Is America Unrivaled? A Repeated Game Analysis[1]

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Abstract

I use a strategic setup to investigate whether unipolarism can indeed persist as a long run equilibrium. In a three-country world, a global power may subsidise two satellites so as to incentivate them not to invest to build up a coalition against it. I single out the conditions under which the one-shot game is a Prisoners' Dilemma where no subsidy is paid and the coalition arises at equilibrium. Then, I revert to the infinitely repeated game and apply the Perfect Folk Theorem to characterise the critical thresholds of dicount factor sustaining unipolarism at the subgame perfect equilibrium.

1 Introduction

According to the realist theory, unipolarism is not bound to be a long run equilibrium.[1] Consequently, the current *status quo* where the US are the only existing global power seems at odds with the dominant view of the theory of the balance of power, establishing that sooner or later a counterbalancing power, perhaps taking the form of a coalition of several states, should arise.

In contrast with this position, Ikenberry (2002)[2] proposes an eclectic alternative view of the issue at stake, whereby the hegemony of the global power is sustained by an international governance system granting every potential competitor a number of appealing benefits, e.g., bilateral trade agreements, military support, etc.

This mixture of liberal and realist attitudes has two major implications. On one side, it limits the power effectively enjoyed by the US as the leader of the international order; on the other, it clearly tends towards a reduction of the incentives to look for alternatives (e.g., a counterbalancing coalition) by means of a growing network of international institutions. This amounts to saying that the system introduced by the US after the end of World War II contains a tradeoff between the amount of effective hegemony exerted by the global power and the degree of confidence that the *status quo* might indeed persist.[3] That is, the insurance against defections is costly. This fact has

[1] To this regard, the existing literature is too wide to allow for an exhaustive listing. See Morgenthau (1948), Waltz (1979) and Gilpin (1981), *inter alia*.

[2] See also Ikenberry (1998-1999, 2001) and Keohane (1989). Horowitz (2001) points out the relevance of appropriately accounting for risk-aversion, economic growth and political rigidities in assessing the performance of the balance-of-power theory.

[3] Quoting Haass (1999, p. 37), "trading some American power for a more stable international system would be a good deal for America and the world".

prompted several doubts on the robustness of the existing system (see, e.g., Kupchan, 2002), as the sustainability of such costs on the part of the US (and their tax payers) is not completely out of question.

The aim of the present paper is precisely to tackle this particular aspect of the issue from a game-theoretic angle. I adopt a stripped down setup modelling a three-country world where the players are a global power and two satellites. To keep things as essential as possible, the strategy space is binary, both for the global power and for each of the satellites. the hegemonic country has to choose whether or not to subsidise the satellites, so as to avoid the formation of a counterbalancing coalition, while the satellites have to decide whether to ally or not, being aware that constructing a coalition is a costly activity. To begin with, I investigate the one-shot game, then I analyse a supergame over an infinite time horizon. The main results of my analysis can be summarised in the following terms. First, the one-shot game produces a unique Nash equilibrium which is also in dominant strategies, where the global power does not distribute the subsidies and the best reply of the satellites consists in building up a coalition. The flavour of this conclusion is clearly in line with the realist theory, whereby unipolarism should not be sustainable as an equilibrium outcome. However, the second result paves the route to a completely different view, in that there always exists an admissible region of the parameter space in which the one-shot game is a Prisoners' Dilemma, i.e., the equilibrium involving the arising of a coalition is Pareto-inefficient. Accordingly, a further investigation through the toolkit of the theory of repeated games appears appropriate. By doing so, I characterise the stability properties that must hold in order for the supergame to admit a subgame perfect equilibrium where the global power subsidies the satellites who, on their part, decide not to invest in order to build up the coalition. This of course does not eliminate altogether the possibility of observing bipolarism at equilibrium. Rather, it illustrates a reasonable scenario where the players involved in the game may rationally select their strategies so as to keep the *status quo* unchanged, eventually forever.

The structure of the paper is the following. The description of the setup and the analysis of the one-shot game are in section 2. Section 3 describes the repeated game. Concluding remarks are in section 4.

