An Equilibrium Approach to the Term Structure of Interest rates with the Interaction between Monetary and Fiscal Policy

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Abstract

In this paper I jointly derive the stochastic process of the price level, the inflation rate, the nominal and real term structures, as function of monetary, fiscal and technological parameters within a general equilibrium framework. The novelty of the present approach is given by the possibility of obtaining closed form solutions for all the variables and by the explicit design of fiscal policy as a crucial parameter in addition to monetary policy. Thus, as stated from FTPL, inflation is not uniquely a monetary phoenomenon, but also fiscal policy plays a crucial role in determining the position of the nominal spot curve and term structure of intereest rates. The risky factors of nominal and real term structures depend upon different factors, when the utility function is strongly separable in both output and real money balances. If not, monetary and fiscal parameters affect only nominal equilibrium. The principal realtionships derived of the model are then simulated for diffrent values of policy parameters. The main conclusion is that fiscal policy parameters play a crucial role in term structure patterns, as recently observed by the perfmance of nominal rates for some countries (like Italy, fore example) after having joined the EMU.

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1 Introduction

The present paper derives an equilibrium model of the term structure of nominal interest rates by considering an explicit role for both fiscal and monetary policy in the determination of the position of the term structure curve. The traditional approaches to the term structure modelling so far considered in the literature have modelled the term structure as directly dependent from two essential factors: technology and inflation or a diffusion process for the growth rate of money supply. Leading contributions for the 'equilibrium' approach to the determination of the term structure have devoted scarce attention to the role of fiscal factors as additional elements to explain the position of the term structure in the plane.

The present paper proposes an integrated model where both monetary and fiscal factors play a significant role - together with the exogenous technological factors - in the determination of the term structure. The inclusion of fiscal factors is here realized by the explicit design of an active role for fiscal policy, by using the properties of the Government Budget Constraint as described by a recent stream of literature denominated Fiscal Theory of the Price Level (FTPL, henceforth). This approach was inaugurated by the contributions by Leeper (1991), Sims (1994), and recently extended by Woodford (1996, 2000), Cochrane (1998, 1999, 2000), Canzoneri, Cumby and Diba (1998). The main results from FTPL state that in order to guarantee a stable path for the inflation rate, it is not enough to define a set of conditions on the rules showing the behavior of the monetary authority, in terms, for example, of a good reputation of being aggressive against expected inflationary sources. Rather, the most important condition to guarantee the existence of a stable price level (and inflation) path is about fiscal policy: the government must set taxes in order to react to the existing stock of the outstanding stock of public debt. This ensures the solvency of the government and a full stability of the inflation path. It is clear, then, that according to the degree of reaction of the tax revenue to the shocks to pubic debt, it is possible to design a full set of fiscal policy rules, each of them will have a different impact on nominal interest rates, price level and inflation.

What are the effects of such policy rules on the term structure of nominal interest rates ? Is it possible to derive the stochastic process of the price level as a function of the relevant fiscal and monetary policy parameters ? These are the main questions that this paper tries to address.

The framework here adopted follows closely the work by Bakshi and Chen (1996), who, by themselves rely on a well consolidated literature on equilibrium models of the term structure of interest rates inaugurated by Cox, Ingersoll and Ross (1985a,b) and extended by Marshall (1992), Sun (1992). The model is solved in closed form and the methodology here chosen allows to highlights the various links existing between the price level, the inflation rate, the equity returns and the nominal and real terms structures to the core economic forces. Monetary and fiscal policy parameters are endogenized in the determination of the equilibrium of the model. On these grounds it is natural to see that inflation is partially a monetary phenomenon, because other elements (such as the variables form the real side of the model, like, f.e. the volatility of output and especially the degree of tightness of fiscal policy parameters), can generate an inflationary pressure.

By introducing an explicit role for fiscal policy, the present model develops the idea that pressures on the (expected) inflation can come also from a loose fiscal policy, i.e. a set of fiscal policies set independently upon the stock of existing bonds. Therefore, a particular set of policies on the management of public debt might have an impact on the inflation rate and on the nominal equilibrium of the model, as pointed out by Cochrane (1998, 1999, 2000), in a different context.

These features allow to disentangle the parameter of the money supply process leading the monetary policy function. From this point of view this is a further step ahead with respect of the existing literature. In fact, in term structure of interest rates literature, we can distinguish at least two types of models: the 'traditional' models, such as the consumption-based CAPM by Breeden (1979), Stulz (1986), and the term structure models by Cox, Ingersoll and Ross (1985a,b), Costantinides (1992), Longstaff and Schwartz (1992), and Sun (1992), where there is not a crucial and really explicit role for money. In these models the nominal equilibrium is obtained through the inclusion of exogenously given processes for the price level and the expected inflation rate.

A second class of models, instead, assume a more explicit role for money. Works in this area has been done by Danthine and Donaldson (1986), Foresi (1990), Marshall (1992), Bakshi and Chen (1996) and Balduzzi (2000). These studies were empirically motivated by the attempt to solve out some of the puzzling correlations found in the empirical literature. None of these papers, however, did endogenize the process of money supply at the core of the model. In this paper, instead, the inclusion of an explicit fiscal policy channel will allow to provide a deeper analysis of the source of monetary dynamics, with which I insert an additional policy dimension.

In this article I assume, as in the tradition of monetary models, that rea1 money balances directly enter into the utility function. As in the most recent set of papers in this area (Sun (1992), Bakshi and Chen (1996) and Balduzzi (2000)), the equilibrium is firstly derived in discrete time, then the continuous time is obtained by taking the limit of the discrete time equilibrium relationships. This allows to easily get the solution in closed form, showing the relationships among the existing variables such as the price level, inflation, equity prices and term structures, all jointly determined.

Two types of equilibria are derived: the first is for a Cobb-Douglas type utility function in consumption and real money balances, with simple stochastic processes for output endowment and money supply. The second equilibrium, instead, assumes a simpler utility function (log-separable in both consumption and real money balances), but with more realistic stochastic processes. In both cases, simple simulations will highlight the underlying dynamic of the model as function of the key policy parameters.

In particular, for the second type of model, a complete set of pictures for the spot curves (nominal and real) and for the nominal term structure, will clearly show the main result of the model: both monetary and fiscal policy parameters affect the spot curve of nominal interest rates and the nominal term structure, in a way coherent with the logic of both monetary theory and of the recent results from the FTPL. In particular, an expected tighter fiscal policy pushes both nominal curves (spot interest rates, and term structure) down: a more aggressive fiscal policy makes the government more solvable. This will allows the government to renew the outstanding debt at a lower nominal interest rate, because of the reduction of the solvency risk premium. In fact, if the government adopts 'Ricardian' or 'Passive' fiscal policies (as they are defined by FTPL theorists), then the investors do not require a premium over the return from government bonds to cover for capital losses, due to the increased probability of government's failure. In this sense, if the government adopts a tighter fiscal policy and is credible in doing so, the value of bonds increase, because of the better protection imposed by a certain expected flow of taxes: the price of bonds raises and their yield decreases.

In the same way, a contractionary monetary policy (here identified by a reduction of the expected growth rate of money supply) will imply a reduction of both nominal spot curve and the term structure of interest rates. The reason is due to the effects induced by the expected inflation: a tighter monetary policy today will imply a lower expected inflation, and a lower expected nominal interest rate, when fiscal policy is set according to the criteria of FTPL.

An important feature of the model is given by the asymmetric behavior of the nominal and real term structure. When a separable log utility function is assumed (in consumption and real money balances), the real term structure is dependent only upon the process leading the 'real side' of the economy, given by the parameters of the diffusion processes assumed for both output and technology, and is independent upon monetary and fiscal policy factors. This, however, is true if monetary shocks are assumed to be independent upon output shocks, and vice-versa. Therefore, the real term structure is perfectly correlated only with the output process, while the nominal term structure is correlated with money growth rate, fiscal policy parameters and the technological parameters. This result is in contrast with the existing model of the term structure of interest rates. In any case, given the flexibility of the framework adopted here, it is possible to amend the model for a more general class of stochastic processes designed to respond to more stringent empirical questions.

The proved dependence of the nominal term structure upon fiscal policy factors can be empirically observed by the huge drop in interest rates registered for many countries after their joining to the EMU[1]. In Italy, for example, after the fiscal retrenchment imposed by *Maastricht Treaty* and the *Stability and Growth Pact* implemented for the joining of the EMU, we observed a remarkable reduction in interest rates from average values of 10 per cent (before 1996/1997) to average of 5 per cent (in 2000). This, despite the fact that monetary policy did not show any dramatic changes. This paper tries to offer a theoretical explanations of similar phenomena where fiscal retrenchments play a crucial role in building up the reputation of the government.

The present paper is organized as follows. The next section introduces the reader to the main assumptions underlying the model together with the description of the portfolio allocation problem faced by the representative agent/investor. Two sections follows on the analysis of monetary and fiscal authorities, respectively. Then, the description of the equilibrium in discrete time follows together with its representation in the continuous time, obtained at the limit. An example of the equilibrium relationships in continuous time is derived for a particular utility function with specific assumptions on the main stochastic processes leading the economy. The main results of the paper are derived by considering a set of more realistic stochastic processes for technology, output and money supply: this will deliver the analytical expressions for the spot rate curves and terms structure equations to be simulated. The simulation results are reported in a separate section. The last section concludes the article. The proof of all the results are collected in the Appendix at the end of the paper.

[1] For a critical analysis about fiscal policy rules adopted by EMU, with the *Stability and Growth Pact* and the *Maastricht Treaty*, see Sims (1999) and Corsetti and Pesenti (1999).

2 The Model

2.1 The Representative Agent

The present model builds on the work by Bakshi and Chen (1996) in structuring an equilibrium model for the term structure of both nominal and real interest rates including the interactions between monetary and fiscal policies.

A representative agent optimally decides the allocation of his (her) portfolio across a wide number of assets, including nominal and real bonds and a set of shares of stocks describing a set of property rights on private-owned companies. The choice setup is described in discrete time intervals of length Δt, then it is passed in continuous time later on .

The utility function of the representative agent is given by:

$$\sum_{t=0}^{\infty} e^{-\beta t} E_0 \left\{ u \left(C_t, \frac{M_t}{P_t} \right) \right\} \Delta t \tag{1}$$

The function (1) depends upon consumption C_t over the interval $[t, t + \Delta t]$, and M_t indicates the nominal money stock, while P_t indicates the CPI (Consumer Price Index), i.e. the general price level of the final consumption good. The instantaneous discount rate is given by $e^{-\beta t}$, where β represents the discount factor. In (1) real money balances M_t/P_t brings directly utility to the representative agent, as in Bakshi and Chen (1996): this choice has been done in order to simplify algebra. As pointed out by Feenstra (1990), we may insert money in a general equilibrium model either through transaction costs in the Representative Agent's budget constraint, or by using Cash in Advance Constraint. In the term structure literature, Marshall (1992) considers a model with transaction costs, while Balduzzi (2000), in a framework similar to the present, adopts a Cash in Advance model. Obviously, the highest empirical reliability is provided by the transaction costs model. In the present framework, the inclusion of real money balances directly in the utility function helps to simplify algebra.

I assume also that the utility function is twice continuously differentiable and concave in both consumption and real balances. Formally, this means that:

$$u_c > 0,\ u_m > 0,\ u_{cc} < 0,\ u_{mm} < 0,\ u_{cm} < 0, u_{cc}u_{mm} - (u_{cm})^2 > 0 \tag{2}$$

where $m_t = \frac{M_t}{P_t}$ is the demand for real money balances. In (2) the subscript to u indicates the arguments with whom the partial derivative is taken. The portfolio allocation problem has been previously studied by Grossmann and Shiller (1982) and Bakshi and Chen (1996).

The optimal choice problem of the representative agent consists in maximizing the utility function (1) subjected to the following intertemporal budget constraint:

$$M_t + \left(P^z_{g,t} + P_t Y_t \Delta t\right) g_t + P_t z_{1,t} + z_{2,t} + \sum_{i=3}^{N} P^z_{i,t} z_{i,t} = P_t C_t \Delta t + M_{t+\Delta t} + P^z_{g,t} g_{t+\Delta t} +$$
$$+ P_t \frac{z_{1,t+\Delta t}}{1 + R_t \Delta t} + \frac{z_{2,t+\Delta t}}{1 + i_t \Delta t} + \sum_{i=3}^{N} P^z_{i,t} z_{i,t+\Delta t} \tag{3}$$

From equation (3) each investor can choose among a wide range of assets traded on the market. At each time t, the agent demands M_t for cash, C_t, for consumption (in real terms), and equity holdings

g_t (shares). The vector $z_t = (z_{1,t}, z_{2,t}, \ldots, z_{N,t})'$ indicates the financial holdings of the representative agent, where $z_{i,t}$, for $i = 1, \ldots, N$ indicates the number of units of financial assets i held from $(t - \Delta t)$ to t. In particular, $z_{1,t}$ is the number of units of risk-free real bonds paying a real interest rate R_t, issued at time t and maturing at $t+\Delta t$. Similarly, $z_{2,t}$ is the number of units of risk-free nominal bonds paying a nominal interest rate i_t, issued at time t and maturing at $t + \Delta t$. Each agent can invest $z_{i,t}$ number of units in risky financial activities (stocks) whose nominal price (including dividend payments) is given by $P_{i,t}$, for $i = 3, \ldots, N$.

Additionally, each representative agent is allowed to invest in one (single) equity share g_t which gives to the holder the property right on all the output Y_t produced through a single technology, which, expressed in units of the final good is given by:

$$\frac{\Delta Y_t}{Y_t} = \frac{Y_{t+\Delta t} - Y_t}{Y_t} = \mu_{y,t}\Delta t + \sigma_{y,t}\Omega_{y,t}\sqrt{\Delta t} \tag{4}$$

where $\mu_{y,t}$ and $\sigma_{y,t}$ are, respectively, the conditional expected value and the standard deviation of the output growth per unit of time while $\{\Omega_{y,t} : t = 0, \Delta t, \ldots\}$ is an i.i.d. standard normal random process. From (4) $\mu_{y,t}$ and $\sigma_{y,t}$ can be time variants, as in Cox, Ingersoll and Ross (1995a,b) and Sun (1992). In the present framework, I consider only a pure endowment economy. Thus, equation (4). indicates the evolution of the stochastic process leading the output endowment.

The real price in terms of the consumption goods of asset i at time t is defined as $p^z_{i,t} = \frac{P^z_{i,t}}{P_t}$. I assume that real asset prices follow a vector diffusion process described by:

$$\frac{\Delta p^z_{i,t}}{p^z_{i,t}} = \mu^z_{i,t}\Delta t + \sigma^z_{i,t}\Omega^z_{i,t}\sqrt{\Delta t} \tag{5}$$

where $\mu^z_{i,t}$ and $\sigma^z_{i,t}$ are, respectively, the conditional expected value and the standard deviation of real return on asset i per unit of time and $\left\{\Omega^z_{i,t} : t = 0, \Delta t, \ldots\right\}$ is a standard normal random process.

