Vertical Integration and Differentiation in an Oligopoly with Process Innovating R&D*

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Abstract

The paper contains first the analysis of the strategic decision as to whether to integrate or not, in a market with differentiation of final products feeding back into the production of intermediate inputs. Cournot competition makes integration a dominant strategy, although it is not Pareto optimal for the industry when goods are close substitutes. Bertrand competition leaves room also for nonintegration and there remains the possibility of asymmetric industry organization, with nonintegrated firms competing with integrated rivals. The analysis is extended to an oligopoly where upstream process R&D takes place. Here, the nonintegrated part of the industry may invest more in process R&D and even perform better than its integrated counterpart.

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1 Introduction

During the last two decades "outsourcing" has become a word frequently spelled by production managers and Ceo's all over the world. Reduced trade barriers, lower domestic and international transport costs, increased standardization of intermediate goods in most industries, easier and more precise transfer of industrial information, more homogeneous firm organization as a result of the demise of socialism as a production system, all these facts seem to have encouraged an increase in the process of vertical disintegration of firms or, as we prefer today, of outsourcing. Vertical disintegration has taken place in a world where consumers and firms seem to put a relevant emphasis on differentiation. Vertical integration has remained so far quite detached from the analysis of product differentiation policies of firms, despite the large amount of literature on this latter topic and the large lot of questions left open. There does not seem to be any relationship between vertical integration and differentiation if the characteristics of the final good do not depend on those of the intermediate inputs and/or if the intermediate inputs can be standardized without decreasing the degree of differentiation of the final good, or its "specificity". On the contrary, there are circumstances in which the relationship between vertical integration and the ability to foster product specificities and/or quality standards is robust and live. In this case, more open markets may increase standardization, mainly of inputs via diffuse outsourcing, eroding the space for differentiation through less vertical integration.

We know that there are productive assets which are specific providing a rationale for vertical integration (Grossman and Hart, 1986). However, the degree of specificity cannot be taken for granted since it depends upon the transaction regime in which assets are embedded. This makes for a close relationship between specificity and the cost of exchanging a product, i.e. the Williamson approach based on transaction costs is quite near (Williamson, 1985, 1986) to the Grossman-Hart specificity scheme.

The degree of vertical integration of firms is quite variable. It changes along time as recent and remote history shows, due to the evolution of the institutions of the economy, as Oliver Williamson emphasized in his "The Economic Institutions of Capitalism", (1985). From Williamson we learn that transaction costs encourage integration when they are on the rise. This explains why former socialist countries developed astonishingly vertically integrated firms since transactions across the planned economy were more expensive and awkward than in any decentralized market system. Yet, also developing countries with costly infant markets need more integrated firms and sometimes even more integrated factories.

Vertical integration varies also across countries and within countries. For instance Japan has less integrated industrial organization than Germany. And, within Italy, the regions Emilia Romagna and Veneto have industrial organizations that use more outsourcing than the rest of Italy.

Finally, across and within industries large differences show up as to the degree of integration. For instance, the chemical sector is more vertically integrated than the computer industry and automotive producer Daimler Chrysler is more vertically integrated than Fiat.

The recent wave of disintegration, due to more efficient markets for intermediate goods on a global scale, has stimulated some fresh contributions (McLaren, 2000; Grossman and Helpman, 2002) casting the integration issue in the new market environment characterized by reduced trade barriers, differentiated goods, economies of scale and imperfect competition. An important result found in Grossman and Helpman (2002) is that in one sector in a particular country we should not be able to observe firms with different levels of vertical integration, since the decisions either to integrate or to outsource are associated to external economies that generate a kind of bandwagon effect. This conclusion seems quite at odd with the casual observation that reveals that sometimes firms operate in the same industries and in the same country with noticeably different levels of integration.

A question remains open. Has the degree of differentiation any effect on the strategic choice to vertically integrate when firms must buy their inputs from specific upstream firms? In this vein, Pepall and Norman (2001) examine the relationship between a differentiated downstream market and a specialized upstream market, showing that vertical foreclosure is not an equilibrium strategy.

Our aim is to answer a similar question by going through the decision to integrate in an oligopolistic environment in which firms produce differentiated goods and take their decision as to integrate in a strategic fashion either competing in prices or quantities. Differentiation in the final stage of production, or downstream (DW), feeds back into the input market organization explaining some of the hold-up problem that exists in disintegrated industrial organizations with specificity. As we shall see Bertrand and Cournot modes of behaviors will give rise to different integration decisions leaving room even for asymmetric choices whereby one integrated firm competes with a non integrated rival.

A subsequent theme that we consider is the relationship of process R&D and integration, an issue on which contributions are quite few (Armour and Teece, 1980; Teece, 1976; Brocas, 2003). In our simple approach R&D takes place in the production of the intermediate input in an oligopoly with many DW firms competing in a Cournot mode selling differentiated goods. Here, again, we find instances of asymmetric structure of the industry, whereby nonintegrated firms compete with integrated rivals.

The paper is organized as follows. In the second section we consider a Cournot duopoly with differentiation and endogenous choice of the industrial mode of integration. In the third section we shift mode of behavior in favour of Bertrand. In the fourth section we analyze R&D in an oligopoly with differentiation of the final good. In the final section concluding remarks are presented.

