CHAOS IN Si MMIC OSCILLATORS

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Abstract - This paper analyzes the dynamics of two Si circuits in millimetric band for which chaotic responses had been experimentally observed. The aim has been to determine, in each case, the particular phenomena leading to the chaotic response, in order to avoid this behavior in future realizations of these circuits. The most influential parameters have been determined, analyzing the routes to chaos under variations of these parameters. Then some frequency-domain techniques have been developed for the detection of the initial bifurcations taking place in these routes from standard harmonic-balance simulators. This should be more efficient in terms of design time and enable the chaos prediction when time domain simulations are not well suited. Two circuits based on an IMPATT diode have been analyzed, one of them being a self-oscillating mixer, obtaining very good agreement with former experimental observations.

I. INTRODUCTION

The microwave circuits of autonomous nature, such as oscillators, frequency dividers or self-oscillating mixers are ruled by non-linear differential equations that often exhibit chaotic solutions. These are neither periodic nor quasi-periodic, having instead a random-like steady-state behavior, which is in fact deterministic [1]. They show an extreme sensitivity to the initial conditions, which makes their time evolution unpredictable. The solution non-periodicity is responsible for the continuity of the spectrum and its typical noise-like appearance. Another characteristic is the fractal dimension of the chaotic limit set when represented in a phase space constituted by a suitable choice of state variables.

The chaotic behavior is usually preceded, as a parameter varies, by a series of bifurcations or qualitative stability changes. Different bifurcation routes have been studied in the bibliography. Some of them are the period-doubling route [1-2], in which the solution period doubles ad infinitum, the quasi-periodic route [3], with consecutive appearance of autonomous fundamentals and the intermittence [1], with apparently periodic behavior interrupted by bursts of chaos, which are longer in time as the parameter evolves. Another possibility is the formation of transverse homoclinic orbits, by collision, for instance, of a limit cycle and a saddle equilibrium point [4].

Although at present the major interest for the microwave circuit designer is to avoid the chaotic solutions, Pecora and Carrol have recently shown [5] the possibility to synchronize two chaotic systems and since then many efforts have been devoted to exploit chaotic signals as broadband carriers. The problem at high frequencies is the extreme sensitivity of the chaotic circuit response to parasitics and variations in the propagation delay, which may prevent the system synchronization. Although the state of the art is still far from these applications in the high microwave range, some advance can be expected in the next few years.

In order to avoid the chaotic responses, an accurate simulation of the nonlinear circuit, prior to manufacturing, is necessary. At low frequency this kind of analysis is carried out in the time domain. However, at microwaves, the long transients and presence of distributed elements often make impractical the time-domain simulation, frequency domain techniques, such as harmonic balance (HB), being used instead. These cannot provide the steady-state chaotic solutions, due to the continuity of the spectra, but the initial bifurcations in the routes to chaos are usually detectable. Actually these routes are closely related with frequency divisions, as in the period-doubling and torus doubling [7] routes, and with the appearance of autonomous fundamentals, as in the quasi-periodic route. In some recent works [3-5], the inclusion of auxiliary generators has enabled the analysis, using standard HB simulators, of the routes to chaos in several microwave circuits. The aim of this work has been to investigate the bifurcation mechanisms, which are responsible of the onset of chaotic responses in two Si MMIC. A second objective has been to develop frequency-domain simulation tools enabling the detection of the initial bifurcations in each route to chaos. This should allow an early prediction of anomalous behavior at the design stage and an efficient correction by using standard HB simulators.

II. ANALYSIS OF TWO IMPATT-BASED CIRCUITS

The IMPATT diodes are very powerful microwave sources, exhibiting a dynamic negative resistance up to high millimetric frequency. Here two IMPATT-based circuits have been analyzed, using an instantaneous device model that consists of two sub-circuits in a series connection [8], one for the *avalanche region* and the other for the *drift region*. The former is basically composed of a nonlinear avalanche inductance L_a and a linear capacitance C_a, in a parallel connection. The drift sub-circuit is composed of a linear capacitance C_d, in parallel with a linear current source, that provides a short-time averager of the current through the avalanche inductance [8]. This nonlinear inductance is given by [2]:

$$L_a = \frac{k_l}{I_a + I_0} \quad with \quad k_l = \frac{l_a v_s}{\alpha'(E_c)} \qquad (1)$$

where l_a is the width of the avalanche region, v_s, the saturated carrier velocity and $\alpha'(E_c)$, the electric-field derivative of the effective ionization rate, evaluated at the static field E_c. Note that the instantaneous avalanche current is expressed as $I_a + I_0$, with the bias current I_0 appearing as an explicit term. The field E_c is calculated from the infinite multiplication condition [8]:

$$\alpha_n(E_c) - \alpha_p(E_c) \exp\left[\alpha_n(E_c) - \alpha_p(E_c)\right] l_a = 0 \qquad (2)$$

where α_n and α_p are respectively the electron and hole ionization rates. The parameter values that have been used here are the same as in [2].

