The Labour Market as a Job-Seeking Contest: Human Capital, Intergenerational Mobility, and Growth*

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Abstract

With a fixed hierarchy of jobs, workers engage in a rent-seeking game based on their individual levels of human capital, in order to obtain high-paying positions in the social division of labour. The macroeconomic effects of the workers' quest for privileged positions are studied through the numerical simulation of an OLG model with credit rationing and neural network expectations. The model shows that a higher turnover and a higher meritocracy degree in the job-allocation mechanism is bound to lead to greater economic growth.

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1 Introduction

In the economic literature, education plays mainly two different roles, and generally a particular model emphasizes only one. The first function of education is to increase the stock of skills and productive knowledge embodied in people; hence education becomes a synonym of human capital accumulation (Becker, 1964). In the 70's, however, another view came forward, which considered education as a means to overcome problems of imperfect information in the labor market (Arrow, 1973; Spence, 1973). Following this view, primary education aims at increasing people skills, while higher education is used mainly as a screening device by firms, assuming that the workers' productivity is due to their innate abilities.[1] If education is just a

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[1]Education can be a signal of different productivity-affecting qualities: "better-educated workers are not a random sample of workers: they have lower propensities to quit or to be absent, are less likely to smoke, drink or use illicit drugs, and are generally

signal, then, with credit market imperfections, it can be the case that better jobs are allocated to people with more money, not necessarily with more skills.[2]

One of the most quoted growth models with human capital is due to Lucas (1988). Building on earlier works by Arrow (1962) and Romer (1989), Lucas mantains the assumption of perfect competition and assumes that education has positive externalities that increase the productivity of all factors.[3] Since individuals do not consider the positive spillover of their investment, investment in human capital is inefficiently low in equilibrium, but it is exactly the social role of education which brings about economic growth in steady state.[4]

In this paper , we present an OLG model characterized by four particular assumptions on:
- the role of education;
- agents' behavior;
- labor compensation;
- the credit market.

The two roles of education mentioned above are merged, emphasizing the *positional* aspect of investment in human capital. Agents make education expenses, increasing their human capital, and this has positive effects on aggregate output. From the individual point of view, however, the aim is not to increase productivity, but to acquire credentials for the competition in the labor market. We do not follow the strategic approach typical of the signalling models: agents make their decisions in a myopic and parametric way, on the basis of opponents' past behavior. This important deviation from mainstream assumptions is due to the fact that positional competition in the labor market can be viewed as a fight "all against all", where each individual can consider his own action as being negligible on the aggregate outcome. The signal role of school achievements does not come from the equilibrium of a game with incomplete information, but it is one of the assumptions of the model. There are different jobs with different pay, and their allocation is such that agents with more human capital have higher chances to get better jobs. Labor compensation depends upon average productivity and upon the hierarchical organization of production: the remuneration structure is institutionally fixed, and does not equate, for each individual, his

healthier. [...] we would expect employers to favor better-educated workers as a means of reducing their costs of sickness and job turnover" (Weiss, 1995).

[2]The problem of "talent allocation" and its implications for growth are analyzed in some recent works (Murphy - Shleifer - Vishny, 1991; Fershtman - Murphy - Weiss, 1996).

[3]On the contrary, in the models belonging to the neo-Schumpeterian approach (Romer, 1990; Grossman - Helpman, 1991), growth is driven by R&D investments by monopolistic firms.

[4]Provided that human capital accumulation is not subject to decreasing returns (see Lucas, 1988).

compensation with his marginal contribution to production.[5] Labor market demand is similar to the demand of a big firm, where there are top and low positions. A fixed number of top positions gives the agents the incentive to engage in a *positional competition* based on human capital (Hirsch, 1976). Schooling expenses play a role which is similar to rent-seeking activities. More education gives more chances to obtain a high-paying job: agents take part in a lottery whose probabilistic weights are determined by their investments in education. Workers engage in what we call a *job-seeking contest*.[6] The last assumption is about credit market imperfections, which do not allow agents to finance their investments through loans. There are two overlapping generations, whose members are altruist with their offspring, and choose consumption and investments (in physical and human capital) subject to liquidity constraints. The presence of bequests, and the impossibility to resort to loans, put only some people in the position to acquire education in the first part of life, building up credentials to get good jobs in the second part of their life.

The paper is organized as follows. Section 2 develops the structure of the model, paying attention to the specific assumptions about the labor market and agents' behavior. Section 3 shows the results of some numerical simulations made with different assumptions on individual preferences and expectations and on the institutional and technological context. Section 4 concludes with some comments.

2 The model

There are two overlapping generations, both of dimension N. Each agent lives and consumes for two periods, and gives birth to a descendant at the beginning of the second period (population is constant). At birth, each agent receives a bequest ($b_{i,t}$ is the bequest of the individual belonging to the dynasty i, received in period t) and has to allocate it between first period consumption $c^y_{i,t}$, investment in education $e_{i,t}$ (which gives rise to human capital accumulation $h_{i,t+1}(e_{i,t})$, which in turn will determine the probabilities of having good pay in $t+1$), and investment in physical capital $k_{i,t}$, whose remuneration, at the end of first period, is R_{t+1}.[7] First period

[5]Compensation schemes which move away from the criterion of marginal productivity can be efficient as well, provided that their structure is an incentive for the competition among workers (see Lazear - Rosen, 1981; Nalebuff - Stiglitz, 1984); we will return to the compensation scheme hypothesis below.

[6]In *rent-seeking* models, higher investments (e.g. in lobbying, corruption, etc.) lead to higher probabilities of success (e.g. a license for a public monopoly). Investments are always relevant but never conclusive (it is always possible that, for fortuitous events, the biggest investor loses anyway). Analogously, in the job-seeking game, better school credentials give more chances to get a good job, but they are not conclusive (as in the real world: we can observe graduate beggars and ignorant millionaires); there is always the possibility (even if very limited) that an agent with a higher education gets a job that is worse than an agent with lower education.

[7]We implicitly assume that the relative price between consumption and the two forms of investment is constant and equal to one.

budget constraint is therefore

$$c_{i,t}^{y} \leq b_{i,t} - e_{i,t} - k_{i,t} \tag{1}$$

In the second period, each individual gets the remuneration of his physical capital (if any), and gets a wage w_{t+1}, whose amount is determined on the basis of a lottery, whose probabilistic weights depend on the investment choices made in the first period by all N agents. He has to allocate these resources between second period consumption ($c_{i,t+1}^{o}$), and the bequest ($b_{i,t+1}$) which must be given to the descendant at the beginning of the period for his own consumption and investments.[8] Second period budget constraint is

$$c_{i,t+1}^{o} \leq R_{t+1} \cdot k_{i,t} + w_{i,t+1} - b_{i,t+1} \tag{2}$$

where $w_{i,t+1}$ can be equal to w_M or w_L, with $w_M > w_L$.

The chronological order of actions is the following: at the beginning of first period the bequest is received and the (young) consumption and investment choices are made; at the beginning of second period, agents receive both the remuneration of physical investment and the remuneration for the labor services they are going to supply; at the beginning of the second period they also consume and give their bequest to descendants.[9]

All choices are made taking factor remunerations as given. In the following section, the assumptions about production and factor remunerations in each period are described.

2.1 Production and factor remunerations

The simplest possible model assumptions about production are made in order to obtain more intelligible results. First of all, the interest rate is assumed to be constant. We can interpret this saying that our economy is small and open to trade, and while human capital is non tradeable, physical

[8]Preferences are homogeneous among generations (not only among dynasties), so the same choices would be made if we assumed that adults make decisions for their offspring, maximizing a utility function which takes offspring's utility into account as well. If $b_{i,t+1}$ appears directly in the adult utility function (and it is given at the beginning of the second period), it is bound to be interpreted as a status-enhancing activity: the descendant can be seen as the person to whom such activities are delegated, as consumption or education investments, that confer prestige on the adult (likewise, Veblen 1899 found that the wife and the sons are the conspicuous consumers of the family). Following this proxy scheme, the youth consumes and gets education confering status on his parent, while the adult works for himself and for his descendant.

