The Monopolist's Optimal R&D Portfolio[1]

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Abstract

The monopolist's incentives towards product and process innovations are evaluated against the social optimum. The main findings are that (i) the incentive to invest in cost-reducing R&D is inversely related to the number of varieties being supplied at equilibrium, under both regimes; (ii) distortions obtain under monopoly, w.r.t. both the number of varieties and the technology. With substitutes (respectively, complements), the monopolist's product range is smaller (respectively, larger) than under social planning. For any given number of goods, the monopolist operates at a higher marginal cost than the planner does.

JEL Classification: L12, O31

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1 Introduction

Firms' investment in a particular technology is usually linked to the kind and number of products they want to supply. Indeed, casual observation suggests that …rms activate R&D portfolios including several innovation projects. Surprisingly enough, this problem has received relatively scanty attention (see Bhattacharya and Mookherjee, 1986; Dasgupta and Maskin, 1987). R&D can be carried out along two di¤erent directions, namely, process and product innovation. To the best of my knowledge, the literature usually treats the two kinds of innovation separately, a noteworthy exception being Rosenkranz (1996). Bonanno and Haworth (1998) consider a market for vertically di¤erentiated goods where …rms can invest either to reduce marginal cost or to introduce a new variety. However, they do not investigate the possibility for …rms to pursue both innovations at the same time.

There exists a wide literature concerning multiproduct …rms.[1] The early studies in this direction justi…ed the supply of product lines on the grounds of production costs. In the theory of contestable markets (Baumol, Panzar and Willig, 1982; Panzar, 1989), the existence of multiproduct …rms is justi…ed by economies of scope.

The role of demand-side factors in in‡uencing the …rms' optimal product range has received much less attention, the reason being that the analysis of this issue can be complicated by the externalities that multiproduct …rms try to internalise. That is, a …rm has to control competition between products belonging to the same line, so as to limit cannibalisation as much as possible. Brander and Eaton (1984) focus upon the interplay between consumer's demand for di¤erentiated goods on one side, and the strategic and technological e¤ects a¤ecting …rms' behaviour, on the other side. Relying on a theoretical model where the analysis con…nes to Cournot competition, Brander and Eaton verify that …rms' strategic decisions as to product range and output level may lead to market equilibria where …rms supply product ranges characterised by a high degree of substitutability. This result is derived under the assumption that each …rm's product range consists in a given number of varieties, and is therefore subject to a fairly natural critique, namely, that …rms may endogenously alter the span of their product range for strategic reasons. This incentive is investigated by Wernerfelt (1986), …nding that the

[1] For an exhaustive overview of the theory of multiproduct …rms in oligopolistic or perfectly competitive environments, see MacDonald and Slivinsky (1987), Okuguchi and Szidarovsky (1990) and De Fraja (1994).

driving forces are the heterogeneity of consumer tastes on one side and the cost of product proliferation on the other. The nested logit approach to the same problem reveals that, in a free entry equilibrium, there are too many …rms but too few varieties per …rm, and the total number of varieties is too small, compared to the social optimum (see Anderson and de Palma, 1992; Anderson, de Palma and Thisse, 1992).

The behaviour of …rms in choosing optimal product lines has been widely investigated in the address models of product di¤erentiation, under both monopoly and oligopoly (see Mussa and Rosen, 1978; Maskin and Riley, 1984; Gabszewicz et al., 1986; Bonanno, 1987; Champsaur and Rochet, 1989, inter alia). The main issue at stake in this strand of research is the monopolist'ss incentive to distort quality and quantity as compared to the social optimum, and his associated attempt at discriminating among customers with di¤erent willingness to pay for quality. A further issue is product proliferation as an entry-deterring device (see, in particular, Judd, 1985; and Bonanno 1987).

The theory of multiproduct …rms has evolved along several lines of research, a relevant one being driven by the idea that consumers may bear switching costs, either real or perceived as such, related to the purchase of product lines. Consequently, consumers' brand loyalty can be so high that they purchase goods from one …rm only, and …rms' pricing behaviour becomes quasi-collusive (see Klemperer, 1992, 1995; Klemperer an Padilla, 1997, inter alia).