A. IMPATT diode loaded by 50 Ohm

Initially this diode is connected to an external current generator Ig, with associated impedance Rg = 50 Ohm. The diode has been used for oscillator design at about 70 GHz, so this is the input-generator frequency f_{in} that has been considered. When increasing Ig from a small signal value, the circuit solution is initially periodic, with the same period T of the input generator. For Ig = 0.18 A, the solution period becomes 2T and this periodicity remains up to the value Ig = 0.42 A, for which the period becomes 4T (Fig. 1). From Ig = 0.48 A, the solution is chaotic (Fig. 2). In fact the period has suffered an infinite number of doublings, with decreasing length of the parameter intervals corresponding to 2^nT period. The ratio between consecutive parameter intervals Δ_{n+1}/Δ_n tends fast to the Feigenbaum constant δ=4.6692, which rules the parameter accumulation. This scenario constitutes a period-doubling route to chaotic behavior and it is observed in many dynamical systems [1]. The expanded view in Fig. 2 shows the fractal structure of the chaotic attractor. The fractality is associated with the non-complete coverage of the geometric figure constituting the attractor [1].

The Lyapunov exponents are a measure of the stability of a limit set. They provide the average rate of expansion or contraction in the limit set neighborhood. In order to give a definition, the fundamental solution matrix should be considered. For an initial condition x_0, this is given by $\phi_t(x_0)$ and its dimension agrees with the number n of state variables. Let $m_i(t)$, with $1 \le i \le n$, be the eigenvalues of $\phi_t(x_0)$. Then the Lyapunov exponents are given by [1]:

$$\lambda_i = \lim_{t \to \infty} \frac{\ln|m_i(t)|}{t} \qquad (3)$$

For the system to be chaotic, at least one the exponents must be bigger than zero [1]. This gives rise to the sensitive dependence on the initial conditions. On the other hand, for the system to be an attractor, the addition of the n Lyapunov exponents must be smaller than zero [1]. In order to calculate the Lyapunov exponents, the circuit nonlinear differential equation has been obtained:

$$(4)$$

$$\dot{I} = -\frac{1}{C_b Rg} I - \frac{1}{Rg}\dot{V}_a - \frac{1}{Rg}\dot{V}_d + \dot{I}g$$

$$\ddot{V}_a = -\frac{1}{C_a C_b Rg} I - \frac{I_0}{C_a k_l} V_a - \frac{1}{C_a Rg}\dot{V}_a - \frac{1}{C_a Rg}\dot{V}_d + \frac{1}{C_a}\left(\dot{I}g + \frac{V_a}{k_l}(C_a.\dot{V}_a - I)\right)$$

$$\ddot{V}_d = -\frac{C_b Rg + \tau_d}{C_d C_b Rg \tau_d} I - \frac{C_a Rg - \tau_d}{C_d Rg \tau_d}\dot{V}_a - \frac{1}{C_d Rg}\dot{V}_d + \frac{1}{C_d}\left(\dot{I}g - \frac{1}{\tau_d}(C_a.\dot{V}_a(t - \tau_d) - I(t - \tau_d))\right)$$

where C_b is the DC block capacitance and I the total diode current. It is a delayed system, due to the presence of τ_d (drift delay). The calculation, according to (3), of the largest Lyapunov exponent is shown in Fig. 3. It stabilizes around a positive value, confirming the chaotic nature of the attractor in Fig.2.

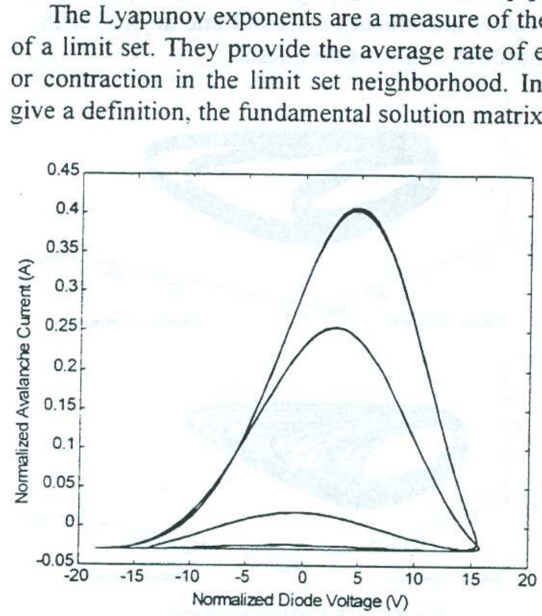

Fig. 1 Division by four (Ig = 0.45 A).