[9]Young people, without collaterals, cannot borrow money, so they cannot move resources from second to first period. When adults, they get a job. Assuming that they are given their pay at the beginning of the second period (and contextually they make consumption and bequest) means that infraperiodal credit is allowed (a common hypothesis in OLG models with credit rationing).

capital movement is completely free. Therefore, the interest rate R is set at being equal to the international interest rate R^*, constant throughout time:

$$R_t = R = R^* \quad \forall t$$

We have a Cobb-Douglas production function, with constant returns to scale in both factors, physical capital K and human capital H:

$$Y_t = A \cdot K_t^{\gamma} \cdot H_t^{1-\gamma} \tag{3}$$

where A is the dimensional parameter. Physical capital is given by:

$$K_t = \sum_{j=1}^{N} k_{j,t} + SMK_t \tag{4}$$

where SMK_t is the balance of capital movements at time t. Human capital at time t is determined by the investments in education made by the N agents at time $t-1$. Young agents can invest in education e_t, to accumulate individual human capital $h_{t+1}(e_t)$. The accumulation function $h_{t+1}(e_t)$ can have different forms, provided that it satisfies the following two conditions:

$$h(0) > 0 \qquad \frac{\partial h_{t+1}(e_t)}{\partial e_t} > 0$$

that is to say, human capital must always be positive (even with no education) and must be increasing with education.[10] We assume that the form is the following:

$$h_{t+1}(e_t) = \tilde{h} + e_t^{\alpha} \tag{5}$$

with $\tilde{h} > 0$ and $\alpha > 0$. The α parameter can be less, equal, or greater than one, so establishing the returns to scale of the accumulation function. Agents who do not invest in education have a human capital equal to $\tilde{h}$, which can be interpreted as the innate abilities or as the abilities coming from compulsory (and free) education. The aggregate amount of human capital is then given by the following expression:

$$H_t = \sum_{j=1}^{N} h_t(e_{j,t-1}) = N \cdot \tilde{h} + \sum_{j=1}^{N} (e_{j,t-1})^{\alpha} \tag{6}$$

While physical capital, which is tradeable, gets a remuneration (R^*) which is always equal to its marginal productivity, human capital is compensated on the basis of the institutional organization of occupations: human

[10] As we will see later in the article, these two conditions ensure that the probability of getting a good job is never zero, and it increases when others decrease their investments in education.

capital always increases the worker's productivity, but it does not necessarily increase his compensation. The neoclassical model is followed only until the distributive shares of the two kind of capital are determined, but then the remuneration of each single worker follows a different criterion. The aggregate income going towards human capital is $W_t = (1-\gamma) \cdot Y_t$; this is then shared out among n top wages (w_M) and $N-n$ low wages (w_L) on the basis of the inequality parameter $\chi = \frac{w_M}{w_L}$ (with $\chi > 1$), which is institutionally determined.[11] So we have:

$$w_{L,t} = \frac{W_t}{N+n\cdot(\chi-1)} \quad \text{and} \quad w_{M,t} = \chi \cdot w_{L,t} = \frac{\chi \cdot W_t}{N+n\cdot(\chi-1)} \tag{7}$$

Given the institutional structure (determined by the parameters N, n, and χ) and given the interest rate R, the fundamental variable is H_t, because the amount of physical capital K_t depends itself upon it:

$$K_t = H_t \cdot \left(\frac{\gamma \cdot A}{R-1}\right)^{\frac{1}{1-\gamma}} \tag{8}$$

So the aggregate amount of wages is:

$$W_t = (1-\gamma) \cdot A^{\frac{1}{1-\gamma}} \cdot \left(\frac{\gamma}{R-1}\right)^{\frac{\gamma}{1-\gamma}} \cdot H_t \tag{9}$$

From (9) and (7) we get the two levels of wages as a function of H_t.

In the numerical simulations, we also consider the case with increasing returns. In this case we have:

$$Y_t = A \cdot K_t^{\gamma} \cdot H_t^{\eta} \qquad \text{with } \gamma + \eta > 1 \tag{10}$$

Following the same hypothesis about factor remuneration as those presented above, the aggregate amount of wages becomes:

$$W_t = (1-\gamma) \cdot A^{\frac{1}{1-\gamma}} \cdot \left(\frac{\gamma}{R-1}\right)^{\frac{\gamma}{1-\gamma}} \cdot H_t^{\frac{\eta}{1-\gamma}} \tag{11}$$

This means that the first factor to be rewarded is physical capital, in order to equate his remuneration to the interest rate R (constant and set abroad). The output left is then distributed to workers on the basis of the established hierarchy.[12] We now see how this hierarchy is determined.

[11] The hypothesis of only two levels of pay, linked by a constant ratio, is certainly a big simplification. The same comment can be made about the hypothesis of a constant structure of occupations (the fixed ratio n/N); about this latter, historical evidence is more reassuring (see Goldin 1994). We will show some numerical simulations of the model, made with different values of χ, but always constant during the observational period.

[12] Only the physical capital is perfectly mobile, so the constraint $R = R^*$ has to be satisfied. The output exhaustion is assured by the assumption that the aggregate amount of wages is residual: it is equal to $(1-\gamma) \cdot Y$, and not to $\eta \cdot Y$.

2.2 Labour market as a lottery: the job-seeking contest

There are N agents competing for n working contracts with wage w_M. They can modify their probability of success investing in education, and accumulating human capital through the accumulation function (5). The probability of obtaining a top-paying (w_M), or a low-paying (w_L) job depends on the investments in education of the different agents. We assume that, for agent i, the probability $P_i(J)$ of getting a wage w_J is a function of the different levels of human capital of all the agents:

$$P_i(J) = f(h_1, h_2, ..., h_N) \quad \text{with} \quad J = M, L \quad 0 < P_i(J) < 1 \tag{12}$$

$$\frac{\partial P_i(M)}{\partial h_i} > 0 \quad \frac{\partial P_i(L)}{\partial h_i} < 0 \quad \frac{\partial P_i(M)}{\partial h_j} < 0 \quad \frac{\partial P_i(L)}{\partial h_j} > 0 \quad \forall j \neq i \quad i = 1, .., N \tag{13}$$

$$\textstyle\sum_{i=1}^{N} P(M) = n \quad \sum_{i=1}^{N} \sum_{J=M,L} P_i(J) = N \tag{14}$$

The probability distribution $P_i(J)$ implies that individual behavior is interdependent, and that everybody has at least a small chance of getting every kind of job. The partial derivatives (13) establish that if agent i increases his investment in education, he has more chances of getting the top job, and the probability of belonging to the low class of workers decreases. Investments by other agents have, *ceteribus paribus*, the opposite effects. Equalities (14) are consistency constraints.

Moreover, we should assume that, if everybody adopts the same investment in education, even equal to zero, then $P_i(M) = n/N \ \ \forall \, i$, since the level of innate human capital is homogeneous.

