Fiscal policies and growth

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Abstract

In this work we present an endogenous growth model where the Government finances a public good by imposing two taxes, one on the return of the accumulative factor and the other on the return of the not accumulative factor. In an economy where individuals have different initial factor endowments, we determine the fiscal policy that maximizes the growth rate, the political equilibrium and, finally, the socially optimal fiscal policy. Because of heterogeneity of individual's endowments maximizing growth rate does not imply maximum welfare; the political equilibrium fiscal policy does not maximize the growth rate, but it could be socially optimal if the inequality aversion degree is sufficiently high.

JEL classification: H3, I3, O4

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1 Introduction

This paper studies optimal taxation in a model of endogenous growth. Our goal is to show that relaxing the representative agent hypothesis changes the standard normative results. Chamley (1986) analyzes the effects of different fiscal policies in a neoclassical growth model. He states that the optimal fiscal policy has not to tax the return on capital if other financing sources are available. Lucas (1990) extends this result to an endogenous growth model with human capital too. Both articles are based on the assumption of representative agent, so that maximum growth rate implies maximum welfare[1]. Recently some authors have introduced endogenous growth models in which individuals have different initial factor endowments (see Alesina and Rodrik (1994) and Bertola (1993)). Moreover these authors have studied the political equilibrium, that is the equilibrium in which the fiscal policy to apply is decided by a majority voting rule, showing that in this equilibrium the economy does not experience the maximum growth rate.

We argue that the conclusions of Lucas (1990) no longer hold in an economy where individuals are heterogeneous in terms of initial factor endowments, because in such an economy maximum growth rate does not imply maximum welfare. It is also necessary to reconsider the conclusions about the political equilibrium: this, indeed, has to be valued in terms of welfare and not in terms of the maximum growth rate. Indeed, the political equilibrium fiscal policy could be socially optimal if the inequality aversion degree is sufficiently high.

The paper is organized as follow.

We present our model in the second section. There are three inputs: the accumulative factor, the not accumulative factor and a factor that can be provided only by Government (Barro 1990). This latter finances its input supply by imposing two type of taxes: one on the return on the cumulative factor and one on the return on the not cumulative factor. Moreover, we introduce an aggregate production function (see Barro (1990)) and, under the assumption of competitive factors markets, we determine the return on factors. Then we analyze the consumer problem; following Alesina and Rodrik (1994) we assume that consumers take the paths of the aggregate variables as given. A key assumption is that every consumer has different initial factor endowments and, therefore, he shows different preferences on the fiscal policy.

In the third section we analyze the fiscal policy that maximizes the growth

[1] In fact, Chamley (1986) claims that his result holds also for an economy where individuals have different factor endowments.

rate and the political equilibrium. We find that to maximize the growth rate the return on the cumulative factor has not to be taxed at all (as in Lucas (1990)). Indeed an increase in the tax on the return on the cumulative factor causes an increase in average propensity to consume and therefore a reduction in the accumulation; in turn this causes a decrease in the growth rate. On the contrary, an increase in the tax on the return on the not cumulative factor has not any effect on its supply, because this factor shows an inelastic supply. Finally we conclude that the political equilibrium fiscal policy coincides with the optimal one for the individual that has a ratio between the two factors endowments (not cumulative and cumulative) that is median as regards to the ratios of all the other individuals.

In the fourth section we get ahead with welfare analysis. We introduce the Lorenz dominance concept and we rank, under very general assumptions about the form of the welfare function, the feasible fiscal policies. We determine two paradigmatic fiscal policies, that maximizing a Rawlsian welfare function and that maximizing the simple sum of all the individual utilities (we call this latter pure utilitarian welfare function). Between these two there is a range of fiscal policies, each one optimal according to a specific inequality aversion degree of our hypothetical social planner.

In the fifth section we compare the socially optimal fiscal policy and the political equilibrium fiscal policy.

In the sixth section we summarize the results of the previous sections and highlight in an organic manner the relationships among the three kinds of fiscal policies we have studied.

Conclusions and references close the paper.

2 The model

In this section we describe the structure of our economy. From now on to simplify the exposition we indicate, without loss of generality, the cumulative factor as capital and the not cumulative factor as labor. We assume that the aggregate production function is[2]:

$$Y = AK^{\alpha}G^{1-\alpha}L^{1-\alpha}, \text{ with } \alpha < 1 \tag{1}$$

where Y is aggregate production, K is the capital, L is the labor, A is a scale parameter and G is a factor provided by the Government, which produces a positive externality on all the other productive factors. We could consider G as productive services supplied by the Government to every firm.

[2]We will always omit the time index if this does not create confusion.

Public expenditure G is financed with a balanced budget:

$$G = \tau r K + \hat{w} \gamma L \tag{2}$$

where τ and γ represent two different taxes on the returns, respectively, on capital and on labor. These factors are payed according to their marginal productivity. This implies that[3]:

$$r = \alpha A^{\frac{1}{\alpha}} \left[\alpha \tau + (1-\alpha)\gamma\right]^{\frac{1-\alpha}{\alpha}} = r(\tau, \gamma) \tag{3}$$

$$\hat{w} = (1-\alpha) A^{\frac{1}{\alpha}} \left[\alpha \tau + (1-\alpha)\gamma\right]^{\frac{1-\alpha}{\alpha}} K = w(\tau, \gamma) K \tag{4}$$

If we substitute (2), (3) and (4) into (1), we get:

$$Y = A^{\frac{1}{\alpha}} \left[\alpha \tau + (1-\alpha)\gamma\right]^{\frac{1-\alpha}{\alpha}} K \tag{5}$$

It is worth noting that we are working with a AK model, that is with a model in which the marginal return on the cumulative factor remains constant, instead of decreasing, as the factors accumulates. Moreover we assume that it is impossible to subsidize the factors and therefore τ and γ are defined over the range $[0, 1]$.

We assume that in our economy there are N consumers with different initial factor endowments. Each consumer maximizes his intertemporal utility, taking the aggregate variables path as given.

