N°42 COLLANA DI TEORIA ECONOMICA

CARTEL STABILITY UNDER VERTICAL DIFFERENTIATION AND CONVEX COSTS: BERTRAND vs COURNOT

LUCA LAMBERTINI

N° 259

CARTEL STABILITY UNDER VERTICAL DIFFERENTIATION AND CONVEX COSTS: BERTRAND vs COURNOT

Luca Lambertini

Dipartimento di Scienze Economiche
Università degli Studi di Bologna

June 1996

Keywords: *vertical differentiation, collusion, cartel stability*
JEL classification: *L 13*

Abstract

The issue of cartel stability is investigated in a vertical differentiation framework with convex variable production costs. The behaviour of firms' critical discount factors as the curvature of the cost function varies is analysed, considering either the noncooperative or the cooperative qualities, and either price or quantity behaviour. It emerges that, if firms aim at stabilizing the cartel, they are better off playing à la Cournot and prefer not to choose the monopoly qualities.

Acknowledgements

This paper draws on the third chapter of my DPhil thesis at Linacre College, University of Oxford. I would like to thank my supervisor, Martin Slater, and my examiners, Norman Ireland and Paul Klemperer, for insightful discussion and suggestions. The responsibility of any errors and shortcomings is exclusively mine.

1. Introduction

On the trace of the seminal contributions due to Friedman (1971, 1977) and Rubinstein (1979) on the endogenous emergence of cooperation over time in competitive settings, the manifold issue of cartel stability has received wide attention in the literature on oligopoly theory. The question how variations in cartel size affects the fortunes of those inside and outside the cartel has been explored by D'Aspremont et al. (1983), Donsimoni (1985) and Donsimoni et al. (1986), reaching the conclusion that stable cartels exist whenever the number of firms is finite. The relative efficiency of Bertrand and Cournot competition in stabilizing cartels composed by firms whose products are imperfect substitutes has been analysed by Deneckere (1983), showing that when substitutability between products is high, collusion is better supported in price-setting games than in quantity-setting games, while the reverse is true in case of low substitutability. Majerus (1988) has proved that this result is not confirmed as the number of firms increases, and Rothschild (1992), in contrast with Deneckere's findings, has shown that in a price-setting duopoly, the greater the degree of substitutability, the greater is the incentive to deviate and therefore the less stable the cartel is, while in quantity-setting games cartel stability is monotonically increasing in the degree of substitutability between products. Finally, the influence of endogenous product differentiation on the stability of collusion in prices has been investigated by Chang (1991), Ross (1992) and Häckner (1994, 1995). The main finding reached by these contributions is that, under vertical differentiation, collusion is more easily sustained, the more similar the products are, while the opposite applies under horizontal differentiation. The consequences of collusion on the extent of optimal differentiation in the horizontal differentiation model have also received attention. Jehiel (1992) has shown that Hotelling's (1929) *minimum differentiation principle* can be restored when firms noncooperatively choose locations and then cooperatively set prices, if a monetary transfer between them is possible. Friedman and Thisse (1993) have considered a repeated price game in the horizontal framework and found out that minimum differentiation obtains if firms collude in the market stage.

In most of these models, although differentiation can be endogenously determined by firms through strategic interaction, the issue of cartel stability is studied by making the degree of differentition vary symmetrically around the ideal midpoint of the interval of technologically feasible or socially preferred varieties, leading to the conclusion that producers may prefer to choose the charatcteristics of their respective goods differently from what profit maximization would suggest, if this help them minimize the probability of defection from the cartel agreement.[1]

Here I want to address the issue of cartel stability under vertical product differentiation and either Bertrand or Cournot behaviour, in order to evaluate the bearings of the degree of convexity of the cost function on firms' ability to collude in both settings. This enriches the traditional debate on firms' preferences regarding the choice of the market variable to enhance cartel stability (Deneckere, 1983; Majerus, 1988, Rothschild, 1992).[2] I shall proceed as follows. First, I describe the market behaviour, either cooperative or noncooperative, in prices or quantities. Then, I determine the optimal deviation strategies in the two setting, provided that the other firm sticks to the implicit collusion agreement. Finally, I determine the optimal qualities alternatively chosen in a cooperative or noncooperative way. All this is done in section 2. Then, in section 3 I describe the behaviour of the critical discount rates in the two settings, as the curvature of the cost function varies, and adopting either the cooperative or the noncooperative quality levels, alternatively. This choice needs an explanation. Since the model is asymmetric the standard comparative statics like those practiced in the models mentioned above cannot be carried out, and one has to evaluate the relative size of the critical discount factors by referring to some *fixed points* selected on the basis of acceptable criteria, provided by profit maximization, either cooperative or noncooperative.