Human capital plays two roles in this model. From a technological point of view, it is a factor of production. From an individual point of view, however, the main role is that of a credential in the labor market. Every single agent invests in human capital aiming at being ahead of others in the positional competition for the scarce number of good jobs. When investing, it is this competition that he keeps in mind, and not the effects of human capital on aggregate income. Investment in education can be seen, from the individual point of view, as a *rent-seeking* activity. In rent-seeking models, the probability of success for each competitor is a function of everybody's expenses. In the traditional case, with only one prize, it is generally assumed that the probability of success for agent i is:

$$P_i(M) = \frac{h_i^r}{\sum_{j=1}^{N} h_j^r} \tag{15}$$

where, as r changes, the marginal cost of influencing probability through modifications of h changes (Tullock, 1980). To simplify the notation, we call

h_i^r the *credential* of agent i. Therefore, the credential production function is the following:

$$h_i^r = \left(\tilde{h} + e_i^\alpha\right)^r \tag{16}$$

Given the distribution of human capital, a greater value of the parameter r involves a smaller probability of success for those whose human capital is lower: increasing r, the probability distribution becomes wider. Hence, r can be interpreted as the *meritocracy degree* of job allocation (in the extreme case, with $r = 0$, human capital plays no role in the allocation, and the probability of success is the same for every agent). Specifying function (16), we make a distinction between human capital accumulation through education and the production of credentials through human capital. These two processes can have different returns to scale. For example, human capital accumulation can have decreasing returns to scale ($\alpha < 1$), while the credential production function can exhibit increasing returns ($r > 1$).[13]

In our model, we have n prizes, each one equal to the difference between the two wages available on the labor market: $w_M - w_L$. In the case of multiple prizes (Berry, 1993), the probability of success for agent i is more complex than that given by (15). As the numerator we have the sum of all possible n-groups of credentials which include h_i. As the denominator we have all possible n-groups, also with the cases where h_i is not included. If, for instance, $N = 4$ and $n = 2$ (two prizes for four competitors), then one agent (e.g. agent 3), can win in three different ways: together with agent 1, with agent 2, or with agent 4. At the same time, the possible results of the competition, including those in which agent 3 loses, are six. Therefore, the probability of success for agent 3 is given by the following:

$$P_3(M) = \frac{(h_3^r + h_1^r) + (h_3^r + h_2^r) + (h_3^r + h_4^r)}{(h_1^r + h_2^r) + (h_1^r + h_3^r) + (h_1^r + h_4^r) + (h_2^r + h_3^r) + (h_2^r + h_4^r) + (h_3^r + h_4^r)}$$

In general, there exist $\begin{pmatrix} N-1 \\ n-1 \end{pmatrix}$ n-groups of addenda (credentials) as the numerator (each one representing one of the different ways an agent can win) and $\begin{pmatrix} N \\ n \end{pmatrix}$ n-groups of addenda as the denominator (each representing one of the possible groups of winners).[14]

[13] In this case, an increase in human capital by a single agent (*ceteribus paribus*) brings about a more than proportionate increase in his probability of success. With $r > 1$, investments in education can be more concentrated (few agents invest big sums, many agents invest only a little) with respect to the efficient allocation, in spite of the decreasing returns in human capital accumulation ($\alpha < 1$).

[14] The combination $\begin{pmatrix} N \\ n \end{pmatrix} = \frac{N!}{n! \cdot (N-n)!}$ represents the possible combinations of N objects taken n at a time.

$$P_i(M) = \frac{\text{Sum of the } \binom{N-1}{n-1} \text{ groups of } n \text{ members including } h_i^\tau}{\text{Sum of all the possible } \binom{N}{n} \text{ groups of } n \text{ members}} \quad (17)$$

Once the n prize winners have been chosen, we have only the $N-n$ losers left, whose pay will be w_L.

Even if the structure of the competition is quite easy from an intuitive perspective, and it can easily be reproduced with the aid of a computer program, it is not easy to give it an analytical representation. In particular, it is quite difficult to obtain the form of the probability weights of the different agents, provided that we are interested in their ex ante values, computed when the effective winners are still unknown.[15] The analysis is complex because each agent, making his decisions, computes his probability weights on the basis of his own decisions about education investments and of an expectation of his competitors' human capital, trying to find the level of investment which maximizes his expected utility.

2.3 Preferences and choice

The intertemporal utility function is assumed to be log-linear:

$$U_{i,t} = \log\left[c_{i,t}^y\right] + \beta \cdot \left(\log\left[c_{i,t+1}^o\right] + \theta \cdot \log\left[b_{i,t+1}\right]\right) \quad (18)$$

where β and θ are, respectively, the subjective discount factor and the individual sensitivity towards bequest. Given the form of the utility function, the two budget constraints (1) and (2) are satisfied with equality. Agents cannot borrow from their second period income, so saving must be non negative ($k_t \geq 0$). Analogously, b_t, b_{t+1}, c_t^y, c_{t+1}^o and e_t cannot be negative either. Ex ante, agents do not know the level of the pay they will get, and this means that, in the first period, their consumption-investment choices are made maximizing the expected value of their utility.[16] Therefore, taking factor remuneration as given, each agent maximizes his intertemporal expected utility:[17]

$$\max_{e_{i,t}, k_{i,t}, b_{i,t+1}} \log\left[c_{i,t}^y\right] + \beta \cdot E_{i,t}\left[\left(\log\left[c_{i,t+1}^o\right] + \theta \cdot \log\left[b_{i,t+1}\right]\right)\right] \quad (19)$$

[15] Berry (1993) solves a model of rent-seeking with multiple winners analytically, assuming homogeneous agents and finding the symmetrical strategic solution. In that case, the solution is quite easy, because the probability of success (the same for everybody) is equal to $\frac{n}{N}$, as we can also see from (17).

[16] With a log-linear utility function we implicitly assume that agents are risk-averse.

[17] Human capital does not compare explicitly in the maximization problem, but we know it modifies the probability weights in the computation of the second period expected utility.

$$
s.t. \begin{cases} c^{y}_{i,t} = b_{i,t} - e_{i,t} - k_{i,t} \\ c^{o}_{i,t+1} = R_{t+1} \cdot k_{i,t} + w_{i,t+1} - b_{i,t+1} \\ k_{i,t} \geq 0 \\ e_{i,t} \geq 0 \end{cases} \tag{20}
$$

The results of positional competition is made known to the N agents at the beginning of the second period $(t+1)$, so they choose c^{o}_{t+1} and b_{t+1} knowing their pay. Then, we can solve the problem backward, at first finding the choice in $t+1$ between consumption and bequest. From the first order condition in the maximization of U_t with respect to b_{t+1}, using the budget constraint, we get the following optimal choices:

$$
b^{J}_{i,t+1} = \frac{\theta}{(1+\theta)} \cdot (R_{t+1} \cdot k_{i,t} + w_J)
$$

$$
c^{o,J}_{i,t+1} = \frac{1}{1+\theta} \cdot (R_{t+1} \cdot k_{i,t} + w_J)
$$

where $c^{o,J}_{i,t+1}$ and $b^{J}_{i,t+1}$ are consumption and bequest if agent i gets high pay $(J = M)$ or low pay $(J = L)$ in $t+1$. Homothetic preferences imply that each agent, aside from the result of the positional competition, will always consume a constant proportion of his second period resources, giving what is left over to his offspring.

Knowing the optimal choices in $t+1$, conditional to the result of the positional competition in t, we can eliminate a choice variable (b_{t+1}) from ex ante optimization; by also substituting the first period budget constraint, and given the optimal bequest, the maximization problem becomes the following:[18]

$$
\begin{aligned}
\max_{e_{i,t}, k_{i,t}} \mathcal{U}^{E}_{i,t} &= \log\left[b_{i,t} - k_{i,t} - e_{i,t}\right] + \beta \cdot \sum_{J=,M,L} P^{E}_{i}(J) \left\{ \log\left[\frac{R_{t+1} \cdot k_{i,t} + w_J}{1+\theta}\right] \right. && (21) \\
&\quad \left. + \theta \cdot \log\left[\frac{\theta \cdot (R_{t+1} \cdot k_{i,t} + w_J)}{(1+\theta)}\right] \right\} && (22) \\
&s.v. \begin{array}{l} k_{i,t} \geq 0 \\ e_{i,t} \geq 0 \end{array}
\end{aligned}
$$

The lagrangian function is

$$
\mathcal{L} = \mathcal{U}^{E}_{i,t} + \lambda \cdot k_{i,t} + \eta \cdot e_{i,t} \tag{23}
$$

[18]Remember that $P^{E}_{i}(J)$, the evaluation of the probability to get a wage that is equal to w_J, is a function of the human capital of all the N agents who are young in period t, and it is therefore a function of $e_{i,t}$ (besides the expectations about the investment in education made by all the other agents).