Let l_i be the labor endowment of the i-th individual and k_i his capital endowment; we define the relative endowment of the i-th individual as:

$$\sigma_i = \frac{K l_i}{k_i}$$

It follows that his income is:

$$y_i = k_i \left[(1-\tau) r(\tau, \gamma) + (1-\gamma) w(\tau, \gamma) \sigma_i\right] \tag{6}$$

We assume that the instantaneous utility function is log-linear[4], so that:

$$U_i = \int_0^\infty e^{-\rho t} \ln c_i dt \tag{7}$$

Given the time paths of both the taxes and the aggregate stock of capital[5], the maximization of U_i, subject to:

[3]For simplicity we have set the total quantity of labor to 1, that is $L = 1$.

[4]Nothing changes if we adopt a CES utility function.

[5]Really the interactions among individuals occur only by the decisions on τ.

$$\dot{k}_i = k_i \left[(1-\tau)r(\tau,\gamma) + (1-\gamma)\, w(\tau,\gamma)\sigma_i\right] - c_i \tag{8}$$

yields to the following optimal consumption path:

$$\frac{\dot{c}_i}{c_i} = (1-\tau)r(\tau,\gamma) - \rho \tag{9}$$

It is worth noting that if τ and γ remain constant, then also the consumption growth rate will be constant. It is possible to demonstrate that (9) represents also the growth rate of K and k_i, that is:

$$\frac{\dot{c}_i}{c_i} = \frac{\dot{k}_i}{k_i} = \frac{\dot{K}}{K} = \eta\,(\tau,\gamma) \tag{10}$$

Therefore $\eta\,(\tau,\gamma)$ is the balanced growth rate.
From (8), (9) and (10) we get:

$$c_i = \left[(1-\gamma)\, w(\tau,\gamma)\sigma_i + \rho\right] k_i \tag{11}$$

This equation shows the instantaneous level of consumption along the optimal path.

3 Fiscal policy: positive analysis

In this section we determine the optimal fiscal policy for the i-th individual, the fiscal policy that maximizes the growth rate and the political equilibrium fiscal policy. The optimal fiscal policy for the i-th individual solves the following problem:

$$\begin{aligned} &\max_{\tau,\gamma} U_i = \int_0^\infty e^{-\rho t} \ln c_i \; dt \\ &s.t. \begin{cases} c_i = \left[(1-\gamma)\, w\,(\tau,\gamma)\,\sigma_i + \rho\right] k_i \\ \dfrac{\dot{k}_i}{k_i} = \dfrac{\dot{K}}{K} = \eta\,(\tau,\gamma) \end{cases} \end{aligned} \tag{12}$$

It is worth noting that we are not assuming that the two taxes are constant. The Hamiltonian for our problem is:

$$H = \ln k_i \left[(1-\gamma)\, w\,(\tau,\gamma)\,\sigma_i + \rho\right] + \lambda k_i \left[(1-\tau)\, r\,(\tau,\gamma) - \rho\right] \tag{13}$$

The necessary and sufficient conditions are:

$$\frac{\partial H}{\partial \tau} = 0 \tag{14}$$

$$\frac{\partial H}{\partial \gamma} = 0 \tag{15}$$

$$\frac{\dot{\lambda}}{\lambda} = \rho - \frac{1}{k_i \lambda} - \eta(\tau, \gamma) \tag{16}$$

A solution to (16) is $\frac{1}{k_i \lambda} = \rho$, which we substitute into the other equations. From (14) and (15), we get:

$$\tau = \frac{(1-\alpha)(1-\gamma)}{\alpha} \tag{17}$$

In the $(\tau,\ \gamma)$ space the feasible solution set defined by (17) consists of a segment, whose bounds are the intersections with the axes. By (14) we get[6]:

$$\tau = (1-\gamma)\left[(1-\alpha) + \rho\frac{(1-\alpha)^2}{\alpha} * \frac{\sigma_i}{(1-\gamma)\, w(\tau,\gamma)\,\sigma_i + \rho}\right] \tag{18}$$

Substituting (17) into (18), we get:

$$\hat{\tau}_i = \max\left[0, \frac{\rho(\sigma_i - 1)}{\alpha A^{\frac{1}{\alpha}} (1-\alpha)^{\frac{1-\alpha}{\alpha}} \sigma_i}\right] \tag{19}$$

The optimal tax on the return on capital for the i-th individual $\hat{\tau}_i$ will be 0 until his relative factor endowments is less or equal to 1 and then increasing for values greater than 1[7]. Moreover τ does not depend on time, so that the optimal fiscal policy is constant. The individual optimal tax on a factor is inversely proportional to the individual relative endowment of that factor; for example, if relative endowment of capital fall (that is σ_i increases), the optimal tax on capital increases (see (19)). Notice also that, given equation (17) holds, w and r are determined independently of τ and γ.

To analyze thoroughly this point we impose, in maximizing the i-th individual's utility, that the two taxes have to be constant (we have seen that

[6]There is a trivial solution of (18), that is $(1,0)$, invariant to σ_i, which comes from the method used to solve the system. To prove this, we notice that if we substitute for this solution in (14), this latter is not verified. Therefore we ignore this solution.

[7]We have assumed that it is impossible to subsidize a factor. In Bertola (1993) we can find a similar result, even if in that work, given the possibility of subsidizing factors, τ can be negative.

for optimal fiscal policy this property holds for every individual). If we solve the integral in (12), we get the following individual indirect utility function:

$$U_i = \frac{1}{\rho}\left\{\ln k_i^0\left(\left(1-\gamma\right)\bar{w}\sigma_i + \rho\right) + \frac{\left(1-\tau\right)\bar{r} - \rho}{\rho}\right\} \tag{20}$$

where $\bar{w}$ and $\bar{r}$ represent the factor gross returns for every fiscal policy belonging to the segment defined by equation (17) and k_i^0 the initial capita endowment. The first term in curly brackets (decreasing in the tax on the return on labor) stands for the level effect , that is the propensity to consume out of capital, while the second term (decreasing in the tax on the return on capital) stands for the growth effect , that is the incentive to accumulate. Therefore, the optimal fiscal policy is determined by trading off these two effects; whereas maximizing the rate of growth amounts to considering the second effect only. Notice that the level effect is a positive function of σ, which explains our previous results about the relationship between preferences over the fiscal policy and relative factor endowments.

