Supplementary Insurance with Ex-Post Moral Hazard: Efficiency and Redistribution*

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Abstract

This paper investigates the topping-up scheme in health insurance when both public and private firms use linear contracts. First, the case with identical consumers is analyzed. The optimal public coverage is derived both when the firms play simultaneously and when they play sequentially. In the former case consumers are over-insured, whereas, in the latter case, the second-best allocation is obtained. Then, consumers' heterogeneity is introduced: consumers differ in their wage rate and labour supply is endogenous. It is assumed that public coverage is uniform and health expenditures are financed by linear taxation. Results show that, in the sequential game, the optimal public coverage is negative and consumers are under-insured.

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1 Introduction

Risk-averse consumers demand health insurance because medical expenses are uncertain. They pay a premium before the realization of the risk and they receive an indemnity if the illness occurs. The consumer's health status is not perfectly observable by the insurer, as a result the indemnity is generally directly related to the health care costs. Thus, health insurance covers the *financial risk* associated with buying medical care.

This paper focuses on the relationship between insurers and consumers when insurers are both private and public. In particular it analyses the effect of ex-post moral hazard on the demand for care when consumers are covered by a mixed insurance scheme. The mixed system explicitly considered in this paper is the topping-up scheme (as opposed to the opting-out) which is actually the most widespread system in developed countries.[1] This system is characterized by public insurance covering a part of the individuals' health expenditure (a package of essentials), and a voluntary private policy topping up the remaining services.

As is it is generally known, ex-post moral-hazard is a consequence of health insurance: insurance coverage reduces the marginal price of care and induces additional consumption. This inefficiency is obviously increased by the presence of supplementary coverage because the latter reduces the marginal price of care even more. Since the mid-1980s developed countries have significantly and progressively increased the role of voluntary insurance in their health systems, and some authors are skeptical about the way private policies have been introduced and mixed schemes have been structured. In particular Blomqvist and Johansson (1997), Selden (1997), Pauly (2000) argue that private supplementary coverage can have spillover effects, increasing the cost of public coverage. But none of these authors has analyzed in detail the inefficiency induced by mixed health insurance coverage. This paper tries to fill this gap showing how, in equilibrium, the public policy changes when public and private insurers move simultaneously or sequentially. In particular, when the firms play simultaneously, both coverages are positive and the standard inefficiency of Nash equilibria is retrieved; whereas, when the firms play sequentially, the public insurer provides zero coverage and the second-best allocation is implemented.

As empirical evidence has shown in the case of the Medicare program in the US and of French mixed coverage (Wolfe and Goddeeris (1991), Ettner (1997) and Henriet and Rochet (1998)), another important problem connected with currently available mixed systems is that supplementary insurance is purchased by wealthier consumers. In particular, in many developed countries public coverage is uniform and limited and richer people complement public insurance with private coverage. As a result, supplementary insurance can seriously affect horizontal equity.

A recent literature has analyzed mixed insurance schemes in order to inves-

[1] The countries in which the opting-out system has developed are Germany, Ireland and the Netherlands. Most other countries are characterized by the topping-up system: Finland, France, Belgium, Sweden, the UK, the US, Canada and Australia.

tigate whether providing public coverage when consumers can supplement in the private market is efficient. Rochet (1991), Cremer and Pestieau (1996) and Henriet and Rochet (1998) have showed that, without ex-post moral hazard, public coverage is welfare improving if public insurance redistributes both from the rich to the poor and from the low- to the high-risk, that is if the correlation between wage rates and morbidity is negative. Petretto (1999) and Boadway *et al.* (2001) and (2004) have extended the previous models to the case of ex-post moral hazard (and, for the last work, adverse selection in the insurance market). The result is now ambiguous: the public insurer may want to set either a tax or a subsidy on health care expenses. Only with quasi-linear utility functions and for a sufficiently high negative correlation between wage rates and morbidity, Boadway *et al.* (2001) and (2004) have found that a positive public coverage is welfare improving.

Some important questions are still open. Let us assume that the correlation between wage rates and morbidity is not sufficiently high such that a positive public coverage is not welfare improving, as according to the above mentioned literature. In this case, how does moral hazard affect the optimal public and private coverage? Which are the consequences of mixed coverage on redistribution? In order to provide an answer to these questions, this work focuses on moral hazard only and considers heterogenous individuals: low- and high-revenue consumers. It is shown that, when moral hazard is sufficiently high, the rich buy more private coverage, and thus over-consume more than the poor. As a consequence, when the two firms play sequentially, the optimal public coverage is negative: health care consumption must be taxed to discourage private policy purchase.

Moreover, this model shows how reverse redistribution in public health care financing can arise. Let us assume that institutional and/or political constraints on public policy exist such that the public insurer *cannot* internalize that consumers also buy the private policy and always supplies a positive public coverage. In addition to this, let us assume that the level of ex post moral hazard is sufficiently high. The result is that the rich *net contribution* to health care financing is lower than the poor one, where net contribution is the fiscal revenue raised from a group minus the health care subsidy paid to such a group.

The plan of this paper is as follows. In section 2 consumers are identical. First, a graphical representation of moral hazard and its effects on expected consumption is proposed (section 2.1). Second, the consequences of moral hazard are analyzed when the public insurer is myopic and does not internalize the effect of private coverage on treatment consumption (section 2.2). Third, in sections 2.3 and 2.4, the equilibrium policies are derived when the public insurer and the private one move simultaneously (Nash equilibrium), and when the public insurer plays the role of the Stackelberg leader (sub-game perfect equilibrium) respectively. In section 3 consumers' heterogeneity is introduced: individuals differ in their wage rate and labor supply is endogenous. Section 3.3 considers the case for reverse redistribution when the public insurer is myopic. Finally, in section 3.4 the equilibrium policies are derived when the public insurer plays the role of the Stackelberg leader. Section 4 concludes.

2 The model with identical consumers

In the following model a part of individuals' health expenditure is covered by public insurance, a part by private insurance, and the last part is out-of-pocket.

In this section consumers are identical and have mass 1. Their state-independent utility is a function of health, the benefits of the health care received, and the income available to be spent on other goods after the cost of treatment has been deducted. In this section income W is exogenous. Consumers are ill with probability p. When ill, they are subject to a negative health shock whose monetary equivalent is $\bar{h}$. Health can be recovered according to a strictly concave function $h(x)$ representing the monetary benefits from health care consumption, where x denotes the quantity of treatment. h is increasing in x, and ranges from 0 to $\bar{h}$. The marginal productivity of x is decreasing and the third derivative of h is positive. The lower bound of x is zero and its upper bound is set at $\bar{x}$ such that $h'(\bar{x}) = 0$.[2] The standard assumption is that $h(x) < \bar{h}$ for every possible level of treatment consumption. Nevertheless, in this paper let us assume that $h(\bar{x}) = \bar{h}$, that is *the upper bound for treatment implies complete recovery.* This allows to compare the first-best allocation (full-insurance with efficient consumption) to the allocation with full coverage and maximal overconsumption (full-insurance with the highest moral hazard level).

To make it simple, it is assumed that technology for medical treatment is linear and subject to constant returns to scale. Marginal cost is constant and normalized at one. Consumers directly purchase on the market the chosen quantity of treatment, which implies that the physician is acting as a perfect agent for his patients.

Using a strictly concave function $U(\cdot)$ to represent the risk-averse consumers' preferences, the expected utility without any insurance is:[3]

$$EU = pU\left[W - x - \bar{h} + h(x)\right] + (1-p)\,U(W) \tag{1}$$

Henceforth aggregate consumption in ill and in healthy states is denoted as C_I and C_0 respectively. The indifference curves represent combinations of wealth in the two states of nature which yield constant expected utility. Indifference curves have slope $\frac{dC_o}{dC_H} = -\frac{1-p}{p}\frac{U'(C_0)}{U'(C_I)}$. From the consumer's budget constraint, whose slope is $-\frac{1-p}{p}$, expected aggregate consumption is $W - p\left[x + \bar{h} - h(x)\right]$. Let us define the *net monetary loss* due to illness as X, where $X \equiv x + \bar{h} - h(x)$.

When the consumer is not insured, he chooses his treatment consumption according to the first-order condition:

$$h'(x) = 1 \tag{2}$$

[2] An upper bound on treatment can be justified by limits on care imposed either by insurers, the government, or providers. Also there may be levels of care beyond which health no longer improves. (Selden 1993)

[3] The same model is used in Barigozzi (2003) to analyze secondary prevention reimbursement. In that model the health recovery function $h(\cdot)$ depends both on treatment and secondary prevention.

Since in the previous expression the marginal cost and the marginal benefit of treatment are equalized, from now on the amount of treatment verifying equation (2) will be referred as x_{FB}. Such an amount is the efficient one, in fact, as it will be shown in few lines, it corresponds to the first-best consumption. Notice that there is no income effect in (2), treatment demand only depends on consumption price.

Before analyzing a standard insurance contract with co-payment, it is useful to consider the first-best allocation of this simple model. First-best insurance can be implemented when the insurance firm perfectly observes the consumer's state of health, in this case it can offer two monetary transfers contingent upon disease: a lump-sum contract. The consumer receive T_I in case of illness and T_0 when healthy, where $pT_I + (1-p)T_0 = 0$. The first-best program is:

$$\begin{cases} \max\limits_{T_I, T_0, x} \quad EU = pU\left[W + T_I - x - \bar{h} + h(x)\right] + (1-p)\,U(W + T_0) \\ \\ s.t.: \quad pT_I + (1-p)T_0 = 0 \end{cases} \tag{FB1}$$

Such a contract leads to full insurance $\left(C_I = C_0 = C^{FB}\right)$. As it is generally known, in full insurance the indifference curves are tangent to the budget constraint. With first-best insurance treatment price is not distorted and consumers choose the efficient quantity of treatment x_{FB}. This implies that, in first-best, aggregate consumption is $C^{FB} = W - p\left[x_{FB} + \bar{h} - h(x_{FB})\right] = W - pX_{FB}$.