Necessary and sufficient conditions for a maximum are:

$$\begin{cases} \dfrac{\partial \mathcal{L}}{\partial k_{i,t}} = 0 \\ \dfrac{\partial \mathcal{L}}{\partial e_{i,t}} = 0 \\ \lambda \cdot k_{i,t} = 0 \\ \eta \cdot e_{i,t} = 0 \\ \lambda \geq 0 \\ \eta \geq 0 \end{cases} \tag{24}$$

We define $U_{i,t}^{o}(M)$ as the utility of agent i (young in t) in the second period of his life in the case where he obtains a wage that is equal to w_M. Combining the first two equations (and simplifying the notation by omitting the i index) we get:

$$\sum_{J=L,M} P_t^E(J) \cdot \left[\frac{(1+\theta)\cdot R_{t+1}}{R_{t+1}\cdot k_t + w_J}\right] - \frac{\partial P_t^E(M)}{\partial e_t} \cdot [U_t^o(M) - U_t^o(L)] + \frac{\lambda - \eta}{\beta} = 0$$

Leaving out corner solutions, the optimal values e_t^* and k_t^* have to satisfy the following condition:

$$\sum_{J=L,M} P_t^E(J)\left|_{e_t^*} \cdot \left[\frac{(1+\theta)\cdot R_{t+1}}{R_{t+1}\cdot k_t^* + w_J}\right] = \frac{\partial P_t^E(M)}{\partial e_t}\right|_{e_t^*} \cdot [U_t^o(M) - U_t^o(L)] \tag{25}$$

The arbitrage condition (25) establishes the equality between the expected marginal utility of more savings and the expected marginal utility coming from more education, which in turn increases the probability of getting a good job. The probability of success for each agent is calculated on the basis of his own investment in education, given the expectations about the investments of all the other agents.

2.4 Expectations

Agents invest in human capital to get a good job. The job-seeking contest, which is the real engine of individual choices regarding education, brings about positive effects on aggregate human capital. We assume that agents make expectations about opponents' behavior (in order to choose the most suitable level of education, given what they expect others will do), but take the levels of the different wages as given (static expectations). Therefore, the aggregate effect of individual investments in education becomes a composition externality, which is summed up with the positional externality due to the fact that the compensation of labor services is based on the relative (and not absolute) level of human capital.

In the numerical simulations which we are going to show, agents try to forecast the opponents levels of human capital to choose their own level of education in order to get a bigger share of aggregate wage income, but while doing so they take this last variable as given. We have expectations only about the levels of human capital of the other competitors. It is upon these expectations that the ex ante evaluation of the probability of getting a good job depends. Expectations are made by all agents by means of a single feed-forward neural network.[19] Neural networks realize multi-variable and multi-values non-linear functions, and they are universal approximators.[20] A neural network is made of different units, each one having an output (z) and more inputs (i). The transformation made by each single unit is the following:

$$z = f\left(\sum_{j=1}^{n} a_j \cdot i_j\right) \tag{26}$$

where the output (z) is the activation level of the unit, f is the activation function (in our case a sigmoid function with values between 0 and 1), while the coefficients (a) stand for the weights of the connections with the units from which the n inputs (i) come. The performance of a neural network depends upon the values of the connective weights (a) between its elementary units. These coefficients are modified during a training phase, through an iterative process in which some input configurations are presented to the neural network, and for every configuration a comparison is made between the network's output and the desired output (in our case, we have a comparison between the expectation made by the neural network and the realizations).

Generally, for the modification of the coefficients, a back-propagation algorithm is adopted (Rumelhart - McClelland, 1986). The difference between the network output (expectation) and the desired output (realization) is multiplied by a *learning factor*, whose value can be varied to solve possible problems of local minima. To reproduce a process, it is not necessary to specify a particular functional form, because the form choice is left to the network which, during the training phase, modifies its coefficients, strenghtening or weakening the connections between its elementary units. At the beginning the values of the coefficients are random, and they are modified after every "experience". For this reason, neural networks are used to simulate every kind of learning process. Generally, using a neural network to make forecasts, the training phase is carried out on a training set of past val-

[19]For an introduction to neural network models see Haykin (1994).

[20]Hornik - Stinchcombe - White (1989) show that a feed-forward neural network with a sigmoid activation function, even with only one hidden layer, can approximate every continuous function with every degree of accuracy, provided that it has a sufficient number of hidden units.

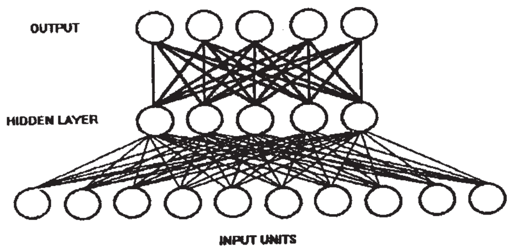

Figure 1: The feed-forward neural network used to forecast the investment in education of all the N agents. The network has $l \cdot N$ input units, N hidden units on a single layer, and N outputs. The picture has been made setting $l = 2$ and $N = 5$. The inputs are the levels of investment of the N agents in the last l periods. As output we obtain the expectations one period ahead.

ues; When the spread between the desired output and the network output is considered to be sufficiently small, the training is stopped, the network coefficients are fixed, and the network is then used to forecast the future values of the time series. In our model, the training set expands in every period, and the training phase never stops. We simulate a process of continuous learning.

To better simulate the learning process, it is possible to vary the intensity of the experiences: increasing the *reverberation factor,* a single experience can be artificially replicated during the training, causing the network to change its connective weights more after every experience. A bigger reverberation factor means a stronger reaction to experiences: in our case it means that expectations are more sensitive to recent experiences. A low reverberation factor means greater inertia in the upgrade of the coefficients.

We adopt a feed-forward neural network with three layers (input, hidden, output). There is only one net which, like an agency, provides every agent with the forecasts.[21] More precisely, each agent "consults" the net to obtain the forecasts about his competitors. The net supplies N output values, among which we also have the forecast about the agent himself. Then the agent has to substitute the forecast about himself with all the possible values

[21] In this model, we have only one net for N agents, so we do not have individual learning. We have an agency which, increasing its observations throughout time, can make better forecasts, furnishing the agents with expectations which evolve homogeneously.

his investment could be, to calculate the expected utility from each level of investment, in order to choose the optimal level of investment from his point of view. On the basis of the last l realizations, the net supplies the next value of the N variables. Therefore, it has $N * l$ input units and N output units. We set the number of hidden units as being equal to the number of outputs, while the number of lags (l) has been set at 2. The structure of the net in the case with $N = 5$ is shown in figure 1. Regarding the neural parameters, we initially set both the learning factor and the reverberation factor equal to 2.[22]

3 The simulation

3.1 The benchmark parametrization

The benchmark values for the parameters are the following:

$N = 10$	$\chi = 3$	$n = 3$
$\tilde{h} = 10$	$\alpha = 0,9$	$r = 3$
$A = 1$	$\gamma = 0,4$	$\eta = 0,7$
$\beta = 0,9$	$\theta = 2$	$R = 1,05$

We have 10 agents who compete for 3 good jobs, whose pay is three times that of the remaining 7 places.[23] The production function has increasing returns ($\gamma+\eta > 1$), while the human capital accumulation function is subject to decreasing returns ($\alpha = 0.9$). On the other hand, there is a high degree of meritocracy ($r = 3$), that is to say, increasing returns in the production of credentials: each agent's human capital (which, without investment in education, is equal to $\tilde{h} = 10$) is transformed into credentials (and then into the probability of getting a high-paying job) through a third power, and this means that the probability weights are much more spread out than the individual levels of human capital. All agents are slightly impatient ($\beta = 0.9$), and rather altruist towards their descendants ($\theta = 2$). The interest rate is set at being 5% ($R = 1.05$).[24]

Since the agents make their decisions on the basis of the choices observed in the previous l ($= 2$) periods, the simulation can start only after 3 periods, whose characteristics have to be set a priori (initial conditions). To

[22] We will also see cases with different reverberation factors. The input values have been normalized to be included in the interval [0,1], which is the range of the activation function. Since the data are subject to growth, we chose the level of the resources of each agent as the normalization variable. Therefore the net forecasts the proportion of inherited resources that each agent puts into education.

