Profit Taxation and Capital Accumulation in Dynamic Oligopoly Models

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Abstract

We illustrate two differential oligopoly games using, respectively, the capital accumulation dynamics *à la* Solow-Nerlove-Arrow, and the capital accumulation dynamics *à la* Ramsey. In both settings, we evaluate the effects of (gross) profit taxation, proving that there exist tax rates yielding the same steady state social welfare as under social planning. Contrary to the static approach, our dynamic analysis shows that, in general, profit taxation affects firms' decisions concerning capital accumulation and sales. In particular, it has pro-competitive effects provided that the extent of delegation is large enough.

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1 Introduction

There exists a relatively large literature on profit taxation in static models of imperfect competition (Levin, 1985; Besley, 1989; Delipalla and Keen, 1992; Dung, 1993; Ushio, 2000). A well established result of this literature is that the taxation of operative profits (defined as the profits gross of fixed costs) is neutral, in that it does not affect first order conditions on market variables.

The dynamic interaction between capital accumulation and taxation has been analysed by Hall and Jorgenson (1967).[1] However, their analysis, as well as the debate stemming from what is now conventionally labelled as Jorgenson's model, is carried out focussing upon monopoly.

In the light of the above mentioned streams of literature, one would like to characterise the influence of taxation on the behaviour of firms in a dynamic setting where strategic interaction is duly accounted for. To this aim, we propose a dynamic capital accumulation game in a Cournot oligopoly, where we consider both the model of reversible investment *à la* Solow-Nerlove-Arrow (1956, 1962), i.e., capital accumulation through costly investment, and the model *à la* Ramsey (1928), i.e., a "corn-corn" growth model, where accumulation is based upon unsold output and coincides with consumption postponement. In both settings, our aim consists in characterising the effects of profit taxation on the steady state behaviour of firms and the associated performance of profits and social welfare. In order to account for the (more realistic) possibility for firms not to be strict profit-seeking agents, we assume, throughout our analysis, that firms may delegate control over their strategic decisions to managers who are interested in expanding output *à la* Vickers (1985; see also Fershtman and Judd, 1987).

Our main results are as follows. First, as shown in Cellini and Lambertini (2001), under both the Solow-Nerlove-Arrow and the Ramsey capital accumulation dynamics, the open-loop Nash equilibrium coincides with the closed-loop memoryless equilibrium, and therefore the former is subgame perfect. This depends upon two features which are common to both settings: (a) the dynamic behaviour of any firm's state variable does not depend on the rivals' control and state variables, which makes the kinematic equations concerning other firms redundant; and (b) for any firm, the first order conditions taken w.r.t. the control variables are independent of the rivals' state

[1] For an exhaustive overview on the effects of uncertainty on investment decisions, with and without taxation, see Dixit and Pindyck (1994).

variables, which entails that the cross effect from rivals' states to own controls (which characterises the closed-loop information structure) disappears.

Second, profit taxation distorts capital accumulation and the associated market performance of firms, in both models, as long as firms are managerial. The distortion disappears in the Ramsey model if firms are strictly entrepreneurial units, i.e., pure profit-seekers. In the Solow-Nerlove-Arrow setting, taxation is always distortionary, independently of the internal organization of firms. This sharply contrasts with the conventional wisdom generated by the static approach to taxation in oligopoly. This difference comes from the fact that, if one takes the more realistic view that capacity accumulation is a dynamic process, then one can verify that indeed the presence of a tax rate on profits enters firms' optimality conditions in a non-neutral way, contrary to what happens in a static model where taxation has only a scale effect on profits.

We also characterise the optimal taxation in both models, from the standpoint of a policy maker aiming at the maximization of social welfare in steady state. In the Solow-Nerlove-Arrow setting, investment in additional capacity involves an instantaneous cost for each firm, and therefore the socially optimal tax rate falls short of the level which would drive the market to the competitive outcome, as this would involve negative steady state profits which, in turn, would lead firms to quit the market. Conversely, in the Ramsey model, investment involves only the intertemporal cost associated with the relocation of unsold output. Accordingly, it is optimal for the policy maker to adopt the tax rate that drives the equilibrium to marginal cost.

The remainder of the paper is structured as follows. The model is laid out in section 2. Section 3 examines the effects of taxation in the Solow-Nerlove-Arrow model, while section 4 carries out the analogous task for the Ramsey model. Section 5 contains concluding remarks.

2 The basic setup

The existing literature on differential games applied to firms' behaviour mainly concentrates on two kinds of solution concepts:[2] the-open loop and the closed-loop equilibria. In the former case, firms precommit their decisions

[2]See Kamien and Schwartz (1981); Başar and Olsder (1982); Mehlmann (1988); Dockner, Jørgensen, Van Long and Sorger (2000).

on the control variables to a path over time and the relevant equilibrium concept is the open-loop Nash equilibrium. In the latter, firms do not precommit on any path and their strategies at any instant may depend on all the preceding history. In this situation, the information set used by firms in setting their strategies at any given time is often simplified to be only the current value of the capital stocks at that time. The relevant equilibrium concept, in this (sub-)case, is the closed-loop memoryless Nash equilibrium, which is strongly time consistent and therefore subgame perfect. When players (firms) adopt the open-loop solution concept, they design the optimal plan at the initial time and then stick to it forever. The resulting open-loop Nash equilibrium is only weakly time consistent and therefore, in general, it is not subgame perfect. A refinement of the closed-loop Nash equilibrium, which is known as the feedback Nash equilibrium, can also be adopted as the solution concept. While in the closed-loop memoryless case the initial and current levels of all state variables are taken into account, in the feedback case only the current stocks of states are considered.[3]