Here, I propose in a model which combines technological and demand factors. My aim is to investigate product and process innovations jointly, in a monopoly model where the representative consumer is characterised by a preference for variety, and goods may be either substitutes or complements. The possibility for the technology to exhibit economies (or diseconomies) of scope is considered, and the …rm's research portfolio is driven by the interplay between scope economies and product substitutability (or complementarity). The monopoly optimum is evaluated against the social optimum. The main …ndings can be summarised as follows. Under both monopoly and social planning, (i) if goods are substitutes (respectively, complements) the …rm …nds it optimal to produce more than one variety if product innovation costs are lower (respectively, higher) than a critical threshold; and (ii) irrespective of whether goods are substitutes or complements, the incentive to reduce marginal production cost is inversely related with the number of varieties. The intuition behind this result is that the …rm may increase her own ability to extract surplus either by reducing marginal cost for a given number of varieties, or by expanding the product range for a given marginal cost. Hence, cost-reducing R&D activities and product innovation are used as substitutes for one another. Finally, (iii) for any product range, the monopolist investment in process innovation is too little as compared to the social optimum. Conversely, for any given marginal cost, the monopolist distort the product range respect to the social optimum. Therefore, the market suffers from distortion along both dimensions of the innovation activity.[2]

The remainder of the paper is organised as follows. Section 2 introduces demand and technology. The optimal R&D portfolio of the monopolist is evaluated in section 3. Section 4 discusses the socially optimal production plan. The planner's optimal R&D portfolio is assessed in section 4. Section 5 provides concluding comments.

2 The model

Consider the following monopoly setting. The firm supplies $n \geq 1$ products, each variety $i = 1; 2; 3 ::: n$ being characterised by the following inverse demand function:[3]

$$p_i = \max \left\{ 0; \alpha - q_i - \gamma \sum_{j \neq i} q_j \right\}; \qquad (1)$$

where $\gamma \in [-1; 1]$ measure complementarity/substitutability between any two products. If $\gamma \in [-1; 0)$ (respectively, $\gamma \in (0; 1]$) goods are demand complements (respectively, substitutes). If $\gamma = 0$; goods are independent of each other and the firm is a monopolist on n independent markets, so that the ensuing analysis reduces to verifying the profitability of each single product on its own market.

For the sake of simplicity, I assume that the output of R&D activity is certain. The total cost borne by the n-good monopolist are:

[2] A similar approach is adopted by Lambertini and Orsini (2000) in a monopoly model with vertical differentiation. Also in this case, there emerge situations where the monopolist offers too few products at a higher marginal cost compared to social planning.

[3] This demand structure was introduced by Bowley (1924), and it has been used, more recently, by several authors (see Spence, 1976; Dixit, 1979; Singh and Vives, 1984; inter alia).

- if $n > 1$;

$$C(n) = c(k)\sum_{i=1} q_i + \theta n F + \xi k^2 \; ; \; \theta > 0 \; ; \; \xi > 0 \tag{2}$$

- if $n = 1$;

$$C(1) = c(k) q + F + \xi k^2 \; ; \; \xi > 0 \tag{3}$$

where:

- $c(k)$ is a constant marginal cost which can be lowered through an R&D effort k in process innovation. The cost of such activity is ξk^2; which entails that process innovation takes place at decreasing returns, with

$$c'(k) \equiv \frac{\partial c(k)}{\partial k} < 0 \; ; \; c''(k) \equiv \frac{\partial^2 c(k)}{\partial k^2} > 0 \; ; \; c(0) = \bar{c} \; ; \; \lim_{k \to \infty} c(k) = 0 \tag{4}$$

- $\theta n F$ is the cost associated to product innovation up to $n > 1$ varieties. If $\theta \in [0;1)$ (resp., $\theta > 1$) we have economies of scope (resp., diseconomies of scope). Notice that here product innovation translates into product proliferation, while leaving unaffected the degree of substitutability or complementarity γ.

Therefore, the relevant profit function is:

$$\pi_M = \sum_{i=1} \left[\alpha - q_i - \gamma \sum_{j \neq i} q_j - c(k) \right] q_i - \theta n F - \xi k^2 \; ; \tag{5}$$

to be maximised w.r.t. the vector $\Omega \equiv \{q_1; ...q_i; ...q_n; k; n\}$. I assume that the monopolist does not practice any form of price discrimination among consumers. To simplify calculations, without loss of generality I solve first the market problem. Then, I will proceed to characterise the monopoly optimum w.r.t. the R&D choices.