2 A differentiated Cournot duopoly

2.1 The no integration case

We consider a market with firms whose production takes place in two stages. In the first an input is produced by two firms operating upstream (UP). In the second a final good is manufactured by a downstream (DW) duopoly using the input produced by UP firms.

Each DW firm is bound to produce a differentiated good and faces an inverse demand function replicating the Singh and Vives (1984) preferences for differentiated goods:

$$p_1 = a - q_1 - s\,q_2 \tag{1}$$

$$p_2 = a - q_2 - s\,q_1 \tag{2}$$

where $p_{1,2}$ stand for the market price of the goods produced by firm 1 and 2 respectively, a is the reservation price, $q_{1,2}$ are quantities sold and $s \in (0,1]$ is the parameter indicating the degree of substitutability between the two goods.

We assume that each firm bears a constant marginal cost c and buys the intermediate goods from the UP firms in a linear combination that reflects the differentiation of the final products of DW firms. Accordingly, the profit functions of the DW firms are:

$$\pi_{DW1} = (p_1 - c - (1-s)g_1 - sg_2)q_1 \tag{3}$$

$$\pi_{DW2} = (p_2 - c - (1-s)g_2 - sg_1)q_2 \tag{4}$$

where g_1 and g_2 are the prices charged by UP duopolists for their respective products. The form of these profit functions reflects the fact that if a DW firm differentiates its product it will not be able to buy the input indifferently from UP 1 and UP 2, but it must buy a fixed portion of input from UP 1 according to the degree of differentiation assumed in the market for the final good, i.e.: s. This formulation captures the feedback of DW differentiation into constrained outsourcing that introduces a hold up problem.

We find the equilibrium outputs by going through the first order conditions (FOCs) of (3) and (4) with respect to the quantities.

$$\begin{cases} \dfrac{\partial \pi_{DW1}}{\partial q_1} = 0 \\ \dfrac{\partial \pi_{DW2}}{\partial q_2} = 0 \end{cases} \tag{5}$$

from which we get:

$$q_1^E = \frac{2g_1 + (a-c)(s-2) - s(2g_1 - g_2) - s^2(g_1 - g_2)}{s^2 - 4} \tag{6}$$

$$q_2^E = \frac{2g_2 + (a-c)(s-2) - s(2g_2 - g_1) - s^2(g_2 - g_1)}{s^2 - 4}. \tag{7}$$

Substitutions produce equilibrium profits, π_{DW1}^E and π_{DW2}^E.

Now consider UP firms producing with constant marginal costs z. Their profits are:

$$\pi_{UP1} = (q_1(1-s) + sq_2)(g_1 - z) \tag{8}$$

$$\pi_{UP2} = (q_2(1-s) + sq_1)(g_2 - z). \tag{9}$$

These two firms maximise their profits in an independent way since their strategic interaction is simply an extension of the competition between DW firms. FOCs with respect to prices g_1 and g_2 are:

$$\begin{cases} \dfrac{\partial \pi_{UP1}}{\partial g_1} = 0 \\ \dfrac{\partial \pi_{UP2}}{\partial g_2} = 0 \end{cases} \tag{10}$$

from which we get optimal prices in reduced form, for the intermediate goods:

$$g_1^* = \frac{(a-c)(2-s) + 2(1+s(s^2+s-2))z}{4+s(-5+2s(1+s))} = g_2^* \tag{11}$$

which are non negative for all $s \in [0,1]$.

Optimal UP profits are:

$$\pi^*_{UP1} = \frac{2(2-s)[1+s(s+s^2-2)](a-c-z)^2}{(2+s)\{4+s[-5+2s(1+s)]\}^2} = \pi^*_{UP2} \tag{12}$$

Then optimal DW profits in reduced form are:

$$\pi^*_{DW1} = \frac{4[1+s(s+s^2-2)]^2(a-c-z)^2}{(2+s)^2\{4+s[-5+2s(1+s)]\}^2} = \pi^*_{DW2}. \tag{13}$$

2.2 Vertical integration

We now go through a vertically integrated duopoly. Firms UP and DW merge and the UP output is transferred to the DW firm at its marginal cost, i.e.: $g_1 = g_2 = z$.

Then profits of the two integrated duopolists are:

$$\pi_{INT1} = (p_1 - c - z)q_1 \tag{14}$$

$$\pi_{INT2} = (p_2 - c - z)q_2. \tag{15}$$

taking FOCs with respect to quantities we get:

$$q_1^* = q_2^* = \frac{a-c-z}{2+s} \tag{16}$$

while optimal profits are:

$$\pi^*_{INT1} = \pi^*_{INT2} = \frac{(a-c-z)^2}{(2+s)^2}. \tag{17}$$

2.3 Comparisons

What is better for firms? The reply comes from comparison of profits in the two cases.