[23] Setting $N = 10$, the net has 20 input units, 10 hidden units and 10 output units: exactly two times the units of the net shown in figure 1.

[24] The parameters N, n, β and R will not change in the simulations we are going to show. Actually, many other exploratory simulations have been carried out, varying all the parameters, in order to find the most interesting cases.

avoid arbitrary distorsions, we assumed the constancy of the state variables for the first 3 periods. In all simulations, initial conditions (assuming a homogeneous distribution of resources) have been set as follows:

$$b_{i,t} = 10 \quad e_{i,t} = 1 \quad c^{y}_{i,t} = \tfrac{2}{3} \cdot (b_{i,t} - e_{i,t}) = 6 \quad \forall\, i \quad t = 1, 2, 3$$

3.1.1 The first 150 periods

In figure 2 we can see the performance of the system regarding the level of the investments in education and the level of output.[25] After an initial period (up to $t = 14$: phase 1) of fast growth of the average level of education (which passes from 1 to 120), we have a phase (from $t = 15$ to $t = 50$: phase 2), in which the investment choices fluctuate regularly, with a cycle of period 3, and with $e_i < 400\ \forall i$. As time passes, forecasts become more accurate, and the agents begin to find the investment in education more convenient: both the width and the period of the fluctuations increase; after 50 periods (and even more after 70 periods) a real *escalation* in education investments takes place (still subject to oscillations: phase 3, starting in $t = 51$), and this brings the level of output to a maximum in $t = 128$ ($Y = 342340$), after which the product still has wide fluctuations, but take place around an average which is much higher than before.[26] In this model, the choices about education made by the N agents are responsible for both the growth and the fluctuations of output. This is due to the double nature of education, which is an instrument for individual competition (negative positional externality) and at the same time an instrument to increase the workers' abilities and therefore the factor productivities (positive externality on human capital).

In the second phase, which ends in $t = 50$, the resources available to the agents are still few, in spite of the fast initial growth; high income agents invest in education, and their investments discourage low income dynasties, who prefer not to invest or only invest a little money in education.[27] Fluctuations are due to the fast growth of investments, which soon discourage the rich dinasties as well. Low income dynasties observe oscillations in the rich dynasties' investments, and so they can forecast which are the most suitable periods to enter the competition. When the well-off dynasties lower their investment, this decrease begins to be correctly forecast by the other agents; then the number of dynasties that try to get a good job through investments in education increases. This fact brings about a small turnover at the top

[25] In figure 2, as in other figures which will follow, ten trajectories are drawn, since ten are the dynasties considered.

[26] Now we focus only on the first 150 periods. We will later see what happens in the sequel.

[27] In other simulations, carried out with the same parametrization, the length of phase 2 (cycles without trend) is variable. It may depend upon the random choice of the neural weights, but the main reason is certainly the probabilistic nature of positional competition: more turnover in the first two phases implies that the third is reached faster.

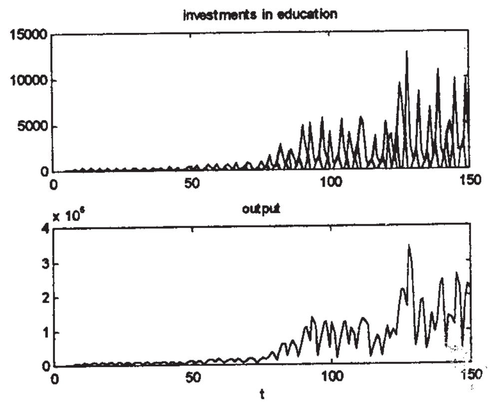

Figure 2: Investments in education and output in the first 150 periods.

of the working hierarchy. The oscillating nature of education investments is such that the low income dynasties never completely give up, because there can be more favourable circumstances in which even a small investment in education can make the difference. Low income dynasties are then attracted by the competition, and with their investments they help (no matter if they succeed or not) to increase the aggregate level of human capital, and consequently the output.[28] Growth seems to be due to the phase-displacement of individual trajectories. Because of the higher mobility, in phase 3 there are not only two possible levels of bequest any more (one for the rich dynasties and one for the poor ones): for each dynasty the bequests begin to depend upon his recent past. This fact makes it possible for more dynasties to compete for the best jobs, and increases the number of dynasties that invest in education, and this in turn increases the amount of investment of all the competitors (also the high income dynasties now have to deal with stronger competition, and they have to further increase their investments in education if they want to properly distinguish themselves. As a result, the aggregate human capital increases, and consequently so do the output and the levels of pay (and the levels of bequests) of both winners and losers. The latter, with bigger inherited resources, put more and more effort into the po-

[28] After phase 2, we observe an increasing trend in every agent's inherited resources (with a maximum in $t = 128$, when the richest dynasty has a bequest of more than 25000). Given the model's assumptions (homotetic preferences), we know that the bequests are always a constant proportion $\frac{\theta}{1+\theta}$ of the second period income of each agent.

sitional competition, increasing their chances of success and increasing the turnover at the top of the social ladder (mobility). It must be noted that the higher growth is not due to a smaller inequality in the different wages (this inequality is determined by the parameter χ, which is constant), but rather it is due to a smaller inequality of the resources that every agent receives at the beginning of his life (and this decrease in inequality is due to the greater mobility: good jobs are not always obtained by the same dynasties). Growth in turn means more starting resources for all, and therefore higher mobility in a self-enforcing process.[29]

3.1.2 Simulation continuation after t = 150

If we carry on the simulation, we can see how the maximum observed in $t = 128$ is actually only a temporary maximum since after a period of wide fluctuations, in $t = 158$ we have another (higher) temporary maximum ($Y = 543930$) and then a sharp fall which opens a phase with exponential growth of bequests. After $t = 160$, we can say that the system starts again from the beginning, but at a much greater speed. After $t = 300$, fluctuations will disappear and all the variables will increase exponentially. In figure 3 the graph of the different dynasties' bequests is drawn.[30]

The average level of bequests reaches a maximum in $t = 158$ ($\bar{b} = 24181$, equal to the starting resources in $t = 159$) and then decreases until $t = 167$ (when it reaches the minimum: $\bar{b} = 163$). From $t = 167$ on, we observe a continuous growth of all the variables (H, Y, w, b, e), at an increasing speed. After the sharp fall in $t = 160$, the economy starts out again with an income level similar wich is similar to the initial conditions ($t = 1$) but it grows at a faster speed, since the resources (bequests) are higher and distributed with more fairness. We can see in detail what happens to investments between $t = 155$ and $t = 159$ (and consequently to human capital and output between $t = 156$ and $t = 160$). In table 1 the values of the ten agents' investments, the average in each period ($\bar{e}_t$), and finally the average level of the expectations made in $t-1$ about the investments in t ($E_{t-1}(\bar{e}_t)$) are reported.

[29] With non-homothetic preferences, this process could be strengthened by saturations in second period consumption, with increasing proportions of income given to offspring.

[30] The graph of figure 3 is only drawn for the first 250 periods to keep an intelligible scale. The graph is of course qualitatively identical to the output graph, which is omitted.

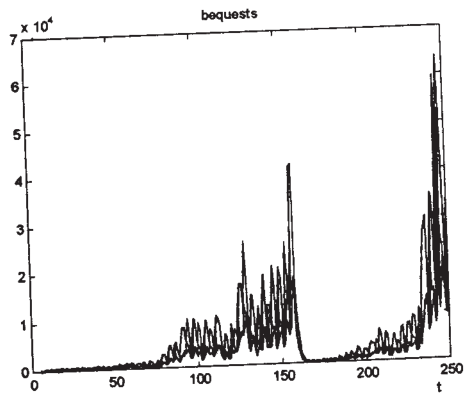

Figure 3: Continuation up to $t = 250$. The bequests reach a local maximum in $t = 158$, with an average value $\bar{b} = 24181$ and a peak $b_{\max} = 41835$, and then fall until $t = 167$, when the average bequest is $\bar{b} = 163$, and the smallest one is $b_{\min} = 81$.