The first order condition (FOC) for profit maximisation w.r.t. variety i is the following:

$$\frac{\partial \pi_M}{\partial q_i} = \alpha - 2\left(q_i + \gamma \sum_{j \neq i} q_j \right) - c(k) = 0 \, : \tag{6}$$

Introducing the symmetry condition $q_i = q_j = q$, the above FOC rewrites:

$$\frac{\partial \Pi_M}{\partial q_i} = \alpha - 2\left[1 + \gamma(n-1)\right] q - c(k) = 0\,, \tag{7}$$

yielding

$$q^* = \frac{\alpha - c(k)}{2\left[1 + \gamma(n-1)\right]} \tag{8}$$

as the optimal output level per-variety. The overall monopoly output in equilibrium is then $Q^* = n\left[\alpha - c(k)\right] / \{2\left[1 + \gamma(n-1)\right]\}$. Equilibrium profits, given the marginal cost and the product line, are:

$$\Pi_M(n;k) = \frac{n\left[\alpha - c(k)\right]^2}{4\left[1 + \gamma(n-1)\right]} - \theta n F - \xi k^2\,. \tag{9}$$

The solution to the profit maximisation problem of the monopolist is closely characterised in the next section.

3 The optimal R&D portfolio under monopoly

Maximisation of $\Pi_M(k;n)$ with respect to $\{k;n\}$ yields the following result.

Proposition 1 In equilibrium, the monopolist's R&D portfolio is given by

$$n^* = \frac{\gamma - 1}{\gamma} + \frac{\left[\alpha - c(k^*)\right]\sqrt{\theta F(1-\gamma)}}{2\theta F\gamma}$$

and

$$c'(k^*) = -\frac{4\xi k^*\left[1 + \gamma(n^*-1)\right]}{n^*\left[\alpha - c(k^*)\right]}$$

provided that

$$c''(k^*) \geq \frac{4\xi\left[1 + \gamma(n^*-1)\right]}{n^*\left[\alpha - c(k^*)\right]} \cdot \left[\frac{4\xi (k^*)^2}{n^*\left[\alpha - c(k^*)\right]^2} - 1\right]$$

and

$$F \in \left[0; \frac{\left[\alpha - c(k^*)\right]^2(1-\gamma)}{4\theta}\right] \quad \forall \gamma \in (0;1]\,; \tag{10}$$

$$F \geq \frac{\left[\alpha - c(k^*)\right]^2(1-\gamma)}{4\theta} \quad \forall \gamma \in [-1;0)\,. \tag{11}$$

Proof. See the Appendix.

Note that either (10) or (11) can be satisfied. Being they mutually exclusive, Proposition 1 entails that, if the monopolist finds it optimal to produce more than one variety in the case of substitutes, he will necessarily find it optimal to supply a single good in the case of complements, and vice versa.

As a complement to Proposition 1, the following Corollary can be stated:

Corollary 1 Under perfect substitutability, i.e., $\gamma = 1$; the monopolist produces at most one variety.

Proof. To prove the above result, it suffices to check from (9) that

$$\lim_{\gamma \to 1} \Pi_M(n;k) = \frac{[\alpha - c(k)]^2}{4} - \theta n F - \xi k^2 : \qquad (12)$$

■

Namely, when products are homogeneous, the most profitable strategy consists in operating a single plant, reducing thus to a minimum the amount of sunk costs. In connection with this fairly intuitive result, I am now in a position to characterise the influence of the relevant demand and cost parameters $\{F; \theta; \gamma\}$ upon the optimal product range n^*: This is summarised in the following:

Proposition 2 Consider $n^* > 1$; i.e., either

$$F \in \left[0; \Phi\right] \; \forall \gamma \in (0;1]$$

or

$$F \geq \Phi \; \forall \gamma \in [-1;0) ;$$

where $\Phi = \dfrac{[\alpha - c(k^*)]^2 (1 - \gamma)}{4\theta}$: Then we have

$$\frac{\partial n^*}{\partial F} < 0 \text{ and } \frac{\partial n^*}{\partial \theta} < 0 \; \forall \gamma \in (0;1] ;$$

$$\frac{\partial n^*}{\partial F} > 0 \text{ and } \frac{\partial n^*}{\partial \theta} > 0 \; \forall \gamma \in [-1;0) :$$

Moreover, for all $\gamma \in (0;1]$:

$$\frac{\partial n^*}{\partial \gamma} < 0 \text{ if } F < \hat{F}\ ;\ \forall\, \gamma \in (0;1]\ ;$$

while for all $\gamma \in [-1;0)$:

$$\frac{\partial n^*}{\partial \gamma} > 0 \text{ if } F > \hat{F}\ :$$

Proof. The effect on n^* of a variation in F or θ is described by:

$$\frac{\partial n^*}{\partial F} = -\frac{[\alpha - c(k^*)]\sqrt{1-\gamma}}{4F\gamma\sqrt{\theta F}} \tag{13}$$

and

$$\frac{\partial n^*}{\partial \theta} = -\frac{[\alpha - c(k^*)]\sqrt{1-\gamma}}{4\theta\gamma\sqrt{\theta F}}\ : \tag{14}$$

The sign of both (13) and (14) is negative (respectively, positive) for all $\gamma \in (0;1]$ (respectively, $\gamma \in [-1;0)$).