To this purpose, just calculate:

$$\pi^*_{INT1} - \pi^*_{DW1} - \pi^*_{UP1} \propto \frac{(s-2)\{-2+s[3+2s(s+s^2-2)]\}}{(2+s)^2\{4+s[-5+2s(1+s)]\}^2} \tag{18}$$

The close scrutiny of (18) reveals that integration gives higher aggregated profits when $s \in [0, 0.862)$, i.e.: when the market for goods is far from homogeneity. As s tends to 1 non integration seems to dominate, i.e. the usual negative externality, discovered first by Spengler (Spengler, 1950), due to the negative influence of the DW price on UP profits disappears and turns into a positive one, making for the following

Lemma 1 *When there is product differentiation the usual negative externality of the DW price upon the UP profit depends on the degree of differentiation. For some interval of the differentiation parameters it may turn into a positive one, making non integration the superior result.*

Proof. Just consider direct demand

$$q_1 = \frac{p_1 + a(s-1) - s\,p_2}{s^2-1}$$

and its twin for q_2. Then obtain

$$\pi_{UP1} = \frac{(a(s-1) + p_2 s^2 - p_1(s^2+s-1))(g_1 - z)}{s^2-1}$$

and then evaluate the sign of

$$\frac{\partial \pi_{UP1}}{\partial p_1}.$$

Since there are two DW firms the externality is divided into two parts and it must be evaluated also vis à vis p_2, i.e.:

$$\frac{\partial \pi_{UP1}}{\partial p_2}.$$

Both derivatives have a sign that depends upon s, consistently with the result coming from the comparison between profits of the integrated and the non integrated case. ■

2.4 The asymmetric cases

So far we have assumed that firms behave in a symmetric way. However, we should also consider the case in which the duopoly is made by one integrated firm competing with a non integrated one.

Suppose that firm 1 integrates while 2 does not. Therefore between firm 2 UP and firm 2 DW there arises a bilateral monopoly.

Consider then profits of firm 1:

$$_{ASY}\pi_{INT1} = (p_1 - c - z)q_1 \tag{19}$$

while profits of the two non integrated firms are:

$$_{ASY}\pi_{DW2} = (p_2 - c - g_2)q_2 \tag{20}$$

$$_{ASY}\pi_{UP2} = (g_2 - z)q_2. \tag{21}$$

We can find the two equilibrium quantities of final goods produced, q_1^E and q_2^E by going through the following FOCs:

$$\begin{cases} \dfrac{\partial_{ASY}\pi_{INT1}}{\partial q_1} = 0 \\ \dfrac{\partial_{ASY}\pi_{DW2}}{\partial q_2} = 0 \end{cases} \tag{22}$$

and solving them simultaneously, obtaining:

$$q_1^E = \frac{2z + (a-c)(s-2) - g_2 s}{s^2 - 4} \tag{23}$$

$$q_2^E = \frac{2g_2 + (a-c)(s-2) - sz}{s^2 - 4}. \tag{24}$$

Then using q_2^E we get $_{ASY}\pi_{UP2}^E$. Taking the FOC on $_{ASY}\pi_{UP2}^E$ with respect to g_2 we get:

$$g_2^* = \frac{(a-c)(2-s) + (2+s)z}{4}. \tag{25}$$

After substitution of g_2^*, q_1^E, q_2^E we get reduced forms optimal profits for the 3 firms operating in the asymmetric case:

$$_{ASY}\pi^*_{INT1} = \frac{(4+s)^2(a-c-z)^2}{16(2+s)^2} \tag{26}$$

$$_{ASY}\pi^*_{UP2} = \frac{(2-s)(a-c-z)^2}{8(2+s)} \tag{27}$$

$$_{ASY}\pi^*_{DW2} = \frac{(a-c-z)^2}{4(2+s)^2}. \tag{28}$$

2.5 Comparisons in the asymmetric and mixed cases

We have now to compute the difference between profits of the integrated firm and those of the disintegrated firms competing with the integrated one. To this purpose we have to evaluate:

$$_{ASY}\pi^*_{INT1} -_{ASY} \pi^*_{UP2} -_{ASY} \pi^*_{DW2} = \frac{(2+3s)(a-c-z)^2}{16(2+s)} \tag{29}$$

which is always non negative implying that the integrated firm is better off than the two disintegrated firms jointly considered. In this case the negative externality comes back and it is independent of the differentiation parameter.

A further comparison is needed to proceed to the evaluation of the decision to integrate. We have to find the sign of the following:

$$_{ASY}\pi^*_{INT1} - \pi^*_{DW1} - \pi^*_{UP1} \tag{30}$$

which is always non negative[1] for all $s \in (0,1]$, i.e.: unilateral integration (asymmetric case) dominates disintegration undertaken in the symmetric case, i.e.: when both duopolists stay non-integrated.

Finally we have to evaluate the difference:

$$\pi^*_{INT1} -_{ASY} \pi^*_{UP2} -_{ASY} \pi^*_{DW2} = \frac{2+s^2}{8(2+s)^2} \tag{31}$$

which is nonnegative for all $s \in (0,1]$.

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[1]The sign of (30) depends upon the sign of:

$$\frac{64 + s^2\{-128 + s[160 + s(-23 + 4s(-25 + s(10+s))))]\}}{16(2+s)^2(4+s(-5+2s(1+s)))^2}$$

which is always non negative for $s \in (0,1]$.

2.6 The reduced form of integration game

Thanks to the above information we are equipped to assemble the reduced form of the corresponding game referred to the decision about whether to integrate or not to integrate vertically.

To do that we introduce the matrix representing the reduced form of the game in normal form (label I stands for *Integration*, NI for *Non Integration*; F1 is firm 1 and F2 firm 2).