In figure 1 the two axes respectively indicate aggregate consumption when the consumer is healthy (C_0) and when she is sick (C_I). In the figure the no-insurance and the first-best allocations are shown. Notice that, in the figure, the net monetary loss $X_{FB} = x_{FB} + \bar{h} - h(x_{FB})$ can be directly read on the vertical axis.

Insert figure 1 about here

Let us assume, now, that the illness status is not perfectly observable either by public or private insurers. As a consequence all insurers offer a contract where the indemnity is directly related to the health care costs. Notice that, in order to obtain reimbursement, consumers must generally show to the insurer a doctor's certification or a hospital/doctor's bill. As a consequence, it is reasonable to assume that health care consumption is ex-post verifiable such that non-linear contracts could be analyzed. Nevertheless, for the sake of tractability, in this paper *linear* contracts will be analyzed.

2.1 With one coverage only: the second-best

First of all, let us consider the case where only the public insurer offers a contract to consumers. The public contract is denoted as (T, α), where T is the actuarially fair public premium ($T = p\alpha x$) and α is a cost-sharing parameter. Hence, $(1-\alpha)$ is consumers' out-of-pocket expense when they buy one unit of treatment. With the contract (T, α), consumers' expected utility becomes:

$$EU = pU\left[W - T - (1-\alpha)\,x - \bar{h} + h(x)\right] + (1-p)\,U(W - T) \tag{3}$$

When choosing treatment quantity x^*, consumers take the premium T and the cost-sharing parameter α as given, such that:

$$x^* = x(\alpha): \quad h'(x) = 1 - \alpha \tag{4}$$

When the cost-sharing parameter α is positive, the consumption price for treatment decreases. This implies that $x^* > x_{FB}$: overconsumption of treatment arises. Here it is the problem of ex-post moral hazard in health insurance.[4] Moreover, by differentiating (4) it follows that $\frac{\partial x}{\partial \alpha} = -\frac{1}{h''(x)} > 0$.[5] Thus, the higher is insurance coverage, the higher is over-consumption.

With a slight abuse of language, from now on $\frac{\partial x}{\partial \alpha}$ will be often referred as the *level of moral hazard* induced by the insurance coverage α.

In order to optimally choose T and α, the public insurer takes into account the choice of treatment made by consumers and solves the following program:

$$\begin{cases} \max\limits_{T,\alpha} \; EU = pU\left[W - T - (1-\alpha)x - \bar{h} + h(x)\right] + (1-p)U(W-T) \\ \\ s.t.: T = p\alpha x \\ \quad\; h'(x) = 1 - \alpha \end{cases}$$

Treatment demand does not depend either on revenue or on aggregate consumption in the illness status, therefore substituting the budget constraint into the public insurer's objective function, the previous program can be rewritten as:

$$\begin{cases} \max\limits_{\alpha} \; EU = pU\left[W - p\alpha x - (1-\alpha)x - \bar{h} + h(x)\right] + (1-p)U(W - p\alpha x) \\ \\ s.t.: h'(x) = 1 - \alpha \end{cases} \tag{P1}$$

From the first order condition with respect to α, an implicit expression for the cost-sharing parameter in second-best can be found:

$$\alpha^{SB} = \frac{(1-p)x\left[U'(C_I) - U'(C_0)\right]}{\frac{\partial x}{\partial \alpha} E\left[U'(C)\right]} \tag{5}$$

Where $E[U'(C)] \equiv pU'(C_I) + (1-p)U'(C_0)$ is expected marginal aggregate consumption.[6]

Definition 1 $\varepsilon_{x,\alpha} \equiv \frac{\partial x}{\partial \alpha}\frac{\alpha}{x} > 0$ *is the coverage elasticity of treatment demand.*

[4] A number of empirical studies have analyzed the impact of cost-sharing on the consumption of health care. Since insurance is thought to induce demand for health care by reducing its marginal price, the price elasticity of demand for care is directly relevant to the moral hazard effect. To date, the most important empirical study is The Rand Health Insurance Experiment which estimated the elasticity of demand at -0.2 (Manning *et al.* (1987))

[5] See also the first lines of appendix 1.

[6] Notice that, because of moral-hazard, neither $\alpha = 0$ nor $\alpha = 1$ are possible solutions for equation (5). In fact for $\alpha = 0$ it is $C_I = W - x_{FB} - \bar{h} + h(x_{FB}) = W - X_{FB}$ and $C_0 = W$, such that $C_0 > C_I$. While for $\alpha = 1$ $C_I = W - T - \bar{h} + h(\bar{x}) = W - p\bar{x}$ and $C_0 = W - p\bar{x}$ such that $C_I = C_0 = C^{FI}$, where FI stands for full-insurance.

Definition 2 $\pi(x) \equiv -x\frac{h'''(x)}{h''(x)} > 0.$[7]

Lemma 1 *(**Concavity**): a sufficient condition for problem P1 to be concave in α is $\varepsilon_{x,\alpha} > \frac{1}{p}(\pi(x))^{-1}$.*

Proof. See the appendix 5.1. ■

Notice that the sufficient condition in lemma 1 implies that problem P1 is well-behaved when moral hazard is sufficiently high.

Lemma 2 *(**Second-best**): when only one firm provides coverage, the second-best allocation is obtained. The second-best coverage α^{SB} is positive and lower than one. Moreover, the lower is moral-hazard, and the higher is consumer's risk aversion, the higher is α^{SB}.*

Proof. In general no over-insurance ($\alpha > 1$) can arise because for $x > \bar{x}$ the marginal benefit from treatment becomes negative, then $C_0 \geq C_I$ and $U'(C_I) - U'(C_0) \geq 0$ holds. The higher is risk aversion, the higher is the difference between the two marginal utilities. Moreover, $\frac{\partial x}{\partial \alpha} > 0$, such that the cost-sharing parameter α is positive. ■

Now let us examine the consequence of moral hazard on consumers' expected aggregate consumption.

Remark 1 *Under ex-post moral hazard, the insurance coverage reduces consumers' expected aggregate consumption.*

Proof. Under the contract (T, α), consumers' expected aggregate consumption becomes $W - p\left[x^* + \bar{h} - h(x^*)\right] = W - pX^*$. Whereas, without any insurance coverage, expected aggregate consumption is $W - pX_{FB}$. The function $h(\cdot)$ is concave and $x^* > x_{FB}$, thus $X^* = x^* + \bar{h} - h(x^*) > X_{FB} = x_{FB} + \bar{h} - h(x_{FB})$ and $W - pX^* < W - pX_{FB}$. ■

Remark 1 implies that the consumers' budget constraint shifts down when insurance coverage is purchased. Figure 1 shows the new budget constraint and the consumers' allocation (the second-best) under the contract (T, α).

The following equation, directly coming from (5), can be interpreted in term of the trade-off between risk-spreading and efficiency:

$$\varepsilon_{x,\alpha^*} = \frac{(1-p)\left[U'(C_I) - U'(C_0)\right]}{E\left[U'(C)\right]} \tag{6}$$

The consumer, moving to the left *on* his budget constraint, reaches partial insurance and his utility increases. On the other hand, the coverage α leads to over-consumption: aggregate consumption decreases and the budget constraint moves down; as a result, consumers' utility falls. The optimal cost-sharing parameter α is such that the marginal benefit (the right hand side of (6)) and

[7] The function $\pi(x) \equiv -x\frac{h'''(x)}{h''(x)}$ recalls the index of relative prudence for the *utility* functions. The economic interpretation in term of the *recovery* function $h(x)$ is difficult to establish.

the marginal cost of insurance coverage (the left hand side of (6)) are equalized. Obviously, in second-best the consumers' utility is lower than in first-best, but it is higher than in the absence of insurance.

It is now interesting to compare consumers' expected utility with no-insurance $(\alpha = 0)$ and in the full-coverage case $(\alpha = 1)$. In fact, if moral hazard is sufficiently high (in other words if $\bar{X} = \bar{x} + \bar{h} - h(\bar{x}) = \bar{x}$ is sufficiently larger than X_{FB}), and/or the consumer's risk aversion is sufficiently low, consumers prefer no-insurance to full-coverage as in figure 2. In other words, the cost of moral hazard in terms of the fall of expected aggregate consumption completely overcomes the benefit from insurance.

Insert figure 2 about here

Definition 3 *$c(L, U)$ is the certainty equivalent of lottery L given utility $U(\cdot)$, where lottery L represents the no-insurance case and it is shown in figure 3.*

Remark 2 *Given the level of moral hazard, there is a $U(\cdot)$ such that $c(L, U) > W - p\bar{X}$ always holds. Alternately, given the utility function $U(\cdot)$, $c(L, U) > W - p\bar{X}$ holds if moral hazard is sufficiently high.*

Proof. In the full-insurance case $\alpha = 1$ and $x = \bar{x}$ such that $T = p\bar{x}$. Aggregate consumption in the two states of nature becomes $C_I = C_0 = C^{FI} = W - p\bar{x}$, where FI stands for full-insurance. Consumers' utility is $U(W - p\bar{x}) = U(W - p\bar{X})$. On the contrary, with no-insurance consumers face the lottery L and expected utility is given by $pU\left[W - x - \bar{h} + h(x)\right] + (1-p)\,U(W)$. Consumers prefer no-insurance to full-coverage if:

$$pU\left[W - x - \bar{h} + h(x)\right] + (1-p)\,U(W) > U(W - p\bar{X})$$

which is equivalent to write:

$$c(L, U) > W - p\bar{X}$$

and which means that consumers prefer the lottery L to the certain amount $W - p\bar{X}$. Recalling that the lower is risk aversion, the higher is $c(L, U)$; whereas the higher is moral hazard, the lower is $U(W - p\bar{X})$, remark 2 can be established. ■

Some comparative statics concerning the cost-sharing parameter will be particularly useful in section 3.