2 The model

For the sake of simplicity, I will focus on a three-country world:[4] one global power (denoted by G) and two satellites, s_1 and s_2. Assume (i) G may pay each of the satellites a given subsidy $\sigma > 0$, lest they get together to form a coalition aimed at diminishing G's hegemonic position; and (ii) each satellite may, if it does not receive the subsidy, invest an amount of resources $k > 0$ to build up the coalition. It is relevant to stress that the alliance is taken not to be a pure public good, in that an amount equal to $2k$ is needed for its construction. To clarify this aspect, I introduce the following:

Assumption *The overall cost of building up a coalition is $2k$. The amount of resources available to each satellite is k.*

This prevents each small country s_i to undertake individually the venture of building up a countervailing power to balance that of G. In turn, this

[4]A three-country world is also considered in some of the existing literature to analyse several issues. For instance, Wagner (2004) uses it to investigate the relation between bargaining and war.

might open a discussion as to whether G could save upon the total amount of subsidies by granting only a single σ to one of the satellites, obtaining in return that the coalition does not arise. Here I will not consider this possibility, the reason being twofold. First, the distribution of subsidies to both satellites is in accordance with facts, as observing the behaviour of the US towards, say, Russia and China suffices to confirm. Second, the model is defined in such a way that it includes only the relevant players, where *relevant* means that they are the only potential competitors that G must take into account. The rest of the world (say, Japan, EU, etc.) which is known not to represent a threat to the global power in this respect, is not modelled.

I am now in a position to introduce the basic elements of the game. The global power faces four alternative perspectives:

1] If it delivers the subsidies and the satellites do not build up a coalition,[5] G's utility is:

$$U_G(\sigma, NC) = c_G - 2\sigma + 2\beta\sigma + P \qquad (1)$$

where:

- NC indicates that there exists no coalition;
- c_G is G's domestic consumption level;
- σ is the size of the subsidy, and therefore 2σ is the total cost borne by G;

[5] The subsidy could, e.g., consist in a pure money transfer or in granting the satellites free access into G's domestic market, i.e., a *free trade* agreement. On the relationship between security and trade, see Dorussen (1999) and Skaperdas and Syropoulos (2001), *inter alia*.

- $2\beta\sigma$ measures the benefit to the global power, generated by the subsidies to the satellites, the marginal benefit being measured by parameter $\beta \in (0,1)$. The relevance of β, on which I will come back later in the analysis, can be preliminarily illustrated on the basis of tow alternative perspectives. Keeping *status quo* unchanged through the distribution of subsidies implies a cost and a return to G. The first is obviously represented by the total amount of subsidies, i.e., the quantity -2σ; the second is a positive component measuring evaluation attached by G to the fact that satellites do not ally against it, $2\beta\sigma$. Hence, clearly it must be $\beta < 1$ for the for the subsidization manoeuvre to entail a net cost, as common sense would suggest. As is going to become apparent in the remainder of the paper, the size of β will play a crucial role in shaping the equilibrium outcome.

- P measures the power level enjoyed by G if no coalition builds up. $U_G(\sigma) > 0$ for all $\sigma \in (0, (c_G + P) / [2(1-\beta)])$.

2] If instead the global power does not deliver any subsidies to the satellite countries, and the latter form a coalition C, then G's payoff is:

$$U_G(0, C) = c_G + P - \beta C \tag{2}$$

where $C \in (0, P)$ measures the level of power held by the alliance between satellites s_1 and s_2. Note that the coalition reduces G's power to an extent measured by parameter β. This amounts to saying that G evaluates in the same way the marginal disutility of any power decrease as well as that associated to paying subsidies.[6]

[6]Note that the consumption level is not sensitive to the presence or absence of the

3] The third situation is that where G pays the subsidies but the satellites decide to invest in a counterbalancing coalition, yielding thus:

$$U_G(\sigma, C) = c_G - 2\sigma + P - \beta C. \tag{3}$$

4] The last possible case is that where the global power does not distribute the subsidies and, this notwithstanding, the satellites do not form an alliance. In this situation, G's utility is:

$$U_G(0, NC) = c_G + P. \tag{4}$$

Now focus upon satellites. Again, four cases can be envisaged:

1'] If they accept the subsidies and do not create a coalition, each satellite attains the following utility:

$$u_i(\sigma, NC) = c_i + \sigma \tag{5}$$

where c_i defines the domestic consumption level in satellite i.