Table 1: *Reduced form of the integration game*

			F2
		I	NI
F1	I	$\pi^*_{INT1}, \quad \pi^*_{INT2}$	${}_{ASY}\pi^*_{INT1},\ {}_{ASY}\pi^*_{UP2} +_{ASY} \pi^*_{DW2}$
	NI	${}_{ASY}\pi^*_{UP1} +_{ASY} \pi^*_{DW1},\ {}_{ASY}\pi^*_{INT2}$	$\pi^*_{DW1} + \pi^*_{UP1},\ \pi^*_{DW2} + \pi^*_{UP2}$

Using the results of the previous section we go through the solution of the above game and spell it in the following:

Proposition 1 *Suppose that competition in the final good is Cournot. Then, integration appears to be always the dominant strategy regardless of the degree of substitutability between final products. However, for* $s \in (0, 0.862]$ *integration maximizes the aggregate payoffs of the firms, while for* $s \in [0.862, 1]$ *the equilibrium is the outcome of a Prisoner's Dilemma.*

Proof. Just go through the comparisons of the above subsections. ■

3 Integration decision in a Bertrand differentiated duopoly

3.1 The disintegrated case

In this section we go through the same game as the one depicted previously, yet adopting the assumption that DW strategic interactions take place in a Bertrand environment.

We then write the demand functions for the two firms in terms of prices, adopting the same preferences as before:

$$q_1 = \frac{a - p_1 - a\ s + p_2 s}{1 - s^2} \tag{32}$$

$$q_2 = \frac{a - p_2 - a\ s + p_1 s}{1 - s^2}. \tag{33}$$

In the case of no integration, profits of DW firms are:

$$\pi_{DW1} = (p_1 - c - (1 - s)g_1 - sg_2)q_1 \tag{34}$$

$$\pi_{DW2} = (p_2 - c - (1 - s)g_2 - sg_1)q_2. \tag{35}$$

¿From FOCs we obtain equilibrium prices:

$$p_1^E = \frac{-2g_1 + 2g_1 s - 3g_2 s - g_1 s^2 + g_2 s^2 - c(2 + s) + a(s + s^2 - 2)}{s^2 - 4} \tag{36}$$

$$p_2^E = \frac{-2g_2 + 2g_2 s - 3g_1 s - g_2 s^2 + g_1 s^2 - c(2 + s) + a(s + s^2 - 2)}{s^2 - 4}. \tag{37}$$

Equilibrium profits are:

$$\pi_{DW1}^E = \frac{-(2g_1 + (g_2 - 2g_1)(s + s^2) + s^3(g_1 - g_2) + (a - c)(s + s^2 - 2))^2}{(s^2 - 4)(s^2 - 1)} \tag{38}$$

$$\pi_{DW2}^E = \frac{-(2g_2 + (g_1 - 2g_2)(s + s^2) + s^3(g_2 - g_1) + (a - c)(s + s^2 - 2))^2}{(s^2 - 4)(s^2 - 1)}. \tag{39}$$

In a similar way we can obtain equilibrium quantities, i.e.: q_1^E and q_2^E. Now consider UP firms' profits:

$$\pi_{UP1} = (g_1 - z)(q_1^E(1 - s) + q_2^E s) \tag{40}$$

$$\pi_{UP2} = (g_2 - z)(q_2^E(1 - s) + q_1^E s). \tag{41}$$

If we go through the FOCs with respect to prices (g_1 and g_2) we get optimal prices for the inputs produced by UP firms:

$$g_1^* = g_2^* = \frac{(a-c)(s+s^2-2)+(-2+s(4+s(-1+2(s-2)s)))z}{-4+s(5+2(s-2)s^2)} \tag{42}$$

which are nonnegative for $s \in (0,1]$. By substituting (42) into (40) and (41) we get reduced form optimal profits, i.e.: π_{UP1}^* and π_{UP2}^*. By repeating the same operation for the profits of DW firms (π_{DW1}^E and π_{DW2}^E) we get π_{DW1}^* and π_{DW2}^*.

3.2 The vertically integrated case

When firms integrate vertically the transfer price of the intermediate good is assumed to be set equal to the marginal cost. Then profits are

$$\pi_{INT1} = (p_1 - c - z)q_1 \tag{43}$$

$$\pi_{INT2} = (p_2 - c - z)q_2. \tag{44}$$

By FOCs we get optimal prices:

$$p_1^* = p_2^* = \frac{a+c+z-as}{2-s}. \tag{45}$$

Optimal profits will be:

$$\pi_{INT1}^* = \frac{(1-s)(a-c-z)^2}{(s-2)^2(1+s)} = \pi_{INT2}^*. \tag{46}$$

Again we may compare the profits of the vertically integrated symmetric case with those of the non integrated case. We still find that integration is superior for low levels of s, while for a higher level of homogeneity non integration is better. Again this is the result of an externality taking place in the non integrated mode of behaviour, from the price of the DW firm to the profits of the UP firm. This externality is negative only in the above range of s, while for larger s it turns positive.

3.3 Asymmetric cases

Here we consider a market with one firm vertically integrated while the rival is vertically disintegrated.

