IMPLICATIONS OF SOLOW'S GROWTH MODEL: A STOCHASTIC APPROACH

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by

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Abstract - *The aim of this paper is to analyse Solow's model, introducing the consideration that steady state labour productivity (in efficiency units) and its determinants are not constant values, for a country over a given period of time. They are, in fact, time series with unit roots. First, the paper shows that Solow's model can be interpreted as an error correction model and it could be consistent with the stochastic nature of the variables. Secondly, implications about integration and cointegration of the relevant series are tested. We use data coming from four countries over the period 1960-88. The error correction mechanism implied by Solow's model never appers to have been operative: convergence of current productivity towards its (stochastic) steady state path does not emerge in any considered case.*

J.E.L. Classification: O40

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1. INTRODUCTION

The aim of this paper is to derive and to test the implications of Solow's (1956, 1957) model, switching the focus from the cross-country predictions about growth, to the pattern of labour productivity in a given country. The main question is: "how is it possible to test Solow's growth model, looking at productivity movements in a country?".

Solow's growth model shows that if the aggregate production function exhibits constant returns to scale and positive, decreasing marginal returns to single factors and if the Inada conditions hold, income and capital levels in efficiency units converge to a steady state. The convergence process is monotonic and the absolute value of the income growth rate is decreasing, when the economic system is approaching the steady state path. Once the steady state has been reached, per capita variables grow at the rate of exogenous technical progress.

Economies sharing identical structural parameters converge to the same per capita income (or labour productivity), so that it is possible to interpret economies observed at different stage of per capita income levels as if they were the same economy observed at different points in time; the less developed economy, moreover, is expected to grow more rapidly and to catch-up with the more developed one. Very often the literature focussed on this point, in order to test Solow's model.

It is well known that the cross-section estimates on actual data give different answer to the question whether catching-up takes place or not, depending on the sample of countries considered, and on the control variables added in regression, notably the investment rate in physical and human capital and the labour growth rate.[1]

In cross-section studies, three points are usually made: (i) the average value of the investment share in GDP, during a given period of time for a country, is taken as the constant propensity to accumulate physical capital; (ii) the average (or the starting) value of some scholarity rate proxies for human capital accumulation rate; (iii) the growth rate of employment in a country is considered a constant during the period under consideration. These three values affect the steady state level of labour productivity, which every economy is approaching to. Assuming them as constants implies that also the steady state levels of output, in efficiency units, is constant for a country during the period of time considered.

This common assumption about the non-variability of the above mentioned factors is far from being correct: data show that these variables vary widely not only across countries, but also, for each country, over time. It will be shown that Dickey-Fuller tests reveal unit roots in the time series of investment rates and in employment growth rates: these determinants of labour productivity equilibrium level are time series integrated of order one. As a consequence, the steady state level of output in efficiency units could be a stochastic process with a unit root too.[2]

It is well known that the current levels of labour productivity (and per capita income) are time series integrated of order one. Unit roots in labour productivity have been sometimes seen as an evidence against Solow's model, since it is impossible that an integrated series converge to some deterministic path, as the standard model predicts. However, if we take into consideration the stochastic nature of the determinants of equilibrium productivity level, integration of current productivity level is no longer a contradiction, but it is consistent with the stochastic nature of other variables.

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1 - Within OECD countries, there is evidence of catching-up effect even if no other variables are added to a regression of labour productivity growth (during a given period of time) upon the level of productivity at the beginning of period, while in a much wider sample of countries (such as Summers-Heston's (1991) data set) the same result is obtained only after controlling for different rate of accumulation of physical (and possibly human) capital and for different employment growth rates; the latter case has been labelled as "conditional convergence". See, for instance, Baumol (1986), Barro (1991), Mankiw, Romer and Weil (1992) and the review on cross-section studies by Levine and Renelt (1992).

2 - "Steady state level" and "equilibrium level" are synonyms in this paper. It is worth stressing that equilibrium level varies over time and across countries --in this stochastic interpretation of Solow's model-- because of changes in its determinants.