	$t = 155$	$t = 156$	$t = 157$	$t = 158$	$t = 159$
e_1	3761	7276	23598	5129	0
e_2	3349	4676	0	0	0
e_3	3021	5424	0	0	0
e_4	3519	4889	0	0	0
e_5	3515	4919	0	0	0
e_6	3511	4949	0	0	0
e_7	3373	5739	23284	5020	0
e_8	3422	5162	0	0	0
e_9	3422	5162	0	0	0
e_{10}	4050	6484	21374	5059	0
$\bar{e}_t$	3418	5531	6825	1520	0
$E_{t-1}(\bar{e}_t)$	1844	2612	10257	9099	4365

Table 1

We notice that after $t = 156$ only the three rich dynasties go on investing in education, reaching a maximum in $t = 157$ and then decreasing in $t = 158$; in $t = 159$ nobody finds it profitable to invest resources in human capital (everybody chooses the investment in physical capital or consumption). This behavior is due to the expectations the agents make about others' choices. They expect there to be levels of investment that are lower than those which will be observed afterwards. Comparing the last two rows of the table, we can notice that, between $t = 155$ and $t = 156$, there is a straight delay in forecasts (actually, this is true for every single expectation, and not only for the reported average). This delay in expectations induces them *ex ante* to evaluate the investment in human capital as being more profitable than what it will be *ex post*. This optimistic evaluation drives each agent to increase his own investment in education.

	$t = 156$	$t = 157$	$t = 158$	$t = 159$	$t = 160$
H	15245	23446	25126	6580	100
Y	303650	501730	543930	113930	860

Table 2

The result is more human capital, more income, higher wages. But expectations are made with a flexible filter, which takes errors into account: the neural network learns how to deal with the data, changing the weights of its connections. After two periods in which investments are underestimated, expectations are sharply increased, and in $t = 156$ we have forecasts about $t = 157$ that are higher than the following realizations. The expectation of high investments by other competitors induces all the low income dynasties to give up: only the three high income dynasties invest in education. Suddenly, in $t = 157$, only three agents compete for the good jobs, investing less than what could be expected.[31] Expectations now overestimate realizations (also individually, not only on average). Since the competition is now lower, in $t = 158$ the three high income dynasties decrease their investments, and so human capital, output and wages decrease either (see table 2). Because of the expectations inertia, agents go on overestimating others' investments, so they think they are not able to compete any more (wages are now lower, and bequests either): in $t = 159$ nobody thinks he is in a position to compete effectively, and the aggregate investment in education is zero.[32]

3.1.3 More responsive expectations

Increasing the reverberation factor to 3, the outcome is quite different, because the expectations evolution is faster, and so is the growth of education investments and the growth of output. The phase of fast growth (in the previous case we called it phase 1) does not stop, and we directly observe an exponential growth.[33] Also in other cases, that we have omitted, we found a great sensitivity to the reverberation factor, which stands for the expectation sensitivity to the last set of observations (in our case, the last two observations, given $l = 2$).

3.2 Constant returns to scale

We can now see the results of a simulation carried out with the same parameter values as the previous one, except η, which is now set equal to 0,6 (then $\gamma+\eta = 1$: production function with constant returns to scale)[34]. Since $\alpha < 1$ (decreasing returns in the human capital accumulation function), we do not obtain long run growth. After a starting phase of fast growth (similar to phase 1 of the benchmark simulation), the output becomes stationary, and goes on fluctuating without any trend[35]. To verify our considerations, the simulation has been carried out for 500 periods, in which we did not find any growth. The investments in education continue to oscillate, but their moving average (on 50 periods) remains stationary. Output never exceeds

[31]The other three agents do not invest in education: they take part to the positional competition only with their innate human capital, $\tilde{h} = 10$, which is almost negligible, given the levels of investment by the three high income agents.

[32]It remains zero for five periods, until $t = 163$. Between $t = 160$ and $t = 164$ human capital is then given only by the innate abilities: $H = \sum \tilde{h} = 100$.

[33]We carried out several simulations, and we obtained different results, because of the stochastic nature of job allocation. In any case, all the simulations with a higher reverberation factor had a faster growth in common.

[34]The reverberation factor is equal to 3. With this parametrization, a different choice about the reverberation factor (e.g. equal to 1 or 2) does not change the qualitative outcome of the simulation.

[35]Given the probabilistic nature of the model, for each parametrization, it is better to carry out not only very long simulations, but also several simulation with the same parameter values, in order to verify the robustness of the observed results.

908 units (maximum reached in $t = 190$). The levels reached by the output depend on the job turnover, which is stochastic and so it can vary in different simulations. Looking at the results of other simulations (carried out with the same parametrization) we verified that, when the turnover is higher, output grows faster, and its average value is greater (this is a result which in some respect agrees with Galor - Tsiddon, 1996).

In figure 4 the levels of wage and output in the first 40 periods are drawn. There is a first phase (until $t = 7$) where expectations are not precise (learning is only at the beginning) and in which the investment in education increases sharply. In this first phase we also have a high mobility in job allocation. Between $t = 7$ and $t = 13$ there is no mobility at all: we have 6 periods in which the jobs are confirmed for each dynasty. As we can see from the wage graph, the phase with less mobility shows a decrease of the investments in education, and therefore of output and wages. We observe the biggest investment in human capital when the agents are not sure about their chances of getting one of the three good jobs, that is to say, when the difference between the agents' investments is small. In the phase without mobility, the high income workers know that the gap between their investment and the investment of the low income dynasties is big, and so they lower their guard, decreasing their own investments in education.[36] This descending phase goes on until the other dynasties become aware that the investment in education can now be effective in changing their chance to get a high-paying job. With the aid of the next two tables, we can analyze the behavior of the ten agents' investment and its consequences on the level of wages in more details, focusing our attention on four periods, from $t = 12$ to $t = 15$.

e	$t = 12$	$t = 13$	$t = 14$	$t = 15$
1	0.0204	0.0204	10.6183	0
2	0.0204	0.0204	18.9265	30.9513
3	17.8446	17.6744	16.7069	30.2496
4	0.0203	0.0203	10.6040	0
5	17.7666	17.7440	16.7177	0
6	0.0204	0.0204	10.6491	0
7	17.7666	17.7440	11.8831	0
8	0.0204	0.0204	10.6183	29.5991
9	0.0204	0.0204	10.6183	0
10	0.0204	0.0204	10.6183	0

Table 3

[36] A necessary condition to obtain this result is that human capital accumulation has decreasing returns to scale ($\alpha < 1$). With incresing returns, as we will see later, also the phases without mobility show an increse in investment.