For expository reasons, let us also define the following stochastic process for the Price level, or CPI, as follows:

$$\frac{\Delta P_t}{P_t} = \mu_{p,t}\Delta t + \sigma_{p,t}\Omega_{p,t}\sqrt{\Delta t} \tag{6}$$

where the drift term $\mu_{p,t}$ and the standard deviation $\sigma_{p,t}$ term are taken as given for the moment, and will be derived later on as function of the core parameters of the economy. As before, $\{\Omega_{p,t} : t = 0, \Delta t, \ldots\}$ is a standard normal random process.

2.2 Monetary Policy

The crucial novelty of the present paper is represented by an explicit design of monetary and fiscal policy interactions. According to the recent literature on the Fiscal Theory of the Price Level, in fact, it is not possible to achieve a stable inflation rate if monetary policy is not accompanies by a fiscal policy which makes taxes as reacting to the level of the outstanding real public debt.

In the follows I will describe the simplest possible framework to introduce monetary policy, which will allow to introduce an active role for fiscal policy also.

Money supply M_t^s is defined by:

$$M_t^s = M_{1t} + M_{2t} \tag{7}$$

According to equation (7) money supply M_t^s is divided up in two components M_{1t}, M_{2t}. The first component M_{1t} indicates the monetary base or high powered money, while M_{2t} indicates the amount of money employed by the government in order to balance its budget. Therefore, M_{2t} represents a source of additional financing for the government. It is known, however, that the amount of government deficit financed by direct money issuance is very low, nevertheless it exists and well documented,as discussed by Walsh (1998). For example, in the United States, the amount of money financing is given by the interest rate proceedings derived from government bond holding in the federal reserve portfolio holdings.

To keep things as simple as possible, let us assume that monetary base M_{1t} is non-stochastic. Thus, the growth rate of monetary base is a deterministic process given by:

$$\frac{\Delta M_{1t}}{M_{1t}} = \frac{M_{1t+\Delta t} - M_{1t}}{M_{1t}} = \mu_{1,t}\Delta t \tag{8}$$

where $\mu_{1,t}$ is the mean of the monetary base growth rate. This type of assumption on monetary policy rule is defined by Leeper as 'Active' monetary policy, meaning the fact that monetary authority sets the amount of money supply independently upon any kind of consideration in terms of expected inflation.

The only stochastic source to total money supply M_t^s come from the process specified for M_{2t}, for which I assume the following Brownian motion with drift:

$$\frac{\Delta M_{2t}}{M_{2t}} = \frac{M_{2t+\Delta t} - M_{2t}}{M_{2t}} = \mu_{2,t}\Delta t + \sigma_{2,t}\Omega_{2,t}\sqrt{\Delta t} \tag{9}$$

where $\mu_{2,t}$ and $\sigma_{2,t}$ are, respectively, the mean and the standard deviation of the stochastic process leading the growth rate of the money supply aggregate M_{2t}, and $\{\Omega_{2,t} : t = 0, \Delta t, \dots\}$ is again an i.i.d. standard normal random process. Combining (8) and (9) with the definition given to money supply (7) we get the stochastic process of money supply given by:

$$\frac{\Delta M_t^s}{M_t^s} = \frac{M_{t+\Delta t}^s - M_t^s}{M_t^s} = \mu_{M,t}\Delta t + \sigma_{M,t}\Omega_{M,t}\sqrt{\Delta t} \tag{10}$$

where:

$$\mu_{M,t} = \mu_{1,t} + \mu_{2,t} \tag{11}$$

$$\sigma_{M,t}\Omega_{M,t} = \sigma_{2,t}\Omega_{2,t} \tag{12}$$

The money supply function is jointly determined by (10)-(12). The drift term given by (11) is the sum of the two components of the monetary policy rule, and the stochastic component (12) is exclusively given by the stochastic fluctuations in the transfers from Central Bank to the Government, $\sigma_{2,t}$.

The stochastic process for money supply given in equation (10) is standard in the literature on the term structure. Here, however, I show a way to endogenize both the drift term and the second order term of (10). This will open up an explicit role for fiscal policy. The trick employed here allows to distinguish between two sources of money supply. The composition of money supply is decided residually from Central Bank, who can fix $\mu_{1,t}$ or $\mu_{2,t}$, say by institutional arrangement[2] and let the other follows automatically.

For further convenience, let us consider the stochastic process of money expressed in real terms. Define real money supply m_t^s as $m_t^s = \frac{M_t^s}{P_t}$, where P_t is the price level (or CPI), whose stochastic process is given by equation (6). Also the other component of money supply M_{1t} and M_{2t} can be expressed in real terms as $m_{1t} = M_{1t}/P_t$, or $m_{2t} = M_{2t}/P_t$. Thus, by taking the limit for $\Delta t \to 0$ and applying Ito's Lemma to the stochastic processes of nominal money supply (8) and (9), we get the following representation for the stochastic processes of real money supply:

$$\frac{dm_t}{m_t} = \mu_{mr,t}dt + \sigma_{mr,t}\Omega_{m,t}\sqrt{dt} \tag{13}$$

$$\frac{dm_{1t}}{m_{1t}} = \mu_{1r,t}dt + \sigma_{1r,t}\Omega_{1,t}\sqrt{dt} \tag{14}$$

$$\frac{dm_{2t}}{m_{2t}} = \mu_{2r,t}dt + \sigma_{2r,t}\Omega_{2,t}\sqrt{dt} \tag{15}$$

where the drift terms are given by (after having dropped for the time subscript index to save notation):

$$\mu_{mr} = \mu_M - \mu_p + 2\sigma_p^2 - \sigma_{pM} \tag{16}$$

$$\mu_{1r} = \mu_1 - \mu_p + 2\sigma_p^2 \tag{17}$$

$$\mu_{2r} = \mu_2 - \mu_p + 2\sigma_p^2 - \sigma_{pm2} \tag{18}$$

and the stochastic terms are given by:

$$\sigma_{mr} = \sigma_M - \sigma_p \tag{19}$$

$$\sigma_{1r} = -\sigma_p \tag{20}$$

$$\sigma_{2r} = \sigma_2 - \sigma_p \tag{21}$$

Starting from the above relationships we can now describe the role of fiscal policy and we will discover how the interactions between fiscal and monetary policy can be introduced in the model.

[2] A type of institutional arrangement of this sort could be for example given by a particular law which fixes an upper bound for deficit financing. In Italy, for example, until 1992 Central Bank was allowed to print money to finance the budget deficit until the 14% of the entire budget deficit specified by the financial law stated in each year. This limit could be also implicitly taken by Central Banks. Incidentally, in almost all G8 countries, Central Banks can play as an active dealer in the government bond market.

2.3 Fiscal Policy

I start by defining the Government Budget Constraint in real terms is given by:

$$\Delta b_t + \Delta m_{2t} = \Delta R_t b_t - \Delta T_t \tag{22}$$

In equation (22) b_t is the stock of nominal public debt issued in period $t - \Delta t$ and maturing in t, while $\Delta T_{t+\Delta t}$ indicates the change in fiscal revenue and $\Delta R_t b_t$ is the burden of interest-rate payment on the outstanding debt. In equation (22) we can identify three source of stochastic volatility: (i) shocks to the stochastic process defined for m_{2t}; (ii) shocks to fiscal revenue ΔT_t and shocks to the burden of interest payments originating from shocks to the stochastic process specified for the real interest rate R_t. Note that I assume that the level of public expenditure net of interest rate payments is zero. This does not affect the results in a dramatic way, and is done in order to simplify algebra.

A large body of literature has recently stressed the importance of introducing in monetary models a set of fiscal policy rules which are at the core of the solvency requirement of the government. According to the Fiscal Theory of the Price Level (FTPL, henceforth) given the types of monetary policy rules adopted in this paper (see the previous section), the level of taxation (or the primary surplus) must be set to promptly react to the outstanding level of public debt in order to avoid an explosionary path for the price level. A typical example of such kind of fiscal policy rule is given by the following equation:

$$\Delta T_t = \phi_1 b_t \Delta t + \phi_1 b_t \sigma_{T,t} \Omega_{T,t} \sqrt{\Delta t} \tag{23}$$

In equation (23) we have a description of the stochastic process leading taxes. The drift of the stochastic process is given a level of taxation exactly proportional to the outstanding level of public debt, the stochastic component is given by a stochastic shock to taxes following a i.i.d. standard normal process $\{\Omega_{T,t} : t = 0, \Delta t, \dots\}$.

The rule described in equation (23) individuates a set of fiscal policies defined as 'Ricardian' or 'Passive', as opposed to 'Active' fiscal policies which sets taxes independently upon the level of the outstanding public debt. These type of rules have been studied by Leeper (1991), Sims (1994) and Woodford (1996, 2000) and Cochrane (1998) as the only to be able to insert a bound on the price level, independently upon monetary policy specification. In rule (23) the tightness on fiscal policy is described by the parameter ϕ_1 for which Sims (1994) established a bound given by: $0 \le \phi_1 < \beta^{-1}$.

The discussion on fiscal policy can be completed by the definition of the following stochastic process for the real rate:

$$\Delta R_t = \mu_{R,t} \Delta t + \sigma_{R,t} \Omega_{R,t} \sqrt{\Delta t} \tag{24}$$

Equation (24) introduces a stochastic process exogenous for the nominal interest with a drift $\mu_{i,t}$ and $\Omega_{i,t}$ as i.i.d. normal variable $\{\Omega_{R,t} : t = 0, \Delta t, \dots\}$, whose expression will be determined later, after having obtained the closed form solution of the model.

If we combine (23) and (24) with (22) we get the following expression for the motion law of the public debt:

$$\Delta b_{t+\Delta t} + \Delta m_{2t+\Delta t} = \left(\mu_{R,t} - \phi_1\right) b_t \Delta t + b_t \sigma_{R,t} \Omega_{R,t} \sqrt{\Delta t} - \phi_1 b_t \sigma_{T,t} \Omega_{T,t} \sqrt{\Delta t} \tag{25}$$

To get a semi-closed solution for the parameter of money supply, let us specify a generic motion law for real public debt by introducing a simple deterministic process given by:

$$\frac{\Delta b_t}{b_t} = \frac{b_{t+\Delta t} - b_t}{b_t} = \mu_{b,t}\Delta t \tag{26}$$

In (26) we have that $\mu_{b,t}$ indicates the expected growth rate of real public debt. From (26) I assumed a trend deterministic component in order to simplify the model. To get a semi reduced form of the public debt equation, let us assume now that the Government aims to maintain a constant ratio of real bonds with respect to money $\psi \equiv b/m_2$. In other words, the parameter ψ indicates the relative importance of bonds over money[3].

To get a reduced form of the model, let us consider all the above relationships in continuous time, by taking the limit for $\Delta t \to 0$. Then, apply Ito's Lemma to the definition of ψ, by using also the stochastic processes for m_t, m_{2t}, and b_t in equations (13), (15), and (26). From these calculations, we easily get (equating the deterministic and the stochastic part of the resulting expressions):

$$\mu_b = \mu_{2r} - \sigma_{2r}^2 \tag{27}$$

Moreover, by using the definition of the stochastic process of b_t and m_{2t} into the government budget constraint (25), and equating the deterministic with the stochastic part, we get:

$$\mu_{2r} = \frac{\left(\mu_R - \phi_1 + \psi\sigma_{2r}^2\right)}{1+\psi} \tag{28}$$

$$\sigma_{2r} = \psi\left(\sigma_R - \phi_1\sigma_T\right) \tag{29}$$

Equations (27) - (29) indicates a set of equilibrium relationships among the drift terms of the relevant stochastic process. To highlight the role of these equation plug (28) into (11) to obtain the following expression for the conditional mean of the diffusion process for the money supply growth (equation (10)):

$$\mu_{mr} = \mu_{1r} + \frac{\left(\mu_R - \phi_1 + \psi\sigma_{2r}^2\right)}{1+\psi} \tag{30}$$

In nominal terms we can rewrite equation (30) as follows:

$$\mu_M = \mu_1 + \sigma_{pM} + \frac{\left(\mu_R - \phi_1 + \psi\sigma_{2r}^2\right)}{1+\psi} \tag{31}$$

Equations (30)-(31) are the crucial for our purposes: both highlight the various links between fiscal and monetary policy. When fiscal policy is tightened (i.e. if ϕ_1 raises), then, the conditional mean of the stochastic process of both nominal and real money growth supply is reduced. Thus, there is a negative relationship between μ_M and ϕ_1 (through the effect of ϕ_1 on μ_{2r}, from (28)).

A final observations pertains the link existing between real and nominal equilibrium, given by the presence of μ_R (the conditional mean of the real interest process). When the model will be solved after having made particular assumptions on the utility function, μ_R will be a function of the 'core' parameter of the economy.

[3] In Woodford (1996), $\psi = 0.1$.

3 The Equilibrium in Discrete Time

I assume that the present economy is populated by a plurality of identical agents. Thus, the behavior of the representative agent is a good proxy of all the agents living in this economy. Moreover, in a representative agent economy optimal consumption, money demand and portfolio holdings must adjust such that the following equilibrium conditions are verified:

$$C_t = Y_t \tag{32}$$

$$M_t = M_t^s \tag{33}$$

$$g_t = 1 \tag{34}$$

$$z_{i,t} = 0, \quad \forall \;\; i = 1, 2, ...N \tag{35}$$

Condition (32) is the usual condition of equality between consumption demand and output endowment, while condition (33) is the usual condition of equality between money demand and supply. Moreover, condition (34) and (35) are the usual equilibrium conditions in the portfolio holdings markets. Thus, solving the optimization process for the representative agent, and using the equilibrium relationships (32)-(35), we find the following set of first order conditions:

$$u_c\left(Y_t, m_t\right) = e^{-\beta \Delta t} E_t \left[u_c\left(Y_{t+\Delta t}, m_{t+\Delta t}\right)\left(1 + R_t \Delta t\right)\right] \tag{36}$$

$$u_c\left(Y_t, m_t\right) = e^{-\beta \Delta t} E_t \left\{\left[u_c\left(Y_{t+\Delta t}, m_{t+\Delta t}\right) + u_m\left(Y_{t+\Delta t}, m_{t+\Delta t}\right)\right] \frac{P_t}{P_{t+\Delta t}}\right\} \tag{37}$$

$$\frac{u_c\left(Y_t, m_t\right)}{P_t} = e^{-\beta \Delta t} E_t \left[\frac{u_c\left(Y_{t+\Delta t}, m_{t+\Delta t}\right)}{P_{t+\Delta t}}\left(1 + i_t \Delta t\right)\right] \tag{38}$$

$$u_c\left(Y_t, m_t\right) = e^{-\beta \Delta t} E_t \left[u_c\left(Y_{t+\Delta t}, m_{t+\Delta t}\right) \frac{p^z_{i,t+\Delta t}}{p^z_{i,t}}\right] \tag{39}$$

Recall that $p^z_{i,t}$ is the real price in terms of the consumption good of asset i at time t. In (36) - (39) I used the fact that $\frac{\Delta p^z_{i,t+\Delta t}}{p^z_{i,t}} = \frac{p^z_{g,t+\Delta t} + Y_{t+\Delta t}}{p^z_{g,t}}$, with $p^z_{g,t} = \frac{P^z_{g,t}}{P_t}$.