The profit of the integrated firm is

$$_{ASYM}\pi_{INT1} = (p_1 - c - z)q_1, \tag{47}$$

while the profit functions of the DW and UP firms, respectively, are:

$$_{ASYM}\pi_{DW2} = (p_2 - c - g_2)q_2 \tag{48}$$

and

$$_{ASYM}\pi_{UP2} = (g_2 - z)q_2. \tag{49}$$

Now we consider simultaneously $_{ASYM}\pi_{INT1}$ and $_{ASYM}\pi_{DW2}$. We calculate the two FOCs with respect to p_1 and p_2 and solve them to get equilibrium prices:

$$p_1^E = \frac{a(2 - s - s^2) + s(c + g_2) + 2(c + z)}{4 - s^2} \tag{50}$$

$$p_2^E = \frac{a(2 - s - s^2) + 2(c + g_2) + s(c + z)}{4 - s^2}. \tag{51}$$

Now, after substitution of equilibrium prices, we get the equilibrium profit function of the UP firm:

$$_{ASYM}\pi_{UP2}^E = \frac{(g_2 - z)\left[s(c + z + s(c + g_2)) + a(2 - s - s^2) - 2(c + g_2)\right]}{4 - 5s^2 + s^4}. \tag{52}$$

The optimal price for the intermediate good obtained from the relative FOC is:

$$g_2^* = \frac{(a - c)(2 - s - s^2) + (2 - s)(1 + s)z}{2(2 - s^2)}. \tag{53}$$

We are now able to write the optimal profits:

$$_{ASYM}\pi_{INT1}^* = \frac{(1 - s)\left[4 - s(2s - 1)\right]^2 (a - c - z)^2}{4(1 + s)(2 - s)^2(2 - s^2)^2} \tag{54}$$

$$_{ASYM}\pi_{DW2}^* = \frac{(1 - s)(a - c - z)^2}{4(1 + s)(2 - s)^2} \tag{55}$$

$$_{ASYM}\pi_{UP2}^* = \frac{(1 - s)(2 + s)(a - c - z)^2}{4(1 + s)(2 - s)(2 - s^2)}. \tag{56}$$

3.4 The game in reduced form

To find the solution of the strategic decision as to integrate vertically or not we have to proceed again to the consideration of the reduced form of the game. A matrix analogous to the one seen in the preceding section may be used and a new proposition is obtained:

Proposition 2 *When DW firms adopt Bertrand competition, we have that, for $s \in (0, 0.778]$, integration is a dominant strategy. For $s \in [0.778, 1)$ we obtain a coordination game where we observe two Nash equilibria on the main diagonal, with firms behaving symmetrically. In the same parameter range there also exists a mixed strategy equilibrium whereby one side of the industry is vertically integrated, while the other side is disintegrated.*

Proof. We just have to make some comparisons. First evaluate:

$$\pi^*_{INT1} -_{ASYM} \pi^*_{UP2} -_{ASYM} \pi^*_{DW2} = -\frac{(s-1)^2}{2(s-2)^2(s^2-2)},$$

which is nonnegative for all $s \in [0, 1)$.

Then consider

$$_{ASYM}\pi^*_{INT1} -_{ASYM} \pi^*_{UP2} -_{ASYM} \pi^*_{DW2} \propto$$

$$\begin{gathered}\{(s-1)[-64+s^2(320+s(-288+s(-281+4s(153+s(-37+s(-97+\\ +s(64+(s-4)s(4s-3))))))))]\}/\\ /\left\{4(s-2)^2(1+s)(s^2-2)^2\left[-4+s(5+2(s-2)s^2)\right]\right\}\end{gathered}$$

which is nonnegative for all $s \in [0, 0.778]$.

As a result we have that the common strategy to integrate (I,I) is dominant for $s \in [0, 0.778]$, while for $s \in [0.778, 1)$ we have a coordination game with two Nash equilibria: both firms do not integrate (NI, NI) and both integrate (I,I).

There obviously exists a mixed strategy equilibrium where firms locate along the secondary diagonal adopting asymmetric strategies, whose probability of adoption may be computed, on the usual assumption that the expected value of the payoffs relative to the first strategy must equal the expected value of payoffs of the second one.

Therefore we can write:

$$\begin{aligned} E(\pi_{I1}) &= \alpha\beta\pi_{I,I} + \alpha(1-\beta)\pi_{I,NI} \\ E(\pi_{NI1}) &= (1-\alpha)\beta(\pi_{NI,I}^{UP} + \pi_{NI,I}^{DW}) + (1-\alpha)(1-\beta)\pi_{NI,NI}, \end{aligned} \tag{57}$$

where α and β are the endogenous probabilities of the integration strategy of the two sides of the industry respectively, while their complements to 1 are the probabilities of the non integration strategy. E is the expectation operator. Imposing $E(\pi_{I1}) = E(\pi_{NI1})$, and its column counterpart, yields:

$$\alpha^* = \beta^* = \frac{\pi_{NI,NI} - \pi_{I,NI}}{\pi_{I,I} - \pi_{I,NI} - \pi_{NI,I}^{UP} - \pi_{NI,I}^{DW} + \pi_{NI,NI}}. \tag{58}$$

We do not consider the case $s = 1$ because of the discontinuity problems arising out the homogeneous Bertrand model. ■

As a partial conclusion of these two sections with the two modes of behavior, Bertrand and Cournot, we can state the equilibria in which firms either integrate or do not integrate depend on the degree of substitutability of final goods that feeds back into UP production with a sort of partial hold-up that depends on substitutability. However, we cannot exclude that there may exist equilibria with firms adopting asymmetric integration policies, when price competition prevails.

Propositions 1 and 2 lead to the following:

Corollary 1 *The interval of s where symmetric integration is a Pareto efficient equilibrium in the Bertrand model is a subset of the corresponding interval in the Cournot model.*

Bertrand competition in both stages dissipates the profit coming from vertical integration at higher levels of product differentiation as compared to the case of DW Cournot competition. In other words, the negative externality associated with the non integration strategy occurs in a reduced parameter area.