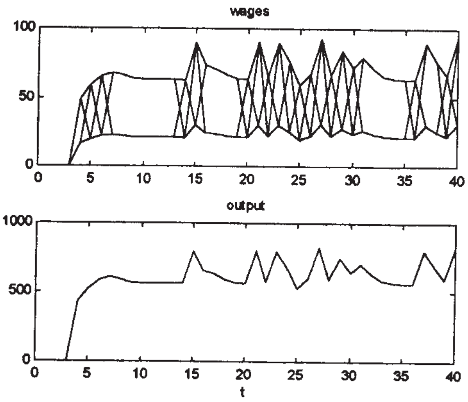

Figure 4: The graphs of the wages and the output in the first 40 periods. It is clear that the phases of stability in the job allocation lead to an output decrease.

w	$t = 12$	$t = 13$	$t = 14$	$t = 15$
1	21.0674	21.0354	21.0135	29.8378
2	21.0674	21.0354	63.0406	89.5135
3	63.2023	63.1063	63.0406	89.5135
4	21.0674	21.0354	21.0135	29.8378
5	63.2023	63.1063	63.0406	29.8378
6	21.0674	21.0354	21.0135	29.8378
7	63.2023	63.1063	21.0135	29.8378
8	21.0674	21.0354	21.0135	89.5135
9	21.0674	21.0354	21.0135	29.8378
10	21.0674	21.0354	21.0135	29.8378

Table 4

Between $t = 7$ and $t = 13$ the low income dynasties' investments are unchanging and nearly negligible ($e < 0.021$). Every agent observes these choices and makes his expectations about their future values, forecasting a lack of competition by the seven low income dynasties. As a result, the three high income dynasties gradually decrease their investments (in $t = 7$ they are equal to 21.8, while in $t = 13$ are reduced to 17.7); one period later we see the repercussion on human capital, on output and on the level of wages, which slightly decrease. Lower investments by the high income dynasties mean a decrease in their credentials (their chances to get a good job) till, in $t = 14$, there is a turnover: the seventh dynasty loses its high-paying job, to the second dynasty's advantage (since human capital accumulation needs one period to take place, the job allocation in $t = 14$ depends upon the investments in education made in $t = 13$). It is worth noting that exactly in $t = 14$ all the low income workers decide to enter back into the competition, investing a big proportion of their resources in education. Remember that in $t = 14$ the investment decisions are made only on the basis of the investments observed up to the previous period. The turnover among agent 2 and agent 7 is not present in their information sets. However, investments have reached such a level that they now find the positional investment profitable. This change of perception happens in the very moment when there is a change at the top of the social ladder. The only agents who in $t = 14$ know with certainty that the games are open again are agents 2 and 7. Their investments are different from others' for two different reasons. Agent 2 has more resources because his parent's high wage has been obtained without any investment in education (the bequest comes from a high wage summed up with the physical capital retribution). For this reason, his investment in education, in $t = 14$, is the highest in the economy. Agent 7 invests in education more than the other six low income agents because the recent history of his dynasty is one of high wages, and this brings about a higher bequest.

3.3 Returns to scale in the accumulation of human capital

The parameter α has a crucial role in the determination of the variables' temporal paths. Keeping the same values for all the parameters but assuming $\alpha = 1$ (constant returns in the accumulation of human capital), the product reaches higher values, but the quality of its evolution is not so much different from the previous case. The exponential growth, which is to be expected with the assumption of constant returns to scale both in the production function and in the accumulation function, takes place only if the parameters A, r, θ are increased. Under standard assumptions about the factor remunerations, growth would be determined by the values of α, γ, η, R, β and θ. In this model, however, also the degree of meritocracy r has an important role. Another variable which affects growth is A: given a constant interest rate and a constant inequality between wages (χ), an increase of the parameter A *ceteris paribus* brings about an increase of wages and an increase in the difference between the two possible levels of wages, which boosts the investments in human capital.

With the same parameter values, only further increasing the parameter α (increasing returns in the accumulation of human capital), the output never ceases to grow, at a great speed.[37] With increasing returns in the accumulation of human capital, agents are bound to invest much more resources in education, and this brings the system towards an unceasing growth, no matter how expectations are made. Contrary to what happened in the previous cases, we also observe an increase of the investment in education in the phases of stable job allocation (no mobility).

It is worth noting that, even with increasing returns in the accumulation of human capital, long run growth is not to be taken for granted. Growth can be limited by decreasing the dimension parameter (A), the meritocracy degree (r), or the weight of bequests in the preferences (θ). For example, it is possible to obtain a long run stationary output even with increasing returns both in the production function and in the accumulation function, provided that the values of r, A and θ are low enough. The following table shows a particular set of parameters:

$N = 10$	$\chi = 3$	$n = 3$
$\bar{h} = 30$	$\alpha = 1.44$	$r = 2$
$A = 0.35$	$\gamma = 0.35$	$\eta = 0.75$
$\beta = 0.9$	$\theta = 1$	$R = 1.05$

We can call this a bordering parametrization, because it is at the boundary between the stationarity and the unceasing growth of all the variables. Slightly increasing α, A, γ, η, r, β or θ, output tends to explode (and the same happens if we decrease R). The stationary outcome of this parametrization is shown in figure 5.[38]

3.4 More retribution inequality

Finally, we can briefly see a simulation carried out with the same parameters as the benchmark, but with χ increased to 10. This change means that the winners' wage is now ten times the losers' one. In figure 6 the investments and the wages are reported. It is apparent that the increased inequality brings about a lower turnover at the top positions: winners' resources are so much higher than that of the losers, that the competition becomes too unfair. As a result, we observe long phases of stability in job allocation (for example, in the first 19 periods, or from $t = 36$ to $t = 46$).Since the returns to scale of the accumulation function are deacreasing, less mobility means

[37] For example, setting $\alpha = 1.5$, the product immediately shows an explosive trend: in just 10 periods it goes over 500.000. Of course, this explosive tendency would be further increased in the case where the returns in the production function are also increasing.

[38] The reverberation factor is equal to 3. We carried out other simulations, with a different reverberation factor, but we did not find remarkable differences. Regarding the asymptotic characteristics, dealing with a numerical simulation, we can only say that, in the observational period, that is to say until $t = 1000$, even making the same simulation several times, we never found a trend in the moving average of the output, after the first phase of fast growth.

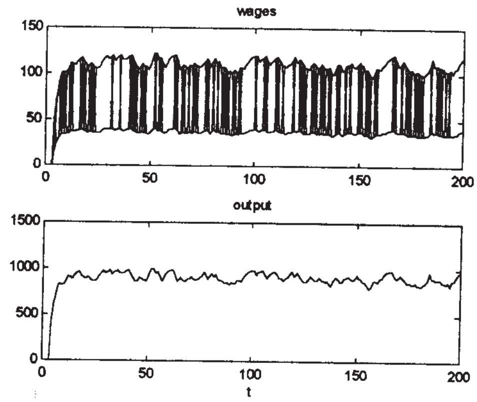

Figure 5: The outcome of the boundary simulation, in which output remains stationary in the long run (with cycles of period 22), although the returns to scale are increasing both in the production function and in the accumulation function.

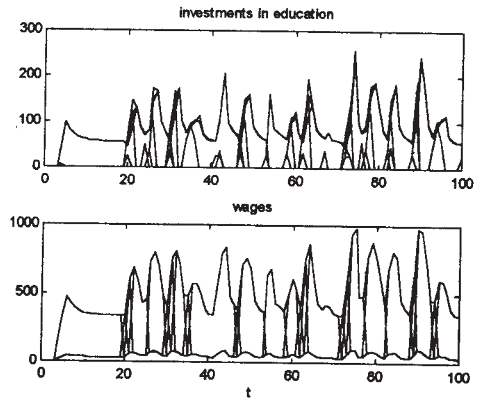

Figure 6: Investments in education and wages with a higher inequality between high and low wages: $\chi = 10$.

less investments, and this affects output negatively.[39] This last variable fluctuates widely, but always remains between 2000 and 6000. Remember that the same parametrization, with an inequality coefficient χ equal to 3, showed an unceasing growth of the output level. This means that bigger prizes, even if they initially bring about a bigger incentive to invest in education, also imply an excessive difference between the resources of the high income and low income dynasties. A difference which is too high discourages the low income dynasties, who do not invest in education, leading to a lower amount of aggregate human capital and output. A strong competition by the low income dynasties is crucial for the performance of the economy. This becomes apparent if we take a detailed look at the results of the simulation between $t = 36$ and $t = 46$. As we already pointed out, in this temporal interval the job allocation does not change at all. However, the path of the investments in education and the output is not monotonous. From figure 6 we can clearly see that from $t = 37$ to $t = 40$ the seven low income dynasties do not invest in education, driving the high income dynasties to gradually decrease their own effort in positional competition. In $t = 41$, forecasting a further decrease (which punctually takes place) of the investments of the high income dynasties, six out of the seven low income dynasties enter the competition, increasing their investments from 0 to 26.46 (see table 5). The three rich dynasties, observing an increase in the positional competition, sharply increase their investments, in order to restore the desired lead between their own credentials and the ones of the low income dynasties. Only after $t = 43$, having observed a retreat, can they again decrease their effort in the competition. Although in this particular case it does not succeed in changing the job allocation, the competition by the low income agents affects the behavior of the high income dynasties, leading to a higher level of output. Low income dynasties, in this case, gain a small increase in their retribution. This is not due to a reallocation of jobs (that does not take place), but to the fact that higher investments in education by all the agents in the economy drive the economy to higher living standards.