In addition to first order conditions (36) - (39), to guarantee the existence of an interior optimum we need the following sufficient conditions:

$$\lim_{T \to \infty} E_t \left\{e^{-\beta \Delta t} \frac{u_c\left(Y_T, m_T\right)}{u_c\left(Y_t, m_t\right)} p^z_{i,t}\right\} = 0 \tag{40}$$

$$\lim_{T \to \infty} E_t \left\{e^{-\beta \Delta t} \frac{u_c\left(Y_T, m_T\right)}{u_c\left(Y_t, m_t\right)} \frac{1}{P_t}\right\} = 0 \tag{41}$$

The trasversality conditions (40) is added in order to rule out bubbles in the price level of risky assets, while the other trasversality conditions (41) is added to rule out bubbles in the general price level. If condition (40) is violated , the agent will be willing to reduce consumption today in exchange of future consumption, without any bound, with the proceeds derived from the investment in risky assets. A similar interpretation works for condition (41): if violated, the agent will be willing to reduce consumption today in exchange of an increased future money service, without any bound at all, if equation (41) is violated.

From the way in which conditions (40)-(41) are presented here, the bound on the utility function is crucial in determining the respect of both conditions. These conditions are important in order to exclude situations where the utility increases (decreases) without bound for an excess (low level) of consumption derived from a steady state increase (decrease) of real money balances, due to a reduction of the Price level.

In this type of model the stability of the price level is guaranteed by the adoption of a particular fiscal policy rule here adopted, like rule (23), as discussed previously. All these considerations should convince the reader of the existence and of the stability of the steady state equilibrium.

4 The Equilibrium in the Continuous time

In what follows I am going to derive the crucial core relationships of the model. It should be noted that the results collected in this section do not necessarily depend upon an particular assumptions made on the utility function. The reader might find some overlapping results between the results following here and what has been previously obtained by Bakshi and Chen (1996) Balduzzi (2000), Breeden (1979,1986) and Stulz (1986). However, it will be useful to collect here all the main results we are going to use in the discussion.

Let us start with the following Proposition defining the risk premium.

Proposition 1 *The equilibrium risk premium for any risky asset over the real interest rate is given by:*

$$\mu_{i,t}^{\tilde{z}} - R_t = -\frac{Y_t u_{cc}}{u_c} cov_t \left(\frac{dp_{i,t}^{\tilde{z}}}{p_{i,t}^{\tilde{z}}}, \frac{dY_t}{Y_t} \right) - \frac{m_t u_{cm}}{u_c} cov_t \left(\frac{dp_{i,t}^{\tilde{z}}}{p_{i,t}^{\tilde{z}}}, \frac{dm_t}{m_t} \right) \tag{42}$$

Proof. See Appendix.

.

From equation (42) we observe that both production and monetary policy risk enter in the definition of the risk premium of asset i over the real interest rate. Additionally, the risk premium is linear in both the covariance between the price of real assets and output and the covariance between the price of real asset and money.

A second proposition will describe how to derive the price level and stochastic process of the inflation rate.

Proposition 2 *In the continuous time limit equilibrium, the commodity price level is given at time t* *by:*

$$\frac{1}{P_t} = E_t \int_t^{\infty} e^{-\beta(s-t)} \frac{u_m(Y_s, m_s)}{u_c(Y_t, m_t)} \frac{1}{P_s} ds \tag{43}$$

The expected inflation rate is given by:

$$\begin{aligned} \pi_t &\equiv \frac{1}{dt} E_t \left\{ \frac{dP_t}{P_t} \right\} = \\ &= i_t - R_t + var_t \left\{ \frac{dP_t}{P_t} \right\} - \frac{u_{cc} Y_t}{u_c} cov_t \left(\frac{dY_t}{Y_t}, \frac{dP_t}{P_t} \right) - \frac{u_{cm} m_t}{u_c} cov_t \left(\frac{dP_t}{P_t}, \frac{dm_t}{m_t} \right) \end{aligned} \tag{44}$$

Proof. See Appendix. ■

From equation (43) we have that the price level equates the future discounted value of all the marginal benefits provided by holding one unit of dollar cash. This equation is one of the key elements to solve for the price level as a function of the stochastic processes of money and output. Note also that the inflation rate is the conditional mean of the stochastic process of the price level given in equation (6). The derivation of the inflation is conducted by setting exactly $\mu_{p,t} = \pi_t$.

After rearranging equation (44) we have that Fischer equation does not hold when second order terms are included.

In fact, the riskiness on bonds given by $\left\{ i_t - \pi_t + var_t \left\{ \frac{dP_t}{P_t} \right\} - R_t \right\}$ becomes a function of the overall riskiness structure of the economy. Nominal risk free assets - like nominal bonds - become 'risky' assets, through the role played by the stochastic process of the price level, which is a function of the risky parameters of the economy, including the monetary and fiscal policy parameters. In the examples provided below, I will make clearer this point through the help of some examples.

Consider now the derivation of real and nominal interest rates.

Proposition 3 *The equilibrium real interest rate is given by:*

$$\begin{aligned} R_t &= \beta - \frac{u_{cc} Y_t}{u_c} \frac{1}{dt} E_t \left\{ \frac{dY_t}{Y_t} \right\} - \frac{1}{2} \frac{u_{cc} Y_t^2}{u_c} var_t \left\{ \frac{dY_t}{Y_t} \right\} - \frac{u_{cm} m_t}{u_c} \frac{1}{dt} E_t \left\{ \frac{dm_t}{m_t} \right\} + \\ &\quad - \frac{1}{2} \frac{u_{cmm} m_t^2}{u_c} var_t \left\{ \frac{dm_t}{m_t} \right\} - \frac{Y_t m_t u_{ccm}}{u_c} cov_t \left(\frac{dY_t}{Y_t}, \frac{dm_t}{m_t} \right) \end{aligned} \tag{45}$$

The equilibrium nominal interest rate is given by:

$$i_t = \frac{u_m(Y_t, m_t)}{u_c(Y_t, m_t)} \tag{46}$$

Proof. See Appendix. ■

From equation (45) we observe that the stochastic process of money supply affects the real interest rate, if the utility function is non-separable in money and output. However, if the utility function is logarithmic in both money and output - like f.e. in Stulz (1986) - (this is equivalent to say that $u_{cm} = u_{ccm} = u_{cmm} = 0$) there is no way by which monetary and fiscal policy can affect real interest rate.

The right hand side of equation (46) is the marginal benefit of holding one additional unit of cash balance, or the marginal rate of substitution between consumption and real cash holdings.

The term structure of interest rate is defined according to the following proposition.

Proposition 4 *(a) The nominal term structure equation is defined as:*

$$\frac{N(t,\tau)}{P_t} = e^{-\beta\tau} E_t \left[\frac{u_c(Y_{t+\tau}, m_{t+\tau})}{u_c(Y_t, m_t)} \frac{1}{P_{t+\tau}} \right] \tag{47}$$

where $N(t,\tau)$ is the time t nominal price of a discount bond paying a dollar in τ periods.
(b) The real term structure is:

$$b(t,\tau) = e^{-\beta\tau} E_t \left[\frac{u_c(Y_{t+\tau}, m_{t+\tau})}{u_c(Y_t, m_t)} \right] \tag{48}$$

where $b(t,\tau)$ is the time t nominal price of a discount bond paying a unit of consumption good in τ periods.

Proof. The results here showed follow directly from the Euler equation (36) ∎

After having established all the above results, we can now consider some example to show how the concepts exposed so far will apply to a specific set of assumptions on the utility function and the stochastic processes.

5 A Specialized Economy

5.1 A Cobb-Douglas Utility Function

In what follows I will explore the characteristics of an equilibrium under a particular specification of the utility function and for the stochastic process of output and money. Taking the limit for $\Delta t \to 0$ of the discrete stochastic process for the output growth as in (4) and money supply (equation (10), we obtain the following respective continuous-time representation:

$$\frac{dY_t}{Y_t} = \mu_{y,t} dt + \sigma_{y,t} \Omega_{y,t} \sqrt{dt} \tag{49}$$

$$\frac{dM_t}{M_t} = \mu_{M,t} dt + \sigma_{M,t} \Omega_{M,t} \sqrt{dt} \tag{50}$$

where $\mu_{M,t}$ is defined as in (30), while the real money supply representation are still given by equations (13)-(15), with parameter definition given in (16)-(21).

Assume that the instantaneous utility function of the representative investor is given by the following Cobb-Douglas type:

$$u\left(C_t, \frac{M_t}{P_t}\right) = \frac{\left[C_t^{\eta} \left(\frac{M_t}{P_t}\right)^{1-\eta} \right]^{1-\varrho}}{1-\varrho} \tag{51}$$

where η is the fraction of utility derived from consumption C_t, and $1-\eta$ derived from real money balances $\frac{M_t}{P_t}$. Moreover, ρ indicates the inverse of the risk aversion, i.e. the intertemporal elasticity of substitution.

If we apply the results from the previous section we get the following set of results, condensed in the following Proposition:

Proposition 5 *Given the utility function* (51) *and the stochastic processes for output and money supply money supply given respectively by* (49) *and* (50) *we derive the following set of results:*
(i) The risk premium is:

$$\mu^{z}_{i,t} - R_t = [1-\eta(1-\rho)]\, cov_t\left(\frac{dp^{z}_{i,t}}{p^{z}_{i,t}}, \frac{dY_t}{Y_t}\right) - (1-\eta)(1-\rho)\, cov_t\left(\frac{dp^{z}_{i,t}}{p^{z}_{i,t}}, \frac{dm_t}{m_t}\right) \tag{52}$$

(ii) The Real Rate of Return is:

$$R_t = \beta - \eta\alpha_1\mu_y - \frac{\alpha_1\left[\eta(1-\rho)-2\right]\sigma^2_y}{2} - \alpha_2\mu_M - \frac{\alpha_2\left[\alpha_2-1\right]\sigma^2_M}{2} - \alpha_1\alpha_2\sigma_y\sigma_M\rho_{y,M} \tag{53}$$

where $\alpha_1 = \eta(1-\rho)-1$; $\alpha_2 = (1-\eta)(1-\rho)$.
(iii) The real price of equity share is:

$$p^{z}_{g,t} = \frac{Y_t}{\beta} \tag{54}$$

(iv) The dynamic of the real price of equity share is governed by:

$$\frac{dp^{a}_{e,t}}{p^{a}_{e,t}} = \mu_{y,t}dt + \sigma_{y,t}\Omega_{y,t}\sqrt{dt} \tag{55}$$

(v) The price level is:

$$P_t = \frac{\eta}{1-\eta}\left(\beta + \mu_{M,t} - \sigma^2_{M,t}\right)\frac{M_t}{Y_t} \tag{56}$$

(vi) The dynamic of the price level is:

$$\frac{dP_t}{P_t} = \pi_t dt + \sigma_{M,t}\Omega_{M,t}\sqrt{dt} - \sigma_{y,t}\Omega_{y,t}\sqrt{dt} \tag{57}$$

(vii) The inflation rate is:

$$\pi = \mu_M - \left(\mu_y - \sigma^2_y\right) - \sigma_y\sigma_M\rho_{y,M} \tag{58}$$

where $\rho_{y,M}$ *is the correlation coefficient between output and money, and* μ_M *is defined as in (31).*

Proof. See Appendix. ■

The reduced form solutions for the variance of the price level is given by:

$$var\left(\frac{dP_t}{P_t}\right) = \sigma^2_M + \sigma^2_y - 2\sigma_y\sigma_M\rho_{y,M} \tag{59}$$

Moreover, a closed form solution of $cov\left(\frac{dP_t}{P_t}, \frac{dY_t}{Y_t}\right)$ is:

$$cov\left(\frac{dP_t}{P_t}, \frac{dY_t}{Y_t}\right) = \sigma_y \sigma_M \rho_{y,M} - \sigma_y^2 \tag{60}$$

Recall from equation (12) that the volatility of money supply is entirely due to the volatility of M_2, the part of money which goes in the Government budget constraint. This implies that $\sigma_y \sigma_2 \rho_{y,2} = \psi\left[\sigma_y \sigma_i \rho_{y,i} - \phi_1 \sigma_y \sigma_T \rho_{y,T}\right]$.

Given these relationships, it is not difficult to find that the reduced form of the variance of the CPI is given by:

$$var\left(\frac{dP_t}{P_t}\right) = \psi^2\left(\sigma_i - \phi_1 \sigma_T\right)^2 + \sigma_y^2 - 2\psi\left(\sigma_y \sigma_i \rho_{y,i} - \phi_1 \sigma_y \sigma_T \rho_{y,T}\right) \tag{61}$$

From (61) we observe that fiscal policy parameters directly enter in the definition of the volatility of the stochastic process leading the growth in the price level: fiscal policy does not only have level effect, but also is of second order importance. Now it comes natural to ask: what is the impact of the fiscal policy parameter ϕ_1 on the volatility of the CPI ? From (61) we get that:

$$\frac{\partial\left[var\left(\frac{dP_t}{P_t}\right)\right]}{\partial \phi_1} \lesseqgtr 0 \Longleftrightarrow \psi\left(\sigma_i - \phi_1 \sigma_T\right)\sigma_T \gtreqless \sigma_y \sigma_T \rho_{y,T}$$

which is equivalent for establishing an upper bound for ϕ_1:

$$\frac{\partial\left[var\left(\frac{dP_t}{P_t}\right)\right]}{\partial \phi_1} \lesseqgtr 0 \Longleftrightarrow \phi_1 \lesseqgtr \frac{\psi\sigma_i - \sigma_y \rho_{y,T}}{\psi \sigma_T} = \overline{\phi_1}$$

Thus, the effect of ϕ_1 on the volatility of the price level strictly depends upon the magnitude chosen for ϕ_1 itself.