2'] If instead, having received no subsidy, the satellite build a coalition, their individual utility becomes:

$$u_i(0, C) = c_i - k + C. \tag{6}$$

To this regard, it is worth stressing that country i pays an individual fee k associated with the investment required to construct the alliance,

subsidy. This will hold for the satellites as well. This entails that here I choose to abstract from any endogenous relationship between domestic welfare and international security, which relates to the 'guns or butter' debate (see Polachek, 1980; Chan and Mintz, 1992; Gowa, 1994; Powell, 1993; Heo, 1998; Skaperdas and Syropuolos, 2001; Carrubba and Singh, 2004).

but then enjoys the full benefit C yielded by the alliance itself. One has to assume that the net payoff generated by the coalition is positive, $C \geq 2k$. This suffices to ensure that $u_i(0, C) > 0$.

3'] In this case, each of the satellites is subsidised but still they decide to form a coalition, whereby the individual payoff accruing to each of them is:

$$u_i(\sigma, C) = c_i + \sigma + C - k. \tag{7}$$

4'] Finally, there remains the case where G does not distribute any subsidy and the satellites do not invest to build an alliance:

$$u_i(0, NC) = c_i. \tag{8}$$

The game can be represented in strategic form as in matrix 1, where G is the row player. Given the additive separability of the payoff functions, I will only represent the payoffs of one of the two satellites as the column player, without loss of generality and without affecting the solution.[7]

		s_i: NC	s_i: C
G	σ	$c_G - 2\sigma(1-\beta) + P; c_i + \sigma$	$c_G - 2\sigma + P - \beta C; c_i + \sigma + C - k$
	0	$c_G + P; c_i$	$c_G + P - \beta C; c_i + C - k$

Matrix 1

Assume the game is one of imperfect, complete and symmetric information, and, for the moment, also assume it is one-shot. Now, in order to

[7] This is acceptable insofar as the alliance is not a public good, i.e., it takes two to tango. The analysis of the game between satellites illustrating this feature of the model is in the appendix.

characterise the equilibrium solution, one has to examine the incentives, respectively, for G to pay the subsidies so as to try and avoid the arising of the coalition, and for the satellites to accept the subsidies and, possibly, not to form the coalition. Observe that:

$$\begin{aligned} U_G(\sigma, NC) < U_G(0, NC) &\Leftrightarrow \beta \in (0,1) \\ U_G(\sigma, C) < U_G(0, C) &\Leftrightarrow \sigma > 0 \end{aligned} \tag{9}$$

i.e., (i) if the satellites do not form a coalition, then it is convenient for the global power not to pay the subsidies for all the admissible values of β, while (ii) in presence of the coalition, it is optimal for G not to distribute any subsidy for all $\sigma > 0$. Therefore:

Lemma 1 *The global power has a strictly dominant strategy, which consists in not paying the subsidies to the satellites, for all admissible levels of parameters β and σ.*

As to the satellite country i, the following holds:

$$\begin{aligned} u_i(\sigma, C) > u_i(\sigma, NC) &\Leftrightarrow C > k \\ u_i(0, C) > u_i(0, NC) &\Leftrightarrow C > k \end{aligned} \tag{10}$$

which entails:

Lemma 2 *Forming a coalition is a strictly dominant strategy for the satellites.*

On the basis of lemmata 1-2, without further proof, I may state:

Proposition 3 *The one-shot game, where the global power must decide whether or not to subsidise the satellites who simultaneously choose whether to build* *up a countervailing coalition or not, has a unique Nash equilibrium identified by the strategy pair* $(0, C)$ *. Such a Nash equilibrium is in strictly dominant strategies.*

The next step consists in verifying whether the present game can be a Prisoners' Dilemma. For this to hold, the additional condition that has to be satisfied is that the equilibrium be Pareto-inefficient, which depends on the relative magnitude of the payoffs appearing along the main diagonal of matrix 1.