The results point to the conclusion that, in order to increase their ability to collude, firms should try to reduce the degree of convexity of the cost function, i.e., the unit production cost

---

1. The possibility of a behaviour that closely mimics collusion to arise in the circular version of the horizontal differentiation model is considered by Salop (1979) and Ireland (1987, ch. 3).

2. The problem of the choice of the market variable has received in itself wide attention in the recent literature. See Singh and Vives (1984); Cheng (1985); Klemperer and Meyer (1986), *inter alia*.

associated with quality. Moreover, they prefer to set the noncooperative qualities and then collude in a Cournot rather than in a Bertrand fashion. The latter result is in line with part of the existing literature on exogenous differentiation (Deneckere, 1983).

2. The model

Two firms, indexed by H and L, operate in a market for a vertically differentiated good, each selling a single variety, q_i, with $q_H \geq q_L$. The analysis is carried out under the assumption of partial market coverage. The general setting is analogous to that adopted by Motta (1993), except for the fact that the degree of curvature of the technology is variable, so that firm i's costs are given by

$$C_i = q_i^n x_i, \quad n > 1, \quad i = H, L, \tag{1}$$

where q_i and x_i denote quality and quantity, respectively. In the two papers just mentioned n=2. The condition on n is adopted to preserve the concavity and solvability of the problem.

Consumers are characterized by a marginal willingness to pay for quality defined by parameter θ, which is uniformly distributed over the interval $[0, \bar{\theta}]$, with density 1. Each consumer either buys one unit of the differentiated good or does not buy at all, in order to maximize the following indirect utility function:

$$U = \theta q_i - p_i, \quad i = H, L. \tag{2}$$

Let θ_H and θ_L denote the levels of the marginal willingness to pay identifying, respectively, the consumer who is indifferent between buyng either the high-quality or the low-quality good and the one who is indifferent between buying the low-quality good and not to buy at all:

$$\theta_H = \frac{p_H - p_L}{q_H - q_L}, \tag{3}$$

$$\theta_L = \frac{p_L}{q_L}. \tag{4}$$

Then, the demands for the two goods are, respectively:

$$x_H = 1 - \theta_H = \frac{(q_H - q_L) - (p_H - p_L)}{(q_H - q_L)} \quad iff \quad 0 < 1 - \theta_H < 1; \tag{5}$$

$$x_H = 0 \quad iff \quad 1 - \theta_H \leq 0; \tag{5'}$$

$$x_H = 1 \quad iff \quad 1 - \theta_H \geq 1; \tag{5''}$$

$$x_L = \theta_H - \theta_L = \frac{q_L p_H - q_H p_L}{q_L(q_H - q_L)} \quad iff \quad 0 < \theta_H - \theta_L < 1; \tag{6}$$

$$x_L = 0 \quad iff \quad \theta_H - \theta_L \leq 0; \tag{6'}$$

$$x_L = 1 \quad iff \quad \theta_H - \theta_L \geq 1; \tag{6''}$$

Thus, the profit functions under Bertrand competition look as follows:

$$\pi_H^B = (p_H - q_H^n)\frac{[(q_H - q_L) - (p_H - p_L)]}{(q_H - q_L)} \tag{7}$$

$$\pi_L^B = (p_L - q_L^n)\frac{(p_H q_L - p_L q_H)}{q_L(q_H - q_L)} \tag{8}$$

Inverting the demand system (5-6), it is possible to investigate also quantity competition.[3] The inverse demand functions are, respectively:

---

3. Under the full coverage assumption, the demand functions cannot be inverted, since total demand is not a function of prices. See Motta (1993, p.116).

$$p_H = q_H - q_L x_L - q_H x_H; \qquad (9$$

$$p_L = q_L(1 - x_L - x_H), \qquad (10$$

while the profit functions are:

$$\pi_H^B = x_H(q_H - q_H^n - q_L x_L - q_H x_H); \qquad (11$$

$$\pi_L^B = x_L(q_L(1 - x_L - x_H) - q_L^n). \qquad (12$$

The superscript C stands for Cournot behaviour.

2.1. Bertrand behaviour

This subsection is devoted to the analysis of price-setting behaviour. Firms optimally set prices in the second stage after having chosen qualities in the previous one. As usual, the solution concept of the noncooperative setting is a subgame perfect equilibrium in two stages, obtained through backward induction. Hence, consider first the market stage. By differentiating the profit functions (7-8) w.r.t. prices, one obtains the following first order conditions (FOCs):

$$\frac{\delta\pi_H^B}{\delta p_H} = \frac{q_H^n + q_H - q_L + p_L - 2p_H}{q_H - q_L} = 0; \qquad (13$$

$$\frac{\delta\pi_L^B}{\delta p_L} = \frac{q_H q_L^n - 2q_H p_L + q_L p_H}{q_L(q_H - q_L)} = 0, \qquad (14$$

from which the following equilibrium prices obtain:[4]