3.1 Maximum growth

Equation (11) shows how the ratio between c_i and k_i varies as the fiscal policy changes along the balanced growth path: the higher the tax on the return on capital, the higher the instantaneous level of consumption. This, in turn, implies a lower growth rate. Indeed, from (10) we can see that the fiscal policy that maximizes the growth rate $\eta\left(\tau,\gamma\right)$ is $\tau^* = 0$ and $\gamma^* = 1$. This result, which agrees with that of Lucas (1990), can be explained by noting that the growth rate of income, and therefore of consumption, is equal to the growth rate of the capital and, because of the dependence of the saving level on its return, a tax on capital reduces the incentives to accumulate (see the next section for a more analytic analysis of this point). This is not true for labor because of it has an inelastic supply. It is worth noting that also in the optimal taxation theory there is a similar result: the factor with less elastic supply has to be charged a higher tax.

3.2 Fiscal policy and average propensity to consume

The analysis of the relationship between the average propensity to consume and the fiscal policy provides us with a very interesting alternative point of view for justifying previous results. By (6) and (11) we can get the consumption level as a function of income:

$$c_i = \frac{(1-\gamma)\,w(\tau,\gamma)\,\sigma_i + \rho}{(1-\gamma)\,w(\tau,\gamma)\,\sigma_i + (1-\tau)\,r(\tau,\gamma)} y_i \tag{21}$$

If we let z_i be the average propensity to consume of the i-th individual, we can say that:

1. an increase in σ_i leads to an increase in the average propensity to consume, which tends to 1 as σ_i goes to infinity (that is when the individual has not any quantity of capital). Formally:

$$\frac{\partial z_i}{\partial \sigma_i} > 0 \text{ and } \lim_{\sigma_i \to \infty} z_i = 1$$

2. the average propensity to consume increases as the tax on the return on capital increases for all fiscal policies satisfying (17). Formally:

$$\frac{\partial z_i}{\partial \tau} < 0 \; \forall (\tau, \gamma) \text{ satisfying (17)}$$

3. the average propensity to consume decreases as the tax on the return on capital increases for all fiscal policies implying a positive growth rate (also (17) is implicitly satisfied). Formally:

$$\frac{\partial z_i}{\partial \gamma} < 0 \text{ if } \eta(\tau, \gamma) > 0$$

The first point shows the relationship between income sources and saving level; we find that individual who gets all his income from labor does not save anything[8]. The second and third points show how taxation affects the average propensity to consume; in particular the fiscal policy that maximizes the growth rate implies the smallest average propensity to consume.

3.3 Political equilibrium

By political equilibrium we mean the fiscal policy deriving from a particular aggregation rule of individual preferences, that is majority voting rule. Though there are two taxes to vote on, we can avoid the problems of the multi-dimensionality of the policy space using equation (17), which allows to reduce the choice to one parameter. Moreover, because consumer preferences are quasi-concave (see (20)), we can apply the median-voter theorem[9].

[8] See Bertola (1993) for a similar conclusion.

[9] See Mueller (1989).

If the σ of the median voter, which we call σ_m, is greater than 1, the political equilibrium fiscal policy does not maximize the growth rate, that is:

$$\text{if } \sigma_m > 1 \Rightarrow \tau^P = \frac{\rho\left(\sigma_m - 1\right)}{\alpha A^{\frac{1}{\alpha}}(1-\alpha)^{\frac{1-\alpha}{\alpha}}\sigma_m} > 0 \tag{22}$$

If we consider $(\sigma_m - 1)$ as an index of inequality and that political parties compete one with other by presenting an economic program which commits them for their future economic choices, then, because τ^P is an increasing function of σ_m, a greater inequality in the distribution of factor property rights determines a lower growth rate. In general terms, we can say that the political equilibrium fiscal policy, for realistic distribution of σ, does not maximize the growth rate.

4 Fiscal policy: normative analysis

The goal of this section is to rank consumption paths in welfare terms. The distribution of σ is crucial to determine the optimal fiscal policy. Analyzing this economy with a representative agent model, that is ignoring the possible heterogeneity among individuals, is equivalent to assuming that every individual had σ equal to 1. This implies that the fiscal policy that maximizes the utility of every individual is the same for everyone and, consequently, it is welfare maximizing. Moreover the same fiscal policy maximizes the economy growth rate (see (19)): there is not trade-off between equity and efficiency. We arrive, therefore, to the same conclusion of Lucas (1990). Indeed, he concluded that the return on capital would not have to be taxed if there had been other disposable sources of financing public expense. This was true from the points of view of both efficiency and welfare. But the heterogeneity of individual endowments implies that this result does not hold any more. In particular, if we do not take into account the possibility of *lump-sum* redistributions and if there is at least an individual that has a relative factor endowment greater than 1, then there is no more a perfect correspondence between welfare and efficiency.

In our economy there are many Pareto optima. The individually optimal fiscal policies represent Pareto optimal allocations for our economy, but they are not the only ones. In particular, because the individual utility function is concave, we can say that the set of Pareto optima of our economy is an interval whose bounds are, for every tax, the optimal fiscal policy of the individuals with, respectively, the highest and the smallest value of σ. Therefore the fiscal policy that maximizes the growth rate is not the optimum optimorum, but only one of the Pareto optima (that is the optimal fiscal policy for the individuals with a σ not greater than 1).

To rank the elements of this set in welfare terms we have to postulate a social welfare function and therefore that the individual utilities are somehow comparable among them. In particular we assume that the welfare function is additive-separable, that is:

$$W = W(x_1, ..., x_N) = \sum_{i=1}^{N} \phi(x_i)$$

where x_i represents the utility index of the i-th individual.