Remark 3 ***(Insurance coverage as a normal good)*** *$U'''(\cdot) < 0$ is a sufficient condition for insurance coverage to be a normal good. Whereas, if $U'''(\cdot) > 0$, a necessary condition is:*

$$C.1: \quad \varepsilon_{x,\alpha} > \frac{(1-p)\left[U''(C_I) - U''(C_0)\right]}{E\left[U''(C)\right]}$$

If $U'''(\cdot) > 0$ and the opposite of $C.1$ holds, than insurance coverage is an inferior good.

Proof. See the appendix 5.2. ■

The standard assumption in Decision Theory is that $U'''(\cdot) > 0$. Without moral hazard, when marginal utility is convex, the higher is consumers' revenue and the lower are risk-aversion and the demand for insurance. In fact, the lower is the revenue, the higher is marginal utility from increasing aggregate consumption in the "bad" state of nature. For this reason insurance is generally considered an "inferior good". On the contrary, concerning supplementary insurance, as mentioned in the introduction, empirical evidence shows that the rich buy more private coverage than the poor. The previous remark explains why. *When moral hazard is sufficiently high, insurance becomes a normal good.* In fact, a high level of moral hazard implies that an increase in coverage leads to a large increase in premium. The premium is paid in both states of nature and, when it is high, it brings to an important fall in expected aggregate consumption. In such a case, the lower is the revenue, the higher is the marginal cost from decreasing aggregate consumption in both states of nature.

On the contrary, when $U'''(\cdot) < 0$, marginal utility is concave such that the higher is consumers' revenue, the higher are risk-aversion and the demand for insurance. Thus, whatever the level of moral hazard, the rich buy more insurance coverage than the poor.

Later on the standard case $U'''(\cdot) > 0$ will be considered.

2.2 Public and private coverage when the public insurer is myopic

Let us assume that, in a second stage, private firms offer a contracts (P, β) where P is the premium and β is the cost-sharing parameter. And let us assume that the private market is competitive so that insurance firms make zero profit and the premium P is actuarially fair. Later on, for the sake of exposition, the representative firm in the insurance market will be referred as *the* private *insurer*.

First of all, let us consider the case where the public insurer is myopic, that means it does not anticipate that the consumer will buy a private coverage. The public insurer's myopia can be motivated by several political or institutional constraints which are not internalized in this simple model but seem important in reality. For example, as shown in section 3.4, when the public insurer moves the first, it would be optimal to impose a negative public coverage (to tax health care consumption). In the real world, it would be really hard for the government to obtain the political agreement to implement such a policy.

With mixed coverage, consumers' expected utility becomes:

$$pU\left[W - T - P - (1 - \alpha - \beta) x - \bar{h} + h(x)\right] + (1 - p) U (W - T - P)$$

Now, the purchased quantity of treatment x^{**} is determined by:

$$x^{**} = x(\alpha + \beta) : \quad h'(x) = 1 - \alpha - \beta \tag{7}$$

When the public insurer is myopic, it solves problem (P1) as in the previous section. Whereas *the private insurer always takes both T and α as given.* In particular, in the case where the public insurer is myopic and also in the case where it acts strategically (both when the public firm has the first mover's advantage and when the two firms play simultaneously), the private firm always solves problem (P2) as below.

$$\begin{cases} \max\limits_{P,\beta} pU\left[W - T - P - (1-\alpha-\beta)\,x - \bar{h} + h\,(x)\right] + (1-p)\,U\,(W - T - P) \\ \\ s.t. : P = p\beta x \\ \qquad h'(x) = 1 - \alpha - \beta \end{cases} \tag{P2}$$

Substituting the premium P in the objective function and rearranging the first-order condition with respect to β, it follows:

$$pxU'(C_I) - p\left(x + \beta\frac{\partial x}{\partial \beta}\right)E\left[U'(C)\right] = 0 \tag{8}$$

where x, C_I and C_0 depend, now, on the mixed coverage. Rearranging (8) it is possible to calculate the derivative of the consumer's expected utility with respect to β when $\beta = 0$:

$$\left.\frac{\partial EU}{\partial \beta}\right|_{\beta=0} = p\,(1-p)\,x\left[U'(C_I) - U'(C_0)\right] > 0. \tag{9}$$

Remark 4 *Given a level of coverage, consumers are better off if they can buy some more coverage from another firm.*

Considering the consumer's point of view, this shows that, once the public contract (α, T) is established, a positive private coverage is welfare-improving. A new contract, which brings the consumer into the shadow area of figure 2, increases his expected utility.

As shown in sections 2.3, a mixed coverage with both α and β non-negative is inefficient. Nevertheless, remark 4 holds because in program (P2) the public premium T is considered as given. In particular, the private insurer does not internalize that an increase in β makes treatment demand increase which, in tour, makes the public premium T raise. Thus, aggregate consumption decreases in both states of nature. As a result, too much coverage is offered by the private firm and a negative externality on the public insurer's contract is produced. Later on it will be referred to as *premium externality.*[8]

Solving (8) for β, the following expression (equivalent to 5) follows for the private coverage:

$$\beta = \frac{(1-p)x\left[U'(C_I) - U'(C_0)\right]}{\frac{\partial x}{\partial \beta}E\left[U'(C)\right]} \tag{11}$$

[8] Notice that, if the premium externality is internalized, first-order condition (8) becomes:

$$pxU'(C_I) - p\left(x + \beta\frac{\partial x}{\partial \beta} + \alpha\frac{\partial x}{\partial \beta}\right)E\left[U'(C)\right] = 0 \tag{10}$$

Given that $\frac{\partial x}{\partial \beta} = \frac{\partial x}{\partial \alpha}$, from (10) and setting $\beta = 0$ it follows an expression equivalent to the first-order condition of problem (P1). Thus, as expected, $\left.\frac{\partial EU}{\partial \beta}\right|_{\beta=0} = 0$ is obtained.

Proceeding as in the previous section, private coverage is positive and the following remark can be established:

Remark 5 *When the public insurer is myopic and, at the same time, the private insurer provides coverage,* $0 < \alpha^{SB} < \alpha^{SB} + \beta < 1$ *holds. Moreover, the lower is moral-hazard, and the higher is consumer's risk aversion, the higher is* $\alpha^{SB} + \beta$.

Proof. The proof comes directly from the public insurer's myopia, from lemma 2 and from remark 4. ■

With the private coverage β, the marginal cost of treatment decreases, it follows that $x^{**} > x^*$: over-consumption increases and the consumers' budget constraint shifts down even more.

When the public insurer is myopic, an important point concerns the externality inflicted on the public insurer by private coverage: later on it will be referred as *coverage externality* in order to distinguish it from the premium externality. In its program (P1) the public insurer does not anticipate the effects of private coverage on consumers' choice. In particular the premium T is calculated upon x^* while, under mixed coverage, the expected consumption is $p\alpha x^{**} > T = p\alpha x^*$. Thus, the public premium T does not pay the public insurer for the expected cost of treatment: the public insurer makes negative profits. In particular the public insurer faces a budget deficit equal to $p\alpha\left(x^{**} - x^*\right)$. Notice that $C_0 = W - T - P$ and $C_I = W - T - P - (1 - \alpha - \beta)\, x^{**} - \bar{h} + h\left(x^{**}\right)$ where $T = p\alpha x^*$ and $P = p\beta x^{**}$. As a result, consumers' expected aggregate consumption with mixed coverage is $W - pX^{**} + p\alpha\left(x^{**} - x^*\right)$ instead of $W - pX^{**}$, and the budget constraint moves down less than it should.

Remark 6 *When the public firm is myopic, consumers' expected aggregate consumption increases of the amount* $p\alpha\left(x^{**} - x^*\right)$. *Such an amount corresponds to a public budget deficit.*

The previous environment could describe some mixed insurance schemes implemented in the real world, the French system being a prime example[9], together with Medicare and Medigap coverage in the US. In these mixed insurance schemes all consumers, those who buy private coverage *and those who do not*, receive the same public coverage $0 < \alpha < 1$.

In the real world full-coverage ($\beta = 1 - \alpha$) is frequent (third payer principle): access to care is completely free for consumers who choose the maximum amount of treatment $\bar{x}$. Notice that, under the assumption $h(\bar{x}) = \bar{h}$, equation (11) is never satisfied for $\beta = 1 - \alpha$. Free access to care can arise *only* if complete recovery is never reached and $C_I < C_0$ for every treatment consumption level. In this case it is possible that $\beta = 1 - \alpha$ verifies (11). Let us assume for a moment that $h(\bar{x}) < \bar{h}$ and $C_I < C_0$ always holds, assuming also that $\beta = 1 - \alpha$ is the

[9] The "ticket modérateur" $(1 - \alpha)$ has always played a great role throughout the history of French health insurance scheme. However, 83% of the population have private insurance that pays all or part of patients' share of the costs, thus lessening its impact.

solution of problem (P2). Then, private firms set the premium $P = p(1-\alpha)\bar{x}$, and the public insurer's deficit reaches its maximum amount: $p\alpha(\bar{x} - x^*)$.

