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# A Finite-Memory approach to the nonlinear modelling of microwave electron devices

**Fabio Filicori*, Giorgio Vannini*,**, Alberto Santarelli**, Vito A. Monaco***

* Dipartimento di Elettronica, Informatica e Sistemistica - Università di Bologna
** Centro di Studi per l'Informatica e i Sistemi di Telecomunicazioni - CNR
Viale Risorgimento, 2 – 40136 Bologna, Italy

**Abstract.** *A technology-independent, mathematical approach is proposed for the look-up-table based nonlinear modeling of electron devices. The model allows for accurate large-signal performance prediction at high operating frequencies, even in the presence of important parasitic and low-frequency dispersive effects. All the nonlinear functions which characterise this black-box model are directly related to conventional measurements which can be carried out with automatic instrumentation. Preliminary experimental results are presented which confirm the validity of the approach.*

## Introduction

A number of mathematical approaches have been proposed [1..7] for the look-up-table based nonlinear modeling of electron devices with the aim of providing accurate large-signal performance prediction directly in terms of commonly available experimental data (i.e., DC characteristics and small-signal AC measurements), without the need for technology-dependent analytical functions to describe the nonlinear device characteristics.

The Nonlinear Integral Model (NIM) proposed in [1,2] meets the above requirements, since it is directly derived, without any constraint on the physical device structure, by truncation of a Volterra-like integral series under the hypothesis of a "short duration" of nonlinear memory effects in voltage-controlled electron devices. All the nonlinear functions which characterise the NIM are directly related to DC characteristics and bias/frequency dependent small-signal admittance parameters. This allows for easy, closed-form identification of the model, without the need for numerical optimisation procedures (with possible local minima problems) or potentially ambiguous computation of the nonlinear characteristics through numerical integration of differential parameters.

The validity of the NIM (i.e., the validity of the short-term memory hypothesis on device behaviour) has been verified [1..4], with good results, for devices not affected by strong parasitics. However when, especially at very high operating frequencies, parasitic phenomena strongly influence the device behaviour, the dynamic response becomes much "slower" (w.r.t. the "intrinsic" device) so that the short-term nonlinear memory hypothesis may not be satisfied. Parasitic de-embedding does not always represent a sufficient solution to this problem, owing to uncertainties in parasitic identification procedures. In such conditions, when strongly nonlinear operation has to be considered, the errors of the NIM may become quite large. To overcome such a limitation a *"Finite-Memory nonlinear Model"* (FMM), following the same "philosophy" of the NIM but with important new concepts, is proposed. In this new approach, the short duration of nonlinear memory effects is not only an *"hypothesis"* on device behaviour, but an *"intrinsic feature"* of the model, where a limitation on memory duration is forced by introducing a finite-time window function with sufficiently short width $T_M$.

## The Finite-Memory nonlinear Model

The time-domain current/voltage relationship of an electron device[1] can be expressed in the form:

$$i(t) = \Psi \left| [v(t-\tau)] \right|_{\tau=0}^{+\infty} \qquad (1)$$

where $\Psi|\cdot|$ is a suitable nonlinear functional or *"line-function"*, according to the symbolism introduced by Volterra [8,9], which indicates the nonlinear dependence of the electron device current $i$ at the generic instant $t$ on the applied voltage $v(t-\tau)$ in a time-interval $0 \leq \tau \leq +\infty$.

By introducing the *"dynamic voltage deviations"*: $e(t,\tau) \doteq v(t-\tau) - v(t)$, which represent the difference between "past" values $v(t-\tau)$ of the applied voltage with respect to the present value $v(t)$, eqn. (1) can be rewritten as:

$$i(t) = \Psi \left| [v(t) + e(t,\tau)] \right|_{\tau=0}^{+\infty} \qquad (2)$$

The above equation is a very general description of the nonlinear dynamic behaviour of an electron device, since (2) has been derived without any simplifying assumption. However, identification of a description like (2) is practically not feasible due to its great generality and complexity.

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[1] For simplicity a single-port electron device will be considered. The model, however, can be directly extended to the case of multiport electron devices.