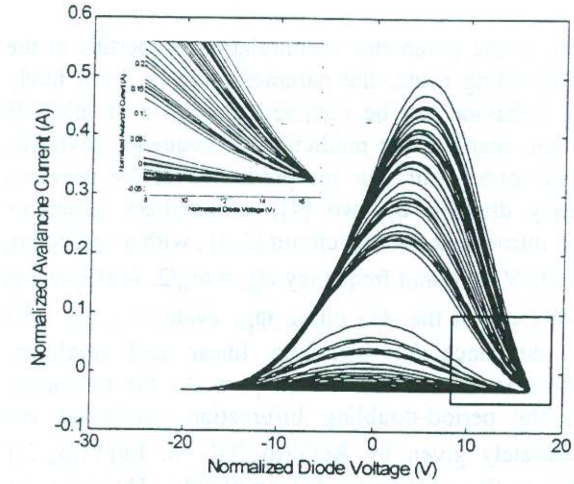

Fig.2 Chaotic attractor (Ig = 0.48 A).

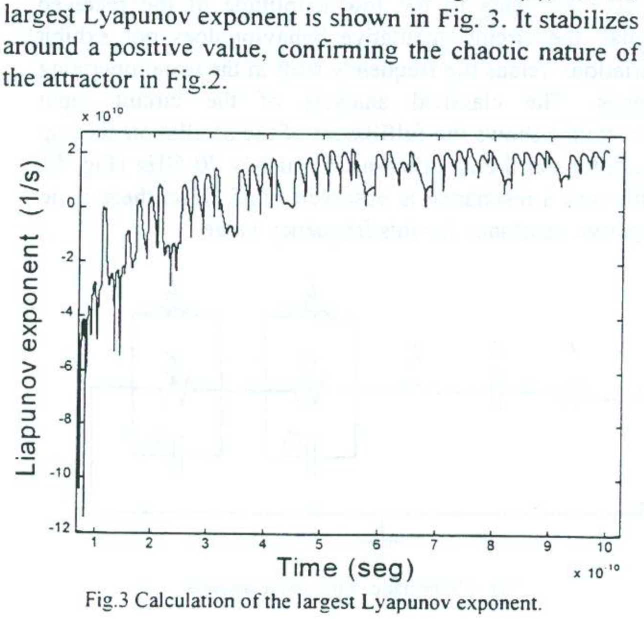

Fig.3 Calculation of the largest Lyapunov exponent.

Due to the parameter accumulation properties in the period-doubling route, the parameter values for a likely chaotic behavior can be obtained through a simple HB technique, enabling the prediction of frequency divisions up to the order four. For the prediction of the previous frequency division by two [4], an auxiliary generator (AG) is introduced into the circuit [3-4] , with a neglecting amplitude $V_{p1} = \varepsilon$ and frequency $\omega_{p1} = \omega_{in}/2$. A HB sweep is carried out in the AG phase φ_{p1}, evaluating the total input admittance Y, including linear and nonlinear contributions, at the observation port. As the parameter varies, the period-doubling bifurcation conditions are approximately given by $Re[Y(\omega_{in}/2)] = 0$, $Im[Y(\omega_{in}/2)] = 0$, due to the neglecting AG amplitude. This may be verified by carrying out the phase sweep for several parameter values. For the parameter values fulfilling $Re[Y(\omega_{in}/2)]<0$, $Im[Y(\omega_{in}/2)]=0$ a period-doubled solution will generally exist. Once the period-doubled solution has been predicted, the corresponding steady state will be obtained through a standard optimization on the variables V_{p1}, φ_{p1}. The goal will be the zero value of the admittance function at the observation port. When this is fulfilled, the auxiliary generator does not perturb the steady state.

The prediction of a possible frequency division by four is carried out by keeping the AG at the value resulting from the former optimization and introducing a second AG, with neglecting amplitude and frequency $\omega_{p2} = \omega_{in}/4$. The second period-doubling is predicted through a phase sweep of the latter generator, checking for the conditions: $Re[Y(\omega_{in}/4)] < 0$, $Im[Y(\omega_{in}/4)] = 0$.