Now examine

$$\frac{\partial n^*}{\partial \gamma} = \frac{4\sqrt{\theta F(1-\gamma)} - [\alpha - c(k^*)](2-\gamma)}{4\gamma^2\sqrt{\theta F(1-\gamma)}}\ : \tag{15}$$

The r.h.s. of (15) is positive for all

$$F > \tilde{F} = \frac{(2-\gamma)^2[\alpha - c(k^*)]^2}{16\theta(1-\gamma)} \tag{16}$$

(and, conversely, negative for all $F \in [0;\tilde{F})$), regardless of whether goods are complements or substitutes. Therefore,

$$\text{for all } \gamma \in (0;1] : \frac{\partial n^*}{\partial \gamma} < 0 \text{ if } F < \min\left\{\tilde{F}\ ;\ \hat{F}\right\}\ ; \tag{17a}$$

$$\text{for all } \gamma \in [-1;0) : \frac{\partial n^*}{\partial \gamma} > 0 \text{ if } F > \max\left\{\tilde{F}\ ;\ \hat{F}\right\}\ : \tag{18}$$

To complete the proof, observe that

$$\tilde{F} > \hat{F} \quad \forall\, \gamma \in (0;1]\ ; \tag{19}$$

$$\tilde{F} < \hat{F} \quad \forall\, \gamma \in [-1;0)\ : \tag{20}$$

Therefore, $\min\left\{\text{Þ}^{e};\text{Þ}^{b}\right\} = \text{Þ}^{b}$ for all $\gamma \in (0;1]$; and $\max\left\{\text{Þ}^{e};\text{Þ}^{b}\right\} = \text{Þ}^{b}$ for all $\gamma \in [-1;0)$: This concludes the proof. ■

The intuition behind the results stated in Proposition 2 largely relies upon the interplay between scope economies (or diseconomies) and product substitutability (or complementarity), as follows. As economies of scope decrease or diseconomies of scope increase (i.e., θ becomes larger), the equilibrium number of varieties shrinks (if goods are substitutes; and conversely if they are complements). The same holds if unit product innovation costs (F) increase. Interpreting the behaviour of n^* as γ varies is also a straightforward task. I can confine myself to consider the case of substitutes (γ positive), as with complements the reverse is true. Here, $\partial n^*/\partial\gamma$ is negative when F is small enough to ensure that $n^* \geq 1$. The explanation appears to be that as the degree of substitutability increases, the firm finds it profitable to shrink the product range as it is more convenient to save on fixed costs rather than to try and extract more surplus from customers through product proliferation, precisely because the monopolist's ability to increase revenues through product proliferation weakens as products becomes more similar to each other. Therefore, as products become more similar, the incentive towards product innovation tends to vanish, and there remains the incentive to reduce marginal cost. To this regard, from (38) we have that

$$\frac{\partial c'(k^*)}{\partial n^*} = \frac{4\xi k^*(1-\gamma)}{(n^*)^2\left[\alpha - c(k^*)\right]} \geq 0 \text{ for all } \gamma \in [-1;1]: \qquad (21)$$

Therefore, I can state the following:

Lemma 1 The monopolist's incentive towards process innovation is decreasing in the number of products supplied in equilibrium.

The interpretation of the above Lemma is immediate. The firm can extract surplus either by enlarging $\alpha - c(k^*)$ for a given number of varieties, or by enlarging the product range for a given marginal cost. Therefore, the maximum incentive towards investment in cost-reducing R&D is observed when a single variety is supplied, as in that situation process innovation is mostly effective.

4 Social optimum

The planner maximises social welfare, defined as the sum of profits and consumer surplus:

$$SW = \Pi + CS = \sum_{i=1}^{n} \left[\alpha - q_i - \gamma \sum_{j \neq i} q_j - c(k) \right] q_i + \tag{22}$$
$$+ \sum_{i=1}^{n} \frac{(\alpha - p_i) q_i}{2} - \theta n F - \xi k^2$$

w.r.t. the vector $\Omega \equiv \{q_1, \ldots q_i, \ldots q_n, k, n\}$. As in the previous case, I first solve the marketing problem, given technology and the product range. The FOC w.r.t. the output level of variety i is:

$$\frac{\partial SW}{\partial q_i} = \alpha - q_i [1 + \gamma (n - 1)] - c(k) = 0 \tag{23}$$

yielding

$$q^{SP} = \frac{\alpha - c(k)}{1 + \gamma (n - 1)} \tag{24}$$

which can be compared to (8) to verify immediately that $q^{SP} = 2q^*$. Obviously $Q^{SP} = 2Q^*$, while $p^{SP} = c(k)$. Therefore,

$$SW^{SP} = CS^{SP} = \frac{n [\alpha - c(k)]^2}{2 [1 + \gamma (n - 1)]} - \theta n F - \xi k^2 \tag{25}$$

while $\Pi^{SP} = 0$. The socially optimal technology and product range are investigated in the next section.