Empirical tests show that the growth rates of labour productivity are a stationary series. Such a series is determined by the lagged difference between the current and the equilibrium levels of labour productivity --according to Solow's model. Thus, for the model to be consistent with the stochastic nature of actual data, current and steady state productivity levels have to be cointegrated so that the stationarity of their difference is consistent with the stationarity of productivity growth rates. This property on cointegration will be regarded as a crucial stochastic implication of Solow's model.

Furthermore, if we regress the growth rate of labour productivity on the lagged difference between actual and equilibrium productivity levels, a significant negative coefficient of this regressor has to be found: according to Solow's model, if the current level is less (greater) than the equilibrium level, the subsequent variation rate will be positive (negative). This is an error correction mechanism, in which a negative value of correction parameter (that is, the parameter linking labour productivity growth rate to previous disequilibrium between current and equilibrium productivity level) is a necessary condition for converging to equilibrium.

Only few of these predictions are supported by the empirical evidence; thus, our conclusion is that the stochastic nature of actual data does not support the error correction mechanism implied by Solow's model.

The rest of the paper is structured as follows: section 2 sketches Solow's model, augmented by human capital, as it appears in Mankiw, Romer and Weil (1992); section 3 focuses on the Error Correction Mechanism implied by Solow's model, given the stochastic nature of time series. Section 4 provides some empirical evidence coming from USA, France, Italy and Japan, over the period 1960-88. Concluding observations are summarized in section 5.

2. AUGMENTED SOLOW'S MODEL

Let us consider the following constant returns to scale production function, for which the Inada conditions hold:

$$Y_{i,t} = K_{i,t}^{\alpha} \cdot H_{i,t}^{\beta} \cdot (\Omega_t L_{i,t})^{1-\alpha-\beta} \quad ;$$

subscripts i and t indicate country and time, respectively; L is labour, K and H are the levels of physical and human capital, while Ω is the technology level, which is the same across countries for given t. Both physical and human capital grow according to the following equations:

$$\frac{dK_{i,t}}{dt} = s_{i,t}^{K} Y_{i,t} - \delta K_{i,t} \quad ,$$

$$\frac{dH_{i,t}}{dt} = s_{i,t}^{H} Y_{i,t} - \delta H_{i,t} \quad .$$

s^K and s^H denote the fraction of output devoted to the accumulation of physical and human capital, respectively; δ is the depreciation rate, which is assumed to be the same for physical and human capital and to be constant across countries and over time. The technology parameters α, β are constant too. The rate of technical progress, g, is also constant and such that:

$$\Omega_{i,t} = \Omega_{i,0} e^{gt} \quad ;$$

finally, the growth rate of labour force is denoted by $n_{i,t}$, so that:

$$L_{i,t} = L_{i,0} e^{n_{i,t}} \quad .$$

$\underline{Given}$ $s^K_{i,t}$, $s^H_{i,t}$, $n_{i,t}$, it exists a steady state level for $K/(\Omega L)$, $H/(\Omega L)$, $Y/(\Omega L)$:[3]

$$\left(\frac{K}{\Omega L}\right)^* = \hat{k}^* = \left(\frac{s_K^{1-\beta} \; s_H^{\beta}}{n+g+\delta}\right)^{\frac{1}{1-\alpha-\beta}} ;$$

$$\left(\frac{H}{\Omega L}\right)^* = \hat{h}^* = \left(\frac{s_K^{\alpha} \; s_H^{1-\alpha}}{n+g+\delta}\right)^{\frac{1}{1-\alpha-\beta}} ;$$

$$\left(\frac{Y}{\Omega L}\right)^* = \hat{y}^* = \frac{s_K^{\frac{\alpha}{1-\alpha-\beta}} s_H^{\frac{\beta}{1-\alpha-\beta}}}{(n+g+\delta)^{\frac{\alpha+\beta}{1-\alpha-\beta}}} .$$