4 Integration and R&D

It seems worthwhile to extend the previous model to consider some form of capital commitment to R&D. To this purpose we consider process innovation, i.e. R&D devoted to decrease the cost of production. Even if we could indifferently analyze R&D in UP or DW production, we confine the innovative activity to the supply of the intermediate good. Extensions are easily available on the basis of the same approach. The analysis is not entirely general since it is just meant to survey cases in which the disintegrated part of an industry may be more efficient even in the presence of oligopolistic markets.

The scenario we investigate in one in which the market for the DW final good is covered by $n+1$ firms, one of which is vertically integrated, while the remaining n DW firms buy the intermediate good from a single UP firm.

The market for the final good is differentiated.

The inverse demand function for the integrated firm is:

$$p_{INT} = a - q_{INT} - s\left(q_i + \sum_{j\neq i} q_j\right) \tag{59}$$

where $q_i + \sum_{j\neq i} q_j$ is the total output of the n DW firms. The inverse demand function of each DW firm is:

$$p_{i,DW} = a - q_i - s\left(q_{INT} + \sum_{j\neq i} q_j\right). \tag{60}$$

The production of the final good takes place at constant returns to scale, as in the previous section.

We assume that process innovating R&D takes place in the UP firm and in the UP division of the integrated firm, according to the following simple technology:

$$z_h = \overline{z} - k_h \ , \ h = INT, UP \tag{61}$$

involving a quadratic cost:

$$c(k_h) = k_h^2. \tag{62}$$

As to the integrated firm, the input transfer takes place at the marginal cost of production, while non-integrated firms buy it from the UP firm at the market price g.

Then, we are able to write the profit function of the integrated firm:

$$\pi_{INT} = (p_{INT} - z_{INT})\, q_{INT} - k_{INT}^2 \tag{63}$$

while on the non-integrated side of the industry profit functions are:

$$\pi_{UP} = (g_{UP} - z_{UP})\left(q_i + \sum_{j\neq i} q_j\right) - k_{UP}^2 \tag{64}$$

$$\pi_i = (p_i - g_{UP})\, q_i. \tag{65}$$

The form of π_{UP} refers to the case where the intermediate good is homogeneous. That is, we are interested in investigating the performance of an industry where final goods embody the same basic technology and product differentiation is meant to meet consumer tastes but do not require specific investments.

Strategic interactions are described by a three stage game. In the first INT and UP compete in R&D by choosing simultaneously the optimal levels of k_h. In the second stage firm UP sets price g_{UP} in the market for the intermediate good. In the third stage all DW's and INT play simultaneously a Cournot equilibrium in the market for the final good. The game is solved by backward induction.

4.1 The characterization of the subgame perfect equilibrium

FOCs can be easily derived w.r.t. q_{INT} and q_i :

$$\begin{cases} \dfrac{\partial \pi_{INT}}{\partial q_{INT}} = 0 \\ \dfrac{\partial \pi_i}{\partial q_i} = 0 \end{cases} \tag{66}$$

yielding equilibrium outputs:[2]

$$q_{INT}^E = \frac{a\,(2-s) + n\; s\; g_{UP} - (\overline{z} - k_{INT})\,[2 + s\,(n-1)]}{(2-s)\,(n\; s+2)} \tag{67}$$

$$q_i^E = \frac{a\,(2-s) + s\,(\overline{z} - k_{INT}) - 2g_{UP}}{(2-s)\,(n\; s+2)} \tag{68}$$

Plugging (68) into (64), we write the relevant objective function of the UP firm at the second stage using symmetry across DW's:

$$\pi_{UP} = n\; q_i^E\,(g_{UP} - \overline{z} + k_{UP}) = \frac{n\,[a\,(2-s) + s\,(\overline{z} - k_{INT}) - 2g_{UP}]\,(g_{UP} - \overline{z} + k_{UP})}{(2-s)\,(n\; s+2)}. \tag{69}$$

[2]In order to solve the system (66), we impose on DW's the symmetry condition $q_j = q_i$ for all j.

From the FOC w.r.t. g_{UP}, we obtain the equilibrium price for the intermediate good:

$$g_{UP}^{E} = \frac{a\,(2-s) + \overline{z}\,(2+s) - 2k_{UP} - s\;k_{INT}}{4}\,. \tag{70}$$

There remains to be solved the Nash equilibrium in R&D efforts at the first stage. The FOCs describing the strategic interaction between UP and INT yield the following optimal investment levels:

$$k_{INT}^{*} = \frac{(a-\overline{z})\left[n^2\,(s-4)\,(1-s)^2\,s + 4n\,(2s^3-10s^2+13s-2) + 16\,(2-s)^2\right]}{n^2\,(15s^3-56s^2+55s-12)\,s + 4n\,(14s^3-52s^2+51s-6) + 48\,(2-s)^2}\,, \tag{71}$$

$$k_{UP}^{*} = \frac{n\,(a-\overline{z})\,[4ns^3 + s^2\,(8-15n) + 4s\,(3n-7) + 24]}{n^2\,(15s^3-56s^2+55s-12)\,s + 4n\,(14s^3-52s^2+51s-6) + 48\,(2-s)^2}\,. \tag{72}$$

