OPTIMAL FISCAL REGIME IN A SPATIAL DUOPOLY

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Abstract

Extending the analysis carried out in Lambertini (1993), we investigate a horizontally differentiated duopoly in which a public authority can either tax or subsidize firms, in order to induce duopolists to choose the socially optimal locations.

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1. Introduction

A wide stream of literature has dealt with the influence of taxation and subsidization on social welfare under imperfect competition (Katz and Rosen (1985), Levin (1985), Besley (1989) and Okuguchi (1993), *inter alia*). The purpose of this paper is to analyse the impact of these policy instruments on the behaviour of firms as well as on social welfare in a horizontally differentiated industry.

Some recent contributions on imperfect competition have taken into consideration the issue of regulating oligopolistic markets for homogeneous goods through public firms (Cremer et al. (1989), De Fraja and Delbono, (1989)). Cremer, Marchand and Thisse (1991) study a horizontally differentiated mixed oligopoly, i.e., in which at least one public firm operates, showing, *inter alia*, that in the duopoly case the presence of a public firm minimizing social costs is sufficient to yield the first best locational configuration.[1]

Extending the analysis contained in Lambertini (1993), we will show that this result can also be obtained in a private duopoly, provided that the public authority adopts a proper taxation/subsidization scheme, which builds on the distance between the socially optimal locations and those noncooperatively chosen by firms. Moreover, the government may want to rely on what might be called a Bertrand threat, i.e., a tax schedule such that firms can avoid paying taxes if and only if they accept to act as perfect competitors.

2. The model

Our starting point is the well known framework described by D'Aspremont et al. (1979). The duopolists sell a physically homogeneous good produced at zero marginal cost. Consumers are

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1. The analysis of a mixed duopoly under vertical product differentiation is in Delbono et al. (1991).

uniformly distributed along a unit interval, and their total density is 1. They have unit demands, and consumption yields a positive constant surplus s. Then, each consumer buys if and only if the net utility derived from consumption is non-negative:

$$U = s - tx^2 - p_i \geq 0, \quad t > 0, \quad i = 1,2; \tag{1}$$

where tx^2 is the transportation cost incurred by a consumer living at distance x from store i, and p_i is the price of good i. We assume that s is large enough for total demand to be always equal to 1. Firm 1 is located at a, while firm 2 is located at $1 - b \geq a$, with $a, b \in R$. The demand functions are, respectively:

$$y_1 = a + \frac{1-a-b}{2} + \frac{p_2 - p_1}{2t(1-a-b)} \tag{2}$$

$$if \quad 0 < a + \frac{1-a-b}{2} + \frac{p_2 - p_1}{2t(1-a-b)} < 1;$$

$$y_1 = 0 \quad if \quad a + \frac{1-a-b}{2} + \frac{p_2 - p_1}{2t(1-a-b)} \leq 0; \tag{2'}$$

$$y_1 = 1 \quad if \quad a + \frac{1-a-b}{2} + \frac{p_2 - p_1}{2t(1-a-b)} \geq 1; \tag{2''}$$

Demand to firm 2 is simply y_2=1-y_2:

$$y_2 = 1 - y_1 = b + \frac{1-a-b}{2} + \frac{p_1 - p_2}{2t(1-a-b)} \tag{3}$$

$$if \quad 0 < b + \frac{1-a-b}{2} + \frac{p_1 - p_2}{2t(1-a-b)} < 1;$$

$$y_2 = 0 \quad if \quad b + \frac{1-a-b}{2} + \frac{p_1 - p_2}{2t(1-a-b)} \leq 0; \tag{3'}$$

$$y_2 = 1 \quad if \quad b + \frac{1-a-b}{2} + \frac{p_1 - p_2}{2t(1-a-b)} \geq 1. \tag{3''}$$

Clearly, for a=1-b, i.e., when sellers locate at the same point, the demand functions are not determined and profits are nil as a consequence of the Bertrand paradox. In the absence of taxation, firms' profit functions are then

$$\pi_i = p_i y_i; \quad i = 1,2. \tag{4}$$

From a social standpoint, the optimal locations are obtained through the following minimization:

$$\min_{a,b} SC = t\left[\int_0^{\alpha_{1,2}} (x-a)^2 dx + \int_{\alpha_{1,2}}^{1} (1-b-x)^2 dx\right], \tag{5}$$

where $\alpha_{1,2}$ denotes the position of the consumer who is indifferent between the two firms. The minimization of social costs given by expression (5) is achieved setting $a = b = 1/4$, which amounts to saying that the maximum distance between any consumer and the nearest firm is being minimized. It is well known that the quadratic transportation cost version of Hotelling's duopoly yields excess differentiation at equilibrium, as compared to the social optimum.[2] Consequently, a public authority could devise a taxation or subsidization policy in order to induce firms to choose the socially efficient