If the following inequalities are simultaneously satisfied, then the game is indeed a Prisoners' Dilemma:

$$U_G(\sigma, NC) > U_G(0, C) \Leftrightarrow c_G - 2\sigma(1-\beta) + P > c_G + P - \beta C \tag{11}$$

$$u_i(\sigma, NC) > u_i(0, C) \Leftrightarrow c_i + \sigma > c_i - k + C. \tag{12}$$

Simplifying condition (11), one obtains:

$$\sigma < \frac{\beta C}{2(1-\beta)} \tag{13}$$

while condition (12) yields:

$$\sigma > C - k. \tag{14}$$

An intuitive interpretation can be given for both. On the one hand, (13) shows that G will find it profitable to deliver the subsidy if it is low enough as compared to the ratio between the damage generated by the coalition, given by βC, and the net marginal benefit of the subsidies measured by $2(1-\beta)$. On the other hand, condition (14) simply states that a satellite will be willing to accept the subsidy and abandon the perspective of investing to build up the coalition if the subsidy is higher than the net gain associated with building up the coalition.

Note that the compatibility between (13) and (14) requires:

$$\frac{\beta C}{2(1-\beta)} > C - k \tag{15}$$

which is equivalent to the following condition:

$$2k(1-\beta) > C(2-3\beta). \tag{16}$$

Whether inequality (16) is met will of course depend on the relative size of the three parameters involved. However, there clearly exist admissible regions of the parameter space where this condition holds. The validity of this assertion is quickly shown, as follows. By assumption, we know that $1-\beta > 0$. Therefore, the l.h.s. of (16) is positive. The r.h.s. may take either sign, depending on the value of β: it is positive for all $\beta \in (0, 2/3)$, while it is negative for all $\beta \in (2/3, 1)$. Therefore, taking any values of β in $(2/3, 1)$ is sufficient (but not necessary) to validate (16).[8] To complete the argument, it suffices to observe that, if (16) holds, G may always choose an appropriate value of σ such that the resulting game is a Prisoners' Dilemma. Hence, I have proved the following result:

Proposition 4 *Examine the size of the subsidy. Two mutually exclusive situations may arise:*

- *Suppose* $2k(1-\beta) > \max\{0, C(2-3\beta)\}$. *Then* $C - k > \frac{\beta C}{2(1-\beta)}$, *and for all* $\sigma \in \left(C-k, \frac{\beta C}{2(1-\beta)}\right)$ *the Nash equilibrium in dominant*

[8] On the other hand, taking $\beta \in (0, 2/3)$ is necessary but by no means sufficient to ensure the opposite result, i.e., that $2k(1-\beta) < C(2-3\beta)$. To see this, consider that, provided $2-3\beta > 0$, this inequality is equivalent to $C/(2k) > (1-\beta)/(2-3\beta)$. Now, while we know that $C/(2k) > 1$ surely, in order for the coalition to be viable, it is also true that $(1-\beta)/(2-3\beta) < 1$ for all $\beta \in (0, 1/2)$. Therefore, if $\beta \in (1/2, 2/3)$, $(1-\beta)/(2-3\beta) > 1$ so that $C/(2k)$ might be lower than $(1-\beta)/(2-3\beta)$ in this range.

strategies identified by $(0, C)$ *is the outcome of a Prisoners' Dilemma. If either* $\sigma > \dfrac{\beta C}{2(1-\beta)}$ *or* $\sigma \in (0,\, C-k)$, *the outcomes* (σ, NC) *and* $(0, C)$ *cannot be Pareto-ranked.*