4. Second order conditions for concavity are also satisfied throughout the calculations contained in the paper, although they are not desplayed for the sake of brevity.

$$p_H = \frac{q_H(q_L^n - 2q_L + 2q_H + 2q_H^n)}{4q_H - q_L}; \quad (15$$

$$p_L = \frac{q_L q_H - q_L^2 + 2q_H q_L^n - q_L q_H^n}{4q_H - q_L}. \quad (16$$

By substituting these expressions into the objective functions (7-8), I obtain the profit functions defined in terms of qualities only:

$$\pi_H^{Bd} = \frac{(2q_H^2 + 2q_H^{n+1} - 2q_H q_L + q_H^n q_L + q_H q_L^n)^2}{(q_H - q_L)(4q_H - q_L)^2}, \quad (17$$

$$\pi_L^{Bd} = \frac{q_H(q_L^2 + 2q_H q_L^n - q_L^{n+1} - q_H q_L - q_H^n q_L)^2}{q_L(q_H - q_L)(4q_H - q_L)^2}, \quad (18$$

where the superscript d denotes the noncooperative solution in prices. I have thus obtained the first magnitude needed for the evaluation of the discount factor defining the critical threshold above which collusion is sustainable over time. Now, if firms noncooperatively choose qualities, they must solve the following FOCs at the first stage of the game, as n varies:

$$\frac{\delta\pi_H^{Bd}}{\delta q_H} = q_H(q_L^n - 2q_L + 2q_H + q_L q_H^{n-1} - 2q_H^n)(2q_L^{n+2} - 4q_L^3 - 10q_H q_L^2 - q_H q_L^{n+1} - 14q_L q_H^2 - 4q_L^n q_H^2 +$$

$$+8q_H^3 + 5q_L^2 q_H^n - 10q_L q_H^{n+1} + 8q_H^{n+2} + 2nq_L^3 q_H^{n-1} - 14nq_L q_H^n + 28nq_L q_H^{n+1} - 16nq_H^{n+2})$$

$$/((q_H - q_L)^2(4q_H - q_L)^3) = 0; \quad (19$$

and

$$\frac{\delta\pi_L^{Bd}}{\delta q_L} = q_H(q_L^2 - q_L^{n+1} - q_H q_L + 2q_H q_L^n - q_L q_H^n)$$

$$(7q_H q_L^3 - 2q_L^{n+3} + 9q_H q_L^{n+2} - 11q_H^2 q_L^2 - 18q_L^{n+1} q_H^2 + 28nq_L^{n+1} q_H^2 - 16nq_L^n q_H^3)$$

$$/(q_L^2(q_H - q_L)^2 (q_L - 4q_H)^3) = 0. \qquad (20$$

Since the system (19-20) cannot be solved analytically, I resorted to numerical calculations, letting n vary in the interval [3/2, 100]. Analogous considerations hold for the remaining cases. The results of the simulation are displayed in table 1.[5]

I can now turn to the two-product monopoly setting. The monopolist or, equivalently, the cartel that offers both varieties is characterized by the following objective:

$$\max_{p_i, q_i} \quad \pi^m = (p_H - q_H^n)x_H + (p_L - q_L^n)x_L. \qquad (21$$

The FOCs w.r.t. prices are

$$\frac{\delta\pi^m}{\delta p_H} = \frac{q_H^n + q_H - q_L - q_L^n + 2p_L - 2p_H}{q_H - q_L} = 0; \qquad (22$$

$$\frac{\delta\pi^m}{\delta p_L} = \frac{q_L q_H^n - q_L^n q_H + 2p_L q_H - 2p_H q_L}{q_L(q_L - q_H)} = 0. \qquad (23$$

Solving the above conditions, the equilibrium prices result as follows:

5. In order to ensure that the quality levels obtained from the system (19-20) (as well as those yielded by (36-37) below) are indeed the equilibrium ones, one should check that there exists no incentive towards leapfrogging. Since such a proof is largely analogous to the one provided by Motta (1993) for n=2, it has been omitted.