It is worth noting that the properties of the welfare function depend on the form of $\phi(.)$. For example, if we are not interested in the inequality of the individual welfare levels, $\phi(.)$ will be linear in its argument (we call this the pure utilitarian welfare function).

4.1 Lorenz dominance

The Lorenz dominance allows us to rank two distributions of individual utilities under very general conditions about the form of $\phi(.)$:

Theorem (Shorrocks)[10]: Given two vectors X and X^*, whose elements are ranked in an increasing order, under the hypothesis that $\phi'(.) \geq 0$ and $\phi''(.) \leq 0$ we have:

$$GL(p) \geq GL^*(p), \forall p \in [0,1] \Leftrightarrow \sum_{j=1}^{N} \phi(x_j) \geq \sum_{j=1}^{N} \phi\left(x_j^*\right) \tag{23}$$

where $GL(p) = \frac{\sum_{j=1}^{S} x_j}{N}$, $p = \frac{S}{N}$ and $S = 1, ..., N$.

The $GL(p)$ represents the (generalized) Lorenz curve and X is said to dominate X^* according to the Lorenz dominance principle. A graphic illustration is given in Figure 1.

[10] See Shorrocks and Foster (1987).

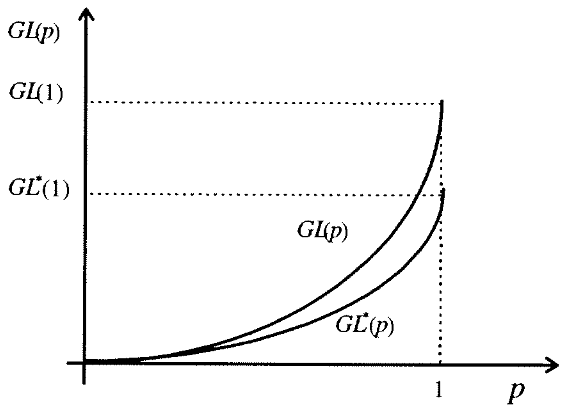

Figure 1

Because the $GL(p)$ curve is always above the $GL^*(p)$ curve, we can apply Shorrocks theorem and conclude that X is preferred in welfare term to X^*. If this condition does not hold we cannot reach an unambiguous conclusion. In particular, if two curves cross only once, we have to impose other constraints on the form of $\phi(.)$ to be able to rank the two corresponding utilities vectors (see Dardanoni and Lambert (1988)).

4.2 Ranking consumption paths

The goal of this section is to use the Shorrocks theorem to rank the consumption paths corresponding to alternative fiscal policies.

As seen all the individually optimal fiscal policies belong to the line segment (17); the same holds for all the Pareto optima which we are interested in; therefore it seems reasonable to limit our search for the socially optimal fiscal policy to this line segment. In this way we can write the balanced consumption path as a function of a single variable, for example τ. Let $\bar{C}_0(\tau) = (\bar{c}_1^0(\tau), ..., \bar{c}_N^0(\tau))$ and $\bar{\eta}(\tau)$ be, respectively, the ranked vector of the individual initial consumptions and the balanced growth rate valued on line segment (17).

We cannot directly apply the Shorrocks theorem because, as shown, we have to rank matrixes (consumption paths) and not vectors. We first consider a single period; we take, without loss of generality, period 0. The Lorenz curve defined by this consumption vector is:

$$GL(p,\tau) = \frac{\sum_{j=1}^{S} \bar{c}_j^0(\tau)}{N} \quad S = 1, ..., N \text{ e } p = \frac{S}{N}$$

where $\bar{c}_j^0(\tau)$ is the j-th individual consumption in period 0. To apply the Shorrocks theorem we have to know how the fiscal policy affects the Lorenz curve. By equations (11) and (17) we can get the individual consumption in period 0 , that is $\bar{c}_j^0 = \bar{r}\tau K^0 l_j + \rho k_j^0$ and the growth rate, that is $\bar{\eta} = (1-\tau)\bar{r} - \rho$.

We can verify that:

$$\frac{\partial GL(p,\tau)}{\partial \tau} \geq 0 \; \forall p \in [0,1] \text{ and } \forall \tau \in [0,1]$$

The result is unambiguous: an increase in the tax on the return on capital, which corresponds to a decrease in the tax on the return on labor, causes a static welfare gain. This conclusion seems to be very intuitive: a fall in the tax on the return on labor causes an increase in the consumption of all individuals and therefore an increase in infratemporal welfare; to put it differently, we have a positive effect level. But we have to consider also the growth effect, which, as seen in a previous section, decreases as τ increases (by equation (17), to every increase in τ corresponds a decrease in γ). The next step is to extend the analysis to all consumption path. Now it is clear that there is a trade-off between the growth rate and initial consumption. The socially optimal fiscal policy would agree with the fiscal policy that maximizes the growth rate if it were possible to compensate the smaller level of initial consumption with future redistribution of consumption (or income); but nothing assures us that this is possible.

As said the Shorrocks theorem cannot be applied in a multidimensional context; to solve this problem we could follow two ways:

1. to extend the results of the Shorrocks theorem to a multidimensional context (see for example Atkinsons and Bourguignon (1982))

2. to use a synthetic index for "concentrating" the whole intertemporal consumptions vector of a single individual, leading again the analysis to an unidimensional context.

We follow the second way.

A natural candidate to represent the synthetic index of the whole intertemporal consumptions vector seems to be the indirect utility function expressed by (20). Therefore the whole consumption path associated to τ will be represented by the following vector:

$$\left[\; U_1\left(\bar{c}_1^0(\tau), \bar{\eta}(\tau)\right) \quad \ldots \quad U_N\left(\bar{c}_N^0(\tau), \bar{\eta}(\tau)\right) \; \right]$$

It is worth noting that individual intertemporal utilities and individual initial consumptions are ranked in an identical manner.