Current research on differential games devotes a considerable amount of attention to identifying classes of games where either the feedback or the closed-loop equilibria degenerate into open-loop equilibria. This interest is motivated by the following reason. Whenever an open-loop equilibrium is a degenerate closed-loop or feedback equilibrium, then the former is also subgame perfect; therefore one can rely on the open-loop equilibrium which, in general, is much easier to derive than feedback and closed-loop ones. Classes of games where this coincidence arises are illustrated in Clemhout and Wan (1974); Reinganum (1982); Mehlmann and Willing (1983); Dockner, Feichtinger and Jørgensen (1985); Fershtman (1987); Fershtman, Kamien and Muller (1992). For an overview, see Mehlmann (1988) and Fershtman, Kamien and Muller (1992).

Here, we consider two well known capital accumulation rules. In both models, the market exists over $t \in [0\,,\,\infty)$, and is served by N firms producing a homogeneous good. Let $q_i(t)$ define the quantity sold by firm i at time t. The marginal production cost is constant and equal to c for both firms. Firms compete *à la* Cournot, the demand function at time t being:

$$p(t) = A - BQ(t)\,,\; Q(t) \equiv \sum_{i=1}^{N} q_i(t)\;. \tag{1}$$

[3]For a clear exposition of the difference among these equilibrium solutions see Başar and Olsder (1982, pp. 318-327, and chapter 6, in particular Proposition 6.1).

In order to produce, firms must accumulate capacity or physical capital $k_i(t)$ over time. The two models we consider in the present paper are characterised by two different kinematic equations for capital accumulation.

A] The Solow (1956) or Nerlove-Arrow (1962) setting, with the relevant dynamic equation being:

$$\frac{\partial k_i(t)}{\partial t} = I_i(t) - \delta k_i(t)\,, \tag{2}$$

where $I_i(t)$ is the investment carried out by firm i at time t, and δ is the constant depreciation rate. The instantaneous cost of investment is $C_i\left[I_i\left(t\right)\right] = b\left[I_i\left(t\right)\right]^2/2$, with $b > 0$. We also assume that firms operate with a decreasing returns technology $q_i(t) = f(k_i(t))$, with $f' \equiv \partial f(k_i(t))/\partial k_i(t) > 0$ and $f'' \equiv \partial^2 f(k_i(t))/\partial k_i(t)^2 < 0$. The demand function rewrites as:[4]

$$p(t) = A - B\sum_{i=1}^{N} f(k_i(t))\,. \tag{3}$$

Here, the control variable is the instantaneous investment $I_i(t)$, while the state variable is obviously $k_i(t)$.

B] The Ramsey (1928) setting, whit the following dynamic equation:

$$\frac{\partial k_i(t)}{\partial t} = f(k_i(t)) - q_i(t) - \delta k_i(t)\,, \tag{4}$$

where $f(k_i(t)) = y_i(t)$ denotes the output produced by firm i at time t. As in setting [A], we assume $f' \equiv \partial f(k_i(t))/\partial k_i(t) > 0$ and $f'' \equiv \partial^2 f(k_i(t))/\partial k_i(t)^2 < 0$. In this case, capital accumulates as a result of intertemporal relocation of unsold output $y_i(t) - q_i(t)$.[5] This can be interpreted in two ways. The first consists in viewing this setup as a

[4]Notice that the assumption $q_i(t) = f(k_i(t))$ entails that firms always operate at full capacity. This, in turn, amounts to saying that this model encompasses the case of Bertrand behaviour under capacity constraints, as in Kreps and Scheinkman (1983), *inter alia*. The open-loop solution of the Nerlove-Arrow differential game in a duopoly model is in Fershtman and Muller (1984) and Reynolds (1987). The latter author also derives the feedback solution through Bellman's value function approach.

[5]In the Ramsey model, firms operate at full capacity in steady state, where any investment is just meant to make up for depreciation.

corn-corn model, where unsold output is reintroduced in the production process. The second consists in thinking of a two-sector economy where there exists an industry producing the capital input which can be traded against the final good at a price equal to one (for further discussion, see Cellini and Lambertini, 2000).

In this model, the control variable is $q_i(t)$, while the state variable remains $k_i(t)$. The demand function is (1).

In the remainder of the paper, we will consider an oligopoly where the control of firms' behaviour is delegated to managers. As in Fershtman (1985), Vickers (1985), Fershtman and Judd (1987) and many others, we assume that delegation contracts are observable and establish that the manager of firm i maximises a combination of profits and output, so that his instantaneous objective function is:

$$M_i(t) = \pi_i(t) + \theta_i q_i(t) \tag{5}$$

where parameter θ_i measures the extent of delegation. If $\theta_i = 0$, the firm is entrepreneurial, i.e., it is run by stockholders so as to strictly maximise profits. Moreover, we assume that firms' profits (gross of investment costs) are taxed at rate τ.