$$p_H = \frac{q_H + q_H^n}{2}; \quad p_L = \frac{q_L + q_L^n}{2}; \tag{24}$$

substituting the above monopoly prices into the objective function in (21), one gets the overall cartel profits. The single firm's cooperative profits are defined as follows:

$$\pi_i^m = \pi_i^d + \frac{(\pi^m - \pi_i^d - \pi_j^d)}{2}, \quad i,j = H,L, \quad i \neq j, \tag{25}$$

i.e., the net gain emerging from cooperation is split in two parts of exactly the same size. This assumption appears reasonable if one considers that firms can adhere to the collusive agreement on equal bases, even though their respective market shares and profits in the noncooperative setting are different.[6] Since these expressions are rather long, they have been confined into the appendix. The cooperative qualities can be calculated by solving numerically the following FOCs:[7]

$$\frac{\delta\pi^m}{\delta q_H} = \frac{(q_L^n - q_H^n - q_L + q_H)(q_H^n + q_H - q_L - q_L^n + 2nq_L q_H^{n-1} - 2nq_H^n)}{4(q_H - q_L)^2}; \tag{26}$$

$$\frac{\delta\pi^m}{\delta q_L} = \frac{q_H(q_H^{n-1} - q_L^{n-1})(q_L^{n-1}q_H + q_H^n - 2q_L^n + 2nq_L^n - 2nq_L^{n-1}q_H)}{4(q_H - q_L)^2}. \tag{27}$$

6. For an exhaustive formal treatment of the bargaining problem, see Osborne and Rubinstein (1994), chs. 7 and 15.

7. It can be easily shown that in the absence of strategic interaction between the two varieties, the monopolist's maximum problem can be solved w.r.t. the four variables involved, simultaneously, yielding the same results as in the case where one first optimizes respect to prices and then qualities. I have chosen the latter method in order to derive the cooperative profits in terms of qualities, which I shall use in calculating the critical discount factors.

Again, the results of the simulation are in table 1.

As a last step, I am now able to concentrate on the gains from cheating. Assume firm i remains loyal to the collusive price agreement; hence, firm j optimal deviation strategy is found by solving her own reaction function at the price stage, i.e. (7) for firm H and (8) for firm L. Then, deviation profits easily obtain as follows:

n	q_H^m	q_H^{Cd}	q_H^{Bd}	q_L^m	q_L^{Cd}	q_L^{Bd}
3/2	0.3214	0.2910	0.3269	0.1228	0.2048	0.1364
2	0.4000	0.3690	0.4097	0.2000	0.2928	0.1994
5	0.6187	0.5975	0.6296	0.4768	0.5539	0.4003
10	0.7459	0.9512	0.7483	0.6564	0.7344	0.4982
15	0.8042	0.9647	0.8021	0.7388	0.7964	0.5122
20	0.8387	0.9722	0.8355	0.7872	0.8329	0.5170
30	0.8786	0.9804	0.8755	0.8424	0.8748	0.5260
40	0.9016	0.9848	0.8989	0.8736	0.8987	0.5328
50	0.9166	0.9876	0.9143	0.8938	0.9143	0.5378
60	0.9274	0.9895	0.9253	0.9081	0.9255	0.5415
80	0.9418	0.9920	0.9402	0.9271	0.9404	0.5468
100	0.9511	0.9935	0.9498	0.9392	0.9500	0.5504

Table 1. Qualities

Bd = noncooperative Bertrand duopoly

Cd = noncooperative Cournot duopoly

m = monopoly or collusive duopoly

$$\pi_H^{Bdev} = \frac{(2q_H - 2q_H^n - q_L + q_L^n)^2}{16(q_H - q_L)}; \qquad (28$$

$$\pi_L^{Bdev} = \frac{(q_H q_L + q_H^n q_L - 2q_H q_L^n)^2}{16 q_H q_L (q_H - q_L)}. \qquad (29$$

This completes the tool kit needed in order to investigate the issue of cartel stability in the price-setting framework. The next subsection is devoted to Cournot behaviour.

2.2. Cournot behaviour

Assume now firms choose quantities in the market stage. By differentiating the profit functions (11-12) w.r.t. x_H and x_L, respectively, the FOCs pertaining to the second stage obtain:

$$\frac{\delta \pi_H^C}{\delta x_H} = q_H - q_H^n - 2q_H x_H - q_L x_L = 0, \qquad (30$$

$$\frac{\delta \pi_L^C}{\delta x_L} = q_L(1 - x_H - x_L) - q_L^n - q_L x_L = 0, \qquad (31$$

from which the equilibrium quantities can be derived:

$$x_H = \frac{2q_H - 2q_H^n - q_L + q_L^n}{4q_H - q_L}; \qquad (32$$

$$x_L = \frac{q_H q_L + q_H^n q_L - 2q_H q_L^n}{q_L(4q_H - q_L)}. \qquad (33$$

After substituting (32-33) into (11-12), the profit functions at the quality stage simplify to:

$$\pi_H^{Cd} = \frac{q_H(2q_H - 2q_H^n - q_L + q_L^n)}{(4q_H - q_L)^2}; \tag{34}$$

$$\pi_L^{Cd} = \frac{(q_H q_L + q_H^n q_L - 2q_H q_L^n)^2}{q_L(4q_H - q_L)^2}. \tag{35}$$