Labour productivity, $y = Y/L$, in steady state (y^*) for given $s^K_{i,t}$, $s^H_{i,t}$, $n_{i,t}$, varies over time according to:

$$\ln y_t{}^* = (\ln \Omega_0 + gt) + \frac{\alpha}{1-\alpha-\beta}\ln s^K + \frac{\beta}{1-\alpha-\beta}\ln s^H - \frac{\alpha+\beta}{1-\alpha-\beta}\ln(n+g+\delta).$$

Note that, for given $s^K_{i,t}$, $s^H_{i,t}$, $n_{i,t}$, it exists one *point* for the steady state level of output in efficiency units, and one *path* for output per worker (or labour productivity). If we assume that s^K, s^H and n can vary both across countries and across time, then the level of the steady state to which any economy is converging, varies both among countries and across time.

In the neighbourhood of the steady state path, labour productivity moves according to the following equation (see Mankiw, Romer and Weil (1992) or Durlauf and Johnson (1992)):

(1)

$$\ln y_{i,t+1} - \ln y_{i,t} = g + \left(1 - e^{-\lambda_{i,t}}\right)\cdot\left[(\ln \Omega_0 + gt) + \frac{\alpha}{1-\alpha-\beta}\ln(s^K_{i,t}) + \frac{\beta}{1-\alpha-\beta}\ln(s^H_{i,t}) - \frac{\alpha+\beta}{1-\alpha-\beta}\ln(n_{i,t}+g+\delta) - \ln y_{i,t}\right],$$

where $\lambda_{i,t} = (n_{i,t} + g + \delta)(1-\alpha-\beta)$.

3 - Hats indicate "in efficiency units"- variables and stars indicate "steady state"- variables.

Inside the squared brackets in equation (1), one finds the difference between steady state value of $y_{i,t}$ (given $s^K_{i,t}$, $s^H_{i,t}$, $n_{i,t}$) and the current value of $y_{i,t}$; the sign and the magnitude of this difference affects the subsequent labour productivity growth rate. If the current level of labour productivity is less than its equilibrium level, current productivity level will rise in the next period; it will decrease when the current level is above the equilibrium.

Notice that the speed of convergence, λ, is not constant, due to the variability of the employment growth rate, n. However, for a given λ, the closer is the economy to the equilibrium value of $y_{i,t}$, the smaller will be the absolute value of labour productivity variation rate due to the disequilibrium. The coefficient $(1-e^{-\lambda_{i,t}})$ indicates how sensitive the growth rate of labour productivity is to the divergence between equilibrium and current level of this productivity. If λ=0 (that is $\alpha+\beta=1$) no steady-state and no convergence mechanism do exist.

It is also worth noting that this meaning of "convergence" differs only partially from that commonly used in the literature (see, for instance, Barro and Sala I Martin (1992)): in the present analysis, "convergence" has to be interpreted as the tendency for each country's labour productivity to move towards its steady state path, and such a path is conditional on the current level of the propensities to accumulate physical and human capital and on the growth rate of employment.

3. THE ERROR CORRECTION MECHANISM IN SOLOW'S MODEL

It is important to stress that as long as the variation of $\ln y_{i,t}$ is determined by the lagged difference between the current and the equilibrium level of $\ln y_{i,t}$, the movement of this variable follows an error correction mechanism.

Generally, models with error correction mechanism are constituted of two parts: a disequilibrium response term, and other "short run" terms, so that the following expression:

$$\Delta y_t = \gamma_1 + \gamma_2 \cdot (y_{t-1} - c' x_{t-1}) + \gamma_3 \cdot \Delta x_t$$

is the most common form of an equation with error correction mechanism (Δ being the first-difference operator, $\boldsymbol{x}$ a vector of variables, $\boldsymbol{c}$ the long term equilibrium parameters vector, such that $y = \boldsymbol{c}'\boldsymbol{x}$ is the expression of equilibrium value of y).