5 The socially optimal R&D portfolio

The planners' FOCs w.r.t. the number of varieties and the cost-reducing R&D effort are:

$$\frac{\partial SW}{\partial k} = - \frac{n [\alpha - c(k)] c'(k)}{1 + \gamma (n - 1)} - 2 \xi k = 0 \, ; \tag{26}$$

$$\frac{\partial SW}{\partial n} = \frac{[\alpha - c(k)]^2}{2 [1 + \gamma (n - 1)]} \cdot \left[\frac{1 + \gamma (2n - 1)}{1 + \gamma (n - 1)} \right] - \theta F = 0 \, . \tag{27}$$

SOCs are:

$$\frac{\partial^2 SW}{\partial k^2} = \frac{n\left[\left(c'(k)\right)^2 - (\alpha - c(k))\, c''(k)\right]}{1+\gamma(n-1)} - 2\xi \leq 0\,; \tag{28}$$

$$\frac{\partial^2 SW}{\partial n^2} = -\frac{(1-\gamma)\gamma\,[\alpha - c(k)]^2}{[1+\gamma(n-1)]^3} \leq 0\,; \tag{29}$$

and

$$\frac{\partial^2 SW(n;k)}{\partial n^2} \cdot \frac{\partial^2 SW(n;k)}{\partial k^2} \geq \frac{\partial^2 SW(n;k)}{\partial k \partial n} \cdot \frac{\partial^2 SW(n;k)}{\partial n \partial k}\,; \tag{30}$$

where

$$\frac{\partial^2 SW(n;k)}{\partial k \partial n} = \frac{\partial^2 SW(n;k)}{\partial n \partial k} = -\frac{(1-\gamma)\,[\alpha - c(k)]\, c'(k)}{[1+\gamma(n-1)]^2}\,. \tag{31}$$

As the procedure and computational details are largely analogous to those characterising the profit-seeking monopolist, without further proof I can state the following:

Proposition 3 In equilibrium, the socially efficient R&D portfolio is given by

$$n^{SP} = \frac{\gamma - 1}{\gamma} + \frac{\left[\alpha - c\left(k^{SP}\right)\right]\sqrt{\theta F(1-\gamma)}}{\theta F \gamma \sqrt{2}}$$

and

$$c'\left(k^{SP}\right) = -\frac{2\xi k^{SP}\left[1+\gamma\left(n^{SP}-1\right)\right]}{n^{SP}\left[\alpha - c\left(k^{SP}\right)\right]}$$

if and only if

$$c''\left(k^{SP}\right) \geq \frac{2\xi\left[1+\gamma\left(n^{SP}-1\right)\right]}{n^{SP}\left[\alpha - c\left(k^{SP}\right)\right]} \cdot \left[\frac{2\xi\left(k^{SP}\right)^2}{n^{SP}\left[\alpha - c\left(k^{SP}\right)\right]^2} - 1\right]$$

and

$$F \in \left[0; \frac{\left[\alpha - c(k^{SP})\right]^2(1-\gamma)}{2\theta}\right] \;\forall \gamma \in (0;1]\,;$$

$$F \geq \frac{\left[\alpha - c(k^{SP})\right]^2(1-\gamma)}{2\theta} \;\forall \gamma \in [-1;0)\,.$$

Notice that the threshold

$$\overline{F} = \frac{\left[\alpha - c(k^{SP})\right]^2 (1 - \gamma)}{2\beta} \tag{32}$$

is the value of the product innovation cost below (resp., above) which n is larger than one when goods are substitutes (resp., complements). As in the monopoly setting, it is easily checked that the planner produces at most one good when $\gamma = 1$.

Moreover, it is immediate to verify that, for any given level of the marginal cost, $\overline{F} = 2\widehat{F}$. Likewise, given k (i.e., given $c(k)$), it can be established that $n^{SP} > n^*$ for all $\gamma \in (0, 1]$, and conversely for all $\gamma \in [-1, 0)$. Therefore, we have the following corollary:

Corollary 2 For a given investment in process innovation, when goods are substitutes (i.e., γ is positive), the social incentive towards product innovation is always larger than the monopolist's. As a result, the planner's product range is wider than the monopolist's. The opposite holds when goods are complements.