[39]The reverberation factor is still equal to 3. Setting it equal to 2 does not change the outcome qualitatively.

	$t=39$	$t=40$	$t=41$	$t=42$	$t=43$
e_1	0	0	0	38	0
e_2	64.24	59.66	59.11	109.98	205.46
e_3	0	0	26.46	25.68	0
e_4	63.38	61.14	58.98	106.33	208.32
e_5	0	0	26.46	25.38	0
e_6	0	0	26.46	25.53	0
e_7	62.28	59.93	58.51	115.20	207
e_8	0	0	26.46	25.53	0
e_9	0	0	26.46	25.53	0
e_{10}	0	0	26.46	25.53	0
$\bar{e}_t$	18.99	18.07	33.54	52.27	62.08
$E_{t-1}(\bar{e}_t)$	23.33	18.97	17.3	28.1	68.32

Table 5

4 Comments

Starting from four hypothesis on agents' behavior and on the functioning of the labor and credit markets, we built up a model where education plays two different roles: from an individual perspective, it increases the credentials to get a good job, while from an aggregate perspective it increases factor productivity. Human capital accumulation emerges in the economy as a result of the individual efforts to win the positional competition in the labor market. For this reason, the fluctuations due to the positional competition are summed up with the growth due to the increase of productivity. The qualitative characteristics of the variables temporal evolution depend upon the way agents make their decisions and upon the values of the parameters which determine the preferences and the technological and institutional context. The structure of the labor retribution (the number of high-paying jobs, the inequality between the two kinds of job) and the way jobs are allocated (the hypothesis on the probabilistic form of positional competition and the meritocracy degree of the system) are crucial in determining the individual incentives to accumulate human capital, and through it, the possibility of economic growth. The labor market has been modelled merging some characters of tournament models and some characters of rent-seeking models, since the difference between the two possible wages has been interpreted as a rent. Well educated agents have more chances of playing a privileged role in the productive system; however, their retribution is not precisely connected to the amount of their own human capital, but it is determined on the basis of the productivity of factors and on the basis of the institutional rules about the allocation of income. Schooling credentials are a means to enter into privileged positions, to which are reserved the biggest shares of aggregate income.

In the model, investment choices fluctuate, and often widely. This can be seen as a departure from the empirical observation of a high correlation of schooling choices among the different generations of the same dynasty. The fluctuations obtained in the simulations can be due on one hand to the lack of self-regulating mechanisms (prices do not move, and even the retribution of physical capital R is fixed). On the other hand they can be due to the hypothesis that different generations are only connected by the amount of bequests, and not by other forms of inertia or dependence between present behavior (sons) and past behavior (parents). Actually, different forms of such a dependence can be observed in the real world, and this can make the evolution of investments smoother among generations. Another cause of instability is introduced in the model by the assumption that only two kinds of jobs are available, and then only two levels of pay. Maybe an increase in the number of levels would reduce instability.[40]

The structure of retributions (determined by the N, n, χ parameters), which is a constant in each simulation, is empirically a variable itself. It can evolve throughout time, changing the individual incentives, and then agents' behavior. The obtained results are also affected by the assumptions about preferences: homothetic preferences make the nature of the connection among several variables obvious: on one hand output and wages are proportionally connected because of the hypothesis about the structure of retribution; on the other hand the constant shares partition of income among consumption (by the elders) and bequests makes the connection between bequests and output quite elementary.[41] The hypothesis about expectations are crucial too. The degree of sensitivity to new experiences has often important consequences on the path of the variables.

Heterogeneity in this model is due to the very hypothesis on factor remuneration, which in each period splits workers in two income categories. The division is never definitive, because jobs are reallocated at each stage, and mobility is always possible. As we noted, the speed of economic growth greatly depends on the turnover in the job allocation. The parameter configurations which lead to a higher *circulation of the élite* also lead to higher levels of output.[42] This circulation is due to the form of the positional competition, that is to say to the number of top positions and to their re-

[40]As an extension of the model, through the introduction of an intermediate class of workers the role of a middle class in a developing country can be analyzed.

[41]Log-linear preferences mean also risk-aversion. We did not analyze other possible attitudes towards risk.

[42]See Checchi - Ichino - Rustichini (1996) for empirical evidence. Lasch (1995) about this point says: "A high level of mobility is not contradictory with a system of stratification which concentrates power and privileges in a dominant élite. On the contrary, the circulation of the élite strengthens the hierarchical principle, because it always provides new talented élite and legitimates their ascent as a function of merit and not of birth".

muneration with respect to low positions. We noticed, for example, how the relationship between compensation inequality (χ) and growth is not monotonous. A low degree of inequality discourages investments, but too much inequality, though stimulating the initial investments in education, excessively breaks apart the resouces of the two kinds of dynasties, making the poor ones not able to compete effectively, and therefore leading to a slower evolution of human capital.[43] The parametrizations which lead to the biggest investments in human capital are then those which make the contest desirable through big prizes in a limited number, but always trying to keep the poor in a position to effectively compete whenever the high income dynasties lower their investments in education. The role of compulsory education (represented by the innate human capital $\tilde{h}$) is controversial as well. In the first place, it increases the initial amount of human capital (freely obtained) and then the output and the aggregate amount of wages, making the contest more desirable. For this reason, a higher level of knowledge usually brings about higher investments and higher growth. Afterwards, however, a higher innate human capital means more capital also for those who do not invest in education; therefore, too much innate abilities can discourage positional competition. At higher levels of innate human capital, agents find it more difficult (expensive) to distinguish themselves through education, since one more unit of education has a smaller influence on the probability of getting a good job.[44] The different simulations carried out varying the innate human capital pointed out the contrasting nature of these two effects.

While the relationship between the $\tilde{h}$, n and χ parameters and economic growth is generally not monotonous, by increasing α, A, γ, η, r, β or θ we always obtain a higher output growth, or its settlement on higher average values. The parameters α and r, which characterize the accumulation function, play an important role. The returns to scale in human capital accumulation affect the evolution of investments. With increasing returns, the high income dynasties increase their positional investments also in the phases without mobility. With decreasing returns, in the phases without turnover the investments in education tend to decrease. We called parameter r the "meritocracy degree" of the economy: if it increases, it becomes more difficult for an agent with low education to obtain a top working position; though it does not affect the amount of human capital given the

[43] In particular, this is true with decreasing returns in the accumulation function ($\alpha < 1$). In the simulations presented, the number of top positions has always been equal to three. Looking at other simulations (here omitted to save space) carried out by varying the parameter n, we noticed that also the relationship between the number of top positions and the strength of positional competition is not univocal. This kind of relationship, between the number and the size of prizes and the incentives to compete, is already the object of the economic literature about tournaments.

[44] If we interpret the innate human capital as the compulsory education, it becomes interesting to study the effects of a continuous increase of it throughout time, since this is the evolution which historically took place.

level of investment in education, it has a great influence on the amount of investments: more meritocratic systems bring about higher investments, in particular by the high income dynasties, and then higher levels of human capital and output.

The model is based on several working hypothesis which move away from standard economic assumptions in growth literature. In the choices of assumptions, we have been guided by a simple criterion of common sense. The aim of the paper has only been to analyze the implications of such assumptions on the temporal evolution of the modelled economy. It is however certainly true that the usefulness of the assumptions themselves has to be tested through the comparison with empirical evidence, a step forward that we hope we can make in the near future.

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