3 The Solow-Nerlove-Arrow model

When capital accumulates according to equation (2), the relevant Hamiltonian for firm i is:

$$\begin{aligned} \mathcal{H}_i(t) &= e^{-\rho t} \left\{ \left[A - Bf(k_i(t)) - B \sum_{i \neq j} f(k_j(t)) - c \right] f(k_i(t)) (1-\tau) - \frac{b}{2} [I_i(t)]^2 \right. \\ &\left. + \theta_i f(k_i(t)) + \lambda_{ii}(t) [I_i(t) - \delta k_i(t)] + \sum_{i \neq j} \lambda_{ij}(t) [I_j(t) - \delta k_j(t)] \right\}, \end{aligned} \tag{6}$$

with initial conditions $k_i(0) = k_{i0}$, $i = 1, 2, 3, ...N$. Necessary conditions for the closed-loop memoryless equilibrium are:

$(i)\ \partial\mathcal{H}_i(t)\partial I_i(t) = 0 \Rightarrow -bI_i(t) + \lambda_{ii}(t) = 0 \Rightarrow \lambda_{ii}(t) = bI_i(t)$
$(ii)\ -\partial\mathcal{H}_i(t)\partial k_i(t) - \sum_{j\neq i}\partial\mathcal{H}_i(t)\partial I_j(t)\partial I_j^*(t)\partial k_i(t) = \partial\lambda_{ii}(t)\partial t - \rho\lambda_{ii}(t) \Rightarrow$
$\Rightarrow \partial\lambda_{ii}(t)\partial t = (\rho+\delta)\,\lambda_{ii}(t) - f'(k_i(t))\,[\theta_i - (1-\tau)\,\Omega]$
$where \Omega \equiv 2Bf(k_i(t)) + B\sum_{j\neq i} f(k_j(t)) - (A-c)$
$(iii)\ -\partial\mathcal{H}_i(t)\partial k_j(t) - \sum_{j\neq i}\partial\mathcal{H}_i(t)\partial I_j(t)\partial I_j^*(t)\partial k_j(t) = \partial\lambda_{ij}(t)\partial t - \rho\lambda_{ij}(t)\ ,$ (7)

with the transversality conditions:

$$t\rightarrow\infty \lim\ \mu_{ij}(t)\cdot k_i(t) = 0\ for all i, j\ . \tag{8}$$

Now observe that, on the basis of (7-i), we have:

$$\partial I_j^*(t)\partial k_i(t) = 0 for all i, j\ . \tag{9}$$

Moreover, condition (7-iii), which yields $\partial\lambda_{ij}(t)/\partial t$, is redundant in that $\lambda_{ij}(t)$ does not appear in the first order conditions (7-i) and (7-ii). This result can be characterised in the following alternative but completely equivalent way, by observing that

$$\partial I_j^*(t)\partial k_i(t) = \frac{\partial^2\mathcal{H}_j(t)}{\partial I_j(t)\partial k_i(t)} \tag{10}$$

where

$$\frac{\partial^2\mathcal{H}_j(t)}{\partial I_j(t)\partial k_i(t)} = 0 for all i, j\ , \tag{11}$$

since the Hamiltonian of firm i is additively separable in control and state variables. Therefore, the open-loop solution is indeed a degenerate closed-loop solution.[6]

The discussion carried out so far establishes:

Under the Solow-Nerlove-Arrow capital accumulation dynamics, the closed-loop memoryless equilibrium coincides with the open-loop equilibrium, which therefore is subgame perfect.

Differentiating (7.i) w.r.t. time we obtain:

$$\frac{\partial I_i(t)}{\partial t} = \frac{1}{b}\cdot\partial\lambda_{ii}(t)\partial t\ . \tag{12}$$

[6]Note that, however, the open-loop solution does not coincide with the feedback solution, where each firm holds a larger capacity and sells more than in the open-loop equilibrium (see Reynolds, 1987).

Then, replace (7-i) into (7-ii), to get the following expression for the dynamics of the costate variable $\lambda_{ii}(t)$:

$$\partial\lambda_{ii}(t)\partial t = b\,(\rho+\delta)\,I_i(t) - f'(k_i(t))\,[\theta_i - (1-\tau)\,\Omega]\ , \tag{13}$$

which can be plugged into (12), that rewrites as:

$$\frac{\partial I_i(t)}{\partial t} = (\rho+\delta)\,I_i(t) + \tag{14}$$

$$-\frac{f'(k_i(t))}{b}\left\{\theta_i - (1-\tau)\left[2Bf(k_i(t)) + B\sum_{j\neq i} f(k_j(t)) - (A-c)\right]\right\}\ ,$$

Invoking symmetry across firms and simplifying, we can rewrite (14):

$$\frac{\partial I(t)}{\partial t} = \frac{1}{b}\left\{b\,(\rho+\delta)\,I(t) - f'(k(t))\,[\theta - (1-\tau)\,(B(N+1)f(k(t)) - (A-c))]\right\}\ , \tag{15}$$

with the r.h.s. being zero at:

$$I(t) = \frac{f'(k(t))}{b\,(\rho+\delta)}\left\{\theta + (1-\tau)\,[(A-c) - B(N+1)f(k(t))]\right\}\ . \tag{16}$$

Expressions (14-16) prove the following result:

In the Solow-Nerlove-Arrow model, any profit tax rate $\tau > 0$ distorts firms' investments and therefore capital accumulation, independently of whether firms are managerial or entrepreneurial.

Even when all firms are strictly profit-seeking agents (that is, $\theta_i = 0$ for all i), the presence of profit taxation is distortionary. The fact that taxation distorts the investment path involves of course that the steady state capacity will also be distorted.