Consider now $(1+n)$ time series, integrated of order 1: $[y, x_1, x_2, \ldots x_n]$. If there exists one vector $A=[A_1 A_2 \ldots A_n]$ such that $z = y-A_1x_1-A_2x_2-\ldots A_nx_n = y-A'\boldsymbol{x}$ is stationary, then $[y, \boldsymbol{x}]$ are cointegrated with rank equal to 1. According to Granger's representation theorem (Granger (1986), Engle and Granger (1987)), y and $\boldsymbol{x}$ are cointegrated if and only if it is possible to express y in the following way:

$$(2) \quad \Delta y_t = \gamma(y - A'x)_{t-1} + b(L)\Delta x_t + \varepsilon_t,$$

where $b(L)$ is a polynomial in lag-operator L, Δ is the first-difference operator and ε is a white noise process. Equation (2) represents an error correction model (ECM). The cointegrating relationship, $y=A'\boldsymbol{x}$, can be interpreted as the long run equilibrium relationship.

If x is a scalar, no more than one scalar A exists such that linear combination of y and x is stationary (a unique cointegrating vector $[-1, A]$ exists); if $\boldsymbol{x}$ is a n-components vector, up to n vectors $[-1, A]$ can exist and n is the "cointegration rank". In order to estimate the parameters of an ECM, Engle-Granger's method assumes that the cointegration rank is 1 and then a two-steps procedure is followed.[4] Johansen's method, by using a maximum likelihood procedure, allows to estimate both the rank and the cointegrating vectors, when more than one cointegrating relationship are possible.[5]

Let us note that equation (1) can be expressed in the following linear form (all variables are in log terms and i-index is omitted):

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4 - First, the long term relation is regressed by using ordinary least squares method; this yields superconsistent estimates of A and stationary fitted residuals $\hat{e} = y - \hat{A}'x$. Secondly, $\hat{e}$ is put in regression (2), instead of $(y-A'\boldsymbol{x})$, and γ and $b(L)$ are estimated.

5 - See Johansen (1988), (1989), (1991) and Johansen and Juselius (1990).

(3) $\Delta y_t \;=\; C+\gamma\cdot[y-A_0-A_1T-A_2s^K-A_3s^H-A_4(n+g+\delta)]_{t-1} \;=\; C+\gamma[y-y^*]_{t-1}$

where A_0 is a constant and T a time trend. This equation describes the movements of labour productivity in a given country, according to Solow's model; the movements follow an error correction mechanism, and the difference between current y and its equilibrium level determines the next growth rate of y. For the correction error mechanism to be "operative" it is necessary that γ is negative.

In order to obtain a complete error correction model (in the form of equation (2)), it is sufficient to add linear short run regressors, capturing the short run movements of y (which have not been taken into account by Solow in his original model),[6] and the stochastic term ε:

(4)

$$\Delta y_t = a_0 + a_1T + a_2s^K_{t-1} + a_3s^H_{t-1} + a_4(n_{t-1}+g+\delta) + \gamma y_{t-1} + b_1\Delta s^K_t + b_2\Delta s^H_t + b_3\Delta(n+g+\delta)_t + \varepsilon_t.$$

Note that $a_2 = -\gamma\cdot A_2$ and $a_3 = -\gamma\cdot A_3$ are positive, and $a_4 = -\gamma\cdot A_4$ negative, according to Solow's theory.

The coefficient of the lagged level of labour productivity plays the crucial role in cross-section estimates. However, since we are dealing with time series regression, it is not correct to interpret the negative sign of this coefficient, γ, as the literature on cross-section estimates does, i.e. as the evidence of catching-up. It is rather the expression of the tendency of labour productivity to move towards its steady-state path, for the economy under consideration; more precisely, the negative sign of γ is a necessary condition for the convergence towards equilibrium: necessary and sufficient condition is $-1 < \gamma < 0$.

6 - There is no specific theory concerning the signs and the magnitude of the coefficients of short run regressors (b_1, b_2, b_3 in equation (4)), since Solow's model does not consider the short run movements of labour productivity. They are, however, an important part of the movements of labour productivity and not taking them into account would lead to a mis-specified regression. Of course, alternative specifications of the short run part of (4) could be considered, given that econometricians are used to deal with these short run components of ECMs in a rather free way.