Let us look at the correlation between inflation and real asset return:

$$\begin{aligned} cov\left(\frac{dp^z_{g,t}}{p^z_{g,t}}, \frac{dP_t}{P_t}\right) &= cov\left(\frac{dM_t}{M_t}, \frac{dY_t}{Y_t}\right) - var\left(\frac{dY_t}{Y_t}\right) = \\ &= \sigma_y \sigma_M \rho_{y,M} - \sigma_y^2 \end{aligned} \tag{62}$$

Thus, by exploiting some of the above relationships, we get:

$$cov\left(\frac{dp^z_{g,t}}{p^z_{g,t}}, \frac{dP_t}{P_t}\right) = \psi\sigma_y \sigma_i \rho_{y,i} - \psi\phi_1 \sigma_y \sigma_T \rho_{y,T} - \sigma_y^2 = \tag{63}$$

$$= \psi\sigma_{yi} - \psi\phi_1 \sigma_{yT} - \sigma_y^2 \tag{64}$$

The covariance between the inflation and the growth rate of real asset pricing is negative if $\psi\sigma_{yi} < \psi\phi_1\sigma_{yT} + \sigma_y^2$. This condition is certainly verified because $\psi < 1$ by construction and in the data the correlation between output and nominal interest rate is much lower than the sum of the output volatility and the correlation between output and taxes multiplied by $\psi\phi_1$. In general, in the asset allocation practice, stocks with a negative correlation with the inflation rate will serve in hedging practice against the inflation. The negative correlation between the real stock price and the inflation rate has been carefully documented by several empirical studies, such as Fama (1981), Geske and Roll (1983) and Marshall (1992).

In particular, we can define the nominal spot rate as follows:

Proposition 6 *The nominal spot interest rate is given by:*

$$i_t = R_t + \pi_t - var\left(\frac{dP_t}{P_t}\right) + \rho\left(\sigma_y^2 - \sigma_M \sigma_y \rho_{M,y}\right) \tag{65}$$

Proof. To get (65), we can directly apply the definition of nominal interest rate obtained after reworking the result given in equation (44), taking the derivative of the utility function (51) and the result given in (60) and (57). ■

From these relationships it is not difficult to verify (by using the definitions given in (30) and (31)) the impact of a tighter fiscal policy on the mean of the stochastic process of money growth supply and nominal interest rate (from (65)): $\partial\mu_M/\partial\phi_1 < 0$ and $\partial i_t/\partial\phi_1 < 0$. Thus, a tighter fiscal policy implies a lower value for both nominal interest rate and the growth rate of money supply.

The approach taken here is very general: all the results nests the case with a separable utility (with logs of its arguments), by setting in all the above relationships $\rho = 1$, as it is common in the literature on the term structure.

To have an idea of the impact effect generated by the fiscal policy parameter ϕ_1 I reported in Figure 1 some static plots for the nominal interest rate, the inflation rate and the volatility of the inflation rate with respect to changes in parameter ϕ_1. To do so, I have chosen a parametrization of the model according to a recent contribution by Balduzzi (2000). The values of the parameters are collected in the following table:

μ_M	μ_R	μ_1	σ_M	σ_y	σ_R	σ_T	$\rho_{y,T}$	$\rho_{y,M}$
2.1	1.15	6.4	3.95	4.82	2.73	2.39	0.03	0.04

Table 1. Parameters for stochastic processes

β	η	ρ	ψ
0.998	0.5	2	0.1

Table 2. Preferences

The parameters collected in Table 1 and 2 are chosen in part from Balduzzi (2000) and in part from the contributions from RBC literature by Cooley and Prescott (1995). In particular, the parameters from Table 1 are the estimated means and stochastic processes for US economy, while parameters from Table 2 are standard. In particular, I have chosen a parameter for fiscal policy ϕ_1 equal to 0.55, which is lower than the upper bound provided by the Real Interest Rate, as suggested by Sims (1994).

By using the parameter values reported in Table 1 and 2 I conducted some static simulations whose results are reported in Figure 1, for the impact effect of changing ϕ_1 on the nominal rate of return i_t, the inflation rate π_t and the volatility of inflation rate $var(\pi_t)$. The picture here reported shows a static set of simulations, ins the sense that the temporal dimension is absent. This is done in order to convince the reader of the impact effect exercised by the fiscal policy parameter. The pictures show that if we raise ϕ_1, i.e. if we adopt a tighter fiscal policy, we observe a steady reduction of the nominal interest rate, the inflation rate and of the volatility of inflation. This result is perfectly in accord with the Fiscal Theory of the Price Level: the adoption of a tighter fiscal policy allows the government to reduce nominal rate payed on nominal bonds, which become more valuable, given the increase of reputation of the government.

The results showed in Figure 1 are somehow obvious and they can be also interpreted along a simple IS-LM from an undergraduate textbook. I decided to represent these results pictorially in order to show the internal coherence of the model.

To include the temporal dimension in the model, we need to add some additional assumptions on the driving stochastic processes. The price to pay for that is to use a simpler utility function.

6 A more realistic economy

In what follows I will introduce a general setup characterizing a more realistic economy setting from which I derive the main equilibrium properties, such as nominal and real term structure, along the same lines discussed previously. The setup chosen here and the relative solution method is close to the work done by Bakshi and Chen (1996) and Balduzzi (2000). It is worth to remember that the approach taken here is in line with the equilibrium models of the term structure of nominal interest rates[4] *à la* Cox, Ingersoll and Ross (1985).

In order to simplify algebra and to have closed form solutions of the model, let us assume a strongly separable log utility function for the representative agent, given by:

$$u\left(C_t, \frac{M_t}{P_t}\right) = \phi \log C_t + (1-\phi)\log m_t \tag{66}$$

where $m_t = M_t/P_t$.

Let us consider now a general stochastic process for output given by:

$$dY_t = (\mu_Y + \eta_Y x_t)\, dt + \sigma_Y \sqrt{x_t} dW_{x,t} \tag{67}$$

where the driving force is represented by the technology factor x_t for which I assume the following mean-reverting diffusion process:

$$dx_t = a\,(b - x_t)\, dt + \sigma_x \sqrt{x_t} dW_{x,t} \tag{68}$$

where $(W_{x,t})_t$ is a unidimensional $\mathbb{Q}$-Brownian motion, and μ_Y, η_Y, σ_Y, a, b, and σ_x are fixed real numbers.

Given the definition of total money supply as in (7), I specify, two different stochastic processes for nominal monetary aggregates M_{1t} and M_{2t}:

$$d\ln M_{1t} = \mu_1^* dt + d\ln(q_t) \tag{69}$$

$$d\ln M_{2t} = \overline{\mu}_2 dt + d\ln(q_t) \tag{70}$$

[4] A similar model can be solved following a more general methodology described in a companion paper by Marzo (2001).

where q_t is the detrended money supply process. Basically, from (69)-(70) we see that the stochastic process for the two types of money supply has two components: a drift term and a stochastic component q_t given by the detrended component. In particular, μ_1^* is assumed to be constant and positive, while $\overline{\mu}_2$ is the drift term for M_{2t}, which will be endogenized by using the Government Budget Constraint. Following Bakshi and Chen (1996), q_t is assumed to follow the stochastic process:

$$\frac{dq_t}{q_t} = k_q \left(\mu_q - q_t\right) dt + \sigma_q \sqrt{q_t} dW_{i,t}, \quad i = 1, 2 \tag{71}$$

where $(W_{i,t})_t$ is a unidimensional $\mathbb{Q}$-Brownian motion independent upon $(W_{x,t})_t$, for each of the two monetary aggregates M_{1t} and M_{2t}.

Therefore, by using the definition of money supply (7) and the above equations (69)-(70) and (71), we find the stochastic process leading nominal money supply given by:

$$\frac{dM_t}{M_t} = \mu_{M,t} dt + \sigma_q \sqrt{q_t} dW_{M,t} \tag{72}$$

where $(W_{M,t})_t$ is a unidimensional $\mathbb{Q}$-Brownian motion independent upon $(W_{x,t})_t$ and $(W_{i,t})_t$. Following exactly the same steps described to get equations (**??**)-(29) and (30)-(**??**), we now obtain the following expressions for the drift term of the growth rate of nominal money supply:

$$\mu_M = \mu_M^* + 2k_q \left(\mu_q - q_t\right) \tag{73}$$

with: $\mu_M^* = \mu_1^* + \sigma_{pM} + \overline{\mu}_2$, and $dW_{M,t} = dW_{1,t} + dW_{2,t}$. Where $\overline{\mu}_2$ is given by:

$$\overline{\mu}_2 = \frac{\left(\mu_R - \phi_1 + \psi \sigma_q^2\right)}{1 + \psi} \tag{74}$$

At this stage, equipped with the above assumptions, we can derive the main results about the stochastic process for the commodity price level and the inflation process. The following Proposition collects the results of interest:

Proposition 7 *Given the utility function of the representative agent as described by equation* (66)*, then the equilibrium price level is given by:*

$$P_t^c = \frac{\phi}{1-\phi} \frac{q_t^2 \left(\beta + \mu_M^*\right) \left(\beta + \mu_M^* + 2k_q \mu_q\right)}{\left(\beta + \mu_M^*\right) + \left(k_q + 3\sigma_q^2\right) q_t 2k_q \mu_q} \frac{M_t}{Y_t} \tag{75}$$

The stochastic process of the CPI is given by:

$$\frac{dP_t^c}{P_t^c} = \pi_t dt + \sigma_q \sqrt{q_t} \left[1 + \frac{\left(\Delta_q \Psi - \Delta \Psi_q\right)}{\Delta \Psi} q_t\right] dW_{M,t} - \sigma_y \sqrt{x_t} dW_{x,t} \tag{76}$$

where the inflation rate is given by:

$$\begin{aligned} \pi_t \quad = \quad & \mu_M^* - \mu_y + \left(\sigma_y^2 - \eta_y\right) x_t + \frac{\left(\Delta_q \Psi - \Delta \Psi_q\right)}{\Delta \Psi} q_t \left(k_q \left(\mu_q - q_t\right) + \frac{\sigma_q^2 q_t}{2}\right) + \\ & + \frac{\left[2\left(\Delta_{qq} \Psi - \Delta \Psi_{qq}\right) - \Delta_q \Psi + \Delta \Psi_q\right] \sigma_q^2 q_t^{3/2}}{2 \Psi^2 \Delta} \end{aligned} \tag{77}$$

with:

$$\Delta(q) = \frac{\phi}{1-\phi}\left[q_t^2(\beta+\mu_M^*)\left(\beta+\mu_M^*+2k_q\mu_q\right)\right] \tag{78}$$

$$\Psi(q) = (\beta+\mu_M^*)+\left(k_q+3\sigma_q^2\right)q_t 2k_q\mu_q \tag{79}$$

and $\Delta_q = \frac{\partial\Delta(q)}{\partial q}$; $\Psi_q = \frac{\partial\Psi(q)}{\partial q}$; $\Delta_{qq} = \frac{\partial\Delta(q)}{\partial q}$.

Proof. See Appendix.

Both the price level (equation (75)) and especially the expression of the inflation rate given in equation (77) are function of both fiscal and monetary parameters in a very complicated way. In the section dedicated to the simulation analysis I will provide an intuitive explanation of the pattern behavior of such variables.

The real price of the equity share is given by the following equation for a logarithmic utility function:

$$p_{z,t} = \frac{Y_t}{\beta}$$

So, in this case, we get that the price of the equity share is:

$$\begin{aligned} P_{z,t} &= p_{z,t}P_t^C = \frac{Y_t}{\beta}P_t^C = \\ &= \frac{\phi}{1-\phi}\frac{q_t^2(\beta+\mu_M^*)\left(\beta+\mu_M^*+2k_q\mu_q\right)}{(\beta+\mu_M^*)+\left(k_q+3\sigma_q^2\right)q_t 2k_q\mu_q}M_t \end{aligned}$$

In the same way, it is not difficult to find the real money demand function, which, by using equation (75):

$$m_t^d = \frac{1-\phi}{\phi}\left[\frac{(\beta+\mu_M^*)+\left(k_q+3\sigma_q^2\right)q_t 2k_q\mu_q}{q_t^2(\beta+\mu_M^*)\left(\beta+\mu_M^*+2k_q\mu_q\right)}\right]Y_t \tag{80}$$

We can now ask what is the impact of changes in the growth rate of money supply in the real demand function as described by equation (80). Thus:

$$\frac{\partial m_t^d}{\partial\mu_M^*} = -Y_t\left(\frac{1-\phi}{\phi}\right)\left[\frac{(\beta+\mu_M^*)^2+\left(k_q+3\sigma_q^2\right)q_t 4k_q\mu_q(1+\beta+\mu_M^*)}{(\beta+\mu_M^*)^2\left(\beta+\mu_M^*+2k_q\mu_q\right)^2}\right]$$

As in Bakshi and Chen (1996), an increase in the expected growth rate of money supply leads to a decrease in the demand for real money balances. This is due to the discounted effect of an higher price level. In fact, an higher growth rate of money implies an higher expected inflation effect, which implies a reduction in real money balances. On the other hand, the effect of a tighter fiscal policy on money demand is exactly the opposite of the effect on money demand, due to the increase of the growth rate of the money supply. In fact:

$$\frac{\partial m_t^d}{\partial\phi_1} = -\frac{\phi}{(1-\phi)}\left[\frac{\partial m_t^d}{\partial\mu_M^*}\right] > 0$$

a tighter fiscal policy will positively affect real money balances, through the reduction of the expected inflation rate. In fact, we have already seen that a tighter fiscal policy implies a reduction on the price level and, consequently, a reduction of the expected inflation rate. This increases the money demand and depresses the demand for interest bearing assets.

What are the consequences on money velocity v_t ?