Now I can compare the social incentive to reduce marginal cost to the private incentive of a monopolist to do so.[4] This can be done for a given number of varieties, n, and it summarised by the difference $|c'(k^*; n)| - |c'(k^{SP}; n)|$, or equivalently by the comparison between (33) and (26). The result is stated in the following:

Corollary 3 For a given number of products, and for all $\gamma \in [-1, 1]$, the social incentive towards process innovation is larger than the private incentive.

Corollaries 2 and 3 imply that, with substitutes, we shall expect the monopolist to produce a smaller product line, at a higher marginal cost than the social planner, i.e., profit incentives distort the monopolist's R&D portfolio in both respects, as compared to the social optimum. This conclusion is reinforced by considering the effect of a variation in parameters $\{F, \gamma, \beta\}$ on n^{SP}, vis à vis their effects on n^* as illustrated in Proposition 2.

[4] Observe that the analogous to Lemma 2 holds under social planning as well, i.e., the planner's investment in cost-reducing R&D reaches a maximum when $n^{SP} = 1$. Hence, it appears that, under both regimes, product variety is a substitute for productive efficiency, and conversely.

Proposition 4 Consider $n^{SP} > 1$; i.e., either

$$F \in \left[0; \overline{F}\right] \; \forall \gamma \in (0;1]$$

or

$$F \geq \overline{F} \; \forall \gamma \in [-1;0) \; ;$$

where $\overline{F} = \dfrac{\left[\alpha - c(k^{SP})\right]^2 (1-\gamma)}{2\theta}$. Then we have

$$\frac{\partial n^{SP}}{\partial F} < 0 \text{ and } \frac{\partial n^{SP}}{\partial \theta} < 0 \; \forall \gamma \in (0;1] \; ;$$

$$\frac{\partial n^{SP}}{\partial F} > 0 \text{ and } \frac{\partial n^{SP}}{\partial \theta} > 0 \; \forall \gamma \in [-1;0) \; :$$

Moreover, for all $\gamma \in (0;1]$:

$$\frac{\partial n^{SP}}{\partial \gamma} < 0 \text{ if } F < \overline{F} \; ; \; \forall \gamma \in (0;1] \; ;$$

while for all $\gamma \in [-1;0)$:

$$\frac{\partial n^{SP}}{\partial \gamma} > 0 \text{ if } F > \overline{F} \; :$$

Proof. See the Appendix.

The interpretation of Proposition 4 is largely analogous to that holding for Proposition 2, except that the incentives to enlarge the product range are always higher for the planner than for the monopolist, for all $\gamma < 1$. This can be quickly verified through the comparison of

$$\frac{\partial n^{SP}}{\partial F} \; ; \; \frac{\partial n^{SP}}{\partial \theta} \; ; \; \frac{\partial n^{SP}}{\partial \gamma}$$

against

$$\frac{\partial n^{*}}{\partial F} \; ; \; \frac{\partial n^{*}}{\partial \theta} \; ; \; \frac{\partial n^{*}}{\partial \gamma} \; :$$

At $\gamma = 1$; both agents supply a single good by operating a single plant, so that the issue of product proliferation vanishes. In such a case, by Corollary

3, it is nevertheless true that the planner's incentive towards investment in process innovation is larger than the monopolist's.

Summing up, we may expect the monopolist to distort investment in both process and product innovation as well as output at the same time, and these distortions obviously interact with one another. In turn, this entails that, if a regulator introduces a policy aimed at correcting one of these distortions, this will creates some undesirable feedback on the remaining variables. For instance, subsidising product innovation (in the case of substitutes) would cause a reduction of the monopolist's investment in process innovation, inducing then an output reduction.

6 Concluding remarks

I have evaluated the monopolist's behaviour along two dimensions of his innovation portfolio, i.e., the R&D activities towards product and process innovation. Then, I have assessed the monopoly equilibrium against the social optimum.

The foregoing analysis reveals that, in addition to the well know output distortion usually associated with the monopolist's optimal marketing decisions, the market also su¤ers from distortions along the two dimensions of innovation, at the same time. that is, the monopolists o¤ers too many (or too few) varieties, produced at a larger marginal cost, than the planner. The di¤erent incentives towards innovation characterising the monopolist and the planner entails that any comparative assessment of the two regimes should take into account that both product range and technology can be expected to di¤er across regimes. Consequently, the task for the regulator is more involved than we were lead to think on the basis of the previous literature, in that the distortions along both dimensions of the monopolist's R&D activity add to the usual output distortion due to monopoly pricing.