Let us assume that the scenario with a myopic public insurer depicts mixed health insurance schemes as they are sometimes available in the real world. The analysis in this section shows that the problem with supplementary insurance is not just that it leads to more overconsumption of care. More important is the coverage externality that can raise with respect to the public insurer. Such a coverage externality makes consumers better off (at least in the short run), as stated by remark 6. If the public contract is not accurately modified, the introduction of supplementary private coverage leads to public deficit, a problem the governments are extremely concerned about and which, paradoxically, has motivated the introduction of a private policy. In fact, in the last decade, fiscal pressure has led many governments to reduce public coverage such that consumers' out-of-pockets expenses for health care increased. In many countries private supplementary coverage is now considered a valuable instrument to smooth consumption in different health status.

2.3 Simultaneous game

Showing a private insurer's program similar to P2 where they added the condition for the balance of the public insurer's budget, Blomqvist and Johansson (1997, page 512) claim that "mixed equilibrium leads to lower welfare than the second-best equilibrium". Thus, they refer to the game where the public and the private insurer play simultaneously; anyway they do not solve the model and the inefficiency of mixed coverage is not analyzed in detail. This analysis is the aim of this section.

As previously explained, consumers and private firms' programs are not affected by the way the public and the private insurer compete. Thus, (7) always describes consumers' choice. Assuming that a large number of competitive private firms and the public insurer simultaneously choose the premium and the cost-sharing parameter, the public insurer solves the following program:

$$\begin{cases} \max\limits_{T,\alpha} p\left[U\left(W-T-P-(1-\alpha-\beta)x\right)-\bar{h}+h(x)\right]+(1-p)U(W-T-P) \\ \\ s.t.: T = p\alpha x \\ \qquad h'(x) = 1-\alpha-\beta \end{cases} \tag{P3}$$

Whereas private firms still solve Programs P2.

As already established, under the assumption $h(\bar{x}) = \bar{h}$, moral hazard implies that full coverage is never an equilibrium and $\alpha + \beta < 1$ necessarily holds. Using the terminology of Industrial Organization, private and public coverage are expected to be *strategic substitutes*: $\frac{\partial^2 EU}{\partial\alpha\partial\beta} < 0$. This implies that when α (respectively β) increases, private coverage (respectively public coverage) becomes less attractive for consumers.

Remark 7 *When the condition in lemma 1 holds, $\varepsilon_{x,\alpha} > \frac{1}{p}$ is a sufficient condition for public and private coverage to be strategic substitutes.*

Proof. See the appendix 5.3. ■

Notice that when $\pi(x)$ is lower than one, the condition in remark 7 implies the condition in lemma 1, otherwise the condition in lemma 1 is sufficient both for concavity and strategic substitutability between public and private coverage.[10]

According to remark 4, given one coverage, expected utility increases when another coverage is added. Thus, in the symmetric Nash equilibrium (α^N, β^N), both private and public coverage are positive. All the considerations made before regarding the optimal coverage still hold such that α^N and β^N are lower than one. Moreover, given that the two coverages are strategic substitutes, $\alpha^N = \beta^N < \alpha^{SB}$ holds.

From the first-order conditions of programs (P2) and (P3) it follows:

$$\text{FOC}\,(\alpha) \quad : \quad U'(C_I)x = \left(x + \alpha\frac{\partial x}{\partial \alpha}\right) E\left[U'(C)\right] \tag{12}$$

$$\text{FOC}\,(\beta) \quad : \quad U'(C_I)x = \left(x + \beta\frac{\partial x}{\partial \beta}\right) E\left[U'(C)\right] \tag{13}$$

where the left-hand sides of (12) and (13) represent marginal benefit and the right-hand sides marginal cost from an increase in coverage. In fact, a higher coverage leads to more treatment consumption and more recovery in the illness status (left-hand sides), and it leads to less aggregated consumption in both states of nature because the insurance premium increases (right-hand sides).

First-order conditions (12) and (13) show that here *two* premium externalities arise: the public firm does not take into account that α also affects the premium of the private contract through treatment consumption, and the private firm does not take into account that β also affects the premium of the public contract. When the premium externalities are internalized, first-order conditions become:

$$\text{FOC}^*\,(\alpha) \quad : \quad U'(C_I)x = \left(x + \alpha\frac{\partial x}{\partial \alpha}\right) E\left[U'(C)\right] + \beta\frac{\partial x}{\partial \alpha} E\left[U'(C)\right] \tag{14}$$

$$\text{FOC}^*\,(\beta) \quad : \quad U'(C_I)x = \left(x + \beta\frac{\partial x}{\partial \beta}\right) E\left[U'(C)\right] + \alpha\frac{\partial x}{\partial \beta} E\left[U'(C)\right] \tag{15}$$

Marginal costs in (12) and (13) are lower than in (14) and (15), showing that the two firms under-estimate the effect of their strategies on expected utility. Thus, aggregate coverage $\left(\alpha^N + \beta^N\right)$ is higher than the second-best coverage α^{SB}.

Proposition 1 *When consumers are homogeneous and public and private firms simultaneously choose insurance coverage, $0 < \alpha^N = \beta^N < \alpha^{SB}$ and $\alpha^N + \beta^N > \alpha^{SB}$: consumers are over-insured.*

[10] For the logaritmic function, $\pi(x)$ is equal to 1 and the two conditions are equivalent.

Proof. The proof results from all the previous discussion. ■

Here the standard inefficiency in Nash equilibria can be retrieved.

2.4 Sequential game

It is plausible to assume that the public insurer can credibly commit to ignore whatever strategy of the private one. Then, analyzing the sequential game between the two firms, it is reasonable to attribute the first-mover advantage to the public insurer.

The timing of the game is as follows: in *stage 1* the public insurer chooses its policy (T, α) without observing either consumers' demand for treatment or private coverage. The public insurer anticipates the effect of its policies both on the insurance market and on consumers' behavior. In *stage 2* the competitive insurance industry sells contracts (P, β) to consumers. Profits are zero such that the premium P is fair. In this stage, (T, α) are considered as given and consumers' behavior is correctly anticipated. In *stage 3* consumers choose the treatment quantity given (T, P, α, β). The equilibrium is assumed to be subgame perfect, so let us solve it by backward induction.

Proposition 2 *When consumers are homogeneous and the public firm has the first-mover's advantage, public coverage is set at zero. Private coverage corresponds to the second-best coverage.*

Proof. See the appendix 5.4. ■

The intuition for the previous proposition is as follows. How it was emphasized by remark (4), whatever is the timing of the game, the premium externality arises because the private firm takes (T, α) as given. Thus, even if the public insurer correctly anticipates consumers' behavior, when a positive public coverage is offered, the premium externality always leads to over-insurance with respect to the second-best. The second-best allocation can be obtained only if the public coverage is zero. In other words, the public insurer anticipates that the private firm reaction function is given by (13); by setting $\alpha = 0$ the premium externality is completly internalized and (13) and (15) are equivalent.

3 Consumers' heterogeneity

In order to investigate the redistributive implications of supplementary insurance under moral hazard, consumer's heterogeneity is here introduced with respect to wage rates. Consumers are characterized by two different productivity levels and their expected utility now is:

$$pU\left[w_i l_i - x_i - \bar{h} + h\left(x_i\right) - v(l_i)\right] + (1-p)\, U\left[w_i l_i - v(l_i)\right]$$

where $i = L, H$ and $w_H > w_L$ are the wage rates. The proportion of high- and low-income individuals in this economy is respectively λ_H and $\lambda_L = 1 - \lambda_H$. The function $v(\cdot)$ represents disutility from labor supply l_i and is increasing and strictly convex. Given that no adverse selection is considered, in order to simplify the notation high and low income groups are characterized by the same risk of illness p. Moreover, when they are ill, they suffer the same exogenous monetary loss $\bar{h}$ and they benefit from health care consumption according to the same function $h(x_i)$.

The public insurer does not observe either consumers' health status and wage rates, or individual demands for aggregated consumption, leisure or insurance. Income $w_i l_i$, preferences, and the distribution of individuals by type i are observable. The public insurer finances the uniform (linear) subsidy α with a linear tax on income. Thus the public insurer's instruments are (t, G, α), where t is the linear tax and G is a lump sum transfer. The public insurer maximizes an utilitarian social welfare function and wants to redistribute from high- to low-types. As before, the competitive insurance industry sells private contracts (P_i, β_i) to consumers. Private insurers do not observe consumers' health status. Profits are zero such that private premiums are fair $(P_i = p\beta_i x_i)$.