B. IMPATT diode loaded by active antenna

The IMPATT diode has been used in the design of a self-oscillating mixer, for automotive radar applications [8]. In this circuit the IMPATT diode is connected to an active antenna (Fig. 4). The self-oscillation frequency is f_a = 70 GHz. Due to the low amplitude of the received signal, the circuit qualitative behavior does not exhibit variations versus the frequency shift in the usual operating ranges. The classical analysis of the circuit input admittance shows the fulfillment of the oscillation start-up conditions at the autonomous frequency 70 GHz (Fig. 5). Although a resonance is observed at 35 GHz, there is no negative resistance for this frequency value.

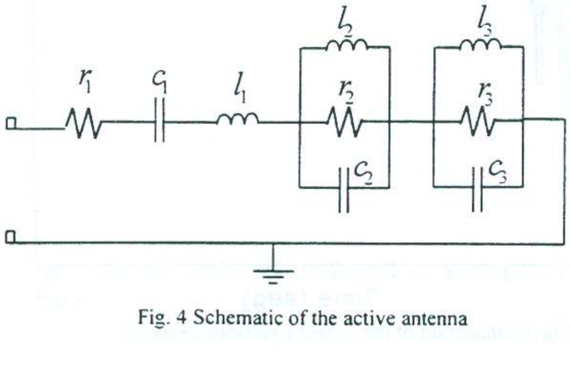

Fig. 4 Schematic of the active antenna

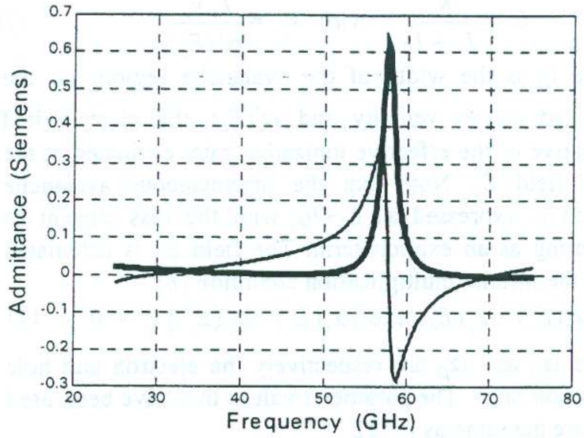

Fig. 5 Real and imaginary parts of SOM small signal admittance. Bold line: real part.

Since there are two incommensurate fundamentals, the solution is quasi-periodic, which in the phase space defined by three state variables gives rise to a 2-torus. When decreasing the antenna resistance r_1, the torus suffers a sequence of doublings, with decreasing parameter intervals, to chaotic behavior (Fig. 6). For this representation, the bias point has been taken as the origin of the coordinate system.

Through the Poincaré-map technique, it is easy to show that the torus-doubling phenomenon corresponds to the frequency division by two of one of the fundamentals. Then, it can be predicted through HB. A two-fundamental Fourier expansion of the circuit state variables is needed, as well as a special strategy in order to avoid the convergence to unstable periodic solutions, for which the intermodulation products involving f_a have not been initialized. The one proposed in [3] is based on the introduction of an auxiliary generator at the autonomous fundamental, fulfilling a non-perturbation condition of the steady state. This is given by the zero value of its input admittance. For the detection of the possible torus doubling, the former auxiliary generator is kept at its steady-state value, introducing a second one at $f_a/2$, with negligible amplitude and variable phase.

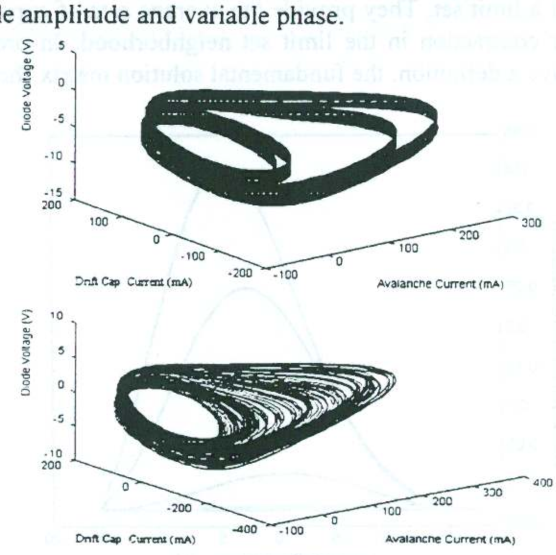

Fig. 6 Torus doubling to chaos

For the torus-doubling prediction, a sweep is carried out in the phase of this generator, in a similar manner to the period-doubling prediction technique.