Therefore the Lorenz curve will be:

$$GL(p,\tau) = \frac{\sum\limits_{j=1}^{S} U_j(.)}{N} = \frac{1}{\rho N}\left[\sum_{j=1}^{S} \ln\left(\bar{r}\tau K^0 l_j + \rho k_j^0\right) + S\frac{\bar{\eta}(\tau)}{\rho}\right]$$

with $S = 1, ..., N$.

If we calculate the first derivative of $GL(p,\tau)$ with respect to τ, we can see how the Lorenz curve changes as fiscal policy varies:

$$\frac{\partial GL(p,\tau)}{\partial \tau} = \frac{1}{\rho N}\left[\sum_{j=1}^{S} \frac{\bar{r}\sigma_j}{\bar{r}\tau\sigma_j + \rho} + \frac{S}{\rho}\frac{\partial \bar{\eta}(\tau)}{\partial \tau}\right] \tag{24}$$

We know that two fiscal policies can be ranked by Shorrocks theorem only if their corresponding Lorenz curves do not cross, that is (24) has to show an unambiguous sign for every $S = 1, ..., N$; if this were not true we should make much more restrictive assumptions about the form of the welfare function.

It is worth noting that for $\tau = 0$ equation (24) yields:

$$\frac{\partial GL\left(\frac{S}{N}, 0\right)}{\partial \tau} > 0 \Leftrightarrow \frac{\sum\limits_{j=1}^{S} \sigma_j}{S} > 1 \quad S = 1, ..., N \tag{25}$$

If we remember that individuals having a σ not greater than 1 prefer a fiscal policy that maximizes the growth rate, the condition expressed by (25) seem to be very intuitive: for every $S = 1, ..., N$ the Lorenz curve shifts up or down as τ becomes greater than 0 depending on the mean value of σ of the first S individuals being, respectively, greater or smaller than 1. Therefore, if (25) holds the maximum growth rate and the maximum welfare do not coincide.

4.3 Rawls and utilitarianism

Now we consider two specific fiscal policies, one maximizing a Rawlsian welfare function and one maximizing a pure utilitarian welfare function (that is, the simple sum of individual utilities).

According to Rawls' principle the welfare level of a society is represented by the utility of the worst-off. In terms of the Lorenz curve this means that we are interested only in the value of the curve corresponding to abscissa $\frac{1}{N}$. Therefore, the tax τ^R, optimal according to Rawls principle, will maximize the value of $GL\left(\frac{1}{N}, \tau\right)$, that is:

$$\tau^R = \underset{\tau \in [0,1]}{\arg\max} \left[GL\left(\frac{1}{N}, \tau\right)\right] \tag{26}$$

This is same as considering only the first element of the vector representing all individual consumption paths and choosing the tax rate correspondingly.

The pure utilitarian principle takes into account only the mean of all the individual utilities, which means that we are interested only in the value of the Lorenz curve corresponding to the abscissa 1. Therefore, the tax τ^U, optimal according to the pure utilitarian principle, will maximize the value of $GL(1,\tau)$, that is:

$$\left.\frac{\partial GL(1,\tau)}{\partial \tau}\right|_{\tau=\tau^U} = 0 \tag{27}$$

It is worth noting that we cannot state which is the greatest of τ^R and τ^U. Indeed, taxation hits individuals according to their relative factor endowments and for every consumption paths we do not know the relative factor endowment of the worst-off. However, using the concavity in τ of the function $GL(1,\tau)$, we can state that:

$$\left.\frac{\partial GL(1,\tau)}{\partial \tau}\right|_{\tau=\tau^R} \leq 0 \Rightarrow \tau^R \geq \tau^U$$

If the above condition holds, then we can state that the range $\left[\tau^U, \tau^R\right]$ represents the set of all possible solutions to the welfare maximization problem according to different degree of inequality aversion (the opposite is true if condition does not hold). It is clear that the tax level maximizing social welfare depends on the form of $\phi(.)$, that is on the degree of inequality aversion of the social planner.

4.4 Income sources and consumption

In this section we analyze what happens when the consumption level depends on income sources, that is on absolute and relative endowments of l_j and k_j. In particular we assume that the consumption level is a positive function of σ, that is:

$$U_j \leq U_{j+1} \Leftrightarrow \sigma_j \geq \sigma_{j+1} \tag{28}$$

The above condition means that individuals having a bigger labor endowment consume less. This seems plausible: the less well-to-do own more unskilled labor and their consumption level is less than that of the individuals owning more capital. This assumption implies that:

$$\frac{\sum_{j=1}^{S} \frac{\bar{r}\sigma_j}{\bar{r}\tau\sigma_j + \rho}}{S} \geq \frac{\sum_{j=1}^{S+1} \frac{\bar{r}\sigma_j}{\bar{r}\tau\sigma_j + \rho}}{S+1} \quad S = 1, ..., N-1 \ \forall \tau \in [0,1]$$

Because τ^U has to satisfy the following equality:

$$\frac{\partial GL(1,\tau)}{\partial \tau} = \sum_{j=1}^{N} \frac{\bar{r}\sigma_j}{\bar{r}\tau^U\sigma_j + \rho} + N\frac{\bar{\eta}'(\tau)|_{\tau=\tau^U}}{\rho} = 0$$

we obtain:

$$\left.\frac{\partial GL\left(\frac{S}{N},\tau\right)}{\partial \tau}\right|_{\tau=\tau^U} \geq 0 \ S = 1, ..., N-1$$

This means that the Lorenz curve is everywhere increasing in τ, but for $p = 1$. Moreover, for every S, we can calculate the tax level τ_S that, if increased, would leave the level of $GL\left(\frac{S}{N},\tau\right)$ unchanged, that is:

$$\left.\frac{\partial GL\left(\frac{S}{N},\tau\right)}{\partial \tau}\right|_{\tau=\tau_S} = 0$$

Because of our assumption (28) these tax levels are ranked in decreasing order, that is $\tau_S \geq \tau_{S+1}$ and, in particular, we have $\tau_1 = \tau^R$.