Given $(t, G, \alpha, \beta_i, P_i)$, consumers maximize their utility with respect to labor and treatment:

$$\max_{x_i, l_i} pU\left[(1-t)\,w_i l_i + G - P_i - (1-\alpha-\beta_i)\,x_i - \bar{h} + h(x_i) - v(l_i)\right]$$
$$+ (1-p)\,U\left[(1-t)\,w_i l_i + G - P_i - v(L_i)\right]$$

Considering consumers' first-order conditions, labor supplies and treatment demand respectively verify:

$$l_i^I = l_i^0 = l_i^*(w_i, t): \quad (1-t)\,w_i = v'(l_i) \tag{16}$$
$$x_i^{**} = x_i(\alpha + \beta_i): \quad h'(x_i) = 1 - \alpha - \beta_i \tag{17}$$

Labor supply is the same in both health status and is, obviously, negatively affected by the tax t $\left(\frac{\partial l_i}{\partial t} < 0\right)$. Moreover, more productive consumers supply more labor $(l_H > l_L)$ and have a higher post-tax revenue: $W_H \equiv (1-t)\,w_H l_H > W_L \equiv (1-t)\,w_L l_L$. As before, there are no income effects in the demand for treatment, as a consequence, *if consumers are not insured or if they have the same private coverage, both types choose the same quantity of treatment.*[11]

Let us consider the utilitarian optimum of the model. When the public insurer observes both the consumers' type and the health status, he solves the following problem:

$$\begin{cases} \max\limits_{T_i^I, T_i^0, l_i, x} \sum_i \lambda_i \left\{ pU\left[w_i l_i + T_i^I - x_i - \bar{h} + h(x_i) - v(l_i)\right]\right\} \\ \qquad\qquad + (1-p)\,U\left[w_i l_i + T_i^0 - v(l_i)\right] \\ \\ s.t.: \quad p\sum_i \lambda_i T_i^I + (1-p)\sum_i \lambda_i T_i^0 = 0 \end{cases}$$

Obviously there is no role for the private market because the utilitarian optimum leads to full insurance: $C_i^I = C_i^0$ for $i = L, H$. Moreover, $0 > T_L^0 > T_H^0$ and

[11] Notice that income effects in treatment demand would reinforce the results. In fact, here the focus is on the case where coverage is a normal good: the high income group buys more coverage and consumes more treatment.

$T_L^I > T_H^I > 0$, that is, the high type consumers pay a higher premium in good health and receive a lower transfer when ill. As in the case without insurance, labor supply and treatment quantity are not distorted: l_i^{FB} is such that $w_i = v'(l_i)$ for $i = L, H$, and x^{FB} is such that $1 = h'(x)$. Notice that, because $\frac{\partial}{\partial l_i}[w_i l_i - v(l_i)] > 0$, in the no-insurance case $C_H^0 > C_L^0$ and $C_H^I > C_L^I$, for $i = L, H$. The no-insurance allocation and the utilitarian optimum are represented in figure 4.

Insert figure 4 about here

Notice that, if the low-income group and/or the wage difference $(w_H - w_L)$ are sufficiently high, or if the risk aversion is sufficiently low, the high-income group is better off with the no-insurance allocation than with the utilitarian optimum.

3.1 With the public insurer only: the second best

When only the public firm provides coverage, the public insurer solves the following problem:

$$\begin{cases} \max\limits_{t,G,\alpha} \sum_i \lambda_i \left\{ pU\left[(1-t)\, w_i l_i + G - (1-\alpha)\, x - \bar{h} + h(x) - v(l_i)\right] \right\} \\ \qquad\qquad + (1-p)\, U\left[(1-t)\, w_i l_i + G - v(L_i)\right] \\ \\ s.t.: \ t\sum_i \lambda_i w_i l_i - G - p\alpha x = 0 \qquad (\delta) \\ \qquad\quad h'(x) = 1-\alpha \end{cases} \tag{P4}$$

where δ is the Lagrangian multiplier for the budget constraint.

Notice that, when public coverage is uniform, both income groups consume the same treatment quantity.

From first-order conditions with respect to G and α :

$$\alpha^{SB} = \frac{(1-p)x\sum_i \lambda_i \left[U'(C_i^I) - U'(C_i^0)\right]}{\frac{\partial x}{\partial \alpha}\sum_i \lambda_i E\left[U_i'(C)\right]}$$

The optimal uniform cost-sharing parameter α^{SB} depends on the *average* difference between marginal utilities. All the considerations on the parameter α made in lemma 2 in section 2.1, also hold here.

3.2 Stage 2: the private insurance market

Given (t, G, α) and anticipating the consumers' choice, private insurers solve the following program:

$$\begin{cases} \max\limits_{\beta_i, P_i} \ pU\left[(1-t)\, w_i l_i + G - P_i - (1-\alpha-\beta_i)\, x_i - \bar{h} + h(x_i) - v(l_i)\right] \\ \qquad\qquad + (1-p)\, U\left[(1-t)\, w_i l_i + G - P_i - v(L_i)\right] \\ \\ s.t.: \quad P_i = p\beta_i x_i \\ \qquad\qquad h'(x_i) = 1-\alpha-\beta_i \end{cases} \tag{P5}$$

Remark 4 in section 2.2 still holds: *consumers always choose a private coverage.* Substituting P_i in the objective function of P5 and calculating the first-order condition with respect to β_i, again private coverage verifies:

$$\beta_i = \frac{(1-p)x_i\left[U'(C_i^I) - U'(C_i^0)\right]}{\frac{\partial x_i}{\partial \beta_i} E\left[U'(C_i)\right]} \tag{18}$$

Corollary 1 directly follows from remarks 3 and 4.

Corollary 1 *Let us assume that $U'''(\cdot) > 0$. When moral hazard is sufficiently high such that $C.1$ holds, the high-income group buys more private coverage than the low-income one ($\beta_H > \beta_L$). When moral hazard is low such that $C.1$ is not satisfied, the opposite holds ($\beta_L > \beta_H$).*

Empirical evidence shows that moral hazard is relatively higher in the case of ambulatory care and specialist services. The previous corollary suggests that the rich are likely to buy more supplementary coverage for such services than the poor. This is in line with Doorslaer *et al.* (2000) who find a pro-rich bias in the use of specialist services.

3.3 Public and private coverage when the public insurer is myopic: reverse redistribution

As in the subsection 2.2, let us consider the consequences of moral hazard on public budget when the public insurer is myopic, that is when it does not anticipate that consumers also purchase a private coverage. The public insurer's budget constraint is as in program P4: $t\sum_i \lambda_i w_i l_i - G - p\alpha x^* = 0$ where $x^* = x(\alpha)$.

Remark 8 *Under condition $C.1$, when the public firm is myopic and the private firm also provides coverage, $0 < \alpha^{SB} < \alpha^{SB} + \beta_i < 1$ holds, with $\beta_H > \beta_L$. Thus $x_i^{**} = x(\alpha + \beta_i)$ with $x_H^{**} > x_L^{**} > x^*$.*

Let us consider again the coverage externality inflicted on the myopic public firm. The tax t is not high enough to cover health care cost: the public firm makes negative profit. In particular the public deficit is now equal to $p\alpha\left[\lambda_H\left(x_H^{**} - x^*\right) + \lambda_L\left(x_L^{**} - x^*\right)\right]$. Notice that *reverse redistribution* arises if:

$$tw_H l_H - p\alpha x_H^{**} < tw_L l_L - p\alpha x_L^{**} \tag{19}$$

where $tw_i l_i - p\alpha x_i^{**}$ is one group's *net contribution* to health care financing, that is the fiscal revenue raised from that group minus the *effective* health care subsidy $p\alpha x_i^{**}$ paid to such a group. Reverse redistribution arises when high types' net contribution is lower than low types' one.[12] Rearranging (19):

[12] Obviously public deficit has to be financed in subsequent periods. If we assume that public deficit will be covered with a lump sum tax or that consumers live only one period, reverse redistribution is not affected by future taxation.

$t\left(w_H l_H - w_L l_L\right) < p\alpha\left(x_H^{**} - x_L^{**}\right)$. Thus the higher is moral hazard and/or the lower is the wage rate difference and the more likely is reverse redistribution. Notice that this analysis does not consider income effects in the demand for treatment. Income effects would increase the difference $\left(x_H^{**} - x_L^{**}\right)$ and make reverse redistribution even more likely.

Some recent works present supplementary coverage as one source of the increase in rich people's medical expenses and the cause of serious inequity in the delivery of medical care (Doorslaer *et al.* 2000 and Henriet and Rochet 1998)[13]. As stated above, this seems to be true in particular for specialist services whose demand is more elastic with respect to coverage and it is characterized by higher income effects. Again this model does not take into account adverse selection. Nevertheless, as mentioned in the introduction, it has been shown that with both adverse selection and moral hazard, public insurance may want to set either a tax or a subsidy on health care expenses (Petretto 1999, Boadway *et al.* 2002, 2004). In the real world mixed health insurance schemes are characterized by a positive public insurance coverage. According to the literature about social insurance and redistribution, a positive public coverage is welfare improving if the negative correlation between wage rates and morbidity is sufficiently high. When such a negative correlation is low and considering health services where moral hazard is high, as shown in this section, reverse redistribution really becomes an issue.

Let us consider expected aggregate consumption. $C_i^0 = (1-t)\,w_i l_i + G - P_i - v\left(l_i\right)$ and $C_i^I = (1-t)\,w_i l_i + G - P_i - v\left(l_i\right) - (1-\alpha-\beta)\,x_i^{**} - \bar{h} + h\left(x_i^{**}\right)$ where $G = tE(wl) - p\alpha x^*$ and $P_i = p\beta_i x_i^{**}$. Expected consumption with mixed coverage and a myopic public insurer is $(1-t)\,w_i l_i + tE(wl) - v\left(l_i\right) - pX_i^{**} + p\alpha\left(x_i^{**} - x^*\right)$ instead of $(1-t)\,w_i l_i + tE(wl) - v\left(l_i\right) - pX_i^{**}$ and, again, the budget constraint moves down less than it should for both income groups. Anyway the coverage externality imposed by the high-income group is higher than the one imposed by the low-income one.