Appendix

Proof of Proposition 1

I look for the solution w.r.t. $\{k, n\}$ of the following system:

$$\frac{\partial \Pi_M(n,k)}{\partial k} = -\frac{n[\alpha - c(k)]c'(k)}{2[1+\gamma(n-1)]} - 2\xi k = 0\,; \tag{33}$$

$$\frac{\partial \Pi_M(n,k)}{\partial n} = \frac{[\alpha - c(k)]^2}{4[1+\gamma(n-1)]} \cdot \left\{1 - \frac{\gamma n}{[1+\gamma(n-1)]}\right\} - \theta F = 0\,; \tag{34}$$

where $c'(k) \equiv \partial c(k)/\partial k$. Second order conditions for (33-34) to yield an internal solution require that the Hessian matrix $H[\Pi_M(n,k)] \leq 0$; that is:

$$\frac{\partial^2 \Pi_M(n,k)}{\partial n^2} = \frac{\gamma(\gamma - 1)[\alpha - c(k)]^2}{2[1+\gamma(n-1)]^3} \leq 0\,; \tag{35}$$

$$\frac{\partial^2 \Pi_M(n,k)}{\partial k^2} = \frac{n\left\{[c'(k)]^2 - [\alpha - c(k)]c''(k)\right\} - 4\xi[1+\gamma(n-1)]}{2[1+\gamma(n-1)]} \leq 0\,; \tag{36}$$

where $c''(k) \equiv \partial^2 c(k)/\partial k^2$; and

$$\frac{\partial^2 \Pi_M(n,k)}{\partial n^2} \cdot \frac{\partial^2 \Pi_M(n,k)}{\partial k^2} \geq \frac{\partial^2 \Pi_M(n,k)}{\partial k \partial n} \cdot \frac{\partial^2 \Pi_M(n,k)}{\partial n \partial k} = \left(\frac{\partial^2 \Pi_M(n,k)}{\partial k \partial n}\right)^2 . \tag{37}$$

Consider first (33) in isolation. For any n, the optimal R&D effort in process innovation, k^*, is implicitly given by:

$$c'(k^*) = -\frac{4\xi k^*[1+\gamma(n-1)]}{n[\alpha - c(k^*)]} . \tag{38}$$

Now consider (34). This FOC has two critical points:

$$n_1 = \frac{\gamma - 1}{\gamma} - \frac{[\alpha - c(k)]\sqrt{\theta F(1-\gamma)}}{2\theta F\gamma} \tag{39}$$

$$n_2 = \frac{\gamma - 1}{\gamma} + \frac{[\alpha - c(k)]\sqrt{\theta F(1-\gamma)}}{2\theta F\gamma} \tag{40}$$

of which only n_2 satisfies (35) for all $\{c(k), \gamma\}$. Therefore, n_2 candidates as the optimal product range.

Consider (38). Substituting the expression for $c'(k^*)$ into (36) and simplifying, I obtain:

$$\frac{\partial^2 \Pi_M(n;k)}{\partial k^2} = 4\lambda\left[1+\gamma\left(n^*-1\right)\right]\left[\frac{4\lambda\left(k^*\right)^2}{n^*\left[\alpha-c(k^*)\right]^2}-1\right]+ \qquad (41)$$

$$-n^*\left[\alpha-c(k^*)\right]c''(k^*)\leq 0;$$

yielding

$$c''(k^*)\geq\frac{4\lambda\left[1+\gamma\left(n^*-1\right)\right]}{n^*\left[\alpha-c(k^*)\right]}\cdot\left[\frac{4\lambda\left(k^*\right)^2}{n^*\left[\alpha-c(k^*)\right]^2}-1\right]. \qquad (42)$$

The SOC concerning the optimal number of goods can be quickly dealt with, by observing that

$$\frac{\gamma\left(\gamma-1\right)\left[\alpha-c(k)\right]^2}{2\left[1+\gamma\left(n^*-1\right)\right]^3}\leq 0$$

is satisfied for all $\gamma\in[-1;1]$.