The optimal noncooperative qualities can be found by solving the following FOCs pertaining to the fist stage of the game:

$$\frac{\delta\pi_H^{Cd}}{\delta q_H} = (2q_H - 2q_H^n - q_L + q_L^n)(8q_H^2 + 8q_H^{n+1} - 16nq_H^{n+1} - 2q_H q_L +$$

$$+2q_H^n q_L + 4nq_H^n q_L + q_L^2 - 4q_H q_L^n - q_L^{n+1})/(4q_H - q_L)^3 = 0; \tag{36}$$

$$\frac{\delta\pi_L^{Cd}}{\delta q_L} = (q_H + q_H^n - 2q_H q_L^{n-1})(4q_H^2 + 4q_H^{n+1} + q_H q_L + q_H^n q_L +$$

$$+8q_H^2 q_L^{n-1} - 16nq_H^2 q_L^{n-1} - 6q_H q_L^n + 4nq_H q_L^n)/(4q_H - q_L)^3 = 0. \tag{37}$$

As in the previous subsection, conditions (36-37) cannot be solved analytically. The results of the numerical simulation are in table 1.

Turn now to the collusive solution. The objective of the cartel is again defined as in expression (21), though here firms maximize joint profits w.r.t. quantities. The FOCs are the following:

$$\frac{\delta\pi^m}{\delta x_H} = q_H - q_H^n - 2q_H x_H - 2q_L x_L = 0, \tag{38}$$

$$\frac{\delta\pi^m}{\delta x_L} = q_L(1 - x_H - x_L) - q_L^n - q_L x_H - q_L x_L = 0. \tag{39}$$

Solving the system (38-39), I get the collusive output levels:

$$x_H^m = \frac{q_H - q_H^n - q_L + q_L^n}{2(q_H - q_L)}; \quad x_L^m = \frac{q_H^n q_L - q_H q_L^n}{2q_L(q_H - q_L)}. \tag{40}$$

Substituting quantities (40) into (21) one obtains the overall cartel profits. The profit accruing to each firm, π_i^m, is defined as in equation (25) above. All these expressions are in the appendix. Here it is woth noting that the global profits accruing to the cartel are the same under both kind of market behaviour, be that Bertrand or Cournot. The reason for this is intuitively clear, since for a given pair of qualities, a monopolist must be indifferent between setting prices and qualities. In turn, this implies that the cartel quality choices, as based on the first order conditions for a maximum, must coincide with those emerging from (26-27). See table 1.

I am now in a position to investigate the optimal cheating behaviour of each firm, provided the other sticks to the collusive output. When a firm decides to deviate from the collusive agreement, her individually optimal production level can be easily calculated by inserting the other firm's cooperative output as defined in (40) and solving. The deviation outputs are the following:

$$x_H^{dev} = \frac{2q_H + 2q_H^n - 2q_L + q_L q_H^{n-1} + q_H q_L^n}{4(q_H - q_L)}; \quad x_L^{dev} = \frac{q_H + q_H^n - q_L - 2q_H q_L^{n-1} + q_L^n}{4(q_H - q_L)}; \tag{41}$$

Substituting the relevant quantity in (41) into the profit function (9) or (10), the following deviation profits obtain:

$$\pi_H^{Cdev} = \frac{(2q_H^2 - 2q_H^{n+1} - 2q_Hq_L + q_H^nq_L + q_Hq_L^n)^2}{16q_H(q_H - q_L)^2}, \quad (42$$

$$\pi_L^{Cdev} = \frac{q_L^2 - q_Hq_L - q_H^nq_L + 2q_Hq_L^n - q_L^{n+1})^2}{16q_H(q_H - q_L)^2}. \quad (43$$

Having completed the analysis of firms' behaviour in both settings, I am now going to tackle the issue of cartel stability.

3. The critical discount factors

The issue of cartel stability is analysed through a usual tool, i.e., by describing the behaviour of firms' critical discount factors as n varies. The discount factors are based on the assumption that after a deviation by one member of the colluding club, the other reverts to the noncooperative Nash price or quantity forever.[8] Since the model is asymmetric, there exist two discount factors under each kind of market behaviour:

$$\alpha_i^B = \frac{\pi_i^{Bdev} - \pi_i^{Bm}}{\pi_i^{Bdev} - \pi_i^{Bd}} \quad i = H, L \quad B = Bertrand \quad (44$$

$$\alpha_i^C = \frac{\pi_i^{Cdev} - \pi_i^{Cm}}{\pi_i^{Cdev} - \pi_i^{Cd}} \quad i = H, L \quad C = Cournot \quad (45$$

where superscripts *m, dev, d* stand for *collusion* (or *monopoly*), *deviation* and *noncooperative duopolistic profits*, respectively. The magnitudes in (44-45) have been calculated by resorting to the relevant cooperative, noncooperative and deviation profits emerged throughout the

8. There exist other, less grim, punishment strategies that can be adopted in order to sustain the cartel agreement. Moreover, it is now usually argued that the grim strategy envisaged here suffers from not being renegotiation-proof. Nevertheless, I assume such a behaviour since it is widely used in the literature on the endogenous emergence of collusion. For a discussion of the concept of renegotiation-proofness, see Fudenberg and Tirole (1991, pp.174-82).

previous section.[9] In such a situation, in order for collusion to be sustainable, duopolists' discount factors must be higher than the highest critical α in (44) and (45), for each value of n.