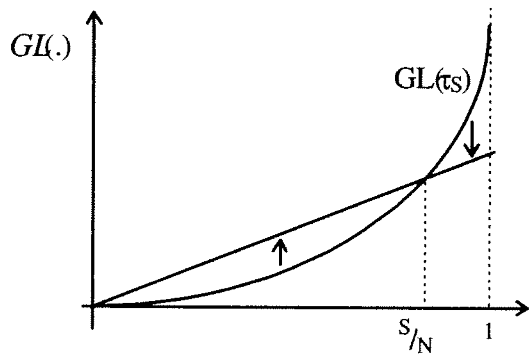

Figure 2

If we consider the $GL(\tau_S)$ curve, that is the curve related to tax τ_S, we note that an increase in the tax causes an upward movement of that part of the $GL(\tau_S)$ curve on the left of the point of abscissa $\frac{S}{N}$ and a downward movement of that part on the right. As Figure 2 shows this implies that the two Lorenz curves, corresponding to two different tax levels, cross in a single point[11]. The intuition that τ^U and τ^R represent the bounds of the range which all possible socially optimal taxes must belong to is verified. Every tax on the return on capital smaller than τ_N does not maximize social welfare because an increase in tax causes a upward movement of the $GL(.)$ curve and this, for Shorrocks theorem, implies a welfare gain. From τ_N onwards only the left part of the $GL(.)$ curve shifts up, while the right part shifts down; this means that some individuals, that is those who consume less, increase their welfare in relative terms, while the welfare of all the other falls. It is worth noting that the average welfare decreases and therefore there is a trade-off between equality and total level of welfare. The higher the tax the less the number of individuals that gain in welfare terms; there will be a tax level such that if τ were increased a little more, then also individual with the smallest welfare level would worsen his condition. This level represents the optimal tax according to the Rawls principle. Therefore the tax maximizing welfare has to belong to the range $\left[\tau^U, \tau^R\right]$; the precise level will depend on the inequality aversion degree of our hypothetical social planner.

Our results agree with those of Dardanoni and Lambert (1988), who, through a further assumption on the welfare function , that is $\phi'''(.) > 0$[12], rank in welfare terms two distributions whose Lorenz curves cross in a single point. In particular, given two distributions X and X^*, in which the first is preferred to the second according to the Rawls principle, while the second is preferred to the first according to the pure utilitarian principle, the first will be preferred to the second in welfare terms if the welfare function presents a high inequality aversion degree and the variance of distribution X is "sufficiently smaller" than the variance of distribution X^*. In economic terms, if we consider the variance of a distribution as a distributive equality index, we can conclude that it could be optimal to give up a higher total utility level for a higher distributive equity if the welfare function shows a high inequality aversion degree.

Now we are interested in analyzing thoroughly the pure utilitarian welfare function because this latter seems to be a good term of comparison for our welfare analysis.

[11] During the 80s some countries, in particular US and UK, reduced taxes on capital. Our model implies that one should then observe an increase in inequality as the growth rate increases. This seems what happened; the Lorenz curve of income distribution of the US actually shifted downward in that part closer to the origin and upward in that part closer to the right limit (Krugman 1994). In the terms of our model, the economy moved to a growth path involving a greater growth rate but also greater inequality.

[12] This assumption reflects the idea of transfer sensibility, see Lambert (1993).

4.5 Pure utilitarian welfare function

The pure utilitarian welfare function takes the form of a simple sum of individual utilities, that is:

$$W = \frac{\ln \prod_{i=1}^{N} k_i^0 \left[(1-\gamma) w(\tau,\gamma) \sigma_i + \rho\right] + N \frac{(1-\tau) r(\tau,\gamma) - \rho}{\rho}}{\rho} \tag{29}$$

We want to maximize W in τ and γ.
From the first order conditions we get:

$$\tau = \frac{(1-\alpha)(1-\gamma)}{\alpha}$$

This confirms our hypothesis that the socially optimal fiscal policy belongs to the line segment (17).

Moreover the first order conditions yield to:

$$\frac{\sum_{i=1}^{N} \frac{\sigma_i}{\bar{r}\sigma_i \tau^U + \rho}}{N} = \frac{1}{\rho} \tag{30}$$

where τ^U is the socially optimal tax. Equation (30) represents the condition for maximizing the value of the Lorenz curve corresponding to abscissa 1.

4.5.1 Welfare and distribution of factor property rights

A preliminary observation regards the logarithmic form of the utility function: if an individual does not own any capital quantity, then the fiscal policy that maximizing the growth rate brings him an utility level equals to $-\infty$. This prevents the fiscal policy that maximizes the growth rate from maximizing social welfare.

For a deeper understanding of the relationship between welfare and the distribution of factor property rights we calculate the total differential of condition (30), that is:

$$d\tau^U = \frac{\sum_{i=1}^{N} \frac{\partial f(.)}{\partial \sigma_i} d\sigma_i}{-\frac{\partial \sum_{i=1}^{N} f(.)}{\partial \tau^U}} \text{ where } f\left(\sigma_i, \tau^U\right) = \frac{\sigma_i}{\bar{r}\tau^U \sigma_i + \rho} \text{ e } \begin{cases} \frac{\partial f(.)}{\partial \sigma_i} > 0 \ \forall i \\ \frac{\partial \sum_{i=1}^{N} f(.)}{\partial \tau^U} < 0 \end{cases} \tag{31}$$

A sufficient condition for having a tax on the return on capital equal to 0 is obtained imposing $\tau^U = 0$ in condition (30). This yields[13]:

$$\frac{\sum_{i=1}^{N} \sigma_i}{N} = 1 \tag{32}$$

If condition (32) holds and we could redistribute factors so as to increase the relative endowment of an individual, keeping all the others' one equal, we would have that the socially optimal tax on the return on capital will be greater than 0 (see (31)). Therefore the mean of the distribution of the σ seems to be a crucial parameter. We could think of the mean as a proxy for the "barycentre" of distribution of the σ.