Remark 9 *Under condition C.1, the coverage externality imposed by each income group on the myopic public insurer is:* $p\alpha\left(x_i^{**} - x^*\right)$, *where* $p\alpha\left(x_H^{**} - x^*\right) > p\alpha\left(x_L^{**} - x^*\right)$. *Reverse redistribution* $\left(tw_H l_H - p\alpha x_H^{**} < tw_L l_L - p\alpha x_L^{**}\right)$ *can arise if moral hazard is sufficiently high and/or the wage rate difference is sufficiently low.*

3.4 Sequential game

In the case of heterogenous consumers, the simultaneous game between the public and the private firm is no more interesting. In fact here there is no symmetry between the two firms. On one side, public coverage is uniform while the private one is not. On the other side, public coverage is financed through linear taxation, while the private one is financed by a type-dependent premium.

[13] Doorslaer *et al.* (2000), page 572, write: "higher income groups may have better or quicker access to certain services because they are more likely to have supplementary private insurance cover, as in Finland, Sweden, the UK and in the US for Medicare patients".

Therefore let us consider only the game where the private firm has the first-mover advantage and correctly anticipates the second stage in which consumers buy private coverage.

Proposition 3 *When consumers are heterogeneous, condition C.1 holds, and the public insurer has the first-mover's advantage:*

(i) public coverage is negative,

(ii) the high-income group purchases more private coverage than the low-income one; both groups are under-insured with respect to the second-best: $\alpha + \beta_i < \alpha^{SB}$.

Proof. (i) See the appendix 5.5.

(ii) According to first-order condition (18) and corollary 1 $\beta_H > \beta_L > 0$. As shown in the appendix 5.5, public coverage leads to less than complete crowding-out of private insurance $\left(-1 < \frac{d\beta_i}{d\alpha} < 0\right)$. Concerning aggregate coverage, this implies that $\frac{\partial(\alpha+\beta_i)}{\partial\alpha} = 1 + \frac{d\beta_i}{d\alpha} > 0$. Thus, even if a reduction in public coverage makes private one increase, *aggregate* coverage always decreases. The previous considerations imply that each income group is less insured than in the second-best. ■

The above-mentioned result is not surprising. The aim of the public insurer is to redistribute from high- to low-income group and insurance coverage is provided by the private market as well. Moreover, public coverage is constrained to be uniform whereas private coverage is type-dependent, this implies that the latter is a more efficient instrument to smooth consumption. It was assumed that moral hazard is sufficiently high to make the high-income group buy more coverage. As a result, high-revenue consumers are better insured and purchase more treatment. A uniform positive public coverage would favor the high-income group more than the low-income one and reverse redistribution could arise. With the taxation of health care expenses, on the contrary, the public insurer indirectly taxes private coverage purchase and increases the level of redistribution. In fact, tax revenue is now given by the sum of income and health care taxation and the following inequality is trivially verified: $tw_H l_H - p\alpha x_H^{**} > tw_L l_L - p\alpha x_L^{**}$.

4 Concluding remarks

This paper investigates the topping-up scheme in health insurance when both public and private firms use linear contracts. The insurance relationship is characterized by ex-post moral-hazard: insurance coverage induces overconsumption of care. Considering a normative approach, in the first part of the paper the optimal public coverage is derived when consumers are homogeneous and, in the second part, when they differ in their wage rate. In the case of homogenous consumers, when the firms play simultaneously, both coverages are positive and consumers are over-insured; whereas, when the firms play sequentially, the public insurer provides zero coverage and the second-best allocation is implemented.

In the case of heterogeneous consumers it is shown that, when moral hazard is sufficiently high, the rich buy more private coverage than the poor, and thus the rich induce more over-consumption of care. As a consequence, when the two firms play sequentially, the optimal public coverage is negative: health care consumption must be taxed to discourage private policy purchase. As a result, consumers are under-insured.

The paper also shows how reverse redistribution in public health care financing can arise. Let us assume that (i) the negative correlation between mortality and wage rate is not high enough to justify a positive public coverage, (ii) institutional and/or political constraints on public policy exist such that the public firm cannot provide a negative coverage and (iii) the level of ex post moral hazard is sufficiently high. Then, the rich net contribution to health care financing (the fiscal revenue raised from the rich minus the health care subsidy paid to them) is lower than the poor one. Unfortunately this scenario could describe some real situations in countries like France and the US for Medicare patients.

Some authors have suggested different ways to make the rich pay for their overconsumption of care. In particular Henriet and Rochet (1998) suggest to apply an income-related deductible to deal with the overly generous coverage of the well-off. Whereas Pauly (2000), referring to Medicare, more radically claims that the public insurance program should be substituted by a voucher for all beneficiaries for paying non-governmental insurers. The US are already partially moving in this direction: some Medicare subscribers can ask for the voucher and use it to buy an HMO plan (this option is called Medicare+Choice). The Medicare HMO plan is able to avoid the inefficiency associated with the supplementation externality of Medigap because it provides all coverage through a single plan and discourages supplementation. Moreover, in order to deal with the income effect, Pauly suggests making the size of both the voucher amount and the minimum covered benefit decrease with income. This would allow a reduction in coverage and moral hazard-related use of care for people who are not poor.[14]

A policy implication of this present paper is that, when the public insurer is constrained to subsidize health care consumption because of political/institutional reasons, the progressivity of contributions to public insurance should be increased with the specific purpose of neutralizing the reverse-redistribution effect of voluntary private coverage. Moreover, tax incentives for the purchase of private supplementary policies should be avoided.

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5 Appendix

5.1 The concavity of problem P1

Before calculating the second order condition of P1, let us consider the demand for treatment as defined by (4). By differentiating (4) it follows that:

$$\frac{dx}{d\alpha} = -\frac{1}{h''(x)} > 0 \tag{20}$$

Thus, $\frac{d^2x}{d\alpha^2} = \frac{d}{d\alpha}\left(-\frac{1}{h''(x)}\right) = \frac{h'''(x)}{[h''(x)]^2}\frac{dx}{d\alpha}$. Which leads to:

$$\frac{d^2x}{d\alpha^2} = -\frac{h'''(x)}{[h''(x)]^3} > 0 \tag{21}$$

The second-order condition of P1 with respect to α can be written as:

$$\begin{aligned} pU'(C_I)\left(-2p\tfrac{\partial x}{\partial \alpha} + \tfrac{\partial x}{\partial \alpha} - p\alpha\tfrac{\partial^2 x}{\partial \alpha^2}\right) + pU''(C_I)\left(-px - p\alpha\tfrac{\partial x}{\partial \alpha} + x\right)^2 \\ + (1-p)U'(C_0)\left(-2p\tfrac{\partial x}{\partial \alpha} - p\alpha\tfrac{\partial^2 x}{\partial \alpha^2}\right) + (1-p)U''(C_0)\left(-px - p\alpha\tfrac{\partial x}{\partial \alpha}\right)^2 \end{aligned} \tag{22}$$

A *sufficient condition* for (22) to be negative is: $\frac{\partial x}{\partial \alpha} - p\alpha\frac{\partial^2 x}{\partial \alpha^2} < 0$. Using (20) and (21) the previous inequality can be rewritten as:

$$\frac{\partial x}{\partial \alpha}\frac{1}{x} < p\frac{\alpha}{x}\frac{\partial x}{\partial \alpha}\frac{h'''(x)}{[h''(x)]^2}$$

or:

$$-\frac{1}{h''(x)}\frac{1}{x} < p\varepsilon_{x,\alpha}\frac{h'''(x)}{[h''(x)]^2} \tag{23}$$

From (23) the condition in lemma 1 can be immediately derived.

5.2 Insurance coverage as a normal good

By totally differentiating the first-order condition of problem P1 with respect to α and W it follows: $\frac{d\alpha}{dW} = -\frac{\frac{dFOC\alpha}{dW}}{\frac{dFOC\alpha}{d\alpha}}$, where the denominator is negative under the condition in lemma 1 and:

$$\frac{dFOC\alpha}{dW} = (1-p)\,x\,[U''(C_I) - U''(C_0)] - \alpha\frac{\partial x}{\partial \alpha}E\,[U''(C)] \tag{24}$$

Since $sign\left(\frac{d\alpha}{dW}\right) = sign\left(\frac{dFOC\alpha}{dW}\right)$, the sign of (24) is crucial. Rearranging (24), $\frac{dFOC\alpha}{dW}$ is positive if:

$$\varepsilon_{x,\alpha} > \frac{(1-p)\left[U''(C_I) - U''(C_0)\right]}{E\left[U''(C)\right]}$$

5.3 Proof of Remark 7

The cross derivative of consumers' expected utility in program P3 is:

$$\begin{array}{l} \frac{\partial^2 EU}{\partial\alpha\partial\beta} = pU'\left(C_I\right)\left(-p\frac{\partial x}{\partial\beta} + \frac{\partial x}{\partial\beta} - p\alpha\frac{\partial^2 x}{\partial\alpha\partial\beta}\right) \\ +pU''\left(C_I\right)\left(-px - p\alpha\frac{\partial x}{\partial\alpha} + x\right)\left(-p\alpha\frac{\partial x}{\partial\beta} + x\right) \\ +\left(1-p\right)U'\left(C_0\right)\left(-p\frac{\partial x}{\partial\beta} - p\alpha\frac{\partial^2 x}{\partial\alpha\partial\beta}\right) \\ +\left(1-p\right)U''\left(C_0\right)\left(-px - p\alpha\frac{\partial x}{\partial\alpha}\right)\left(-p\alpha\frac{\partial x}{\partial\beta}\right) \end{array} \quad (25)$$

Notice that, considering consumers' demand for treatment, $\frac{\partial x}{\partial\beta} = \frac{\partial x}{\partial\alpha} > 0$ and $\frac{\partial^2 x}{\partial\alpha\partial\beta} = \frac{\partial^2 x}{\partial\alpha^2} > 0$. The third and the fourth term in (25) are negative. The first term is negative when the condition in remark 1 is verified. The second term can be rewritten as:

$$pU''\left(C_I\right)\left(-p\alpha\frac{\partial x}{\partial\beta} + x\right)^2 - p^2 x U''\left(C_I\right)\left(-p\alpha\frac{\partial x}{\partial\alpha} + x\right)$$

which is negative if $-p\alpha\frac{\partial x}{\partial\alpha} + x < 0$. Rearranging the previous inequality it follows that $\varepsilon_{x,\alpha} > \frac{1}{p}$ is a sufficient condition to have $\frac{\partial^2 EU}{\partial\alpha\partial\beta} < 0$.