I have proceeded as follows. Letting n vary between 3/2 and 100, I have calculated the value of each α considering qualities as being set either cooperatively or non cooperatively (see table 1). This is because cooperation appears as a long run choice; hence, in deciding to collude, besides choosing (i) between Cournot and Bertrand behaviour, and (ii) the optimal degree of curvature of technology, firms might also decide to choose the same qualities that a monopolist with two plants would set or, alternatively, they may cooperate in the market varible after having noncooperatively chosen their respective quality levels.

The numerical results obtained through the simulation on the discount factors are displayed in table 2 and illustrated in figures 1 and 2. In figure 1, qualities are cooperatively set, so that this may be defined as a two-stage cooperative setting, while in figure 2 the behaviour of the critical discount factor is computed assuming that qualities are noncooperatively set, and collusion involves the market variable only.

The behaviour of the discount factors is largely the same under both types of market behaviour, with the exception that, under Cournot competition, the discount factor of the low-quality firm rapidly falls below zero when the monopoly qualities are being offered. Consider figure 1. All discount factors, except α_L^{Cm}, are increasing in n, with the two magnitudes referring to Bertrand behaviour that asymptotically tend to one. This implies that the collusive agreement become increasingly difficult as the convexity of the cost function increases, since the relevant discount factor to be considered is the higher of the two for each type of market behaviour. Analogous considerations hold if one looks at figure 2.

It appears at first sight that in correspondence of the lower bound of the interval (n=3/2), i.e., when costs are only slightly convex, the minimax value of α is given by $\alpha_H^C = 0.5438$, i.e., the critical discount factor of the high-quality firm in the Cournot setting when qualities are the noncooperative ones. This entails that if firms aim at stabilizing collusion, or equivalently

9. The expressions describing α_i^J, $i=H,L$, $J=B,C$, can be written by using the appropriate profits that can be found throughout the text and in the appendix.

minimizing the probability of defection, they will find it advantageous (i) to set quantities rather than prices in the market stage, and (ii) not to set monopoly qualities, choosing instead to produce the noncooperative ones. Moreover, since $\alpha_H^J > \alpha_L^J$ and the higher α is everywhere increasing in n independently of the kind of market competition being considered, it appears that if firms might consider choosing n from a *technological menu*, they would choose it as low as possible.

The variation of the curvature of the cost function may also be interpreted as a consequence of investments in R&D, although such a process has not been explicitly modelled here. This perspective allows for some considerations on the incentive to create a research joint venture in order to reduce the curvature of technology and, ultimately, enhance the sustainability of collusion at the market stage. To the best of my knowledge, the only contribution we avail of, where the issue of cooperation in R&D under vertical product differentiation is due to Rosenkranz (1995), though in her model cooperation is aimed at product innovation, given the linear production technology common to all firms. Here, instead, the technological innovation consisting in a reduction of the degree of curvature of the cost function can be considered as both a process and a product innovation, in that the reduction of the curvature lowers production cost and, at the same time, entails a change in the quality level of the varieties being supplied. Almost paradoxically, in the present setting cooperation appears advantageous as far as production technology and market behaviour are concerned, but not for the quality stage, where cooperation would maximize single period profits without minimizing the probability of defection from the cartel by either firm.

n	α_H^{Bd}	α_L^{Bd}	α_H^{Cd}	α_L^{Cd}	α_H^{Bm}	α_L^{Bm}	α_H^{Cm}	α_L^{Cm}
3/2	0.5912	0.5333	0.5432	0.4999	0.5949	0.5077	0.6470	0.2269
2	0.6137	0.5484	0.5436	0.5047	0.6198	0.5501	0.6617	0.1707
5	0.6830	0.5851	0.5433	0.5116	0.7334	0.7061	0.6926	-0.0027
10	0.7168	0.5587	0.3132	0.8244	0.8257	0.8144	0.7049	-0.1060
15	0.7176	0.4947	0.4883	0.8429	0.8699	0.8643	0.7090	-0.1478
20	0.7226	0.4519	0.6095	0.8569	0.8967	0.8932	0.7110	-0.1691
30	0.7235	0.4102	0.7294	0.8772	0.9268	0.9251	0.7139	-0.2054
40	0.7240	0.3904	0.7899	0.8915	0.9433	0.9423	0.7140	-0.2012
50	0.7242	0.3787	0.8268	0.9023	0.9538	0.9531	0.7148	-0.2285
60	0.7245	0.3709	0.8519	0.9109	0.9609	0.9604	0.7153	-0.2186
80	0.7247	0.3614	0.8842	0.9237	0.9702	0.9699	0.7162	-0.2307
100	0.7249	0.3557	0.9042	0.9329	0.9759	0.9757	0.7180	-0.2599