4. EMPIRICAL EVIDENCE

For the proposed interpretation of Solow's model to be meaningful and consistent with the data, it is necessary that s^K, s^H, n and y are integrated series, with one cointegrating vector. It is well known that the level of labour productivity contains a unit root. In what follows, formal tests for unit roots in the series are presented for four countries: USA, France, Italy and Japan.

Data come from the Summers-Heston (1991) PWT5 data set (for y_t, s^K_t, n_t) and from UNESCO and WORLD BANK sources (for s^H_t).

Y denotes the log of real GDP per worker at constant international price (variable 19 in Summers-Heston's data set); *SK* is the log of investment share in GDP (Summers-Heston's variable 4, divided by 100); *SH* is the log of secondary school enrollment rate divided by 1000 and it proxies for the propensity to human capital accumulation[7]; *NGD* is the empirical counterpart of $\log(n_t + g + \delta)$ and it is the log of the sum of employment growth rate plus a guess estimate of technological progress rate plus depreciation rate ($g+\delta=0.05$ is assumed).

Table 1 shows the results of Augmented Dickey-Fuller test for each series; all the series accept the null-hypothesis of unit root presence, at the 95% confidence level, except *SK* in USA. First-differences series, *DY, DSH, DSK, DNGD* always refuse the null hypothesis that unit roots are present, so that the series in table 1 are integrated of order one.

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7 - See Mankiw, Romer and Weil (1992, p. 419), for example, about this commonly used proxy. We use the data on secondary school enrollment rates provided by Mankiw, Romer and Weil (1992). The missing data are obtained by linear interpolation. This means that a data generation process with a segmented trend has been used. It is well known that a time series with a segmented deterministic trend can represent a data generation process containing a unit root and no deterministic trend; thus, not surprisingly, ADF statistic accepts a unit root in *SH*.

TABLE 1 - Unit roots in the series (ADF(1) statistics).

VARIABLE	USA	FRANCE	ITALY	JAPAN
Y	-2.56	-1.61	-1.46	-1.10
SK	-4.82	-2.01	-3.09	-2.21
SH	0.64	-1.51	-2.13	-1.75
NGD	-0.98	-0.96	-1.90	-1.54

Notes: The figures are the Student-t statistics of coefficient c in regression: $\Delta x_t = a + bT + cx_{t-1} + d\Delta x_{t-1}$. Critical value of Dickey Fuller test is -3.58 (confidence 95%).

As a consequence of the stochastic nature of *SK*, *SH* and *NGD*, the steady state growth path (i.e., the equilibrium level of output in efficiency units) can be a stochastic process with a unit root too. The idea that steady state growth path is stochastic, rather than stable, is also suggested by Quah (1993). He makes this point using a non parametric approach and his conclusion is that the data show instability in the underlying long-run growth patterns within each country. Our result is even stronger: the steady state level of output in efficiency units, in each country, can be a stochastic process integrated of order one.

Let us now consider cointegration among the variables, by using Johansen's (1988) technique. The associated VAR model is cut after one lagged period (twice lagged variables are not statistically significant) and it contains a deterministic linear trend. Both tests based on maximum eigenvalue statistics, and tests based on trace statistics are reported in Table 2/A; these tests lead to conclude that the series are cointegrated with rank equal to 1, in USA and France, while no cointegration appears in Italy and Japan. Table 2/B reports the (unique) cointegrating vector for USA and for France, after normalizing the coefficient of *Y* to -1, so that the values are the coefficients of long term relation between *Y* and its determinants.

TABLE 2/A Tests on cointegration rank.