On the contrary, if we keep the mean constant and increase the variance of the distribution (see again (31)), the level of the socially optimal tax on the return on capital decreases. Therefore variance is also a crucial parameter. In terms of the growth effect and level effect we could say the gains of a higher growth rate exceed the gains of a higher infratemporal distributive equity.

4.5.2 Two individual economies

Now we analyze a particular example, that is a two individual economy. By equation (31) we can get all the pairs of individual relative factor endowments that, in order to maximize welfare, imply the same tax on the return on capital, that is:

$$\frac{d\sigma_2}{d\sigma_1} = -\left(\frac{\bar{r}\tau^U\sigma_2 + \rho}{\bar{r}\tau^U\sigma_1 + \rho}\right)^2 \tag{33}$$

Equation (33) represents the slope of the iso-tax curve in the relative factor endowments space. The slope of the iso-tax curve corresponding to $\tau^U = 0$ is always equal to -1, therefore this iso-tax curve appears as a line segment. For τ^U greater than 0, the following relation holds:

$$\text{if } \sigma_1 > \sigma_2 \Rightarrow \left|\frac{d\sigma_1}{d\sigma_2}\right| < 1$$

$$\text{if } \sigma_1 = \sigma_2 \Rightarrow \left|\frac{d\sigma_1}{d\sigma_2}\right| = 1$$

$$\text{if } \sigma_1 < \sigma_2 \Rightarrow \left|\frac{d\sigma_1}{d\sigma_2}\right| > 1 \tag{34}$$

[13]It is worth noting that condition (32) could be derived also by (25).

Figure 3 shows, in the relative factor endowments space, different iso-tax curves.

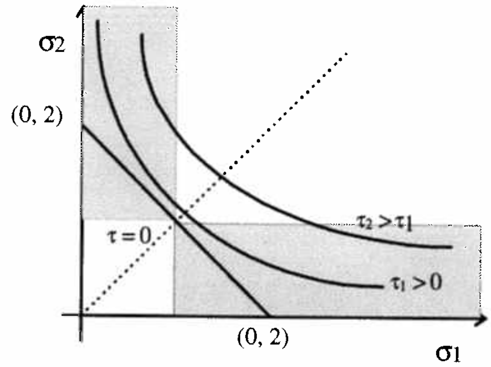

Figura 3

All the feasible individual relative factor endowments σ_1 and σ_2 have to belong to the shaded area of Figure 3 to satisfy the resource constraints $l_1 + l_2 = 1$ and $k_1 + k_2 = K$. Because individual relative factor endowments are equal on the first quadrant bisector, by (30) it is possible to calculate the socially optimal tax on the return on capital for every point of the same bisector. In this way we can assign to every iso-tax curve the tax to which it corresponds: this tax increases as we go away from the origin. The concavity of the iso-tax curve highlights the effects of an increase in the variance of the distribution of the σ.

The extension to an economy with more than two individuals does not seem difficult. Indeed, we can identify all the feasible endowments in the same way. Moreover, in spaces of greater dimensions it is always possible to suppose that the variations in the distribution occur between single pair of individuals, so that the space included between the hyperplane defined by $\frac{\sum_{i=1}^{N} \sigma_i}{N} = 1$ and the origin is the locus in which the socially optimal tax on the return on capital is equal to 0. Outside of this hyperplane the socially optimal tax will be positive and increasing as long as we go away from the origin.

4.5.3 An example

It is worth analyzing the case in which individuals own different quantities of a factor (usually capital) and equal quantities of the other one (usually labor)[14].

[14] In Alesina and Rodrik (1994) there is such an hypothesis.

We redefine our analysis only in terms of capital, that is:

$$\frac{d\sigma_2}{d\sigma_1} = \frac{d\left(\frac{k_2}{K}\right)}{d\left(\frac{k_1}{K}\right)} * \left(\frac{k_1}{k_2}\right)^2 \text{ if } l_1 = l_2 = \hat{l} = \frac{1}{2} \text{ for the hypothesis } L = 1$$

By equation (33) and the above relation we can calculate the slope of iso-tax curves in the $\left(\frac{k_1}{K}, \frac{k_2}{K}\right)$ space.

$$\frac{d\left(\frac{k_2}{K}\right)}{d\left(\frac{k_1}{K}\right)} = -\left(\frac{\bar{r}\tau\hat{l} + \rho\left(\frac{k_2}{K}\right)}{\bar{r}\tau\hat{l} + \rho\left(\frac{k_1}{K}\right)}\right)^2$$

The resource constraint in a two individual economy is:

$$\frac{k_1}{K} + \frac{k_2}{K} = 1$$

All the feasible property right distributions have to belong to this line segment. We notice that this constraint implies that condition (32) is verified only in the case of an uniform distribution of the σ, elsewhere we have always a mean of the σ greater than 1[15]. Therefore the socially optimal tax on the return on capital is always greater than 0, except for the case of an uniform distribution of resources. Figure 4 shows the iso-tax curves in the $\left(\frac{k_1}{K}, \frac{k_2}{K}\right)$ space.

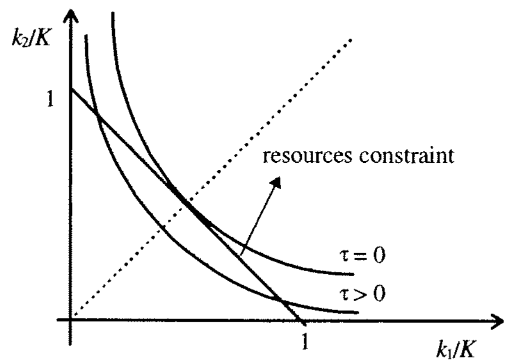

Figure 4

[15] Notice that $\hat{l} = \frac{1}{2}$.