Table 2. Critical discount factors

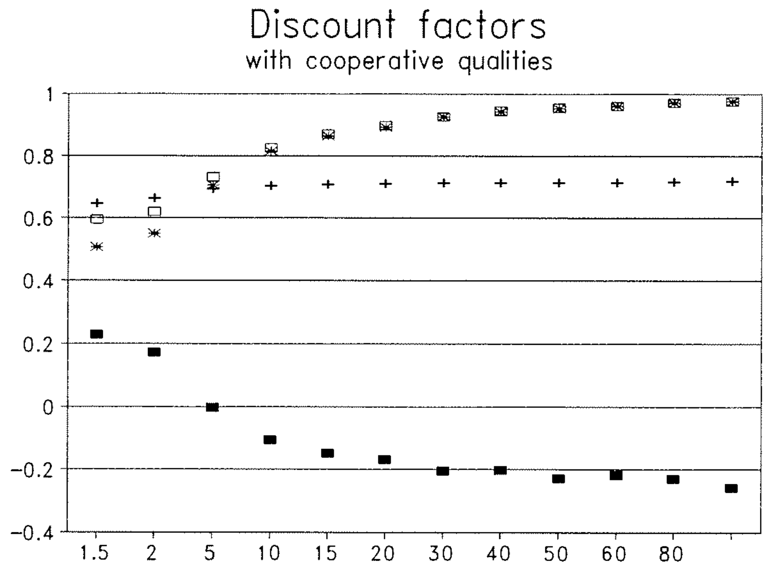

Figure 1. Critical discount factors (cooperative qualities)
Symbols: □=α_H^B; *=α_L^B; +=α_H^C; ■=α_L^C.

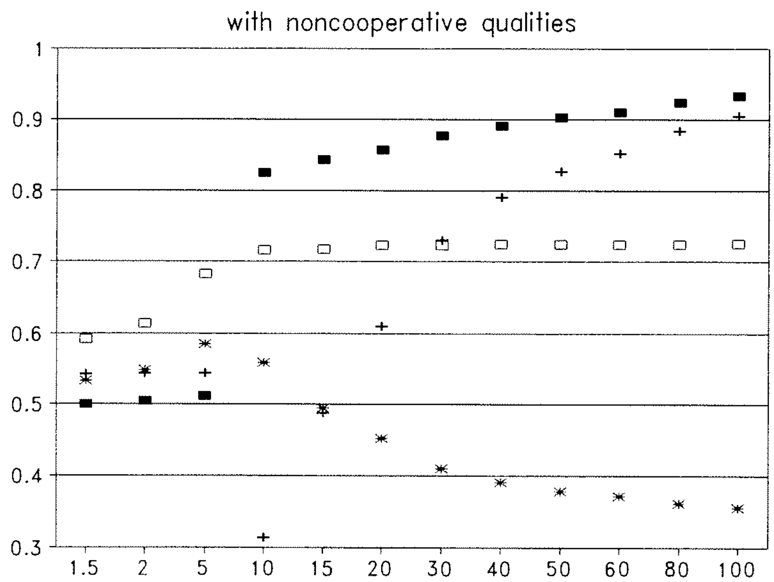

Figure 2. Critical discount factors (noncooperative qualities). Symbols as above.

4. Conclusions

I have investigated the issue of cartel stability in the market stage, under vertical differentiation and partial market coverage, in a model where the curvature of technology, represented by variable costs only, is variable. In such a framework, in order to enhance their ability to collude, firms can choose between (i) Bertrand or Cournot behaviour; (ii) the cooperative or noncooperative qualities; and, finally, (iii) they may pick up that particular technology that minimizes the probability of defection by any firm. As it turns out on the basis of the above analysis, they prefer to compete in quantities rather than in prices, as already found by Deneckere (1983), in an exogenous differentiation model, after having set the noncooperative qualities rather than the noncooperative ones. This leads one to claim that the single-period maximization of collusive profits, attainable through the choice of the monopoly qualities, does not coincide with the maximization of long-run profits in that it leaves too much room for deviation from the cartel agreement, while on the contrary a higher stability is reached by giving up a slice of single-period monopolistic profits. Moreover, firms choose the lowest possible degree of curvature, in that it yields the lowest possible value of the relevant discount factor. Since a reduction of the curvature of technology is in the common interest of both firms, this may represent a strong incentive to set up a research joint venture. To a certain extent, the outcome of the latter appears as both a process and a product innovation, since the change in the convexity of the cost function brings about a change in the quality levels of the varieties being produced as well.