In order to solve the model explicitly, we now examine the case where $f(k(t)) = k(t)$.[7] Under the assumption of a technology characterised by constant returns to scale, expression (16) rewrites as:

$$I(t) = \frac{(1-\tau)\,[(A-c) - B(N+1)k(t)] + \theta}{b\,(\rho+\delta)}\ . \tag{17}$$

[7]This setting is also investigated in Fershtman and Muller (1984) and Calzolari and Lambertini (2001), *inter alia*.

By substituting (17) into (2) and imposing $\partial k(t)/\partial t = 0$, we obtain the steady state capacity:

$$k^* = \frac{(1-\tau)(A-c)+\theta}{B(N+1)(1-\tau)+b(\rho+\delta)\delta} \tag{18}$$

with $I^* = \delta k^*$. By checking the stability condition, one can easily verify that the pair $\{I^*, k^*\}$ identifies a saddle point for all $\tau \in [0, 1]$.

As one would expect from the outset, k^* is everywhere increasing in θ. This obviously entails that, as we know from Vickers (1985), Fershtman (1985) *et alii*, the market becomes more competitive as the extent of delegation increases. Moreover, k^* is everywhere increasing in the number of firms.

This formulation allows us to evaluate the effect on steady state capacity of a change in the tax rate. From (18), we have that

$$\begin{aligned} \frac{\partial k^*}{\partial \tau} &= \frac{B(N+1)\theta - b(A-c)(\rho+\delta)\delta}{[B(N+1)(1-\tau)+b(\rho+\delta)\delta]^2} > 0 \\ forall\theta &> \overline{\theta} \equiv \frac{b(A-c)(\rho+\delta)\delta}{B(N+1)} > 0\,. \end{aligned} \tag{19}$$

This proves the following Corollary to Proposition 2:

If firms are strictly profit-seeking units, or the extent of delegation is small enough, profit taxation reduces the steady state capacity. Otherwise, if the extent of delegation is larger than a critical threshold, taxing profits leads to an increase in the long run capacity of firms.

This result can be interpreted in the following terms. If firms are strictly entrepreneurial, i.e., $\theta = 0$ for all of them, taxation reduces the steady state capacity in that capital accumulation is costly. As soon as firms delegate control to managers interested in output expansion, a countervailing effect is operating. However, for this to overcome the negative effect exerted by taxation, the extent of delegation must be large enough.

We are now in a position to address the issue of choosing the optimal tax rate, from the standpoint of a policy maker aiming at the maximisation of social welfare. To this purpose, we first characterise the behaviour of a social planner operating N firms (without taxation).

Instantaneous consumer surplus is:

$$CS(t) = [A - p(t)]\frac{\sum_{i=1}^{N} k_i(t)}{2} = \frac{B}{2}\left[\sum_{i=1}^{N} k_i(t)\right]^2 . \tag{20}$$

This must be added to instantaneous industry profits $\Pi(t) = \sum_{i=1}^{N} \pi_i(t)$ to obtain the relevant expression for instantaneous social welfare, $SW(t) = CS(t) + \Pi(t)$. Hence, the Hamiltonian of the social planner is:

$$\mathcal{H}_{SP}(t) = e^{-\rho t} \left\{ \left[A - Bk_i(t) - B \sum_{i \neq j} k_j(t) - c + \theta_i \right] k_i(t) - \frac{b}{2} \left[I_i(t) \right]^2 + \right.$$
$$\sum_{j \neq i} \left[\left(A - Bk_j(t) - B \sum_{m \neq j} k_m(t) - c + \theta_j \right) k_j(t) - \frac{b}{2} \left[I_j(t) \right]^2 \right] +$$
$$\left. \frac{B}{2} \left[\sum_{i=1}^{N} k_i(t) \right]^2 + \lambda_i(t) \left[I_i(t) - \delta k_i(t) \right] + \sum_{j \neq i} \lambda_j(t) \left[I_j(t) - \delta k_j(t) \right] \right\} \tag{21}$$

The first order conditions are:[8]

$$\partial \mathcal{H}_{SP}(t) \partial I_i(t) = -bI_i(t) + \lambda_i(t) = 0 \Rightarrow \lambda_i(t) = bI_i(t) and \frac{\partial I_i(t)}{\partial t} \propto \frac{\partial \lambda_i(t)}{\partial t} \; ; \tag{22}$$

$$\partial \mathcal{H}_{SP}(t) \partial k_i(t) = \frac{\partial \lambda_i(t)}{\partial t} - \rho \lambda_i(t) \; . \tag{23}$$

The latter, after imposing the symmetry condition across firms, yields:

$$\frac{\partial \lambda(t)}{\partial t} = bI(t) (\rho + \delta) - (A - c + \theta) + BNk(t) \tag{24}$$

which allows us to write:

$$\frac{\partial I_i(t)}{\partial t} \propto bI(t) (\rho + \delta) - (A - c + \theta) + BNk(t) = 0 \tag{25}$$

in

$$I(t) = \frac{(A - c + \theta) - BNk(t)}{b(\rho + \delta)} \; . \tag{26}$$

This expression can be plugged into $\partial k(t)/\partial t$ to check immediately that the steady state level of capacity at the social optimum is:

$$k_{SP} = \frac{A - c + \theta}{BN + b\delta(\rho + \delta)} \; . \tag{27}$$

[8] We omit initial conditions and transversality conditions for the sake of brevity.