Let us consider the following definition of money velocity:

$$v_t = \frac{M_t}{P_t Y_t} = \frac{1-\phi}{\phi}\left[\frac{(\beta+\mu_M^*)+\left(k_q+3\sigma_q^2\right)q_t 2k_q\mu_q}{q_t^2\left(\beta+\mu_M^*\right)\left(\beta+\mu_M^*+2k_q\mu_q\right)}\right]$$

As in Bakshi and Chen (1996), it is not difficult to verify that money velocity is strictly increasing in the drift of the diffusion process of money supply μ_M^*. However, money velocity is a decreasing function of the technological parameter q_t:

$$\frac{\partial v_t}{\partial q_t} = \frac{-(1-\phi)}{\phi q_t^3\left(\beta+\mu_M^*\right)\left(\beta+\mu_M^*+2k_q\mu_q\right)} < 0$$

It is worth to remark that the technology here is linked to output productivity and does not have to be intended as a technology to speed up the transactions. The stochastic process leading money velocity is given by:

$$\frac{dv_t}{v_t} = \left[\theta_1 - \theta_2 k_q\left(\mu_q - q_t\right)\right]dt + \theta_3\sigma_q q_t^{-1/2} dW_{M,t} \tag{81}$$

where:

$$\begin{aligned}
\theta_1 &= \frac{2\left[2k_q\mu_q q_t\left(k_q+3\sigma_q^2\right)+3\left(\beta+\mu_M^*\right)\right]\sigma_q^2}{\left[(\beta+\mu_M^*)+\left(k_q+3\sigma_q^2\right)q_t 2k_q\mu_q\right]q_t}\\
\theta_2 &= \frac{2\left[\beta+\mu_M^*+k_q\mu_q q_t\left(k_q+3\sigma_q^2\right)\right]}{\left[(\beta+\mu_M^*)+\left(k_q+3\sigma_q^2\right)q_t 2k_q\mu_q\right]q_t}\\
\theta_3 &= \frac{2\left[\beta+\mu_M^*+k_q\mu_q q_t\left(k_q+3\sigma_q^2\right)\right]\sigma_q q_t^{-1/2}}{(\beta+\mu_M^*)+\left(k_q+3\sigma_q^2\right)2k_q\mu_q}
\end{aligned}$$

The stochastic process for money velocity given by equation (81), has a central tendency toward the drift of the diffusion process of the technology.

With these results at hand, we can easily find the covariance between the real stock prices and the inflation rate. Let us start by considering the following findings:

$$\begin{aligned}
cov_t\left(\frac{dp_{z,t}}{p_{z,t}},\frac{dP_t}{P_t^C}\right) &= cov_t\left(\frac{dY_t}{Y_t},\frac{dP_t}{P_t^C}\right) =\\
&= 1+\frac{(\Delta_q\Psi-\Delta\Psi_q)}{\Delta\Psi}q_t cov_t\left(\frac{dY_t}{Y_t},\frac{dM_t}{M_t}\right) - var\left(Y_t\right) =\\
&\leq cov_t\left(\frac{dY_t}{Y_t},\frac{dM_t}{M_t}\right) - var_t\left(\frac{dY_t}{Y_t}\right)
\end{aligned}$$

According to (81), we find that the correlation between the real stock price and the inflation rate is negative if the volatility of output growth rate is bigger than the correlation between the growth rate of output and the drift term of the diffusion process of the money supply. In general, (at least for US data), it is true that $var_t\left(\frac{dY_t}{Y_t}\right) > cov_t\left(\frac{dY_t}{Y_t},\frac{dM_t}{M_t}\right)$. In particular, this is especially true for the recent sample periods where the correlations between the growth rate of nominal money supply and the growth rate of output has been diminishing for a considerable amount of time.

The next steps include the analysis of the term structure of both real and nominal interest rates.

7 The Term Structure of Interest Rates

In what follows I will derive the analytical expressions for both nominal and real terms structure equations with some related results.

7.1 The Real Term Structure

Let us start by establishing the real price of a pure discount bond.

Proposition 8 *The equilibrium real interest rate in the continuous time limit is given by:*

$$R_t = \beta + \mu_y + \left(\eta_y - \sigma_y^2\right) x_t \tag{82}$$

Also, the real price of a pure discount bond that pays one unit of the good in τ periods is given by:

$$h(t,\tau) = \exp\left[-d_1(\tau) - d_2(\tau) x_t\right] \tag{83}$$

where:

$$\begin{aligned}
d_1(\tau) &= \left(\beta + \mu_y\right)\tau + \frac{2ab}{\sigma_x^2}\left\{\ln\left[1 + \frac{\left(1 - e^{-d_3\tau}\right)\left(a + \sigma_x\sigma_y - d_3\right)}{2d_3}\right] + \frac{\tau}{2}\left[d_3 - \left(a + \sigma_x\sigma_y\right)\right]\right\} \\
d_2(\tau) &= \frac{2\left(\eta_y - \sigma_y^2\right)\left(1 - e^{-d_3\tau}\right)}{2d_3 + \left[a + \sigma_x\sigma_y - d_3\right]\left(1 - e^{-d_3\tau}\right)} \\
d_3 &= \sqrt{\left(a + \sigma_x\sigma_y\right)^2 + 2\sigma_x^2\left(\eta_y - \sigma_y^2\right)}
\end{aligned}$$

Proof. See Appendix.

From Proposition 8, we get that the real spot rate is increasing in both the time preference parameter β and the expected output growth. By applying Ito's lemma to equation (82) we get the following dynamic equation for the real rate:

$$dR_t = a\left(\gamma_R - R_t\right)dt + \sigma_R\sqrt{x_t}dW_{x,t} \tag{84}$$

where $\gamma_R \equiv \beta + \mu_y + \left(\eta_y - \sigma_y^2\right)$, and $\sigma_R \equiv \left(\eta_y - \sigma_y^2\right)\sigma_x$. From these results we get that the real interest rate follows a mean reverting square root process, with the same speed of adjustment, but with a different long run mean. Additionally, can now properly define the drift term of the stochastic process of the real interest rate entering also in equations (30) and (31). Thus:

$$\begin{aligned}
\mu_R &= a\left(\gamma_R - R_t\right) = \\
&= a\left(\eta_y - \sigma_y^2\right)\left(1 - x_t\right)
\end{aligned} \tag{85}$$

where I made use the definition of γ_R. Thus, through μ_R given in (85) in we have that the technology process parameters affect also the nominal equilibrium of the economy, as displayed in (30) and (31). This fact is independent upon the choice of the utility function, and the link is provided by the role of the Government Budget Constraint. This is in accord with the traditional models of the term structure of interest rates, but is in contrast with the results from Bakshi and Chen (1996), where nominal and real term structures appear to be totally detached one from the other.

Given the above results it is not difficult to obtain the equation for the term structure for real interest rates $r(t,\tau)$:

$$r(t,\tau) = \frac{d_1(\tau)}{\tau} + \frac{d_2(\tau)}{\tau} x_t \tag{86}$$

with $d_1(\tau)$ and $d_2(\tau)$ defined as in Proposition 8.

The patterns for the term structure for real interest rates $r(t,\tau)$ defined in equation (86) will be visualized with the help of simulations, after having assigned certain parameters values.

7.2 The nominal term structure

Given the properties of the utility function under (66), we get the following results for the nominal term structure of interest rate, collected in the following lemma:

Lemma 9 *In the continuous time limit the nominal interest rate is:*

$$i_t = \frac{q_t^2 (\beta + \mu_M^*) \left(\beta + \mu_M^* + 2k_q\mu_q\right)}{(\beta + \mu_M^*) + \left(k_q + 3\sigma_q^2\right) q_t 2k_q\mu_q} \tag{87}$$

The nominal price of a discount bond paying one dollar in τ periods is given by:

$$\begin{aligned} Nb(t,\tau) &= f_1 \left\{ f_2 e^{-2k_q\mu_q\tau} \frac{2k_q^2\mu_q^2}{q_t} (1 - f_2) + 2f_2^2 k_q^2 \mu_q^2 \right\} + \\ &\quad + f_1 \frac{(\beta + \mu_M^*)}{q_t^3} e^{-3k_q\mu_q\tau} (1 - f_2) \end{aligned} \tag{88}$$

where:

$$\begin{aligned} f_1(\tau) &= \frac{\exp\left[-(\beta + \mu_M^*)\tau\right]}{(\beta + \mu_M^*) + \left(k_q + 3\sigma_q^2\right) 2k_q\mu_q} q_t^3 \\ f_2 &= \frac{\left(k_q + 3\sigma_q^2\right)}{k_q\mu_q} \end{aligned}$$

Proof. See the Appendix.

From the above results, we easily get the following dynamic equation for the nominal interest rate, by applying Ito's Lemma to equation (87):

$$\frac{di_t}{i_t} = \mu_{i,t}(q)\, dt + \sigma_{i,t}(q)\, \sigma_q q_t^{1/2} dW_{M,t}$$

where

$$\begin{aligned} \mu_{i,t}(q) &= \left[\frac{(2g_0 + g_1) k_q \left(\mu_q - q_t\right)}{(g_0 + g_1 q_t)} + \frac{\left[2g_0 (1 - q_t) + 2g_1 q_t - g_1 q_t^2\right]}{(g_0 + g_1 q_t)^2} \sigma_q^2 \right] \\ \sigma_{i,t}(q) &= \frac{(2g_0 + g_1 q_t)}{(g_0 + g_1 q_t)} \sigma_q q_t^{1/2} \end{aligned}$$

Given the more complex structure of the model assumed here, it is not easily possible to link the structure of the above process to standard stochastic processes for the nominal interest rates, as f.e. is done in Bakshi and Chen (1996). For this reason, in the following section I am going to consider some simulations to highlight the main relationships existing among the variables.

Finally, the nominal term structure of interest rates is given by:

$$N(t,\tau) = -\frac{1}{\tau}\ln[Nb(t,\tau)] \tag{89}$$

with $Nb(t,\tau)$ given in equation (88).

8 Simulation Results

The results here reported in an analytic form can be better understood by considering some simulations which will shed light on the mechanics of the transmission channels among the variables. To do so, I decided to parametrize the model by following the work by Balduzzi (2000) who estimated the parameters of the stochastic process leading the model by using methods of moments. The parameters chosen for this model are collected in Table 3:

ϕ	k_q	μ_q	a	b	μ_Y	η_Y	σ_Y	σ_x	σ_q
0.01	0.56	0.11	0.86	0.06	1.3	0.7	0.06	0.03	0.02

Table 3

The parameter β is set equal to 0.998, as considered in the previous model, and the fiscal policy parameter is set equal to $\phi_1 = 0.55$. However, it should be remembered that the main goal of the present paper is to show the joint role of monetary and fiscal policy parameters in the determination of the term structure of interest rates. Thus, I do not pretend that the model parametrization here considered is the best choice in absolute terms, rather it should be regarded as a working hypothesis. The search for the model which best could fit the data is not the goal of the present work.

In order to run the simulations, I integrated the diffusion processes for money supply, output growth, technology and the stochastic process describing the stochastic trend in money supply, and I initialized them by setting an initial condition equal to $x_0 = Y_0 = M_0 = q_0 = 1$.

In figures 3-4 I reported the pictures to detect the effect of changes in these the price level and the inflation rate for changes in the fiscal policy parameter ϕ_1 and the monetary policy parameter μ_1. The simulations reported in Figures 3 and 4 are conducted within an *a-temporal* context, without considering the intertemporal dimension. This is done in order to show the impact effect on both the price level and the inflation rate induced by changes in parameters ϕ_1 and μ_1. In Figure 3 we observe that a progressive increase in ϕ_1 creates a steady reduction of the level of the price index and, at the same time, of the inflation rate. Thus, even in this case with a more complex structure we find a confirmation of the results found in the previous section: a tighter fiscal policy has a deflationary impact. Differently from the simple case considered before, these results cannot be observed by directly referring to the equations in the model, so the simulation well visualize the impact effect due to fiscal and monetary policy.

In Figure 4, I show the effects on the same variables after changing the monetary policy parameter μ_1, keeping the fiscal policy parameter fixed at 0.55, as benchmark parameter. In this case, we observe an almost reversed effect: the increase of the drift term of the stochastic process of money supply growth held by private agents has an inflationary impact, as expected. These static simulations are coherent with what we would expect from a simple macroeconomic model.

When we include the temporal dimension into the model the results are more interesting. In Figures 5-7 I reported the pictures for the nominal and real spot interest rate, together with the nominal term structure, for different values of the monetary and fiscal policy parameters.

In Figure 5, I considered how the change in the drift of the diffusion process affects the nominal curve and the position of the term structure of interest rate[5]. In both panels reported in Figure 5, the continuous dark line is drawn for the benchmark values of the drift for the diffusion process, by using for μ_1 a value equal to $\mu_1 = 6.4$. The dashed line is drawn for a lower value such as $\mu_1 = 5$, while the dot-dashed line is drawn for $\mu_1 = 8$.

Let us concentrate on the top panel. The dotted line represents the spot real interest rates, which is not influenced at all by changes in the monetary policy parameter, given the utility function assumed in (66). If we concentrate on the nominal curves, we observe that position of the spot cure in the plane is affected by the size of μ_1. In particular, a monetary contraction, given by a reduction in μ_1 shifts the curve of nominal rate downwards (dot-dashed line), while a monetary expansion shifts the curve upwards (dashed line). These results are due to the expected inflation effect: a monetary expansion today will imply an increase of the expected inflation (see equation (77)), this will make government bonds less attractive, making their price decreasing and the nominal interest rate increasing.

In the bottom panel of Figure 5 I reported the effects of changing the drift of the diffusion process of the growth rate of nominal money supply μ_1 in the term structure of nominal interest rates. In particular, we observe that even in this case different values of the parameter μ_1 shifts the position of the term structure equation: contractionary monetary policy shifts the entire term structure downwards (dotted/dashed line), while a monetary policy expansion moves the curve upwards. The results showed in Figure 5 are coherent with the traditional approaches for the term structure highlighted by Cox, Ingersoll and Ross (1985): the term structure depends from both technological parameters and monetary policy parameters. However, this model has an additional feature: movements of the term structure and of the spot curve of nominal rates can be also justified by looking at the impact of the fiscal policy parameter ϕ_1. It is clear, however, that all these movements are due to the expected change in a particular monetary policy or fiscal policy parameter.

As before, in Figure 6, I have reported the curve of the nominal spot rates for different values of the fiscal policy parameter ϕ_1. In particular, the dark line is drawn for a values of $\phi_1 = 0.55$, assumed to be the benchmark value[6]. Moreover, the dashed line is drawn for $\phi_1 = 0.55$, the dashed dotted line is for $\phi_1 = 0.55$, the dotted line is for $\phi_1 = 0.55$. The bold line indicates the real rates, not influenced by fiscal policy parameters,a as discussed previously for monetary policy parameter. Basically, if we expects the Government to adopt a tighter fiscal policy, by making the level of taxes reacting more vigorously

[5] The simulation goes for 9 periods with a step of 0.01.

[6] In all the simulations reported in Figures 6 and 7, I assumed a benchmark value for the monetary policy parameter, set equal to its estimated mean value, given by $\mu_1 = 6.4$.

to the outstanding level of public debt, (ϕ_1 raises), the spot rate curve will move down. Instead, when we expect a softer fiscal policy (low ϕ_1), the curve will shift up. This is because the market will require an higher level of the nominal interest rate in order to be willing to buy bonds, issued by a government which adopted a loose fiscal policy, in order to compensate the investor from expected capital losses due to solvency troubles of the government. Instead, if fiscal policy is expected to be tight (high level of ϕ_1), the lower level of the curve is implied by the increase in the price of government bonds, which now are more valuable. This implies a reduction of nominal interest rate paid on the outstanding level of government bonds.