Usually, in an electron device, the memory effects have a practically finite, short duration. This important feature can be exploited to derive a simplified, practically usable electron device model. In particular, by introducing a suitable time-windowing function $w(T_M,\tau)$, which limits the functional dependence on $e(t,\tau)$ in eqn. (2) to a practically finite time $T_M$, we can obtain the *"finite-memory-time model"*:

$$i(t) = \Psi \left| [v(t) + w(T_M,\tau)e(t,\tau)] \right|_{\tau=0}^{+\infty} + \Delta i_M \tag{3}$$

Clearly, for $T_M \to \infty$, the *"memory truncation error"* $\Delta i_M \to 0$.

To allow for practically feasible model identification, further simplifications are necessary. The still high complexity of (3) can be substantially reduced under the hypothesis of a short-duration of the memory effects (i.e., small $T_M$). In fact, under such conditions, the dynamic deviations $e(t,\tau)$ can be small[2] even in the presence of large signals $v(t-\tau)$. On this basis we can introduce the concept of a device with a relatively "short memory" under large-signal operation. For this class of devices, since the dynamic deviations are small, the nonlinear functional dependence on $e(t,\tau)$ in (3) can be linearised and described in terms of a linear convolution w.r.t. $e(t,\tau)$:

$$i(t) = F_{DC}[v(t)] + \int_0^{+\infty} w(T_M,\tau)g[v(t),\tau]e(t,\tau)d\tau + \Delta i_M + \Delta i_N \tag{4}$$

where $F_{DC}$ is the DC characteristic of the device and $g[v(t),\tau]$ its nonlinearly voltage controlled impulse response. More precisely, the integral expression in (4) is the first term of a multidimensional integral series where the higher order terms have been neglected under the hypothesis of small dynamic deviations $e(t,\tau)$. So, the term $\Delta i_N$ represents the *"nonlinear series truncation error"* due to the linearised approximation for small dynamic deviations. Clearly, $\lim_{T_M \to 0} \Delta i_N = 0$ since, when $T_M$ approaches zero, the dynamic deviations $e(t,\tau)$ become vanishing small, and the convolution description becomes exact.

It is worth noting the strong analogy between (4) and the small-signal description of a device by means of the linear convolution integral. In fact, the short-term memory concept (i.e., small dynamic deviations) enables the description in terms of a convolution integral with respect to the dynamic deviations to be adopted even under large signals, likewise the small-signal hypothesis allows for the description in terms of a convolution integral with respect to the small signals applied.

Equation (4) represents a Finite-Memory Nonlinear Model which can correctly describe the behaviour of a given electron device provided that suitable values of $T_M$ can be found for which $\Delta i_M$ and $\Delta i_N$ are both small enough. This happens in small-signal (or mildly nonlinear) operating conditions, as the series truncation error $\Delta i_N$ is certainly small, while $\Delta i_M$ can be made small by simply chosing $T_M$ large enough.

In strongly nonlinear operation, instead, in order to have a small $\Delta i_N$, a relatively small $T_M$ must necessarily be chosen. In such conditions, the actual duration of the memory effects within the device must be not too much longer than $T_M$, to have also small $\Delta i_M$. Both accurate physics-based simulations and experimental evidence [2,3] have shown that this normally happens for devices with small parasitic effects (e.g., "intrinsic devices") and negligible low-frequency dispersive phenomena. Otherwise, to achieve good accuracy in high-frequency strongly nonlinear operation, both parasitic and low-frequency dispersive effects should be dealt with separately in the model extraction procedure.

When considering discrete-spectrum signals, the voltage $v$ can be described by a Fourier series as a sum of spectral components $V_k$, so that, according to well-known properties of the Fourier transform, eqn. (4) can be expressed, after simple mathematical developments, in the Harmonic-Balance-oriented form:

$$i(t) = F_{DC}[v(t)] + \sum_{k=-\infty}^{+\infty} \widetilde{Y}[T_M, v(t), \omega_k] V_k e^{j\omega_k t} + \Delta i_M + \Delta i_N \tag{5}$$

where $\widetilde{Y}$ is a nonlinearly voltage-controlled dynamic admittance, which describes only the purely dynamic phenomena (that is deviations from purely static behaviour defined by $F_{DC}$) in large-signal, high-frequency operating conditions.