Now, using the above expressions, we can obtain the optimal final output, corresponding price levels, as well as profits:

$$q_{INT}^{*} = \frac{4\,[4(s-2) + n(s-1)^2]\,(s-2)(2+n\,s)(a-\overline{z})}{48\,(2-s)^2 + 4n(s-2)\,[3+2s(7s-12)] + n^2 s\,\{s\,[55+s(15s-56)] - 12\}} \tag{73}$$

$$q_{UP}^{*} = \frac{4\,[4(s-2) + n(s-1)^2]\,(s-2)(2+n\,s)(a-\overline{z})}{48\,(2-s)^2 + 4n(s-2)\,[3+2s(7s-12)] + n^2 s\,\{s\,[55+s(15s-56)] - 12\}}. \tag{74}$$

Now we are ready to compare the level of R&D commitment between the integrated firm and the disintegrated firm:

$$k_{INT}^{*} - k_{UP}^{*} = \frac{(a-\overline{z})\,(s-2)\,[n^2 s\,(s(s-8)+8) + 8n\;s\,(s(s-4)+2) + 16\,(s-2)]}{48\,(2-s)^2 + 4n(s-2)\,[3+2s(7s-12)] + n^2 s\,\{s\,[55+s(15s-56)] - 12\}}. \tag{75}$$

We are then able to prove the following:

Lemma 2 *For all $s \in [0, 0.305)$ we have*

- $k_{INT}^{*} > k_{UP}^{*}$ *for all $n \in [1, \widetilde{n}_{+})$ and for all $n > \overline{n}_{+}$;*
- $k_{INT}^{*} < k_{UP}^{*}$ *for all $n \in (\widetilde{n}_{+}, \overline{n}_{+})$.*

For all $s \in (0.305, 1]$ we have

- $k_{INT}^* > k_{UP}^*$ *for all* $n \in [1, \widetilde{n}_+)$;
- $k_{INT}^* < k_{UP}^*$ *for all* $n > \widetilde{n}_+$.

Proof. The numerator (75) is equal to zero if :

$$\widetilde{n} = \frac{4\left(s(4-s) - 2 \pm \sqrt{2(2-s^2(2-s))}\right)}{s(8+s(s-8))}. \tag{76}$$

The smaller root is always negative for substitutes, while the larger root is larger or equal to 2 for all $s \in [0,1]$. The denominator of (75) is nil at

$$\overline{n} = \frac{2(s-2)\left(2s(12-7s^2) - 3 \pm \sqrt{9+16s^4}\right)}{s(15s^3 - 56s^2 + 55s - 12)} \tag{77}$$

Also in this case the smaller root is negative for substitutes. The larger root is strictly positive for $s \in [0, 0.305)$, while it is negative in the remainder of the substitutability range.

For all $s \in [0, 0.305)$ we have $\overline{n}_+ > \widetilde{n}_+$. ■

This Lemma shows that there are admissible parameter ranges and industry structures where the UP firm invests more than the INT firm. As a result of larger R&D commitment the more investment prone firm may do worse than the less investment prone and the industry organization may be liable to change.

If we proceed to a comparative assessment of the equilibrium profits, we have to span

$$\pi_{INT}^* - \pi_{UP}^* - n\ \pi_i^* \tag{78}$$

over the space $s \in [0,1]$ and $n \geq 1$. It can be ascertained, through numerical calculation, that there are admissible intervals of s and n where $\pi_{INT}^* < \pi_{UP}^* + n\ \pi_i^*$. This inequality is analysed in the Appendix. For instance, if $n = 2$:

$$\pi_{INT}^* < \pi_{UP}^* + n\ \pi_i^* \text{ for all } s \in [0, 0.465). \tag{79}$$

It can also be verified that $n = 1$ is insufficient to produce $\pi_{INT}^* < \pi_{UP}^* + n\ \pi_i^*$.

These arguments prove the following:

Proposition 3 *For all* $n \geq 2$, *there exists some admissible degree of substitutability* s *such that the non-integrated part of the market is performing better than the integrated part.*

When either (i) $n = 1$ for all degrees of substitutability, or (ii) $s = 0$ for all n, $\pi^*_{INT} > \pi^*_{UP} + n\ \pi^*_i$ because of the usual predominance of the negative externality (Spengler, 1950; Perry, 1989).

Unlike in the analysis carried out in the previous two sections, here we are not endogenizing market structure. Therefore, we may briefly inspect an alternative market scenario. Suppose on the non-integrated side of the industry there is a population of UP firms and only one DW firm; on the integrated side, there is a single INT firm, as above. As long as the number of UP firms is at least equal to two, Bertrand competition between them entails that the price of the intermediate good falls to its marginal production cost. In such a case, the DW firm enjoys the same degree of productive efficiency as the INT firm. Should the DW firm integrate with one of the UP firms, the input would be transferred internally at marginal production cost and the equilibrium is observationally equivalent to that emerging in the previous case. Hence, in the presence of a large number of UP firms supplying a single DW firm, there is no incentive towards vertical integration and the industry may be partially vertically disintegrated at equilibrium.

5 Concluding remarks

We have gone through the vertical integration decisions in markets for differentiated final goods.