5.4 Proof of proposition 2

In the third stage, treatment demand is given by equation (7) and $x^{**} = x(\alpha + \beta)$. The indirect utility function is $v = v(T + P, \alpha + \beta)$. Applying the envelope theorem it follows:

$$\frac{\partial v}{\partial T} = \frac{\partial v}{\partial P} = -E\left[U'(C)\right], \quad \frac{\partial v}{\partial\alpha} = \frac{\partial v}{\partial\beta} = xpU'(C_I) \quad (26)$$

In the second stage the private insurer solves program (P2) where contract (α, T) is taken as given and consumers' behavior is correctly anticipated. Program (P2) can be written using the indirect utility function $v = v(T + P, \alpha + \beta)$:

$$\max_{\beta,P} \pounds = v(T + P, \alpha + \beta) + \lambda\left[P - p\beta x\left(\alpha + \beta\right)\right] \quad (27)$$

The first-order conditions are:

$$\begin{array}{ll} P: & \frac{\partial v}{\partial P} + \lambda = 0 \\ \beta: & \frac{\partial v}{\partial\beta} - \lambda\left(px + p\beta\frac{\partial x}{\partial\beta}\right) = 0 \end{array} \quad (28)$$

Since the solution to problem (27) gives $\beta(\alpha, T)$ and $P(\alpha, T)$, the maximum value function for this problem is defined as $V(\alpha, T)$. By the envelope theorem, from (26) and the first-order conditions (28), it follows the properties of $V(\alpha, T)$:

$$\begin{array}{l} \frac{\partial V}{\partial T} = \frac{\partial v}{\partial T} = \frac{\partial v}{\partial P} = -\lambda \\ \frac{\partial V}{\partial \alpha} = \frac{\partial v}{\partial \alpha} - \lambda p \beta \frac{\partial x}{\partial \alpha} = \frac{\partial v}{\partial \beta} - \lambda p \beta \frac{\partial x}{\partial \beta} = \lambda p x \end{array} \tag{29}$$

Finally in the third stage the public insurer solves program:

$$\max_{\alpha, T} \pounds = V(T, \alpha) + \gamma \left[T - p \alpha x \left(\alpha, T \right) \right]$$

The first-order conditions are:

$$\begin{array}{ll} T: & \frac{\partial V}{\partial T} + \gamma \left(1 - p \alpha \frac{\partial x}{\partial T} \right) = 0 \\ \alpha: & \frac{\partial V}{\partial \alpha} - \gamma \left(p x + p \alpha \frac{\partial x}{\partial \alpha} \right) = 0 \end{array} \tag{30}$$

Using (29) and rearranging (30):

$$\alpha \left(\frac{\partial x}{\partial \alpha} + p x \frac{\partial x}{\partial T} \right) = 0 \tag{31}$$

Notice that $\frac{\partial x}{\partial T} = -\frac{\partial x}{\partial W}$, then $\frac{\partial x}{\partial \alpha} + p x \frac{\partial x}{\partial T} = \frac{\partial x}{\partial \alpha} - p x \frac{\partial x}{\partial W}$ where $\frac{\partial x^C}{\partial \alpha} \equiv \frac{\partial x}{\partial \alpha} - p x \frac{\partial x}{\partial W}$ corresponds to the derivative of the compensated demand for treatment. In fact, treatment demand is $x^{**} = x \left[\alpha + \beta \left(\alpha, T \right) \right]$, and an increase in α affects x both directly and indirectly through a change in β. By differentiating treatment demand it follows that $\frac{\partial x}{\partial \alpha} + p x \frac{\partial x}{\partial T} = -\frac{1}{H''(x)} \left(1 + \frac{\partial \beta}{\partial \alpha} - p x \frac{\partial \beta}{\partial W} \right)$ which is different from zero.[15] As a consequence, from (31) $\alpha = 0$.

5.5 Proof of proposition 3

In the third stage, labor supply is given by equation (16) and treatment demand by (17). The indirect utility function is $v_i = v_i(t, G - P_i, \alpha + \beta_i)$ where $i = L, H$. Applying the envelope theorem:

$$\begin{array}{l} \frac{\partial v_i}{\partial t} = -w_i l_i E \left[U'(C_i) \right] \\ \frac{\partial v_i}{\partial G} = -\frac{\partial v_i}{\partial P} = E \left[U'(C_i) \right] \\ \frac{\partial v_i}{\partial \alpha} = \frac{\partial v_i}{\partial \beta_i} = x_i p U'(C_i^I) \end{array} \tag{32}$$

In the second stage, private insurers solve program (P5) where (t, G, α) are taken as given and consumers' behavior is correctly anticipated. Program (P5) can be written using the indirect utility function $v_i = v_i(t, G - P_i, \alpha + \beta_i)$:

$$\max_{\beta_i, P_i} \pounds_i = v_i(t, G - P_i, \alpha + \beta_i) + \mu_i \left[P_i - p \beta_i x_i \left(\alpha + \beta_i \right) \right] \tag{33}$$

The first-order conditions are:

$$\begin{array}{ll} P_i: & \frac{\partial v_i}{\partial P_i} + \mu_i = 0 \\ \beta_i: & \frac{\partial v_i}{\partial \beta_i} - \mu_i \left(p x_i + p \beta_i \frac{\partial x_i}{\partial \beta_i} \right) = 0 \end{array} \tag{34}$$

[15] A similar expression can be found in Boadway *et al.* (2001).

Since the solution to problem (P5) gives $\beta_i(t, G, \alpha)$ and $P_i(t, G, \alpha)$, the maximum value function for this problem is defined as $V_i(t, G, \alpha)$. From equations (32) and (34), using the envelope theorem it follows:

$$\begin{array}{l} \frac{\partial V_i}{\partial t} = \frac{\partial v_i}{\partial t} = -w_i l_i E\left[U'(C_i)\right] - \mu_i p \beta_i \frac{\partial x_i}{\partial t} \\ \frac{\partial V_i}{\partial G} = \frac{\partial v_i}{\partial G} = -\frac{\partial v_i}{\partial P_i} = E\left[U'(C_i)\right] = \mu_i \\ \frac{\partial V_i}{\partial \alpha} = \frac{\partial v_i}{\partial \alpha} - \mu_i p \beta_i \frac{\partial x_i}{\partial \alpha} = \frac{\partial v_i}{\partial \beta_i} - \mu_i p \beta_i \frac{\partial x_i}{\partial \beta_i} = \mu_i p x_i \end{array}$$

Finally, in the third stage the public insurer solves program:

$$\max_{t,G,\alpha} \pounds = \sum_i \lambda_i V_i(t, G, \alpha) + \gamma \left[t \sum_i \lambda_i w_i l_i(t) - G - p\alpha \sum_i \lambda_i x_i(t, G, \alpha) \right] \quad (35)$$

The first-order conditions are:

$$\begin{array}{ll} t: & \sum_i \lambda_i \frac{\partial V_i}{\partial t} + \gamma \left[\sum_i \lambda_i w_i l_i + t \sum_i \lambda_i w_i \frac{\partial l_i}{\partial t} - p\alpha \sum_i \lambda_i \frac{\partial x_i}{\partial t} \right] = 0 \\ G: & \sum_i \lambda_i \frac{\partial V_i}{\partial G} + \gamma \left[-1 - p\alpha \sum_i \lambda_i \frac{\partial x_i}{\partial G} \right] = 0 \\ \alpha: & \sum_i \lambda_i \frac{\partial V_i}{\partial \alpha} + \gamma \left[-p \sum_i \lambda_i x_i - p\alpha \sum_i \lambda_i \frac{\partial x_i}{\partial \alpha} \right] = 0 \end{array} \quad (36)$$

Rearranging first-order condition with respect to G:

$$\sum_i \lambda_i \left[\frac{\mu_i}{\gamma} - 1 - p\alpha \frac{\partial x_i}{\partial G} \right] = 0$$

Let us define $b_i \equiv \frac{\mu_i}{\gamma} - p\alpha \frac{\partial x_i}{\partial G}$ the net marginal social utility of income for type-i consumers. From the previous equation:

$$E(b) = 1 \quad (37)$$

It is well known in the optimal taxation theory that when $b_H < b_L$ redistributing income from type-H to type-L is socially desirable.