The width of the avalanche region l_a is another sensitive parameter. In Fig. 7 the immittance diagram has enabled the prediction of a torus doubling from about l_a = 0.105 μm. The optimization of the two auxiliary generators, in order to make them fulfill a non-perturbation condition, permits to obtain the steady-state doubled torus. The spectra from time domain and harmonic balance simulations can be compared in Fig. 8. Then it is also possible, through the application of a HB continuation technique, to determine the evolution of the doubled torus versus the parameter. In Fig. 9, the harmonic components at f_{in}, f_a and $f_a/2$ have been traced versus l_a. The torus-doubling bifurcation is given by the onset of the $f_a/2$ component and the corresponding l_a value is in good agreement with the prediction from the AG phase sweep.

III. CONCLUSIONS

This paper analyzes the nonlinear dynamics of two circuits based on Si IMPATT diodes for which chaotic responses had been experimentally encountered. The mechanisms leading to chaos in each circuit have been determined, obtaining the sense of variation of the most influential parameters in order to avoid the chaotic solutions in future realizations of these circuits. Some frequency domain techniques for the detection of the observed routes to chaos have also been provided, which should enable a more efficient prediction of the anomalous behavior. Time and frequency domain simulations showed an excellent agreement.

IV. ACKNOWLEDGEMENT

The authors are grateful to the Technical University of Munich for providing the model of the active antenna.

The authors are grateful to Johannes F. Luy from DAIMLER-CHRYSLER Research Center for supplying circuit information.

The authors would like to acknowledge the financial contribution to this work from the Commission of the European Communities through the TMR program.

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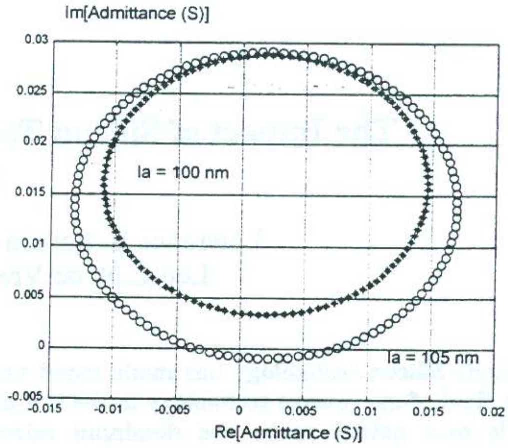

Fig. 7 Torus doubling prediction

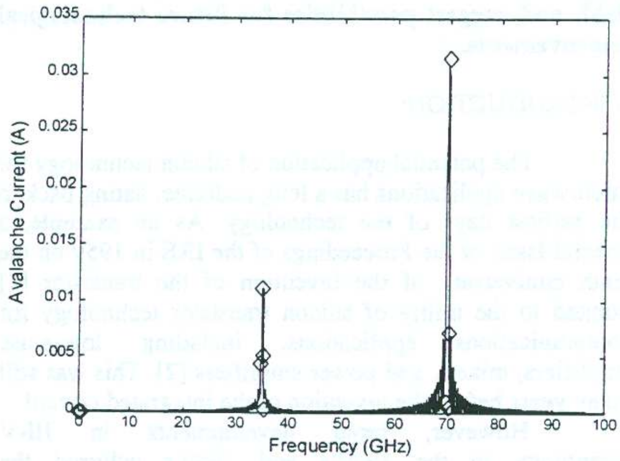

Fig. 8 Spectrum of doubled torus. Points: HB.

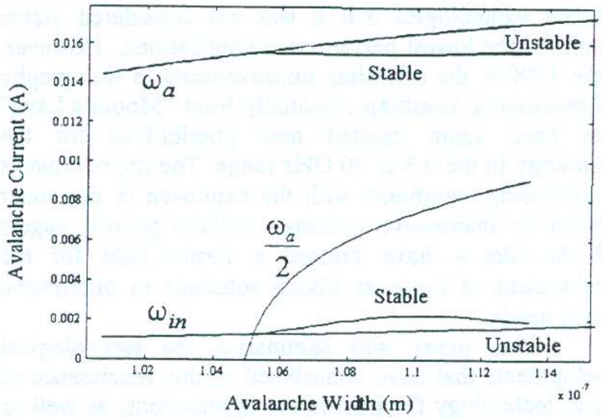

Fig. 9 Torus bifurcation diagram.