A GENERALIZED MEASURE OF COMPETITION

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Abstract

Using the conditional probabilities of the passage of customers from one firm to another, a new, implementable measure of market competition is obtained which generalizes the classical Hirschman-Herfindahl index.

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1. Introduction.

The degree of competition is usually considered a structural aspect of the market, i.e. a feature changing sufficiently slowly in time to represent a basic, unobservable characteristic. In this case, when other observable characteristics (barriers, concentration, etc.) are correlated with the degree of competition, the former can be used to measure the latter.

A leading example is the Hirschman-Herfindahl index of concentration:

$$HHI = \sum_{i=1}^{N} y_i^2$$

y_i^2 being the square of the market share of the i-th firm in the market ([1]). Simple theories of monopolistic pricing show that HHI is positively related to non-competitive profits [see Cowling and Waterson (1976)] so that it can provide a measure of non-competitive behavior or, at least, a threshold to suspect it (see the 1982 Merger Guidelines of the U.S. Department of Justice).

Competition, however, is essentially a dynamic process so that it may be interesting to try to grasp its 'intensity' in a simple but properly dynamic context. This approach was

[1] HHI is equivalent to $(cv^2+1)/N$ where cv is the coefficient of variation of market shares. It, therefore, ranges between 1/N (equidistribution) and 1 (a single firm in the market).

taken in a seminal paper by Stigler (1964) and, surprisingly, it conduced again to HHI ([2]).

In this paper, using a simple, two period model, we shall show that a more general result and a new implementable measure of competition can be obtained for which HHI represents a special (and implausible) case.

2. The model.

Let us consider, for clarity sake, the simplest case of a two-period market with two firms ([3]), F_1 and F_2 selling a differentiated good at time 1 and at time 2 ('years').

If $p(\hat{F}_i, F_j)$ is the joint probability of being customer of F_j at year 1 and of F_i at year 2 (year 2 is indicated by a hat), the 2x2 probability table is the following:

[HERE TABLE 1]

Note that the marginal probability of being customer of Firm 1 at year 2 is the sum of the joint probability of being already one of its clients at year 1 and stay with it plus the joint probability of being a client of the competitor at

[2] In Stigler's paper the index is correlated with the likelihood of effective collusion.

[3] The extension to the general case of N firms is trivial.

time 1 and then change [4]:

$$p(\hat{F}_1) = p(F_1, \hat{F}_1) + p(F_2, \hat{F}_1) \tag{1}$$

Using the conditional probabilities:

$$\begin{aligned} p(\hat{F}_1) &= p(F_1)p(\hat{F}_1|F_1) + p(F_2)p(\hat{F}_1|F_2) \\ &\equiv p(F_1)\ f_{11} \qquad + p(F_2)\ f_{12} \end{aligned} \tag{2}$$

and for Firm 2:

$$\begin{aligned} p(\hat{F}_2) &= p(F_1)p(\hat{F}_2|F_1) + p(F_2)p(\hat{F}_2|F_2) \\ &= p(F_1)\ f_{21} \qquad + p(F_2)\ f_{22} \end{aligned} \tag{3}$$

In matrix form:

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{4}$$

with the obvious conditions:

$$\sum_i y_i=1, \quad \sum_j x_j=1, \quad \sum_i f_{ij}=1\ \forall\ j, \quad 0 \le f_{ij} \le 1 \tag{5}$$

where y_i and x_j are respectively year 2 and year 1 market shares and, following Stigler (1964), they are assumed as proxies of the marginal probabilities: $p(\hat{F}_i)=y_i$, $p(F_j)=x_j$.

The parameters f_{ij} are the transition probabilities from Firm j to Firm i. In particular, the diagonal of the transition matrix contains the probabilities to stay with the

[4] Note that the entry in year 2 of new customers (a new column in table 1) or new firms (a new row in table 1) poses no special problem to the model.

same firm both in year 1 and year 2. They measure, therefore, the fidelity level of the customer relationship and in this sense they are correlated with market power. If market power can be excluded from the demand side, a high fidelity level means that flows among firms are small, customers are 'captured' by their suppliers and the latter benefits by a monopoly degree. On average, if the diagonal probabilities decrease it means that the market competition has increased, raising the flows of customers among firms.

This suggests a simple measure of non-competition defined by a weighted average of the diagonal conditional probabilities. Using the y_i weights we have the following fidelity index (with N firms):

$$FI = \sum_{i=1}^{N} f_{ii} y_i = \sum_{i=1}^{N} p(\hat{F}_i | F_i) p(\hat{F}_i) \quad (6)$$

Note that the value 1/N is obtained when the conditional probabilities are all equals: every year the consumer chooses his/her supplier as the result of a coin toss so that there is perfect mobility of customers, no market power is accumulated by firms and the market is, in a dynamic, probabilistic sense, perfectly competitive.