As far as model identification is concerned, the function $F_{DC}$ can be directly measured, since it simply represents the DC characteristic of the device. The dynamic admittance $\widetilde{Y}$, instead, can be easily identified [13] according to the formula:

$$\widetilde{Y}[T_M, V_B, \omega] = \int_{-\infty}^{+\infty} W(T_M,\xi) \left\{ Y[V_B, \omega - \xi] - Y[V_B, -\xi] \right\} d\xi \tag{6}$$

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[2] It can be shown [9] that, for a given signal $v$ with fundamental frequency $f_1$, the peak-to-peak amplitude of the dynamic deviations has an upper limit given by the product $\rho T_M f_1 V_{pp}$, where $\rho$ is a parameter associated to the voltage signal shape and is independent of its amplitude and frequency. For instance, for a sinusoidal voltage waveform $\rho = \pi$.

where $\widetilde{Y}$ is obtained by applying a simple frequency-domain linear convolution of conventional small-signal Y-parameters[3] (measured at different bias conditions $V_B$) with a suitable frequency windowing function $W(T_M, \omega)$ which corresponds, in the time-domain, to the truncation of memory effects of duration longer than $T_M$. Practically, in (6) a Gaussian-like windowing function $W(T_M, \omega)$ is adopted which has a limited frequency width so that the integration interval is finite.

It should be noted that the integral operator in (6) actually involves a "weighted" averaging of measured $Y$ parameters, which also introduces a beneficial "smoothing" effect on noise-like errors in the frequency-domain measurements.

Once the functions $F_{DC}$ and $\widetilde{Y}$ have been identified on a suitable grid of bias conditions and in the frequency range of interest, eqn. (5) can be directly used to compute the FMM response in the framework of Harmonic-Balance tools for circuit analysis. To this end, suitable interpolation techniques are needed [14] to evaluate the functions $F_{DC}$ and $\widetilde{Y}$ for each frequency and set of voltages occurring in the HB analysis.

Equation (5) defines, for different values of the parameter $T_M$ ranging from 0 to $\infty$, a family of models among which a number of existing approaches (see Fig.1) can be singled out as special cases of the FMM.

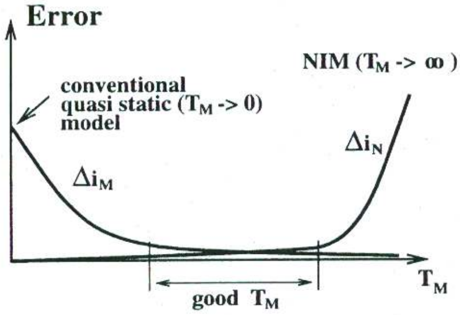

Figure 1: Qualitative behaviour of the series truncation error $\Delta i_N$ and memory truncation error $\Delta i_M$ associated with the FMM.

When $T_M \rightarrow \infty$ (i.e., $\Delta i_M = 0$ but possibly large $\Delta i_N$), for instance, the new FMM (5) coincides with the NIM proposed in [1,2]. For $T_M \rightarrow 0$, (i.e., $\Delta i_N = 0$ but possibly large $\Delta i_M$), instead, the FMM practically coincides with conventional quasi-static models[4] where the currents are represented as the sum of static conductive terms and displacement ones defined as time derivatives of quasi-static charges.

In practice, by chosing a suitable, small yet finite value of $T_M$ a *mildly* non quasi-static model is obtained which can provide more accurate and "robust" (w.r.t. measurement errors) results than the extreme cases mentioned above (i.e., both $\Delta i_N$ and $\Delta i_M$ small). It should be noted that when, for a given device and a given set of possible operating conditions, a suitable value of $T_M$ has been chosen, the general-purpose technology-independent model (5) has been, in practice, "tailored" on the specific nonlinear dynamics of that device.

For transistor modelling, eqn. (1) must be considered as a two-dimensional functional of the port voltages $v_1$ and $v_2$. Following the same procedure above described a two-port model is obtained which can still be described by eqns. (5), (6), considered in a vectorial form. Moreover, in the case of III-V FETs, low-frequency dispersive effects, due to surface state densities and bulk traps, should be taken into account. Since these effects are characterised by slow dynamics, which would limit the validity of the simplifications based on the short-term memory concept, they must be dealt with separately [10,11]. Similar considerations can be made for non negligible parasitics effects which slow down the device response. More details on these issues can be found in [12,13].