Similar considerations can be done for the term structure curve represented in Figure 7. The picture shows that the position in the plane of the term structure strictly depends upon the value assumed for ϕ_1. In particular, it is evident that if the fiscal pressure decreases (i.e. if ϕ_1 decreases), the curve of the nominal term structure zero coupon bond shifts down.

The picture shows that the position in the plane of the term structure strictly depends upon the value assumed for ϕ_1. In particular, it is evident that if the fiscal parameter ϕ_1 tax rate decreases, the curve of the nominal term structure shifts up, even if for very high values of ϕ_1 the curve appears to loose its sensitiveness to this parameter. The behavior of the curve is coherent with both the results from Fiscal Theory and the empirical evidence. In fact, if the tax rate raises, this means that it will be possible to reduce the number of issue of new public debt in order to finance the current position of the government. This will call for a lower interest rate, and for a bigger government credibility.

These results clearly show that the position of the term structure in the plane depends also upon fiscal policy parameters in the same way as for monetary policy parameters. Thus, both fiscal and monetary policy are important for position of the term structure, on the plane. Empirically, this has been verified as true in the experience of Italy after 1996-1997 episode of fiscal retrenchment which shifted down the position of the nominal term structure after a crucial fiscal consolidation, necessary to get into the EU. These results have extended the traditional term structure approach by Cox, Ingersoll and Ross (1985a,b), and Bakshi and Chen (1996) by including a specific role for fiscal policy, by exploiting the role of the government budget constraint.

9 Concluding Remarks

In this paper I addressed a basic question: given the importance of fiscal retrenchment episodes in the recent past of some European countries and observed contemporaneous drop of nominal interest rates, is it possible to construct a general equilibrium model for the term structure of interest rate which would depend upon both monetary and fiscal policy parameters ? Borrowing many features from Bakshi and Chen (1996) and Balduzzi (2000), I constructed a model which, through the explicit design of fiscal policy as suggested by the contributions of the Fiscal Theory of the Price Level (FTPL), shows that inflation is not necessarily monetary phenomena. In this framework, fiscal policy together with monetary policy plays a crucial rule in the determination of the position of the nominal term structure. An expected tighter fiscal policy shifts down the nominal term structure curve and the nominal spot curve. In an extreme simple case with a strongly separable utility function in both output and real money balances, nominal and real term structures depend upon different set of parameters: the real curve depends only upon the technological factors, but not upon policy (monetary and fiscal) policy parameters. However, the nominal curve depends also upon the parameters which determine the drift of the stochastic process of the real interest rate.

All the variables of the model (such as price level inflation, nominal and real interest rates) were jointly derived as function of the 'core' parameter of the economy, within a general equilibrium approach. The results appears to be in line with the recent experience of some countries, like f.e. Italy, which after a fiscal retrenchment and the adoption of particular fiscal policy rules, showed a marked reduction in nominal interest rates.

These results generalize two types of literature that were considered independent so far: the literature on the term structure and asset prices in a general equilibrium framework and the Fiscal Theory of the Price Level. Given the simplicity and the generality of the model, there is enough room to further generalize the model to allow different diffusion processes leading the money growth supply, output and technology, as those assumed by the various models of the term structure of interest rates.

Appendix

Proof of Proposition 1.

Subtract equation (36) from equation (39). Then manipulate the resulting expression and use the definition of the stochastic process for $p_{i,t}^{z}$ given in (5) to get:

$$e^{-\beta\Delta t}E_t\left\{\frac{u_c\left(Y_{t+\Delta t},m_{t+\Delta t}\right)}{u_c\left(Y_t,m_t\right)}\left[\left(\mu_{i,t}^{z}-R_t\right)\Delta t+\sigma_{i,t}^{z}\Omega_{i,t}^{z}\sqrt{\Delta t}\right]\right\}=0 \tag{90}$$

Take now a Taylor expansion of the equation (90) around the steady state, we obtain:

$$\begin{aligned}e^{-\beta\Delta t}E_t\Bigg\{\left[\left(\mu_{i,t}^{z}-R_t\right)\Delta t+\sigma_{i,t}^{z}\Omega_{i,t}^{z}\sqrt{\Delta t}\right]\cdot\Bigg[1+\frac{u_c\left(Y_{t+\Delta t},m_{t+\Delta t}\right)Y_t}{u_c\left(Y_t,m_t\right)}\frac{\Delta Y_t}{Y_t}+\\+\frac{u_{cm}\left(Y_{t+\Delta t},m_{t+\Delta t}\right)m_t}{u_c\left(Y_t,m_t\right)}\frac{\Delta m_t}{m_t}\Bigg]\Bigg\}\frac{1}{\Delta t}=0\end{aligned} \tag{91}$$

letting $\Delta t\rightarrow 0$ in (91) and applying the Ito's multiplication rule, we easily derive (42). ■

Proof of Proposition 2.

Rewrite the first order condition (37) as follows:

$$\frac{1}{P_t}=e^{-\beta\Delta t}E_t\left\{\left[\frac{u_c\left(Y_{t+\Delta t},m_{t+\Delta t}\right)}{u_c\left(Y_t,m_t\right)}\frac{1}{P_{t+\Delta t}}+\frac{u_m\left(Y_{t+\Delta t},m_{t+\Delta t}\right)}{u_c\left(Y_t,m_t\right)}\right]\frac{\Delta t}{P_{t+\Delta t}}\right\} \tag{92}$$

then, iterate equation (92) to get:

$$\frac{1}{P_t}=E_t\left\{\sum_{j=1}^{\infty}e^{-\beta(j\Delta t)}\frac{u_m\left(Y_{t+j\Delta t},m_{t+j\Delta t}\right)}{u_c\left(Y_t,m_t\right)}\frac{\Delta t}{P_{t+j\Delta t}}\right\} \tag{93}$$

Taking the limit of the equation (93) we get the result under (43).

To obtain the expression for the inflation rate, divide the two first order conditions (36) and (38), to get:

$$E_t\left\{\frac{u_c\left(Y_{t+\Delta t},m_{t+\Delta t}\right)}{u_c\left(Y_t,m_t\right)}\left(1+R_t\Delta t\right)\right\}=E_t\left[u_c\left(Y_{t+\Delta t},m_{t+\Delta t}\right)\left(1+i_t\Delta t\right)\frac{P_t}{P_{t+\Delta t}}\right] \tag{94}$$

Expand equation (94), in Taylor's series to get, after rearrangement:

$$\left(i_t-R_t\right)\Delta t=\left[1+\frac{u_{cc}Y_t}{u_c}\left(\frac{\Delta Y_t}{Y_t}\right)+\frac{u_{cm}}{u_c}m_t\left(\frac{\Delta m_t}{m_t}\right)\right]\left[\frac{\Delta P_t}{P_t}-\left(\frac{\Delta P_t}{P_t}\right)^2\right]+o\left(\Delta t\right)^{3/2} \tag{95}$$

Thus, by taking the limit of equation (95), for $\Delta t\rightarrow 0$ we have:

$$i_t-R_t=\frac{1}{dt}E_t\left\{\frac{dP_t}{P_t}\right\}-var_t\left\{\frac{dP_t}{P_t}\right\}+\frac{u_{cc}Y_t}{u_c}cov_t\left(\frac{dY_t}{Y_t},\frac{dP_t}{P_t^C}\right)+\frac{u_{cm}m_t}{u_c}cov_t\left(\frac{dP_t}{P_t},\frac{dm_t}{m_t}\right)$$

Thus, by using $\pi_t=\frac{1}{dt}E_t\left\{\frac{dP_t}{P_t}\right\}$, and rearranging we easily get the equation (44). ■

Proof of Proposition 3.

Consider the First Order Condition in equation (36) and take a Taylor expansion around the equilibrium, to get:

$$u_c(Y_t,m_t)(1+R_t\Delta t) = E_t\left\{\left[u_c(Y_t,m_t)+u_{cc}(Y_t,m_t)\Delta Y_t+u_{cm}(Y_t,m_t)\Delta m_t+\frac{u_{ccc}(Y_t,m_t)}{2}(\Delta Y_t)^2+\right.\right.$$
$$\left.\left.+\frac{u_{ccm}(Y_t,m_t)}{2}\Delta Y_t\Delta m_t+\frac{u_{cmm}(Y_t,m_t)}{2}(\Delta m_t)^2\right](1+R_t\Delta t)\right\}+o(\Delta t)^{3/2} \tag{96}$$

Thus, simplifying, we get:

$$R_t = \beta-\frac{1}{\Delta t}E_t\left\{\frac{u_{cc}(Y_t,m_t)Y_t}{u_c(Y_t,m_t)}\left(\frac{\Delta Y_t}{Y_t}\right)+\frac{u_{cm}m_t}{u_c}\left(\frac{\Delta m_t}{m_t}\right)+\frac{u_{ccc}Y_t^2}{2u_c}\left(\frac{\Delta Y_t}{Y_t}\right)^2+\right.$$
$$\left.+\frac{u_{ccm}Y_tm_t}{2u_c}\left(\frac{\Delta Y_t}{Y_t}\right)\left(\frac{\Delta m_t}{m_t}\right)+\frac{u_{cmm}Y_tm_t^2}{2u_c}\left(\frac{\Delta m_t}{m_t}\right)^2\right\}+o(\Delta t)^{3/2} \tag{97}$$

Recall that:

$$\begin{aligned}\frac{1}{dt}E_t\left\{\left(\frac{dY_t}{Y_t}\right)^2\right\} &= var_t\left(\frac{dY_t}{Y_t}\right);\\ \frac{1}{dt}E_t\left\{\left(\frac{dm_t}{m_t}\right)^2\right\} &= var_t\left(\frac{dm_t}{m_t}\right);\end{aligned}$$

If we now take the limit of equation (97) for $\Delta t\to 0$ and apply the Ito's Lemma, we finally get (45). ■

Proof of Proposition 4.

Subtract First Order Condition (38) from (37) to get:

$$E_t\left[\frac{u_m(Y_{t+\Delta t},m_{t+\Delta t})}{u_c(Y_t,m_t)}\frac{P_t}{P_{t+\Delta t}}\right]=E_t\left[\frac{u_c(Y_{t+\Delta t},m_{t+\Delta t})}{u_c(Y_t,m_t)}\frac{P_t}{P_{t+\Delta t}}i_t\right] \tag{98}$$

Consider now a Taylor expansion of the price ratio, to get:

$$\frac{P_t}{P_{t+\Delta t}}=1-\frac{\Delta P_t}{P_t}+\left(\frac{\Delta P_t}{P_t}\right)^2+o(\Delta t)^{3/2} \tag{99}$$

Expand also in Taylor series the LHS of equation (98) so that:

$$\begin{aligned}E_t\left[\frac{u_c(Y_{t+\Delta t},m_{t+\Delta t})}{u_c(Y_t,m_t)}\frac{P_t}{P_{t+\Delta t}}i_t\right] &= E_t\left\{\underbrace{\left[1+\frac{u_{cc}Y_t}{u_c}\left(\frac{\Delta Y_t}{Y_t}\right)+\frac{u_{cm}m_t}{u_c}\left(\frac{\Delta m_t}{m_t}\right)\right]}\left(1-\frac{\Delta P_t}{P_t}\right)\right.\\ &\quad\left.+o(\Delta t)^{3/2}\right\}i_t\end{aligned} \tag{100}$$

Note that the term underbraced in the first line of equation (100) tends to 1, as $\Delta t\to 0$, after having applied the Ito's multiplication rule. Moreover, we have that:

$$\lim_{\Delta t\to 0}\left\{\frac{u_m(Y_{t+\Delta t},m_{t+\Delta t})}{u_c(Y_t,m_t)}\frac{P_t}{P_{t+\Delta t}}\right\}=\frac{u_m(Y_t,m_t)}{u_c(Y_t,m_t)} \tag{101}$$

which proves the proposition. ■

Proof of Proposition 5

(i) To prove result (52) it is enough to apply the result in Proposition 1, by substituting the derivative of the utility function into their respective expressions.

(ii) To get real rate in (53) we just need to apply the results from Proposition 2, after having plugged the expression for the derivative into equation (45).

(iii) To prove result (53) recall that from Proposition 2 and equation (43) we have a formula directly usable for the present case. Taking the appropriate derivatives of the utility function (51) in equation (43) we get:

$$p_{g,t}^{z} = E_t \int_t^{\infty} e^{-\beta(s-t)} \frac{C_s^{\eta} m_s^{1-\eta}}{C_t^{\eta-1} m_t^{1-\eta}} ds = \tag{102}$$

$$= \frac{1}{Y_t^{\eta-1} m_t^{1-\eta}} E_t \int_t^{\infty} e^{-\beta(s-t)} Y_s^{\eta} m_s^{1-\eta} ds \tag{103}$$

where I also inserted the equilibrium condition $C_t = Y_t$. After taking the integral of (103) we get exactly the result given in equation (54).

(iv) The dynamics of $p_{g,t}^{z}$ can be obtained by simply applying the Ito's Lemma to equation (54).

(v) To obtain the price level in (56) recall that from Proposition 3 a general definition of the price level is given by equation (43). In our case, this implies that:

$$\frac{1}{P_t} = \left(\frac{1-\eta}{\eta}\right) \frac{1}{Y_t^{\eta(1-\rho)-1} m_t^{(1-\eta)(1-\rho)}} E_t \int_t^{\infty} e^{-\beta(s-t)} Y_s^{\eta(1-\rho)} m_s^{(1-\eta)(1-\rho)} \frac{1}{P_s} ds \tag{104}$$

which can be rewritten as:

$$\frac{1}{P_t Y_t} = \left(\frac{1-\eta}{\eta}\right) \frac{1}{Y_t^{\eta(1-\rho)} m_t^{(1-\eta)(1-\rho)}} \int_t^{\infty} e^{-\beta(s-t)} Y_s^{\eta(1-\rho)} m_s^{(1-\eta)(1-\rho)} E_t \left[\frac{1}{M(s)}\right] ds \tag{105}$$

To calculate the expectation term $E_t \left[\frac{1}{M(s)}\right]$ in (105) we need to integrate the stochastic process for $M(t)$ given by equation (50) so:

$$M(s) = M(t) \exp\left\{ - \left[\left(\mu_M - \frac{\sigma_M^2}{2}\right)(s-t) - \sigma_M \int_t^s d\omega_{M,t} \right] \right\} \tag{106}$$

where $d\omega_{M,t} = \Omega_{M,t}\sqrt{dt}$. So that:

$$E_t \left[\frac{1}{M(s)} \mid M(t)\right] = \frac{1}{M(t)} \exp\left\{ - \left[\left(\mu_M - \sigma_M^2\right)(s-t) \right] \right\} \tag{107}$$

Plugging equation (107) into (105) and integrate the resulting equation, we get exactly the result given by (56).

