \times 10^{-21}$ s, $C_2 = 10C_1$,..., $C_5 = 10C_4$, while the lorentzian corner frequencies are determined by the equivalent trap lifetime defined in [16].

In order to derive the modulated stationary models, we define $\tilde{h}_k(\omega_{\mathrm{ss}}) = (1 + \omega_{\mathrm{ss}}^2 \tau_k^2)^{-1/2}$, and $f_k(t) = \sqrt{C_k} I(t)$, where $I(t)$ is the periodic current flowing into the sample. According to (2) and (3), we can derive the MF and FM compact noise models as:

$$\left(\mathbf{S}_{i_{\mathrm{MF}},i_{\mathrm{MF}},k}(\omega)\right)_{m,l} = C_k \frac{(I^2)_{m-l}}{\sqrt{[1+(\omega_m^+\tau_k)^2][1+(\omega_l^+\tau_k)^2]}}, \tag{5}$$

$$\left(\mathbf{S}_{i_{\mathrm{FM}},i_{\mathrm{FM}},k}(\omega)\right)_{m,l} = C_k \sum_{n=-\infty}^{+\infty} \frac{I_{m-n} I_{n-l}}{1+(\omega_n^+\tau_k)^2}, \tag{6}$$

where $(I^2)_m$ is the m-th harmonic component of $I^2(t)$. The comparison with the physics-based cyclostationary simulations in Fig. 3 (for the 3rd trap only) and 4 (for the total noise spectrum) clearly show that the FM scheme is exact, while the MF modulation underestimates noise at upper sidebands. This is in agreement with the result in [8], where a direct GR mechanism was considered.

A completely different result is found by considering a pn junction diode, as discussed in [17]. If the stationary diode noise has a behaviour significantly different from the pure shot noise at the frequency of the applied signal and of its

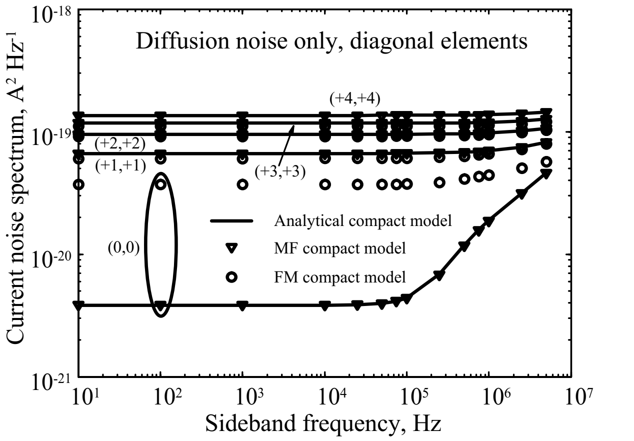

Fig. 5. Sideband frequency dependence of the diagonal elements of the noise current SCM due to diffusion noise for the pn diode analytical and two heuristic compact models (from [17]). Device cross section is normalized to 1 cm^2.

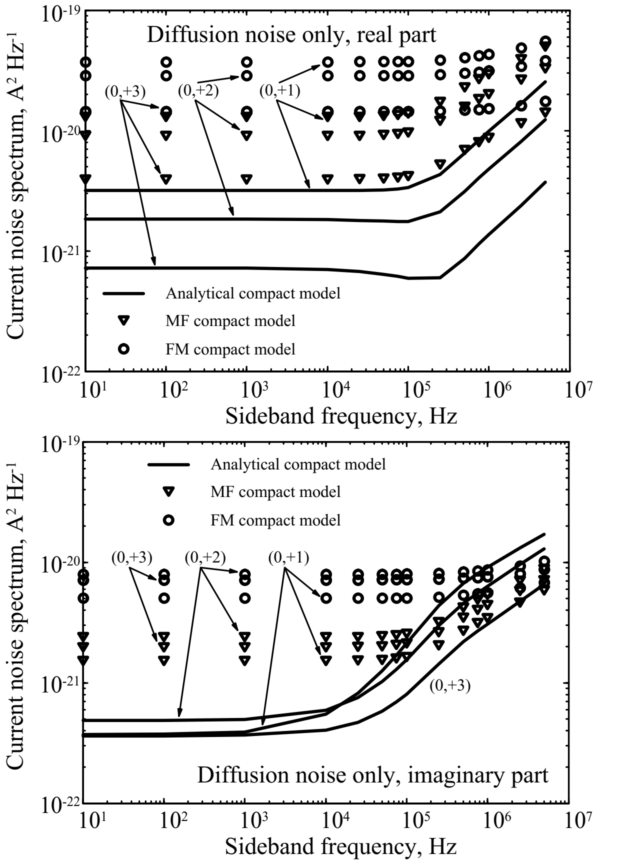

Fig. 6. Sideband frequency dependence of the off-diagonal elements of the noise current SCM due to diffusion noise for the pn diode analytical and two heuristic compact models (from [17]).

relevant harmonics, the previous modulation schemes can be applied to the analytical stationary compact model derived in [12], yielding the (quite complex) expressions reported in [17]. Fig. 5 and 6 show the FM and MF modulated compact models compared to the reference solution, provided here by an exact, fully analytical cyclostationary compact model [12]: the MF scheme yields exact results for the diagonal elements (Fig. 5), while the FM model significantly overestimates the baseband noise. On the other hand, the off-diagonal terms (Fig. 6) are not correctly reproduced by either heuristic model.

IV. Conclusion

We have discussed the available cyclostationary noise modelling techniques, starting from the physics-based simulation approach. Heuristic compact models, derived from the modulation of stationary noise expressions, have also been discussed, presenting two possible modulation schemes. A comparison with physical models, exploited as a reference solution, allows to point out that a unique and exact choice for the modulated compact model is lacking; rather, a device and noise source dependent strategy seems to be required.

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