First, we have analyzed the incentives to integrate when the differentiation of final goods has a feedback on upstream supply decisions. In this case, the form of downstream market competition matters. If downstream firms compete in quantities, the only equilibrium outcome is one where the whole industry is vertically integrated. However, when substitutability is sufficiently high, full integration is the outcome of a Prisoner's Dilemma, because of the changing role of the externality that plagues the non integrated arrangement when imperfect competition is there. If instead Bertrand competition is adopted, full integration is the unique equilibrium only for sufficiently low substitutability. When goods are close substitutes, there exist two equilibria in pure strategies, i.e., either full integration or full disintegration obtains. The range of the differentiation of final products wherein vertical integration is Pareto superior for firms is narrower in Bertrand than in Cournot. As a result, there exists a positive probability that firms adopt asymmetric strategies as to integration. This adds new scenarios to the ones depicted by Grossman and Helpman (2002), where only symmetric strategies are observed in equilibrium.

Second, we have introduced R&D expenditure among the strategic decision variables. We have gone through the theoretical description of a scenario where one integrated firm competes DW with many Cournot rivals and UP with a unique provider of the input. The UP independent firm and the UP division of the integrated firm carry out independent R&D activities to reduce the marginal production cost of the intermediate good. We have found that, for sufficiently low levels of substitutability between any two varieties of the final good the performance of the non-integrated side of the industry is superior to that of the integrated firm, provided that at least two independent DW firms are in the market.

Appendix

The sign of the expression for $\pi_{INT}^* - \pi_{UP}^* - n\pi_i^*$ is equal to the sign of:

$$((-4n(24+4(3n-7)s+(8-15n)s^2+4ns^3)^2+n(n-8+4s-4ns+2ns^2)(24+4(3n-7)s+(8-15n)s^2+4ns^3)^2+(4(s-2)+n(s-1)^2)^2(192+192(n-1)s+16(3-13n+3n^2)s^2-56(n-1)ns^3+15n^2s^4)))/(48(s-2)^2+4n(51s-52s^2+14s^3-6)+n^2s(15s^3-56s^2+55s-12))^2 \tag{a1}$$

which can be plotted over $s \in [0,1]$ and $n \in [1,100]$ to yield Figure 1 below.

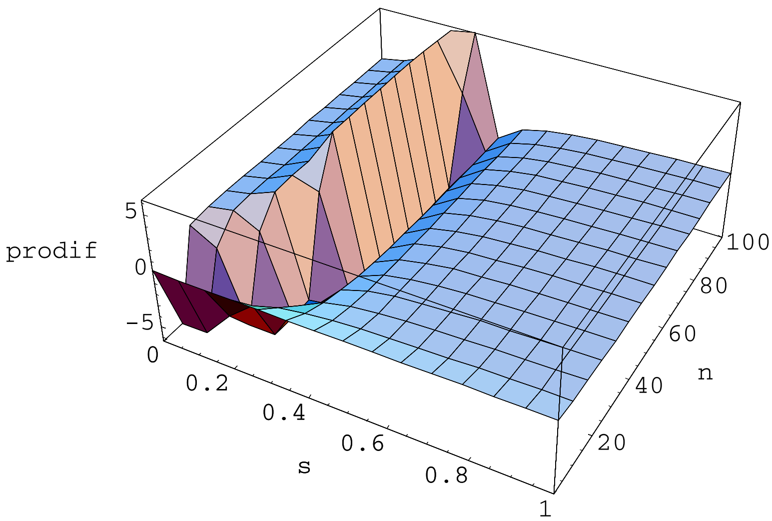

Figure 1: $\pi_{INT}^* - \pi_{UP}^* - n\pi_i^*$

References

[1] Armour, H.O., and Teece, D.J. (1980) Vertical integration and technological innovation. *Review of Economics and Statistics*, 62, 490-94.

[2] Brocas, I. (2003) Vertical Integration and the incentive to innovate. *International Journal of Industrial Organisation*, **21**, 457-605

[3] Grossman, S.J. and Hart O. (1986) The Costs and Benefits of Ownership: a Theory of Vertical and Lateral Integration. *Journal of Political Economy*, **XCIV**, 691-719.

[4] Grossman, G.M., Helpman, E. (2002) Integration Versus Outsourcing in Industry Equilibrium. *Quarterly Journal of Economics*,**117**, 85-120.

[5] McLaren, J. (2000) "Globalization" and Vertical Structure. *American Economic Review*, **XC**, 1239-1254.

[6] Pepall, L. and Norman, G. (2001) Product Differentiation and Upstream-Downstream Relations. *Journal of Economics and Management* Strategy, **10**, 201-33.

[7] Perry, M.K. (1989) Vertical Integration: Determinants and Effects. In R. Schmalensee and R. Willig (eds), *Handbook of Industrial Organization*, vol. 1, pp. 103-255, Amsterdam, North-Holland.

[8] Singh, N. and Vives, X. (1984) Price and Quantity Competition in a Differentiated Duopoly. *Rand Journal of Economics*, **15**, 546-54.

[9] Spengler, J. (1950), Vertical Integration and Antitrust Policy. *Journal of Political Economy*, **58**, 347-52.

[10] Teece, D.J. (1976) *Vertical Integration and Vertical Divestiture in the US Oil Industry.* Institute for Energy Studies, Stanford CA.

[11] Williamson, O.E. (1971) The Vertical Integration of Production: market failure Considerations. *American Economic Review*, **61**, 112-123.

[12] Williamson, O.E. (1985) *The Economic Institutions of Capitalism.* New York NY, Free Press

[13] Williamson, O.E. (1986) *Economic Organisation.* Brighton, Wheatsheaf Books.