The corresponding socially optimal investment in steady state is $I_{SP} = \delta k_{SP}$. At the socially optimal equilibrium, the individual firm's profits are:

$$\pi_{SP} = \frac{b\delta\,(\delta + 2\rho)\,(A - c + \theta)^2}{2\,[BN + b\delta\,(\rho + \delta)]^2} > 0\ , \tag{28}$$

while social welfare is:

$$SW_{SP} = \frac{N\,[BN + b\delta\,(\delta + 2\rho)]\,(A - c + \theta)^2}{2\,[BN + b\delta\,(\rho + \delta)]^2}\ . \tag{29}$$

Now, if we compare $\{I_{SP}\, ,\, k_{SP}\}$ with $\{I^*\, ,\, k^*\}$, we can easily find that $I^* = I_{SP}$ and $k^* = k_{SP}$ at

$$\tau_{SP} = \frac{B\,(A - c + \theta)}{(A - c)\,[B - b\delta\,(\rho + \delta)] + B\theta\,(N + 1)}\ . \tag{30}$$

A sufficient condition for $\tau_{SP} > 0$ is that intertemporal parameters δ and ρ be small enough. Moreover, it must be that $\tau_{SP} \leq 1$, which requires:

$$\theta \geq \widehat{\theta} \equiv \frac{b\,(A - c)\,(\rho + \delta)\,\delta}{BN} > \overline{\theta}\,. \tag{31}$$

This proves the following:

If the policy maker adopts the tax rate τ_{SP}, the steady state oligopoly equilibrium yields the same level of social welfare attainable under social planning. In correspondence of τ_{SP}, firms' equilibrium profits are positive.

Depending upon the level of θ, the sign of $\partial k^*/\partial t$ and $\partial I^*/\partial t$ can be either negative or positive if evaluated at τ_{SP}. It is just the case of observing the obvious feature that the presence of an investment cost prevents the planner as well as the policy maker to reach the perfectly competitive equilibrium with marginal cost pricing.

4 The Ramsey model

Under the dynamic constraint (4), the Hamiltonian of firm i is:

$$\begin{aligned}\mathcal{H}_i(t) \;=\; & \Big\{ e^{-\rho t}\,[A - Bq_i(t) - BQ_{-i}(t) - c]\,(1 - \tau)\,q_i(t) + \theta_i q_i(t) + \\ & + \lambda_{ii}(t)\,[f(k_i(t)) - q_i(t) - \delta k_i(t)] + \sum_{j \neq i} \lambda_{ij}(t)\,[f(k_j(t)) - q_j(t) - \delta k_j(t)] \Big\}\ ,\end{aligned} \tag{32}$$

where $Q_{-i}(t) = \sum_{j \neq i} q_j(t)$.[9]

[9] Initial conditions and transversality conditions are omitted for the sake of brevity.

The first order condition concerning the control variable is:

$$\frac{\partial \mathcal{H}_i(t)}{\partial q_i(t)} = [A - 2Bq_i(t) - BQ_{-i}(t) - c]\,(1-\tau) + \theta_i - \lambda_{ii}(t) = 0\ . \tag{33}$$

Now examine at the co-state equation of firm i calculated for the state variable of firm i herself, for the closed-loop solution of the game:

$$\begin{aligned} -\frac{\partial \mathcal{H}_i(t)}{\partial k_i(t)} - \sum_{j\neq i} \frac{\partial \mathcal{H}_i(t)}{\partial q_j(t)} \frac{\partial q_j^*(t)}{\partial k_i(t)} &= \frac{\partial \lambda_{ii}(t)}{\partial t} - \rho\lambda_{ii}(t) \Rightarrow \\ \frac{\partial \lambda_{ii}(t)}{\partial t} &= \lambda_{ii}(t)\,[\rho + \delta - f(k_i(t))] \end{aligned} \tag{34}$$

with

$$\frac{\partial q_j^*(t)}{\partial k_i(t)} = 0 \tag{35}$$

as it emerges from the best reply function obtained from the analogous to (33):

$$q_j^*(t) = \frac{[A - BQ_{-j}(t) - c]\,(1-\tau) + \theta_i - \lambda_{jj}(t)}{2B\,(1-\tau)}\ ; \tag{36}$$

Moreover, (36) also suffices to establish that the co-state equation:

$$-\frac{\partial \mathcal{H}_i(t)}{\partial k_j(t)} - \sum_{j\neq i} \frac{\partial \mathcal{H}_i(t)}{\partial q_j(t)} \frac{\partial q_j^*(t)}{\partial k_j(t)} = \frac{\partial \lambda_{ij}(t)}{\partial t} - \rho\lambda_{ij}(t) \tag{37}$$

pertaining to the state variable of the generic rival j is indeed redundant since $\mu_{ij}(t) = \lambda_{ij}(t)e^{-\rho t}$ does not appear in firm i's first order condition (33) on the control variable. This amounts to saying that, in the Ramsey game, the open-loop solution is a degenerate closed-loop solution because the best reply function of firm i does not contain the state variable pertaining to the same firm or any of her rivals. Therefore, we have proved the following analogous to Proposition 1:

Under the Ramsey capital accumulation dynamics, the closed-loop memoryless equilibrium coincides with the open-loop equilibrium, which therefore is subgame perfect.