(vi) To get the dynamics of the price level (57) it is enough to apply the Ito's Lemma to equation (56).

(vii) The inflation rate (58) is the drift term of the stochastic process found in (vi). ■

Proof of Proposition 7.

I start by showing how to get (75). From (72) we have that:

$$\ln M_t = \mu_M^* + 2\ln q_t$$

so that:

$$M_t = e^{\mu_M^*} q_t^2$$

inverting it:

$$\frac{1}{M_t} = e^{-\mu_M^*} q_t^{-2}$$

Define $G(q) = \frac{1}{q_t^2}$. Thus by using Ito's Lemma, we get:

$$d\left[\frac{e^{2k_q\mu_q}}{q_t^2}\right] = \frac{2}{q_t}\left(k_q + 3\sigma_q^2\right) e^{2k_q\mu_q t}dt - \frac{2\sigma_q}{q_t\sqrt{q_t}}dW_{M,t}$$

Thus, let us consider the expected value, as follows:

$$\begin{aligned}
E_t\left[\frac{1}{q_s^2}\right] &= E_t\left[\frac{1}{q_s^2} \mid q_t\right] = \\
&= E_t\left[e^{-2k_q\mu_q s}\left\{\int_t^s d\left[\frac{e^{2k_q\mu_q z}}{q_z^2}\right] + \frac{e^{2k_q\mu_q t}}{q_t^2}\right\} \mid q_t\right] = \\
&= \frac{e^{-2k_q\mu_q(s-t)}}{q_t} + \frac{\left(k_q + 3\sigma_q^2\right)}{q_t k_q \mu_q}\left(1 - e^{-2k_q\mu_q(s-t)}\right)
\end{aligned}$$

Finally, from the FOC of the problem of the representative agent, we get:

$$\frac{1}{P_t^C Y_t} = \frac{1-\phi}{\phi}\int_t^\infty E_t\left[\frac{1}{q_s^2}\right] e^{-\beta(s-t)-\mu_M^*}ds$$

which, after solving for the integral, we get exactly the result under (76).

To get the expression of the inflation rate, it is enough to apply Ito's lemma to (76) by setting $V\left(M_t, Y_t, q_t\right) = \frac{\Delta(q_t)}{\Psi(q_t)}\frac{M_t}{Y_t}$, so that:

$$dP_t^c = G_M dM_t + G_Y dY_t + G_q dq_t + \frac{1}{2}\left[G_{YY}\left(dY_t\right)^2 + G_{qq}\left(dq_t\right)^2 + G_{Mq}\left(dM_t\right)\left(dq_t\right)\right]$$

thus, performing the calculations by taking into account (78), (79) and the definitions of the stochastic processes for Y_t, q_t and M_t (67), (71), (72) we get exactly the result given by (77). ■

Proof of Proposition 8.

I follows closely the approach by Bakshi and Chen (1996). After having integrated the stochastic process for output, taking advantage of the utility function relationships we get:

$$\begin{aligned}
h(t,\tau) &= e^{-\beta\tau}E_t\left(\frac{Y_t}{Y_{t+\tau}}\right) = \\
&= e^{-[\beta+\mu_y]\tau}E_t\left\{\exp\left[-\left(\eta_y - \frac{1}{2}\sigma_y^2\right)\int_t^{t+\tau} x_s ds - \sigma_y\int_t^{t+\tau}\sqrt{x_s}dw_{x,s}\right]\right\} = \\
&= e^{-[\beta+\mu_y]\tau}\widetilde{h}(t,\tau)
\end{aligned}$$

Thus, according to Richard (1978), the term $\widetilde{h}(t,\tau)$ is a solution to the following PDE:

$$\frac{1}{2}\sigma_x^2 x_t \widetilde{h}_{xx} + \left[a(b - x_t) - \sigma_x \sigma_y x_t\right]\widetilde{h}_x - \widetilde{h}_\tau - \left(\eta_y - \sigma_y^2\right) x_t \widetilde{h} = 0$$

Thus, by using a standard separation of variables technique, the unique solution subjected to the boundary condition, $\widetilde{h}(t+\tau, 0) = 1$ is the real bond price formula in Proposition 8, equation (83). Finally, equation (82) is then obtained by taking the limit of $R_t = \lim_{\tau \to 0}\left\{-\frac{1}{\tau}\ln\left[h(t,\tau)\right]\right\}$. ■

Proof of Proposition 9.

Recall that from the equilibrium conditions we get:

$$\frac{N(t,\tau)}{P_t^c} = e^{-\beta\tau} E_t\left[\frac{u_c(Y_{t+\tau}, m_{t+\tau})}{u_c(Y_t, m_t)}\frac{1}{P_{t+\tau}^c}\right]$$

Thus, by using the properties of the log-utility function given in (66), we get:

$$N(t,\tau) = e^{-\beta\tau} P_t^c Y_t E_t\left[\frac{1}{P_{t+\tau}^c Y_{t+\tau}}\right]$$

Now from (75) we get:

$$\frac{1}{P_t^c Y_t} = \frac{1-\phi}{\phi}\left[\frac{(\beta + \mu_M^*) + \left(k_q + 3\sigma_q^2\right) q_t 2k_q\mu_q}{q_t^2(\beta + \mu_M^*)\left(\beta + \mu_M^* + 2k_q\mu_q\right)}\right]\frac{1}{M_t}$$

From the definition of the stochastic process for M_t, we can integrate to get: $M_t = q_t e^{\mu_M^* t}$. Therefore, using these features and rearranging, we obtain the following expression for the term structure of nominal interest rates:

$$N(t,\tau) = \frac{e^{-(\beta+\mu_M^*)\tau} q_t^2}{\left\{\frac{\beta+\mu_M^*}{q_t} + \left(k_q + 3\sigma_q^2\right) 2k_q\mu_q\right\}} E_t\left\{\left(k_q + 3\sigma_q^2\right)\frac{2k_q\mu_q}{q_{t+\tau}^2} + \frac{\beta + \mu_M^*}{q_{t+\tau}^3}\right\}$$

Thus, integrate $E_t\left[\frac{1}{q_{t+\tau}^2} \mid q_t\right]$, $E_t\left[\frac{1}{q_{t+\tau}^3} \mid q_t\right]$ and rearrange the resulting expression, we finally obtain the expression under (88). ■

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Figure 1: i, π, var(π), for ϕ_1, Cobb Douglas Utility