Using (37) and rearranging first-order condition with respect to α:

$$E\left[bx\right] - E(x) - \alpha \sum_i \lambda_i \left(\frac{\partial x_i}{\partial \alpha} - p x_i \frac{\partial x_i}{\partial G} \right) = 0 \quad (38)$$

where, as in appendix 5.4, $\frac{\partial x_i^C}{\partial \alpha} \equiv \frac{\partial x_i}{\partial \alpha} - p x_i \frac{\partial x_i}{\partial G}$ corresponds to the derivative of the compensated demand for treatment for type-i consumers. Equation (38) can be rewritten as:

$$\alpha = \frac{cov\left[b, x\right]}{\sum_i \lambda_i \frac{\partial x_i^C}{\partial \alpha}} \quad (39)$$

Presumably it is $b_H < b_L$, while, under condition $C.1$, $x_H > x_L$. Thus $cov\left[b, x\right] < 0$.

Let us analyse the sign of $\frac{\partial x_i^C}{\partial \alpha}$. What follows is adapted from Boadway *et al.* (2001).

From appendix 5.4 derives that $\frac{\partial x_i^C}{\partial \alpha} \equiv \frac{\partial x_i}{\partial \alpha} - p x_i \frac{\partial x_i}{\partial G} = -\frac{1}{H''(x)} \left(1 + \frac{d\beta_i}{d\alpha} - p x_i \frac{d\beta_i}{dG} \right)$.

In order to find the sign of $\frac{\partial x_i^C}{\partial \alpha}$ it is necessary to calculate the expressions for $\frac{d\beta_i}{d\alpha}$

and $\frac{d\beta_i}{dG}$ from the second stage, that is from the maximization of expected utility by the private firm. Notice that the result concerning strategic substitutability of section 2.3 can be used even here. In fact, also in the sequential game the private firm maximizes consumers' expected utility given (t, G, α). Under the condition in remark 7, $\frac{\partial^2 EU}{\partial\alpha\partial\beta} < 0$ holds *when the derivative of private coverage with respect to the public one is considered*. Thus, $\frac{d\beta_i}{d\alpha} < 0$. While, according to remark 3, $\frac{d\beta_i}{dG} > 0$. First-order condition (18) can be rewritten as:

$$\Delta_i = U'(C_i^I)x_i - E\left[U'\left(C_i\right)\right]\left(x_i + \beta_i \frac{\partial x_i}{\partial \beta_i}\right) \tag{40}$$

By totally differentiating (40) it follows:

$$\frac{\partial \Delta_i}{\partial \beta_i} d\beta_i + \frac{\partial \Delta_i}{\partial \alpha} d\alpha + \frac{\partial \Delta_i}{\partial G} dG = 0$$

such that:

$$\frac{d\beta_i}{d\alpha} = -\frac{\frac{\partial \Delta_i}{\partial \alpha}}{\frac{\partial \Delta_i}{\partial \beta_i}} \quad \text{and} \quad \frac{d\beta_i}{dG} = -\frac{\frac{\partial \Delta_i}{\partial G}}{\frac{\partial \Delta_i}{\partial \beta_i}}$$

where $\frac{\partial \Delta_i}{\partial \beta_i} < 0$ when condition in lemma 1 holds, such that $\frac{\partial \Delta_i}{\partial \alpha} < 0$. Taking into account that in stage three $\frac{dx_i}{d\alpha} = \frac{dx_i}{d\beta_i}$ and $\frac{d^2 x_i}{d\alpha^2} = \frac{d^2 x_i}{d\beta_i^2} = \frac{d^2 x_i}{d\alpha d\beta_i}$:

$$\frac{\partial \Delta_i}{\partial G} = U''(C_i^I)x_i - E\left[U''\left(C_i\right)\right]\left(x_i + \beta_i \frac{\partial x_i}{\partial \beta_i}\right) > 0 \tag{41}$$

$$\begin{aligned} \frac{\partial \Delta_i}{\partial \alpha} &= p\beta_i \frac{dx_i}{d\alpha}\frac{\partial \Delta_i}{\partial G} + \frac{dx_i}{d\alpha} U'\left(C_i^I\right) + x_i U''\left(C_i^I\right)\left[x_i - p\left(x_i + \beta_i \frac{\partial x_i}{\partial \beta_i}\right)\right] \\ &-E\left[U'\left(C_i\right)\right]\left(\frac{dx_i}{d\alpha} + \beta_i \frac{\partial^2 x_i}{\partial \alpha \partial \beta_i}\right) \end{aligned} \tag{42}$$

$$\frac{\partial \Delta_i}{\partial \beta_i} = \frac{\partial \Delta_i}{\partial \alpha} - px_i \frac{\partial \Delta_i}{\partial G} - \frac{\partial x_i}{\partial \beta_i} E\left[U'\left(C_i\right)\right] \tag{43}$$

From (43) $\frac{\partial \Delta_i}{\partial \beta_i} < \frac{\partial \Delta_i}{\partial \alpha} < 0$. Thus, $-1 < \frac{d\beta_i}{d\alpha} < 0$: public coverage leads to less than complete crowding-out of private insurance. Notice that, concerning aggregate coverage, $\frac{\partial(\alpha+\beta_i)}{\partial\alpha} = 1 + \frac{d\beta_i}{d\alpha} > 0$ holds. This means that, even if an increase in public coverage makes private coverage decrease, aggregate coverage increases as well.

Finally, using the expression in (41), (42) and (43), $1 + \frac{d\beta_i}{d\alpha} - px\frac{d\beta_i}{dG}$ can be rewritten as:

$$1 - \frac{\frac{\partial \Delta_i}{\partial \alpha}}{\frac{\partial \Delta_i}{\partial \beta_i}} + px_i \frac{\frac{\partial \Delta_i}{\partial G}}{\frac{\partial \Delta_i}{\partial \beta_i}} = \frac{-\frac{\partial x_i}{\partial \beta_i} E\left[U'\left(C_i\right)\right]}{\frac{\partial \Delta_i}{\partial \beta_i}} > 0.$$

As a consequence $\frac{\partial x_i^C}{\partial \alpha}$ is negative as the denominator in (39). Thus public coverage is negative.

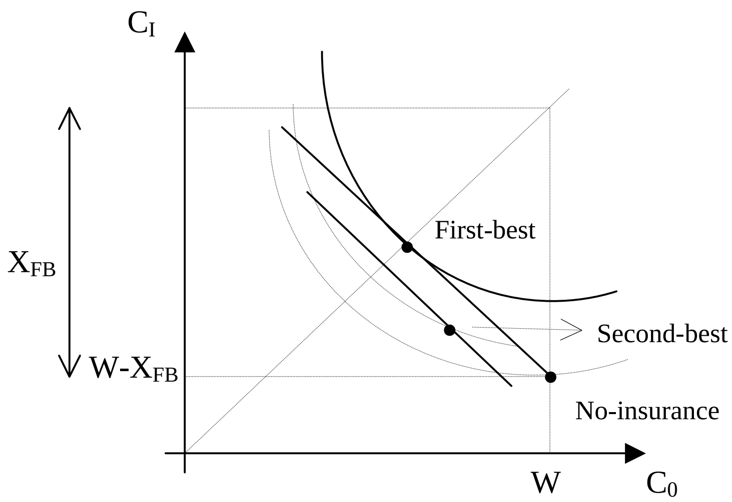

Figure 1: the efficient quantity of treatment and the second-best contract.

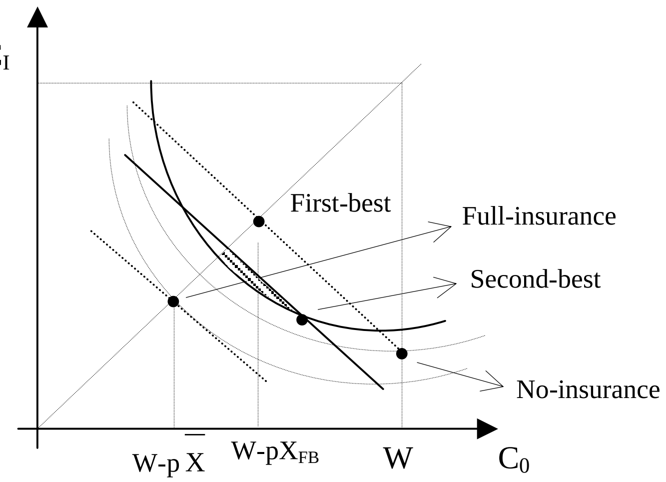

Figure 2: the amount of treatment in first-best and in full-insurance.

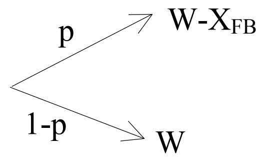

Figure 3: lottery *L*

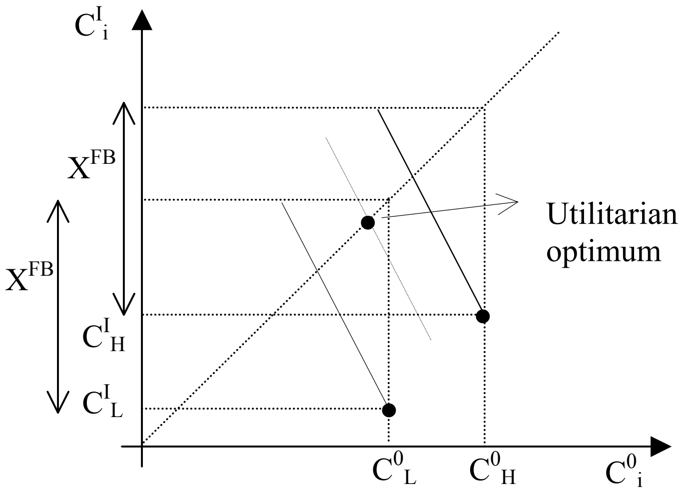

Figure 4: the utilitarian optimum with heterogeneous consumers.