Accordingly, we set $\lambda_{ii}(t) = \lambda_i(t)$, and $\lambda_{ij}(t) = 0$ for all $j \neq i$. Then, using (36), we can write:

$$\frac{dq_i^*(t)}{dt} = \frac{1}{2B(1-\tau)}\left[-(1-\tau)B\sum_{j\neq i}\frac{\partial q_j(t)}{\partial t} - \frac{\partial \lambda_i(t)}{\partial t}\right] \tag{38}$$

and

$$\lambda_i(t) = (1-\tau)\left[A - 2Bq_i(t) - BQ_{-j}(t) - c\right] + \theta_i \ . \tag{39}$$

Now we can impose the symmetry condition $q_i(t) = q(t)$ and $\theta_i = \theta$ for all i, and using (34) and (39) we can rewrite (38) as follows:

$$\frac{dq^*(t)}{dt} \propto \left[(1-\tau)\left(A - c - q(t)(N+1)B\right) + \theta\right]\left[f'(k) - \rho - \delta\right] \tag{40}$$

which is equal to zero in correspondence of the following steady state solutions:

$$q^{SS} = \frac{(A-c)(1-\tau)+\theta}{B(N+1)(1-\tau)} \ ; \ f'(k) = \rho + \delta \ . \tag{41}$$

To ease the exposition, define:

$$\widehat{k} \equiv \left\{k : f'\left(\widehat{k}\right) = \rho + \delta\right\} \ . \tag{42}$$

That is, $\widehat{k}$ is the level of capacity associated with the Ramsey steady state equilibrium where the marginal productivity of capital is equal to the sum of depreciation and discount rates.

The phase diagram of the present model can be drawn in the space $\{k, q\}$. The locus $\dot{q} \equiv dq/dt = 0$ is given by the solutions in (41). The two loci partition the space $\{k, q\}$ into four regions, where the dynamics of q is summarised by the vertical arrows. The locus $\dot{k} \equiv dk/dt = 0$ as well as the dynamics of k, depicted by horizontal arrows, derive from (4). Steady state equilibria, denoted by $E1$, $E2$ along the horizontal arm, and $E3$ along the vertical one, are identified by the intersections between loci.

Figure 1: The Ramsey model

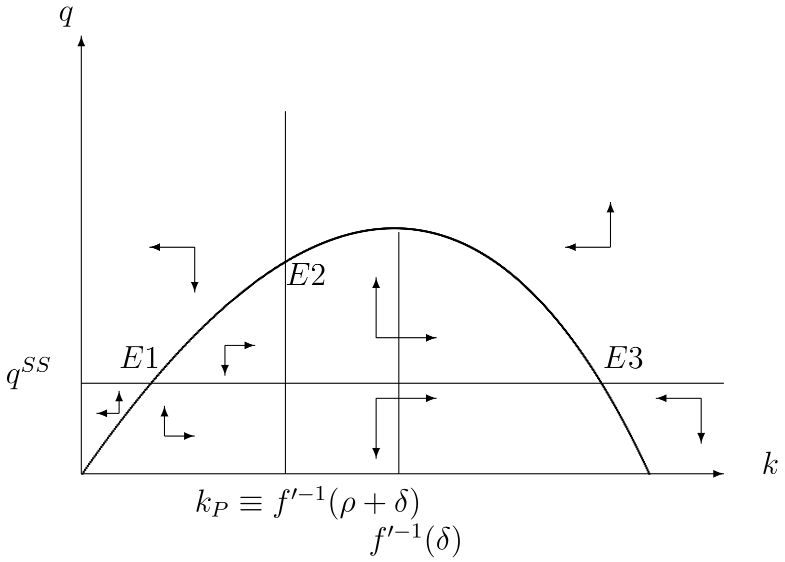

Figure 1 describes only one out of five possible configurations, due to the fact that the position of the vertical line $f'(k) = \rho + \delta$ is independent of demand parameters, while the locus $q = [(A-c)(1-\tau)+\theta] / [B(N+1)(1-\tau)]$ shifts upwards (downwards) as $A - c$ and/or θ (B and N) increases. Therefore, we obtain one out of five possible regimes:

1. There exist three steady state points, with $k_{E1} < k_{E2} < k_{E3}$ (figure 1).
2. There exist two steady state points, with $k_{E1} = k_{E2} < k_{E3}$.
3. There exist three steady state points, with $k_{E2} < k_{E1} < k_{E3}$.
4. There exist two steady state points, with $k_{E2} < k_{E1} = k_{E3}$.
5. There exists a unique steady state equilibrium point, corresponding to $E2$.

The vertical locus $f'(k) = \rho + \delta$ is a constraint on optimal capital, determined by firms' intertemporal preferences, i.e., their common discount rate, and depreciation. Accordingly, maximum output level in steady state would be that corresponding to (i) $\rho = 0$, and (ii) a capacity such that $f'(k) = \delta$.

Yet, a positive discounting (i.e., impatience) leads firms to install a smaller capacity at the long-run equilibrium. This is the *optimal capital constraint* $\widehat{k}$. When the market size $A - c$ and the extent of delegation θ are very large (or B and N are low), points $E1$ and $E3$ either do not exist (regime 5) or fall to the right of $E2$ (regimes 2, 3 and 4). In such a case, the capital constraint is operative and firms choose the capital accumulation corresponding to $E2$.