Now look at the condition (37). This can be written as:

$$\frac{\partial^2 \Pi_M(n;k)}{\partial n^2}\cdot\frac{\partial^2 \Pi_M(n;k)}{\partial k^2}-\left(\frac{\partial^2 \Pi_M(n;k)}{\partial k\partial n}\right)^2= \qquad (43)$$

$$=\frac{\gamma\left(1-\gamma\right)\left[\alpha-c(k^*)\right]}{4\left[1+\gamma\left(n^*-1\right)\right]}\cdot\left\{\frac{4\lambda\left[1+\gamma\left(n^*-1\right)\right]\left[n^*\left[\alpha-c(k^*)\right]^2-4\lambda\left(k^*\right)^2\right]}{n^*\left[\alpha-c(k^*)\right]^2}+\right.$$

$$\left.+n^*\left[\alpha-c(k^*)\right]c''(k^*)\right\}-4\cdot\left\{\frac{\left[\lambda(1-\gamma)k^*\right]}{n^*\left[1+\gamma\left(n^*-1\right)\right]}\right\}^2\geq 0.$$

The above inequality is satisfied for

$$c''(k^*)\geq\frac{4\lambda\left[1+\gamma\left(n^*-1\right)\right]}{\gamma\left\{n^*\left[\alpha-c(k^*)\right]\right\}^3}\cdot\left\{4\lambda\left(k^*\right)^2\left[(1-\gamma)^2+\gamma n^*(2-2\gamma+\gamma n^*)\right]+\right. \qquad (44)$$

$$\left.-\gamma n^*\left[\alpha-c(k^*)\right]^2\right\}.$$

Finally, note that

$$\frac{4\lambda\left[1+\gamma\left(n^*-1\right)\right]}{n^*\left[\alpha-c(k^*)\right]}\cdot\left[\frac{4\lambda\left(k^*\right)^2}{n^*\left[\alpha-c(k^*)\right]^2}-1\right] \qquad (45)$$

is larger than

$$\frac{4\lambda[1+\gamma(n^*-1)]\left\{4\lambda(k^*)^2[(1-\gamma)^2+\gamma n^*(2-2\gamma+\gamma n^*)]+-\gamma n^*[\alpha-c(k^*)]^2\right\}}{\gamma\{n^*[\alpha-c(k^*)]\}^3} \tag{46}$$

over the whole admissible range of parameters. Therefore, (42) is also sufficient to ensure global optimality.

For the above solution to be economically acceptable, it must also be that $n^* \geq 1$; i.e.,

$$\frac{[\alpha-c(k^*)]\sqrt{\theta F(1-\gamma)}-2\theta F}{2\theta F\gamma} \geq 0 \tag{47}$$

yielding

$$F \in \left[0;\ \frac{[\alpha-c(k^*)]^2(1-\gamma)}{4\theta}\right]\ \forall\gamma\in(0;1]\,; \tag{48}$$

$$F \geq \frac{[\alpha-c(k^*)]^2(1-\gamma)}{4\theta}\ \forall\gamma\in[-1;0)\,: \tag{49}$$

This concludes the proof. ■

Proof of Proposition 4

The proof proceeds along the same lines as for Proposition 2. The effects on n^{SP} of a variation in F or θ are described by:

$$\frac{\partial n^{SP}}{\partial F} = -\frac{[\alpha-c(k^*)]\sqrt{1-\gamma}}{2F\gamma\sqrt{2\theta F}} \tag{50}$$

and

$$\frac{\partial n^{SP}}{\partial \theta} = -\frac{[\alpha-c(k^*)]\sqrt{1-\gamma}}{2\theta\gamma\sqrt{2\theta F}}\,: \tag{51}$$

The sign of both (50) and (51) is negative (respectively, positive) for all $\gamma\in(0;1]$ (respectively, $\gamma\in[-1;0)$).

Then, we have that

$$\frac{\partial n^{SP}}{\partial\gamma} > 0 \text{ for all } F > \underline{F} = \frac{(2-\gamma)^2[\alpha-c(k^*)]^2}{16\theta(1-\gamma)} \tag{52}$$

(and, conversely, negative for all $F \in [0; \underline{F})$), regardless of whether goods are complements or substitutes. Hence,

$$\text{for all } \gamma \in (0;1]: \ \frac{\partial n^{SP}}{\partial \gamma} < 0 \text{ if } F < \min\left\{\underline{F}; \overline{F}\right\}; \tag{53a}$$

$$\text{for all } \gamma \in [-1;0): \ \frac{\partial n^{SP}}{\partial \gamma} > 0 \text{ if } F > \max\left\{\underline{F}; \overline{F}\right\}: \tag{54}$$

To complete the proof, observe that

$$\underline{F} > \overline{F} \quad \forall \gamma \in (0;1]; \tag{55}$$

$$\underline{F} < \overline{F} \quad \forall \gamma \in [-1;0): \tag{56}$$

Therefore, $\min\left\{\underline{F}; \overline{F}\right\} = \overline{F}$ for all $\gamma \in (0;1]$; and $\max\left\{\underline{F}; \overline{F}\right\} = \overline{F}$ for all $\gamma \in [-1;0)$: This concludes the proof. ■

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