- *Suppose instead* $C(2-3\beta) > 2k(1-\beta) > 0$. *Then* $C - k < \dfrac{\beta C}{2(1-\beta)}$, *and for all* $\sigma \in \left(\dfrac{\beta C}{2(1-\beta)},\, C-k\right)$ *the Nash equilibrium in dominant strategies identified by* $(0, C)$ *is Pareto-efficient. If either* $\sigma > C - k$ *or* $\sigma \in \left(0,\, \dfrac{\beta C}{2(1-\beta)}\right)$, *the outcomes* (σ, NC) *and* $(0, C)$ *cannot be Pareto-ranked.*

The second part of the above proposition describes the case where the obvious solution is one where satellites do ally, the global power does not pay any subsidies to try and keep the *status quo* unaltered, and ultimately there are no regrets on either side. More interesting is the alternative perspective where we observe a Prisoners' Dilemma game, because in such a case, as is well known, a repeated game framework may offer a way out of the Pareto inefficient Nash equilibrium generated by the one-shot game. This is done in the next section.

3 The supergame

To model the behaviour of players in the supergame, I will resort here to the so-called Perfect Folk Theorem based upon grim trigger strategies (Friedman, 1971).[9]

[9] Alternatively I could use Axelrod's (1981, 1984) tit for tat strategies, reaching qualitatively similar conclusions.

Assume $\frac{C}{2k} > \frac{\beta - 1}{3\beta - 2}$ and $\sigma \in \left(\frac{\beta - 1}{3\beta - 2}, \frac{C}{2k} \right)$, so that the constituent game described by matrix 1 is indeed a Prisoners' Dilemma. It is important to stress that G may intentionally choose a level of σ in this interval, in order to make the stage game a Prisoners' Dilemma. The relevance of this aspect lies in the fact that, by doing so, the global power may deliberately induce the development of a supergame that may have an equilibrium outcome which differs completely from the one characterising the one-shot game. It is also worth emphasising that only G is able to manipulate the strategic incentives underlying the game, while satellites cannot do it. To some extent, this tells that one of the prerogatives of the global power is precisely to affect at will the nature of the game, if this appears to be profitable for her.

Let the stage game repeat over discrete time $t \in [0, \infty)$.[10] Players share the same discount factor $\delta \in [0, 1]$. The rules of the Perfect Folk Theorem establish that players stick to the Pareto-efficient path as long as no deviation is detected. As soon as this happens, then they revert to the dominant (Nash equilibrium) strategy forever. Formally, players are instructed to adhere to these prescriptions:

a] At $t = 0$, G plays σ and s_i, $i = 1, 2$, plays NC.

b] At any $t \geq 1$, both keep playing according to rule [a], unless any deviation has been detected at $t - 1$, in which case they have to play the respective dominant strategies yielding the Nash equilibrium of the one shot-game.

[10] It could be objected that governments do not last indefinitely. However, the present analysis is designed with the view that they may be sufficiently forward looking to closely mimic the behaviour of a single player living forever. Incidentally, this is in accordance with observation, at least with respect to the issue at hand.

The second part of rule [b] entails that, after any defection from the initial (collusive) path, players revert to the one-shot Nash equilibrium forever.[11] Therefore, the strategy pair (σ, NC) is a subgame perfect equilibrium of the supergame if and only if, for both players, the discounted flow of payoffs associated to the collusive outcome (σ, NC) is at least as large as the discounted flow of payoffs associated to the alternative perspective where either player defects once and then both revert to the one-shot noncooperative equilibrium forever. That is, the outcome (σ, NC) is sustainable if and only if the following inequalities are both satisfied:

$$\frac{U_G(\sigma, NC)}{1-\delta} \geq U_G(0, NC) + \frac{\delta U_G(0, C)}{1-\delta}\,; \tag{17}$$

$$\frac{u_i(\sigma, NC)}{1-\delta} \geq u_i(\sigma, C) + \frac{\delta u_i(0, C)}{1-\delta}\,. \tag{18}$$

From (17), one obtains:

$$\delta \geq \frac{2(1-\beta)\sigma}{\beta C} \equiv \delta_G \tag{19}$$

while (18) yields:

$$\delta \geq \frac{C-k}{\sigma} \equiv \delta_s \tag{20}$$

with $\delta_G, \delta_s \in (0,1)$ on the basis of conditions (13) and (14). A straightforward examination of δ_G and δ_s reveals what follows:

Lemma 5 *δ_G increases monotonically in σ, while the opposite holds for δ_s.*

[11] Nash reversion has been criticised and improved upon in the game theory literature. In games defined in continuous strategies, one should use optimal punishments as the most efficient instrument to deter defections (see Abreu, 1986; and Fudenberg and Maskin, 1986, inter alia). However, optimal punishments cannot be designed if players' strategy space is discrete.