Notice that, as $E1$ and $E3$ entail the same levels of sales, point $E3$ is surely inefficient in that it requires a higher amount of capital. $E1$ corresponds to the optimal quantity emerging from the static version of the game. It is hardly the case of emphasising that this solution encompasses both monopoly and perfect competition (as, in the limit, $N \to \infty$).

The stability analysis of the above system reveals that:[10]

Regime 1. $E1$ is a saddle point, while $E2$ is an unstable focus. $E3$ is again a saddle point, with the horizontal line as the stable arm.

Regime 2. $E1$ coincides with $E2$, so that we have only two steady states which are both are saddle points. In $E1 = E2$, the saddle path approaches the saddle point from the left only, while in $E3$ the stable arm is again the horizontal line.

Regime 3. $E2$ is a saddle, $E1$ is an unstable focus. $E3$ is a saddle point, as in regimes 1 and 2.

Regime 4. Here, $E1$ and $E3$ coincide. $E3$ remains a saddle, while $E1 = E3$ is a saddle whose converging arm proceeds from the right along the horizontal line.

Regime 5. Here, there exists a unique steady state point, $E2$, which is a saddle point.

We can sum up the above discussion as follows. The unique efficient and non-unstable steady state point is $E2$ if $k_{E2} < k_{E1}$, while it is $E1$ if the opposite inequality holds. Such a point is always a saddle. Individual equilibrium output is q^{SS} if the equilibrium is in $E1$, or the level corresponding to the optimal capital constraint $\widehat{k}$ if the equilibrium is point $E2$. The reason is that, if the capacity at which marginal instantaneous profit is nil is larger

[10] See Cellini and Lambertini (1998, 2002) for details concerning the Jacobian matrix of the dynamic system.

than the optimal capital constraint, the latter becomes binding. Otherwise, the capital constraint is irrelevant, and firms' decisions in each period are driven by the unconstrained maximisation of single-period profits only.

Now, examining the steady state solutions in (41) allows us to state, without further proof, the following result:

In the market-driven steady state, any profit tax rate $\tau > 0$ distorts q^{SS} and k_{E1} if firms are managerial, i.e., iff $\theta > 0$. If instead firms are strictly entrepreneurial, i.e., $\theta = 0$, profit taxation is neutral. In the Ramsey equilibrium where $f'(k) = \rho + \delta$, profit taxation is always neutral.

Consider the market-driven solution. The effects of profit taxation on the equilibrium output level are summarised by:

$$\frac{\partial q^{SS}}{\partial \tau} = \frac{\theta}{B\,(N+1)\,(1-\tau)^2} > 0 \;\forall\; \theta > 0\,. \tag{43}$$

Since output is proportional to capacity, $\partial k_{E1}/\partial\tau \propto \partial q^{SS}/\partial\tau$. This proves a relevant Corollary to Proposition 4:

If firms are managerial, the optimal levels of output and capacity are monotonically increasing in τ.

In this model, capacity accumulation involves the intertemporal relocation of production only. Therefore, being absent any instantaneous costs, the introduction of a profit tax rate has an expansionary effect on capital accumulation whenever firms are managerial, the reason being that firms expand sales so as to try to recover through larger market shares some of the profits extracted by the policy maker. Since this incentive exists for all firms, the effect of taxation is definitely pro-competitive in that it translates into an overall expansion of the industry output and a reduction in the price level.

Given the Ramsey accumulation mechanism, by which the unsold output increases capacity, firms do not bear any fixed cost and the social optimum involves firms producing an aggregate output that must be sufficiently large to drive the market price to marginal cost, with zero profits. This can be obtained by setting:

$$\overline{\tau} = 1 - \frac{N\theta}{A-c}\,. \tag{44}$$

Clearly, $\overline{\tau}$ is decreasing in the extent of delegation θ.

5 Conclusions

We have investigated two dynamic oligopoly games in order to characterise the effects of gross profit taxation on the performance of firms in steady state. In particular, we have established that, if capacity accumulation is modelled as a dynamic investment process, then taxation exerts distortionary effects on the amount of capital and the output level at equilibrium. In the Solow-Nerlove-Arrow approach, the steady state capacity and sales are non-monotone in the size of the tax rate. On the contrary, taxation leads to a decrease in optimal capacity and sales in the Ramsey approach, provided that firms are managerial. If instead firms are under the direct control of stockholders, then taxation is neutral in the Ramsey model.

Finally, we have also investigated the welfare-maximising tax rates in both cases, finding that the Ramsey accumulation rule allows for a perfectly competitive outcome, while the presence of instantaneous costs of investments forces the Solow-Nerlove-Arrow model to fall short of such objective.

References

[1] Başar, T. and G.J. Olsder (1982, 1995^2), *Dynamic Noncooperative Game Theory*, San Diego, Academic Press.

[2] Besley, T. (1989), "Commodity Taxation and Imperfect Competition: A Note on the Effects of Entry", *Journal of Public Economics*, **40**, 359-67.

[3] Calzolari, G. and L. Lambertini (2001), "Tariffs vs Quotas in a Model of Trade with Capital Accumulation", in R. Neck (ed.), *Proceedings of the IFAC Symposium on Modelling and Control of Economic Systems (SME 2001)*, Pergamon Press, forthcoming.