At the other extreme, FI can correctly identify a situation in which a number of producers, possibly many with equal market shares (low HHI), determine a non-competitive, monopolistic configuration of the market, in which the probability of stay with the same supplier is near to 1 (high FI).

Moreover, note that in the case of independence we have $p(\hat{F}_i|F_i) = p(\hat{F}_i)$, meaning that becoming customer of Firm i at time 2 is independent of being already one of its customers at time 1. It is clearly an implausible assumption and in this case the fidelity index reduces to HHI:

$$HHI = \sum_{i=1}^{N} y_i^2 = \sum_{i=1}^{N} p(\hat{F}_i)p(\hat{F}_i) \qquad (7)$$

3. Implementation.

In order to implement model (4) we have to estimate the conditional probabilities in the transition matrix, under the constraints given in (5).

Fortunately, the problem of estimating conditional probabilities from marginals (the 'ecological regression' problem) has been object of many contributions in the social sciences, starting from Miller (1952), Duncan and Davis (1953) and Goodman (1953, 1959), as well as in econometrics, from Telser (1963) to Theil and Rey (1966) and Lee and Judge (1972).

Let us assume that the extra-information needed to estimate the transition probabilities comes from H different geographical markets ([5]) where the 2 firms compete, under the maintained hypothesis that the transition probabilities do not vary in the considered areas.

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[5] Another possibility, considered in the econometric literature, is information from the time dimension. The analysis is similar.

Introducing an error term ε, the first line (Firm 1) in (4) becomes, in the H areas:

$$
\begin{aligned}
y_{1,1} &= x_{1,1}\ f_{11} + x_{2,1}\ f_{12} + \varepsilon_{1,1} \\
&\cdots\cdots\cdots \\
y_{1,h} &= x_{1,h}\ f_{11} + x_{2,h}\ f_{12} + \varepsilon_{1,h} \\
&\cdots\cdots\cdots \\
y_{1,H} &= x_{1,H}\ f_{11} + x_{2,H}\ f_{12} + \varepsilon_{1,H}
\end{aligned}
\tag{8}
$$

i.e. in matrix form

$Y_1 = X \quad b_1 + \varepsilon_1$

with $\varepsilon_1 \sim D(0, \sigma_{11} I_H)$, homoskedastic error term

and the complete system (in the general case of N firms) becomes:

$$
\underset{HNx1}{\begin{bmatrix} Y_1 \\ \cdot \\ \cdot \\ Y_j \\ \cdot \\ \cdot \\ Y_N \end{bmatrix}} = \underset{HNxN^2}{\begin{bmatrix} X & 0 & . & . & . & 0 \\ 0 & X & 0 & . & . & 0 \\ . & . & . & . & . & . \\ . & . & . & X & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ 0 & 0 & . & . & 0 & X \end{bmatrix}} \underset{N^2x1}{\begin{bmatrix} f_1 \\ \cdot \\ \cdot \\ f_j \\ \cdot \\ \cdot \\ f_N \end{bmatrix}} + \underset{HNx1}{\begin{bmatrix} \epsilon_1 \\ \cdot \\ \cdot \\ \epsilon_j \\ \cdot \\ \cdot \\ \epsilon_N \end{bmatrix}} \tag{9}
$$

i.e.

$$
Y = \mathbf{X} f + \epsilon \quad \epsilon \sim D(0, \Sigma \otimes I_H), \quad \Sigma = [\sigma_{ij}], \quad \sigma_{ij} = E(\epsilon_{ih}\epsilon_{jh}) \tag{10}
$$

with the constraints $\sum_{i=1}^{N} f_{ij} = 1 \ \forall j \quad 0 \le f_{ij} \le 1$

Note that Σ is singular because the errors are linearly dependent:

$$\sum_{j=1}^{N} y_{j,h}=1 \ , \sum_{j=1}^{N} x_{j,h}=1 \quad \text{so that} \quad \sum_{j=1}^{N} \epsilon_{j,h}=0 \quad \forall h \tag{11}$$

The system (10) is a special case of Zellner's SUR equations with identical regressors and cross-equation parameter restrictions. An estimation method is suggested by Judge et al. (1985, p.499) and Theil and Rey (1966). Following this method, a first application to the Italian banking markets has been developed by the author obtaining different levels and behaviors of the fidelity and Herfindahl indexes. In particular, differently with respect to previous alternative measure, the fidelity index resulted to be non correlated with HHI.

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TABLE 1

		YEAR 1		
		Firm 1	**Firm 2**	
YEAR 2	**Firm 1**	$p(\hat{F}_1, F_1)$	$p(\hat{F}_1, F_2)$	$p(\hat{F}_1)$
	Firm 2	$p(\hat{F}_2, F_1)$	$p(\hat{F}_2, F_2)$	$p(\hat{F}_2)$
		$p(F_1)$	$p(F_2)$	1