To prove Lemma 5, it suffices to observe that $\partial\delta_G/\partial\sigma > 0$ while $\partial\delta_s/\partial\sigma < 0$ always. The intuitive explanation of these facts is that, as σ becomes larger, it becomes increasingly costly for G to pay the subsidies, while it becomes more attractive to each of the satellites to accept the subsidies and renounce to build the coalition.

Moreover:

$$\delta_G - \delta_s \propto 2(1-\beta)\sigma^2 - \beta(C-k)C \tag{21}$$

which is positive iff

$$\sigma > \sqrt{\frac{\beta(C-k)C}{1(1-\beta)}}\,. \tag{22}$$

Comparing the expression in the r.h.s. of (22) with the lower and upper bound of the interval for σ wherein we observe a Prisoners' Dilemma, we find that

$$\sqrt{\frac{\beta(C-k)C}{1(1-\beta)}} \in \left(C-k\,,\,\frac{\beta C}{2(1-\beta)}\right) \tag{23}$$

provided that (16) is satisfied. This allows me to formulate the main result:

Theorem 6 *Take* $2k(1-\beta) > \max\{0, C(2-3\beta)\}$, *so that the game is defined by matrix 1 is a Prisoners' Dilemma. Then, consider the infinitely repeated game whose constituent game is such a Prisoners' Dilemma. Using the Perfect Folk Theorem, the resulting critical thresholds of the discount factors are:*

$$1 > \delta_G > \delta_s > 0 \text{ for all } \sigma \in \left(\sqrt{\frac{\beta(C-k)C}{1(1-\beta)}}\,,\,\frac{\beta C}{2(1-\beta)}\right);$$

$$1 > \delta_s > \delta_G > 0 \text{ for all } \sigma \in \left(C-k\,,\,\sqrt{\frac{\beta(C-k)C}{1(1-\beta)}}\right).$$

If $\delta \geq \max\{\delta_G, \delta_s\}$, *then the strategy pair* (σ, NC) *is sustainable at the subgame perfect equilibrium of the repeated game. For all* $\delta \in [0, \max\{\delta_G, \delta_s\})$, *the subgame perfect equilibrium is* $(0, C)$.

It is worth observing that, for comparatively low levels of the subsidy, the most demanding threshold is associated with the time preferences of the global power; conversely, for sufficiently high values of σ, the decisive threshold is δ_s. A possible interpretation of this outcome could be that, if σ is large enough, then the perspective of cheating becomes very appealing for each of the satellites. By doing so each of them receives, if only for one period, a very rewarding subsidy without bearing the cost of an alliance.

As a last remark, observe that the repeated game still allows for the Nash equilibrium of the constituent game to repeat forever, that is, the above Theorem illustrates, as is always the case with repeated games, the existence of multiple equilibria, one of which is that yielded by the Prisoners' Dilemma. However, the point made by the present analysis is that the persistence of a *status quo* characterised by unipolarism cannot be rationally ruled out.

4 Concluding remarks

I have investigated a simple game-theoretical model examining the issue of the stability of unipolarism. The foregoing analysis has highlighted that, while the one-shot game has a clearcut solution prescribing the arising of a countervailing coalition, the persistence of the *status quo* as a long run equilibrium ultimately depends on the time preferences of the countries (or their policy makers) involved in the repeated game. Therefore, on the basis of the present modelization, the contradiction between the views expressed by the realist theory and observation seems to vanish, since a multiplicity of equally plausible (or rationalisable) equilibria arises, depending upon how much forward looking the players are in assessing their respective incentives and consequently in selecting their optimal strategies. The analysis of the repeated Prisoners' Dilemma game has allowed me to single out specific conditions under which the prediction of the model is in accordance with our observation of the current state of the world.