The iso-tax curves which cross the bisector below the resources constraint correspond to socially optimal taxes on the return on capital greater than 0. Because as we move along the bisector from the centre towards an axe we cross iso-tax curves near and near to the origin, then the socially optimal tax on the return on capital is increasing as we go away from the uniform distribution point.

It is worth noting that the example just analyzed implicitly satisfies the hypothesis of section 4.4 about the relationship between consumption level and factors endowments. Therefore, if it is impossible to make *lump-sum* redistributions, under the hypotheses of equal distribution in labor and unequal distribution in capital we can conclude that the socially optimal fiscal policy is different from the fiscal policy that maximizes the growth rate for a wide class of welfare functions.

5 Political equilibrium versus maximum welfare

The goal of this section is to compare the socially optimal fiscal policy with the political equilibrium. We know that both have to belong to the line segment (17). From (17) and (18) we have:

$$\frac{\sigma_m}{\bar{r}\sigma_m\tau^P+\rho}=\frac{1}{\rho} \tag{35}$$

We define $g_i(\tau)=\dfrac{\sigma_i}{\bar{r}\tau\sigma_i+\rho}$ the variable obtained by the concave transformation (35) on σ and σ_m and μ_σ, respectively, the median and the mean of the σ. We divide all the possible distributions of σ into three classes:

1. symmetric distribution, that is $\sigma_m=\mu_\sigma$.
2. distribution for which $\sigma_m>\mu_\sigma$.
3. distribution for which $\sigma_m<\mu_\sigma$.

Let consider the first class. Because the transformation on the variable σ satisfies the hypotheses of the Jensen theorem [16] and the mean and the median are equal, we can conclude that:

$$E\left[g_i(\tau)\right]<g_m(\tau)\ \forall\tau$$

[16]Jensen theorem states that, given a distribution y and a trasformation $h(y)$ of this, with $h'(y)\geq 0$ and $h''(y)\leq 0$, then $E[h(y)]<h(E[y])$.

where $E[.]$ represents the mean operator. But for (30) and (35) the following relationship has to hold:

$$E\left[g_i\left(\tau^U\right)\right] = g_m\left(\tau^P\right)$$

from which $\tau^U < \tau^P$.

An example of such a type of distribution is the normal[17]. From an economic point of view this means that the relative factor endowments are concentrated around the mean value and therefore there is an uniform distribution in the property rights such that the socially optimal τ is smaller than the political equilibrium one.

With regard to the second class, by an analysis similar to the previous one, we can find that $\tau^U < \tau^P$. This type of distribution implies that most of individuals own a very high relative endowment of labor. For example let take the following density function:

$$f(\sigma) = \frac{\theta+1}{\sigma^{\max}}\left(\frac{\sigma}{\sigma^{\max}}\right)$$

where $\sigma \in [0, \sigma^{\max}]$ and, for $\theta > 0$, $\sigma_m > \mu_\sigma$. This density function could represent a paretian distribution of individual incomes if individuals with the greatest labor endowments have also the smallest income and therefore the lowest consumption levels. This scenario satisfies the hypotheses of section 4.4 and empirically it seems to be the more plausible case.

The analysis of the third class presents some difficulties. Indeed it is impossible to apply the Jensen theorem to get a general result. We could show that different distributions, all satisfying the condition of point 3, imply a different relationship between τ^U and τ^P.

Generally speaking we can say that the transformation of the variable σ implies a greater penalization of the mean as regard than median. However if the difference between these two is very high, then it is possible that the mean remains always greater than the median after the transformation. This large difference between the mean and the median could mean that in the economy there is a set of individuals whose cumulative factor endowments are very high as regards to those of the other individuals; because of this high distributive inequality the maximization of social welfare requires a very high tax on the return on capital, so high to be greater than the one preferred by the median elector.

[17]We ignore its negative realizations.

6 Maximum growth, political equilibrium and welfare

The goal of this section is to give an unitary look on what done up to now. For simplicity we only consider the tax on the return on capital. We have just seen that, under plausible hypotheses on the distribution of the σ, $\tau^U < \tau^P$. Moreover if we suppose the level of σ is related to the consumption level, then the socially optimal tax increases as the inequality aversion degree increases. In particular, the set of socially optimal taxes has a lower bound corresponding to the pure utilitarian socially optimal tax and an upper bound corresponding to the Rawlsian socially optimal tax. Therefore the socially optimal tax has to belong to the range $\left[\tau^U, \tau^R\right]$. Moreover also the political equilibrium tax belongs to this range; indeed, this represents the optimal tax for the individual with the median σ, that, for our hypothesis, is higher than the σ of the worst-off. Therefore the Rawlsian socially optimal tax is higher than the political equilibrium one, that is $\tau^R > \tau^P$. The tax that maximizes the growth rate τ^* is represented by the lower bound of the feasible range, that is 0. In short we have found that the following inequality holds:

$$\tau^R > \tau^P \geq \tau^U \geq \tau^*$$

It is worth noting that the political equilibrium tax could be socially optimal if the inequality aversion degree were sufficiently high. In the welfare analysis the tax that maximizes the growth rate is important only in the representative agent case. The fiscal policy decided by a majority voting rule does not seem so inefficient as the conclusions of Lucas (1990) suggested.

7 Conclusions

In this paper we analyzed an economy in which individuals have different factor endowments and there is a factor, indispensable for production, provided by the Government and financed by a taxation on the returns on factors. Our analysis, both positive and normative, showed that, if it were impossible to make lump-sum redistributions, the maximum growth rate and the maximum welfare do not coincide. To sacrifice a part of the growth rate in order to have a greater equality could be optimal in welfare terms. We defined and calculated, through the median voter theorem, the political equilibrium, that is the equilibrium in which the fiscal policy is decided by a majority voting rule. We showed that in this equilibrium the growth rate probably will not be the maximum. Then we compared the political equilibrium and the socially optimal fiscal policies; we highlighted that the relationship between them depends on the form of the distribution of the σ and that the political equilibrium fiscal policy could be socially optimal if in the economy there were a sufficient degree of inequality aversion.

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