[4] Cellini, R. and L. Lambertini (1998), "A Dynamic Model of Differentiated Oligopoly with Capital Accumulation", *Journal of Economic Theory*, **83**, 145-55.

[5] Cellini, R. and L. Lambertini (2000), "Non-Linear Market Demand and Capital Accumulation in a Differential Oligopoly Game", working paper no. 370, Department of Economics, University of Bologna.

[6] Cellini, R. and L. Lambertini (2001), "Differential Oligopoly Games where the Closed-Loop Memoryless and Open-Loop Equilibria Coincide", mimeo, Dipartimento di Scienze Economiche, Università degli Studi di Bologna.

[7] Cellini, R. and L. Lambertini (2002), "Differential Oligopoly Games", in R. Bianchi and L. Lambertini (eds.), *Technology, Information and Market Dynamics: Topics in Advanced Industrial Organization*, Cheltenham, Edward Elgar, forthcoming.

[8] Clemhout, S. and H.Y. Wan, Jr. (1974), "A Class of Trilinear Differential Games", *Journal of Optimization Theory and Applications*, **14**, 419-24.

[9] Delipalla, S. and M. Keen (1992), "The Comparison between *Ad Valorem* and Specific Taxation under Imperfect Competition", *Journal of Public Economics*, **49**, 351-67.

[10] Dixit, A.K. and R.S. Pindyck (1994), *Investment under Uncertainty*, Princeton, NJ, Princeton University Press.

[11] Driskill, R.A. and S. McCafferty (1989), "Dynamic Duopoly with Adjustment Costs: A Differential Game Approach", *Journal of Economic Theory*, **49**, 324-38.

[12] Dockner, E.J., G. Feichtinger and S. Jørgensen (1985), "Tractable Classes of Nonzero-Sum Open-Loop Nash Differential Games: Theory and Examples", *Journal of Optimization Theory and Applications*, **45**, 179-97.

[13] Dockner, E.J, S. Jørgensen, N. Van Long and G. Sorger (2000), *Differential Games in Economics and Management Science*, Cambridge, Cambridge University Press.

[14] Dung, T.H. (1993), "Optimal Taxation and Heterogeneous Oligopoly", *Canadian Journal of Economics*, *26*, 933-47.

[15] Fershtman, C. (1985), "Internal Organizations and Managerial Incentives as Strategic Variables in a Competitive Environment", *International Journal of Industrial Organization*, **3**, 245-53.

[16] Fershtman, C. (1987), "Identification of Classes of Differential Games for Which the Open-Loop is a Degenerated Feedback Nash Equilibrium", *Journal of Optimization Theory and Applications*, **55**, 217-31.

[17] Fershtman, C. and K. Judd (1987), "Equilibrium Incentives in Oligopoly", *American Economic Review*, **77**, 927-40.

[18] Fershtman, C. and E. Muller (1984), "Capital Accumulation Games of Infinite Duration", *Journal of Economic Theory*, **33**, 322-39.

[19] Fershtman, C., M. Kamien and E. Muller (1992), "Integral Games: Theory and Applications", in Feichtinger, G. (ed.), *Dynamic Economic Models and Optimal Control*, Amsterdam, North-Holland, 297-311.

[20] Hall, R.E. and D.W. Jorgenson (1967), "Tax Policy and Investment Behaviour", *American Economic Review*, **57**, 391-414.

[21] Kamien, M.I. and N.L. Schwartz (1981), *Dynamic Optimization*, Amsterdam, North-Holland.

[22] Kreps, D. and J. Scheinkman (1983), "Quantity Precommitment and Bertrand Competition Yield Cournot Outcomes", *Bell Journal of Economics*, **14**, 326-37.

[23] Levin, D. (1985), "Taxation within Cournot Oligopoly", *Journal of Public Economics*, **27**, 281-90.

[24] Mehlmann, A. (1988), *Applied Differential Games*, New York, Plenum Press.

[25] Mehlmann, A. and R. Willing (1983), "On Nonunique Closed-Loop Nash Equilibria for a Class of Differential Games with a Unique and Degenerate Feedback Solution", *Journal of Optimization Theory and Applications*, **41**, 463-72.

[26] Nerlove, M. and K.J. Arrow (1962), "Optimal Advertising Policy under Dynamic Conditions", *Economica*, **29**, 129-42.

[27] Ramsey, F.P. (1928), "A Mathematical Theory of Saving", *Economic Journal*, **38**, 543-59. Reprinted in Stiglitz, J.E. and H. Uzawa (1969, eds.), *Readings in the Modern Theory of Economic Growth*, Cambridge, MA, MIT Press.

[28] Reinganum, J. (1982), "A Class of Differential Games for Which the Closed Loop and Open Loop Nash Equilibria Coincide", *Journal of Optimization Theory and Applications*, **36**, 253-62.

[29] Reynolds, S.S. (1987), "Capacity Investment, Preemption and Commitment in an Infinite Horizon Model", *International Economic Review*, **28**, 69-88.

[30] Solow, R. (1956), "A Contribution to the Theory of Economic Growth", *Quarterly Journal of Economics*, **70**, 65-94.

[31] Ushio, Y. (2000), "Welfare Effects of Commodity Taxation in Cournot Oligopoly", *Japanese Economic Review*, **51**, 268-73.

[32] Vickers, J. (1985), "Delegation and the Theory of the Firm", *Economic Journal*, **95** (Conference Papers), 138-47.