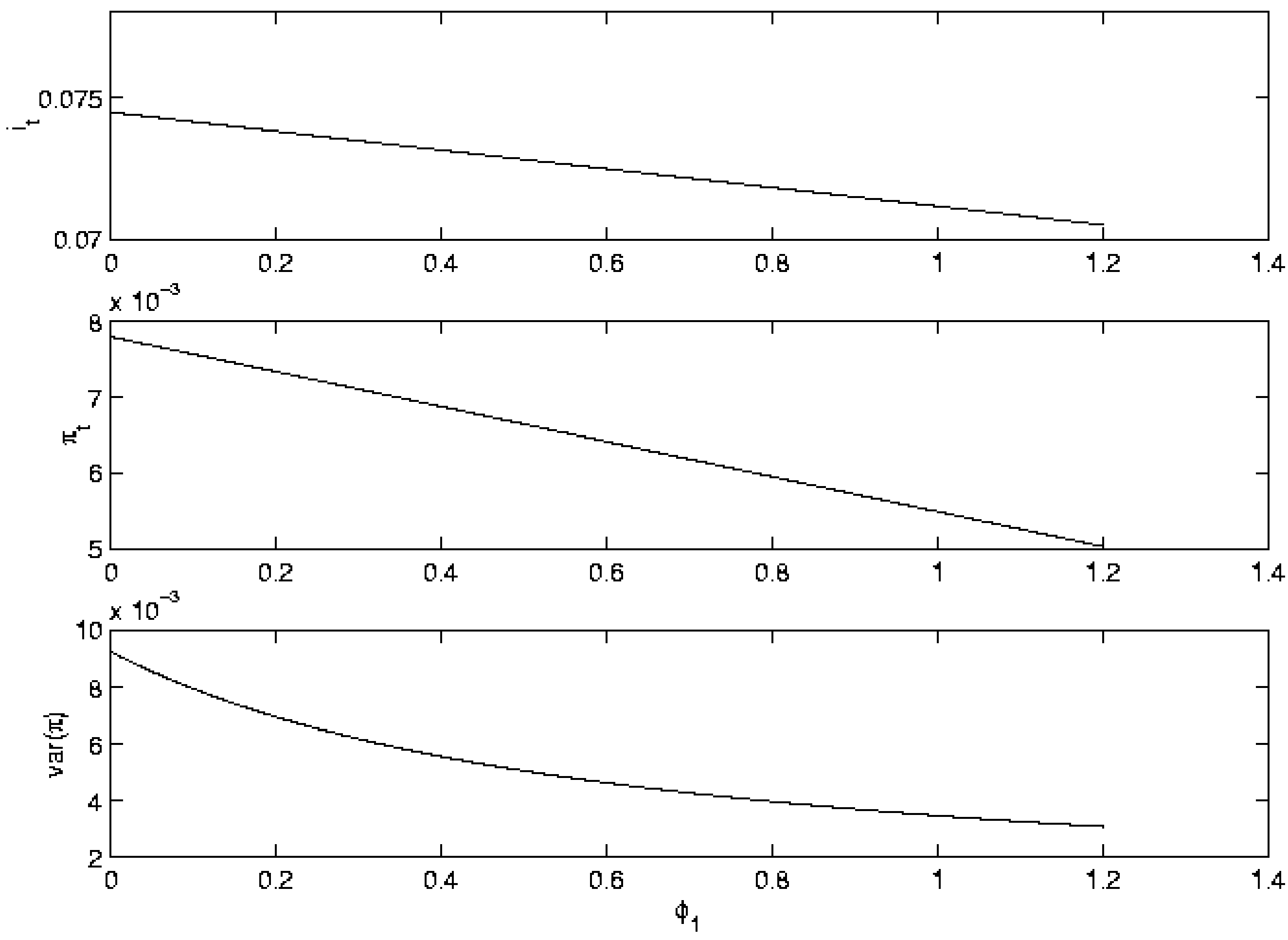

Figure 2: Nominal Interest Rate: static simulation for ϕ_1

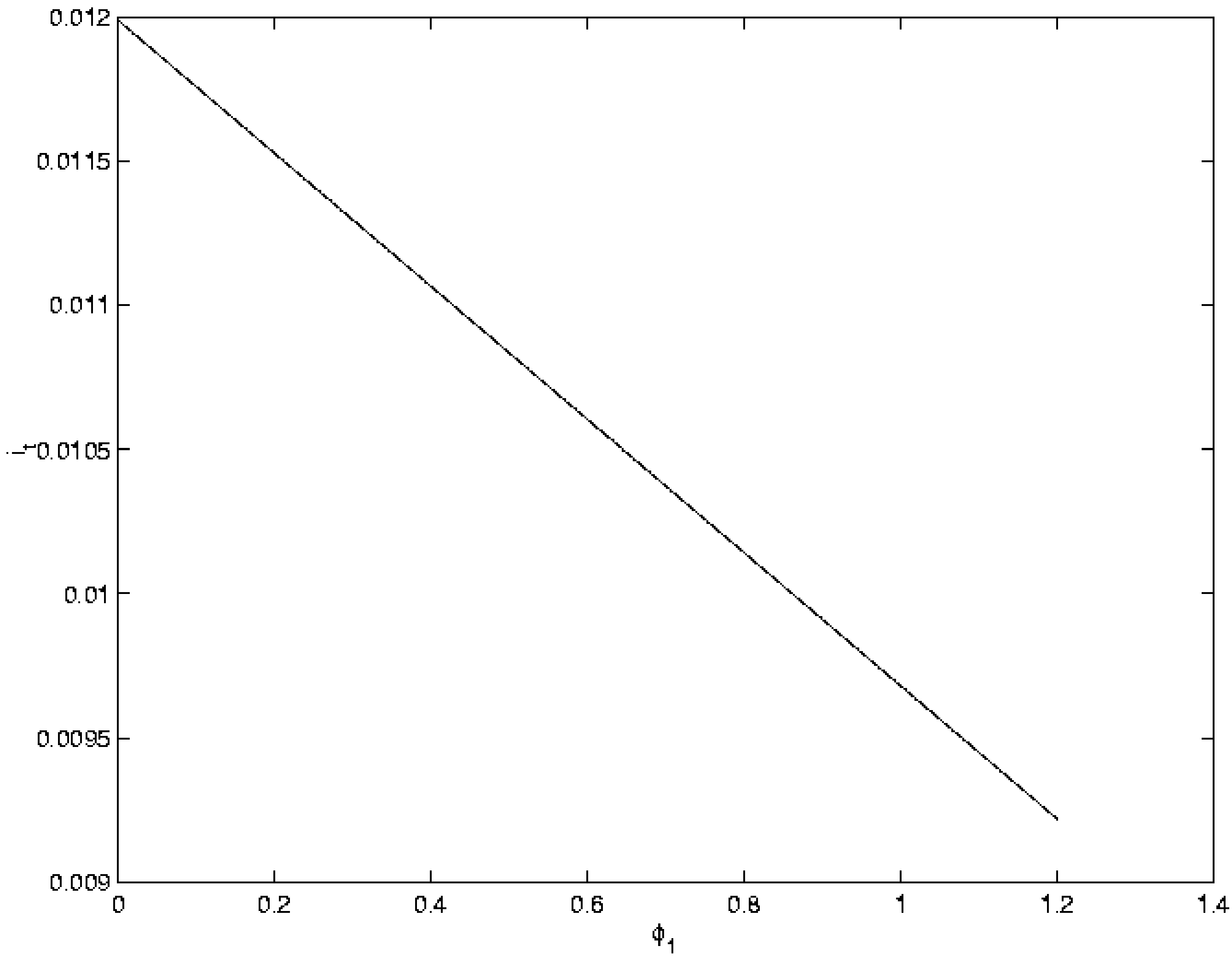

Figure 3: Effects of changes in ϕ_1 on P and π

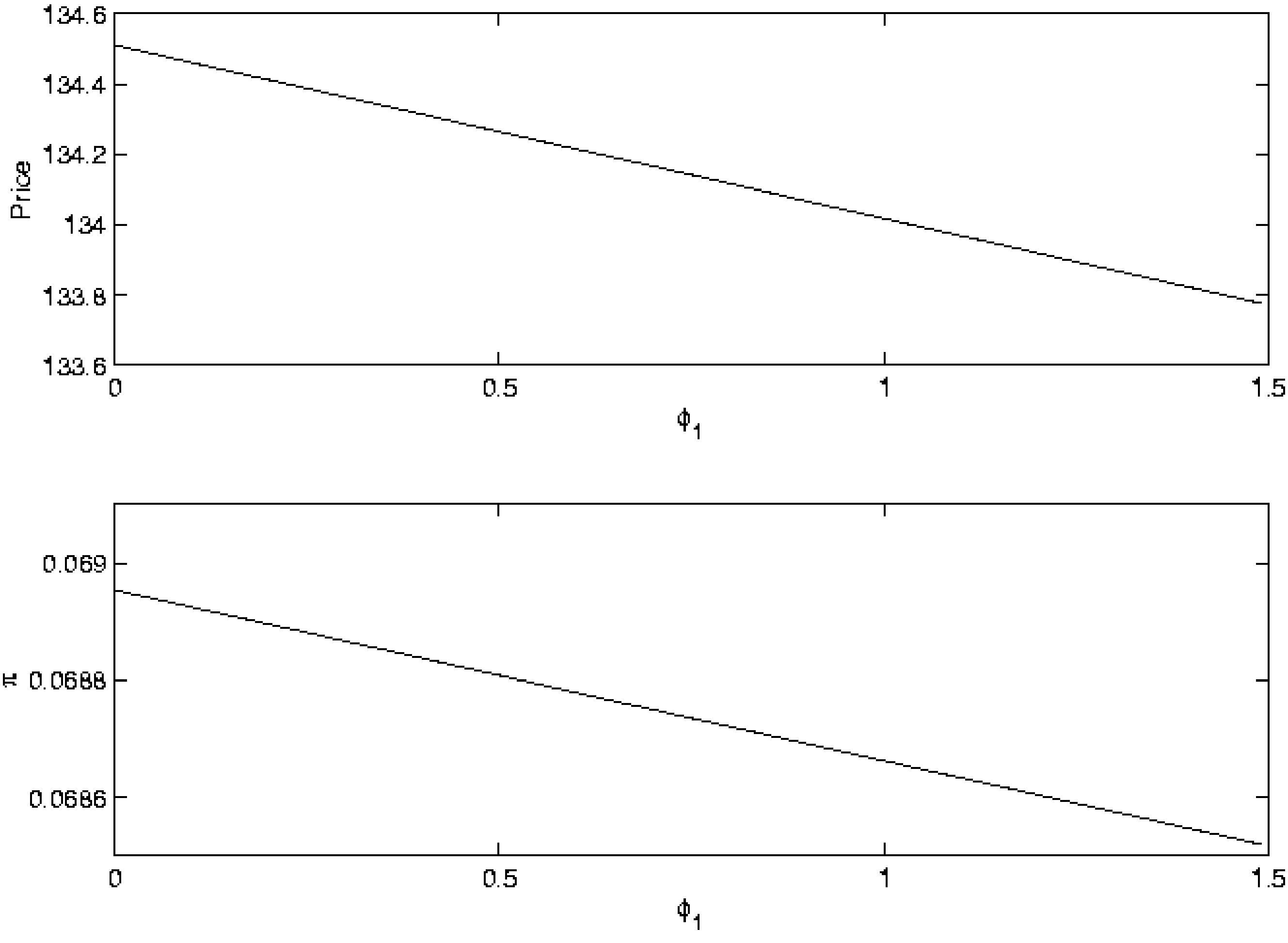

Figure 4: Effects of changes in μ_1 on P and π

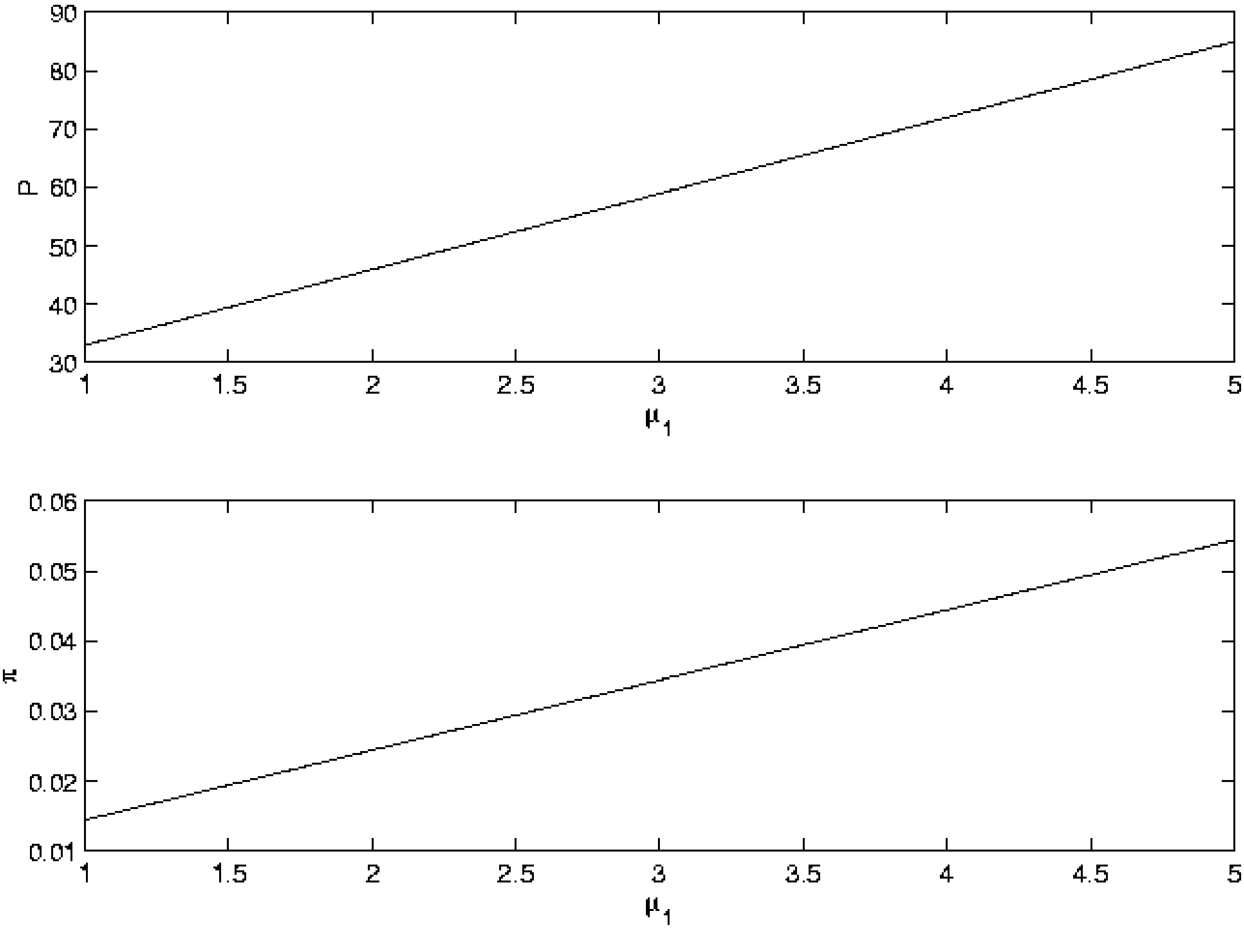

Figure 5: Effects of changing μ_1

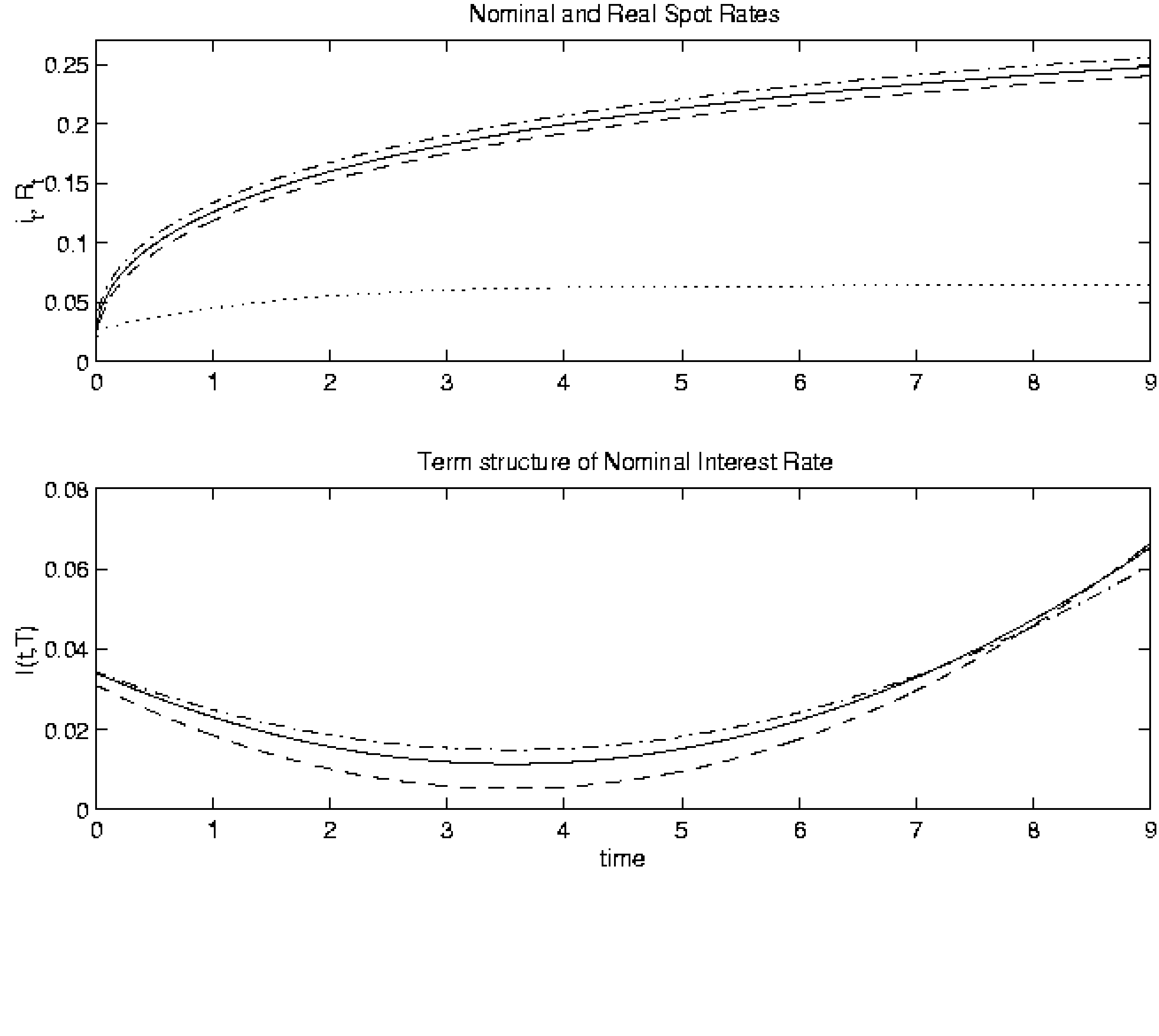

Figure 6: Nominal and Real spot Interest Rates

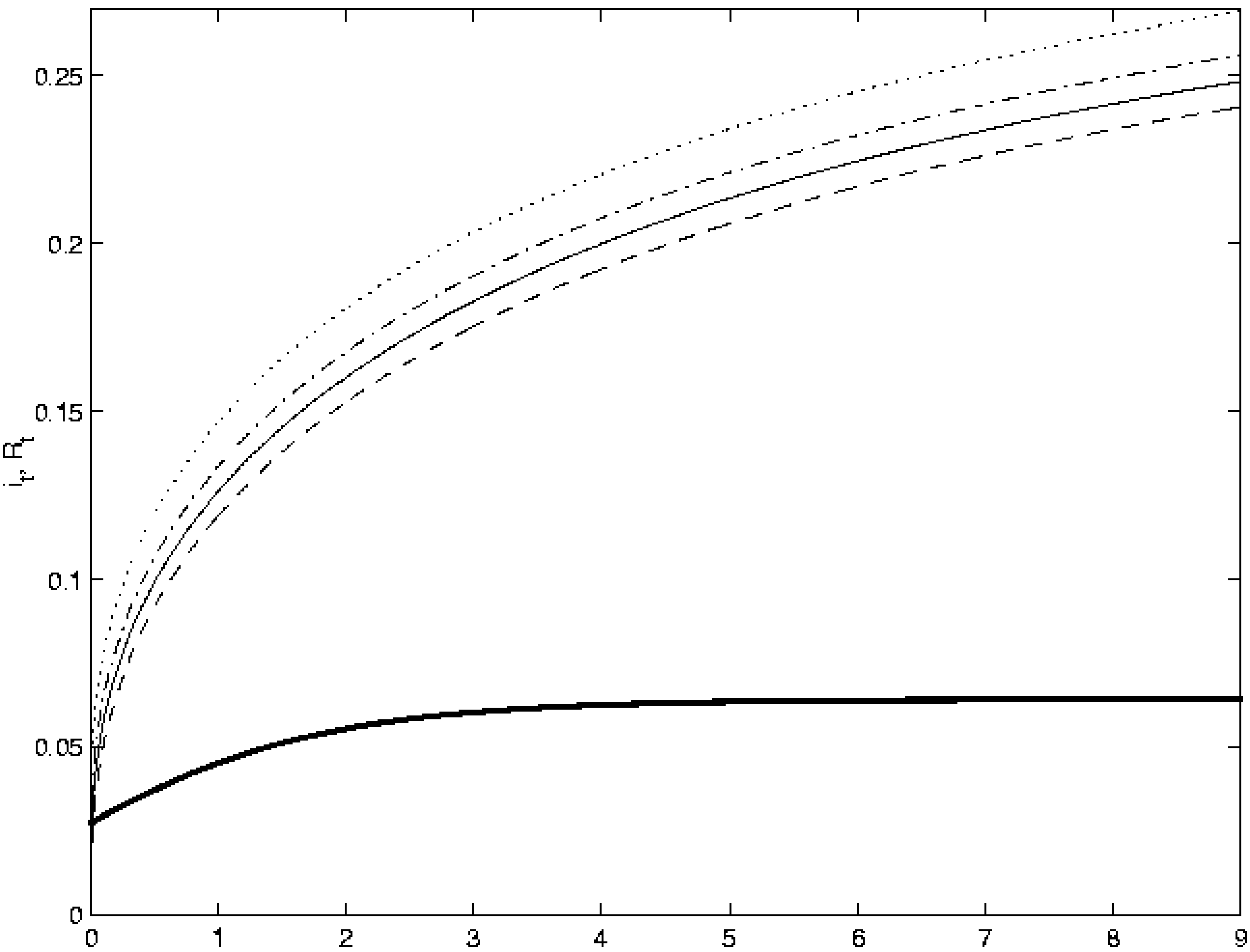

Figure 7: Term Structure of Nominal Interest Rates