Appendix

Here I examine the strategic interaction between satellites, that have to choose whether to undertake the construction of the coalition or not. To do so, I take a fully noncooperative perspective. Two alternative games can be envisaged, depending on whether the global power subsidises the satellites or not. Matrix 2 describes the first case.

		s_2: 0	s_2: k
s_1	0	$c_1+\sigma; c_2+\sigma$	$c_1+\sigma; c_2+\sigma-k$
	k	$c_1+\sigma-k; c_2+\sigma$	$c_1+\sigma+C-k; c_2+\sigma+C-k$

Matrix 2

Clearly, the game is symmetric along the main diagonal and there is no dominant strategy, with

$$\begin{gathered} c_i+\sigma > c_i+\sigma-k; \\ c_i+\sigma+C-k > c_i+\sigma. \end{gathered} \tag{a1}$$

Therefore, matrix 1 describes a coordination game with two symmetric equilibria, $(0,0)$ and (k,k), which can be Pareto-ranked: $(k,k) \succ (0,0)$, as both countries strictly prefer to ally. However, if the game is played under imperfect information, i.e., simultaneously, then there is no particular reason to believe that the satellites will select (k,k).[12] Completely different considerations can be put forward under perfect information, i.e., if the game is played sequentially with either player at the root of the game tree. The extensive form is in figure 1.

[12] Except, possibly, by invoking the focal point refinement (Schelling, 1960).

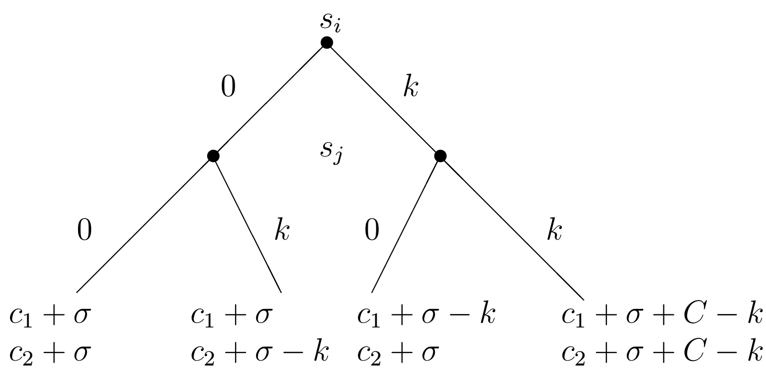

Figure 1. The extensive form of matrix 2

It is easily checked by backward induction that the subgame perfect equilibrium is (k, k). To see this, examine each subgame in isolation, starting from the terminal nodes. If satellite j knows to be in the left singleton (because i has chosen strategy 0), then the optimal choice is to play 0. The resulting payoffs are $c_i + \sigma$ for both. Otherwise, in correspondence of the right singleton, j's best reply to k consists in choosing k, the associated payoffs being $c_i+\sigma+C-k$. Hence, i is aware that choosing 0 it will attain $c_1+\sigma$, while choosing k it will attain $c_1 + \sigma + C - k > c_i + \sigma$. As a result, the optimal choice for the player located at the root of the game tree is to invest k, inducing an analogous behaviour by the country located at the intermediate nodes.

The alternative case where G does not pay any subsidies can be quickly dealt with, as the game is qualitatively equivalent except for the absence of

σ in all of the payoffs involved, as clarified by matrix 3.

		s_2	
		0	k
s_1	0	$c_1; c_2$	$c_1; c_2 - k$
	k	$c_1 - k; c_2$	$c_1 + C - k; c_2 + C - k$

Matrix 3

Therefore, under imperfect information we have again two Nash equilibria in undominated strategies, $(0, 0)$ and (k, k), the latter being selected as the unique subgame perfect equilibrium by backward induction, if the game is solved under perfect information.

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