Appendix: collusive profits

The following expressions define the collusive profits accruing to the high and the low-quality firm. They are invariant w.r.t. the two alternative market variables, price or quantity:

$$\pi_H^m = (8q_H^3 - 16q_H^{n+2} + 8q_H^{2n+1} - 13q_H^2 q_L - 5q_H^{2n} q_L +$$
$$+18q_H^{n+1} q_L + 5q_H q_L^2 - 2q_H^n q_L^2 + 8q_H^2 q_L^n - 8q_H^{n+1} q_L^n +$$
$$+3q_H q_L^{2n} - 8q_H q_L^{n+1} + 2q_H^n q_L^{n+1})/(32q_H^2 - 40q_H q_L + 8q_L^2); \qquad (a1$$

$$\pi_L^m = (3q_H^2 q_L^2 + 3q_H^{2n} q_L^2 + 2q_H^{n+1} q_L^2 - 3q_H q_L^3 - 2q_H^n q_L^3 +$$
$$+8q_H^2 q_L^{2n} - 8q_H^2 q_L^{n+1} - 8q_H^{n+1} q_L^{n+1} + 8q_H q_L^{n+2} +$$
$$+2q_H^n q_L^{n+2} - 5q_H q_L^{2n+1})/(32q_H^2 q_L - 40q_H q_L^2 + 8q_L^3). \qquad (a2$$

References

Chang M. (1991), "The Effects of Product Differentiation on Collusive Pricing", *International Journal of Industrial Organization*, 9, pp.453-69.

Cheng, L. (1985), "Comparing Bertrand and Cournot Equilibria: A Geometric Approach", *Rand Journal of Economics*, 16, pp.146-52.

D'Aspremont, C., A. Jacquemin, J.J. Gabszewicz and J. Weymark (1983), "On the Stability of Collusive Price Leadership", *Canadian Economic Journal*, 16, pp.17-25.

Deneckere, R. (1983), "Duopoly Supergames with Product Differentiation", *Economics Letters*, 11, pp.37-42.

Donsimoni, M.-P. (1985), "Stable Heterogeneous Cartels", *International Journal of Industrial Organization*, 3, pp.451-67.

Donsimoni, M.-P., N. Economides and H.M. Polemarchakis (1986), "Stable Cartels", *International Economic Review*, 27, pp.317-27.

Friedman, J.W. (1971), "A Noncooperative Equilibrium for Supergames", *Review of Economic Studies*, 38, pp.1-12.

Friedman, J.W. (1977), *Oligopoly and the Theory of Games*, Amsterdam, North-Holland.

Fudenberg, D. and J. Tirole (1991), *Game Theory*, Cambridge, MA, MIT Press.

Häckner, J. (1994), "Collusive Pricing in Markets for Vertically Differentiated Products", *International Journal of Industrial Organization*, 12, pp.155-77.

Häckner, J. (1995), "Endogenous Product Design in an Infinitely Repeated Game", *International Journal of Industrial Organization*, 13, pp.277-99.

Ireland, N. (1987), *Product Differentiation and Non-Price Competition*, Oxford, Basil Blackwell.

Klemperer, P. and M. Meyer (1986), "Price Competition vs Quantity Competition: The Role of Uncertainty", *Rand Journal of Economics*, 17, pp.618-38.

Majerus, D.W. (1988), "Price vs Quantity Competition in Oligopoly Supergames", *Economics Letters*, 27, pp.293-7.

Motta, M. (1993), "Endogenous Quality Choice: Price vs Quantity Competition", *Journal of Industrial Economics*, 41, pp.113-32.

Osborne, M.J. and A. Rubinstein (1994), *A Course in Game Theory*, Cambridge, MA, MIT Press.

Rosenkranz, S. (1995), "Innovation and Cooperation under Vertical Product Differentiation", *International Journal of Industrial Organization*, 13, pp.1-22.

Ross, T.W. (1992), "Cartel Stability and Product Differentiation", *International Journal of Industrial Organization*, 10, pp.1-13.

Rothschild, R. (1992), "On the Sustainability of Collusion in Differentiated Duopolies", *Economics Letters*, 40, pp.33-7.

Rubinstein, A. (1979), "Equilibrium in Supergames with the Overtaking Criterion", *Journal of Economic Theory*, 21, pp.1-9.

Salop, S. (1979), "Monopolistic Competition with Outside Goods", *Bell Journal of Economics*, 10, pp.141-56.

Singh, A. and X. Vives (1984), "Price and Quantity Competition in a Differentiated Duopoly", *Rand Journal of Economics*, 15, pp.546-54.