2. For an exhaustive survey of location models, see Gabszewicz and Thisse (1992).

locations. As intuition suggests, it can be shown that neither an excise nor an ad valorem tax can affect firms' location. Thus, the government has to revert to a policy in which the latter variable is explicitly taken into account. The most general schedule is the following:

$$T_i = k(l-j)^2, \quad k \in R, \quad l = a,b, \quad j \in [0, 1/2], \quad i = 1,2, \qquad (6$$

i.e., according to the sign of k, firms pay no taxes or receives no subsidy if and only if they locate at j and $1-j$, respectively. If $j=1/2$, firms avoid paying taxes if they locate in the middle of the city. This would obviously entail zero profit, due to the Bertrand paradox. Under the fiscal regime described by (6), we will show the results summarized in the following

Proposition. For all $j \in [0, 1/2]$, there exists an optimal ratio k/t minimizing social transportation costs. If $j \in [0, 1/4[$, firms are being subsidized, while if $j \in]1/4, 1/2]$, firms are being taxed.

Proof. If the public authority adopts rule (6), each duopolist's profit function is given by:

$$\pi_i = p_i y_i - k(l-j)^2, \quad l = a,b, \quad j \in [0, 1/2], \quad i = 1,2 \qquad (7$$

Firms noncooperatively maximize (7) in a two-stage game in which first they simultaneously choose locations, and then they simultaneously compete in prices. The game is solved through backward induction. The solution concept is a subgame perfect equilibrium in locations and prices.

The equilibrium prices are:[3]

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3. Notice that the introduction of such a taxation scheme bears no consequence on the equilibrium prices described by (8-9). Cfr. D'Aspremont et al. (1979).

$$p_1^* = t(1-a-b)\left(1+\frac{a-b}{3}\right), \quad (8$$

$$p_2^* = t(1-a-b)\left(1+\frac{b-a}{3}\right), \quad (9$$

Substituting (8-9) into the profit functions (7), we obtain:

$$\pi_1 = \frac{t}{18}(a-b+3)^2(1-a-b) - k(a-j)^2, \quad (10$$

$$\pi_2 = \frac{t}{18}(a-b-3)^2(1-a-b) - k(b-j)^2. \quad (11$$

The equilibrium locations are obtained by solving the first order conditions relative to the location stage w.r.t. a and b:

$$a^* = b^* = j + (j+1/4)\left(\frac{t^3}{3kt^2+t^3}\right), \quad (12$$

which reduces to

$$a^* = b^* = \frac{4j(3r+1)-(4j+1)}{4(3r+1)}, \quad (13$$

where $r = \frac{k}{t}$. Consequently,

$$a^* = b^* = 1/4 \quad iff \quad r = \frac{2}{3(4j-1)}. \quad (14$$

Notice that the behaviour of the optimal value of r above is hyperbolic, with

$$\lim_{j \to 0} r = -2/3; \quad \lim_{j \to 1/4_-} r = -\infty;$$

$$\lim_{j \to 1/4_+} r = \infty; \quad \lim_{j \to 1/2} r = 2/3, \qquad (15$$

implying that if $j \in [0, 1/4[$, firms are being subsidized, while if $j \in]1/4, 1/2]$, firms are being taxed. *Q.E.D.*

As a corollary, we may note that, for j=1/4 the payment between firms and goverment is nil, but this merely corresponds to imposing firms to choose the socially optimal locations, by setting the rate k to infinity.[4] The extremely scarce feasibility of such a policy is apparent. Furthermore, it appears that:

$$\sum_i \pi_i + \sum_i T_i = \sum_i p_i y_i = \frac{t}{2}, \quad \forall r \quad \ni \quad a^* = b^* = 1/4, \quad i = 1,2. \qquad (16$$

This implies that the choice between taxation and subsidization may depend on the overall conditions of the economy, i.e., on factors external to the market under analysis. If instead one confine his attention to a partial equilibrium perspective, provided that the total transportation costs are being minimized by the choice of the socially optimal locations by the duopolists, the public

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4. Under subsidization, this holds in terms of absolute value, since k, and thus also r, is negative.

authority finds it advantageous to set j=1/2. This is made clear by noting that firms' profits are decreasing while the total tax revenue accruing to the government increases as j increases over the whole admissible interval, reaching a corner maximum at j=1/2, where $\sum_i T_i = t/12$.

3. Conclusions

We have analysed the behaviour of a horizontally differentiated duopoly in which firms locate along a linear city of finite length, subject to taxation/subsidization by the public authority. It turns out that the nearer to the socially optimal locations the authority starts taxing (respectively, subsidizing) firms, the higher (in absolute value, in the case of subsidization) must be the ratio between tax (subsidy) and transportation cost rates in order for the differentiation degree to be socially optimal at equilibrium. Alternatively, the adoption of either a subsidy scheme starting from the opposite city boundaries or a tax scheme starting from the middle of the city ensures the optimal degree of differentiation for a reasonably low value of the ratio between subsidy or tax and transportation cost rates, in the latter case maximizing at the same time the revenue accruing to the government.

Finally, according to the results obtained by Cremer and Thisse (1991), who show that, provided that the market is completely covered at equilibrium, any model *à la* Hotelling can be considered as a special case of vertical differentiation models, the same conclusions also hold under vertical differentiation and Bertrand competition.

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