## Preliminary experimental results

Preliminary experimental validation was carried out by comparing the performance predicted by the above described modelling approach and large-signal measurements on a $0.6 \mu m \times 600 \mu m$ GaAs MESFET.

In particular, DC characteristics and scattering parameters of the MESFET were measured on a grid of 200 bias points up to a frequency of 40GHz. After parasitic de-embedding, the voltage-controlled dynamic admittance matrix $\widetilde{Y}$ of the FMM was identified according to a convolution formula like eqn. (6) but including also the dispersive term (see Ref. [13]), In particular, the FMM model was identified by using both a windowing function with $T_M = \infty$ (this corresponds to the NIM model [1,2]) and $T_M = 10$ *psec* in (6).

Since no memory truncation is introduced with the NIM model, the small-signal device behaviour is exactly reproduced. However, under large-signal operations, the accuracy of the NIM was found to be not accurate enough. This is reasonably due to uncertainties in the parasitic de-embedding which leads to an intrinsic device

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[3] The Y-parameters can be obtained through simple transformations of scattering parameters which are more easily measured at microwave frequencies.

[4] In fact, it can be shown that $\lim_{T_M \rightarrow 0} \widetilde{Y}[T_M, v, \omega] = j\omega C(v)$ where $C(v)$ is a voltage-controlled capacitance corresponding to the derivative of a quasi-static voltage-controlled charge.

where the memory effects are still important enough so that for $T_M = \infty$ the series truncation error $\Delta i_N$ in (5) is so large to limit the model accuracy.

As previously said, the application of the FMM approach with a suitable, finite $T_M$ should reduce the series truncation error $\Delta i_N$, having at the same time small memory truncation error $\Delta i_M$. For the GaAs MESFET considered, a value of $T_M = 10$ *psec* was easily found which satisfies the above condition. In particular, the good agreement [13] obtained between measured and simulated small-signal scattering parameters, confirms the memory truncation error $\Delta i_M$ introduced is negligible, since the small-signal frequency response of the device is not substantially modified. Moreover, the accuracy of large-signal performance prediction of the FMM (the results in Fig.2 and Table 1 were obtained applying in nonlinear simulations the highly accurate interpolation techniques described in [14]) also indicates that the series truncation error $\Delta i_N$ has been substantially reduced w.r.t. the NIM approach.

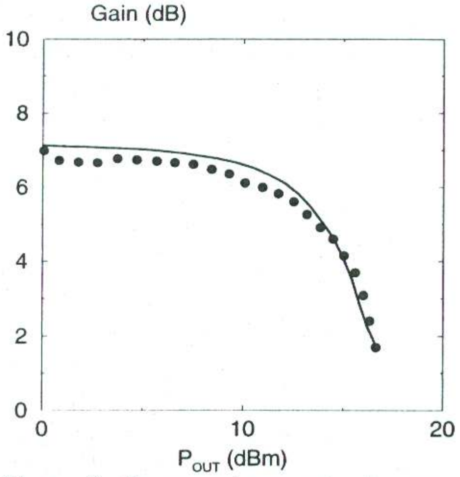

Figure 2: Power gain vs output power for a GaAs MESFET at 5GHz. The agreement between measurements (●) and the performance predicted through the FMM (—) is good.

| $P_{IN}$ (dBm) | Measured $P_{OUT}$ (dBm) $k=1$ | $k=2$ | $k=3$ | FMM $P_{OUT}$ (dBm) $k=1$ | $k=2$ | $k=3$ |
|---|---|---|---|---|---|---|
| 0 | 6.6 | -25.3 | -36.6 | 6.8 | -27.8 | -38.8 |
| 5 | 11.0 | -13.4 | -22.7 | 11.3 | -14.4 | -23.0 |
| 10 | 14.5 | -3.3 | -12.0 | 14.5 | -2.6 | -12.6 |
| 15 | 16.7 | 2.0 | -3.3 | 16.7 | 2.2 | -3.6 |

Table 1: Comparison between harmonics of the output power measured and predicted by the FMM for a GaAs MESFET. The fundamental frequency is 5GHz.

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