(Test based on Maximal Eigenvalue of the Stochastic Matrix)

Null Hypothesis	USA	France	Italy	Japan	95% crit. v.
r=0 vs. r=1	34.38	22.20	15.64	15.50	27.06
r<=1 vs. r=2	18.03	16.73	11.72	11.43	20.96
r<=2 vs. r=3	7.98	7.58	7.25	3.73	14.07
r<=3 vs. r=4	2.49	2.67	3.61	1.08	3.76

(Test based on Trace of the Stochastic Matrix)

Null Hypothesis	USA	France	Italy	Japan	95% crit. v.
r=0 vs. r=1	62.89	49.19	38.23	31.75	47.21
r<=1 vs. r=2	28.51	26.99	22.58	16.15	29.68
r<=2 vs. r=3	10.48	10.25	10.87	4.82	15.41
r<=3 vs. r=4	2.49	2.67	3.61	1.08	3.76

TABLE 2/B - Coefficient in long term cointegating relationship between Y and its determinants (Johansen's method).

VARIABLE	USA	FRANCE
SK	2.566 *	0.939
SH	1.054	-0.697 *
NGD	-2.759 *	2.183 *

Note: Star indicates that χ^2 test rejects the null-hypothesis that the coefficient is equal to zero (95% confidence level).

Note that the coefficients are *not* completely consistent with the predictions of Solow's model, in the countries where cointegrating relationships appear: the estimated values are statistically significant and correctly signed, only for two out of the six cases; moreover, coefficients widely differ amongst countries, indicating that a different cointegration vector exists for each country.

The estimates based on Johansen's method do not support Solow's model: series are either not cointegrated or cointegrated with a vector far from the prediction of Solow's theory.[8]

On the other hand, the long run regression using OLS (i.e. the first step in Engle-Granger's procedure) denotes *no cointegration* among the series, in any considered country: ADF(1) test for the fitted residuals of static regression[9] gives -2.2 for USA, -3.17 for France, -2.4 for Italy and -2.88 for Japan (critical value at the 95% level of significance is -5.01).

However, conclusions based only on the results of unit root tests are rather weak (see Campbell-Perron (1991) about pitfalls connected to unit root tests); in particular, it has been forcefully suggested that univariate time series analysis may lead to misleading result in small samples. Thus, we will run regressions on model with error correction mechanism and the results concerning the sign and significance of error correction factor will be used to get a stronger conclusion about cointegration.

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8 - Similar results are also obtained by considering the associated VAR model with twice lagged variables, that leads to cointegration rank equal to 2: no meaningful cointegrating vectors appear.

9 - We run static regression of *Y* upon *SK*, *SH*, *NGD*, constant and deterministic trend.

Both linear and non-linear specifications are considered.

Let us start with the non linear specification. For each country, non-linear-least-square estimation are calculated on the following regression:

$$\Delta \ln y_t = a_1 + \left[1 - e^{-(n_t + a_1 + a_5)(1 - a_2 - a_3)}\right] \cdot \left[a_4 + a_1 T + \frac{a_2}{1 - a_2 - a_3} \ln s^K_{t-1} + \frac{a_3}{1 - a_2 - a_3} \ln s^H_{t-1} - \frac{a_2 + a_3}{1 - a_2 - a_3} \ln(n_{t-1} + a_1 + a_5) - \ln y_{t-1} \right] +$$

$$+ a_6 \Delta \ln s^K_t + a_7 \Delta \ln s^H_t + a_8 \Delta \ln n_t .$$

Notice that a_2 correspond to α and a_3 to β of the equations in section 2; a_1 is the exogenous technical progress growth rate and a_5 is the depreciation rate of capitals; a_6, a_7, a_8 describe short run movements.

Estimated parameters and diagnostic tests are reported in Table 3; also the implied (average) speeds of convergence, $\lambda = (n_t + g + \delta)(1 - \alpha - \beta)$, and the estimated error correction factors, $(1 - e^{-\lambda})$, are in Table 3: λ and $(1 - e^{-\lambda})$ are calculated, by taking the average value of n_t for each country. The results concerning USA do not appear, because calculations do not converge in that case. Note that diagnostic tests do not show any particular problems and the R^2 values are pretty good.

TABLE 3 - Non linear least square estimations. (Dependent variable is ΔY).

VARIABLE	FRANCE	ITALY	JAPAN
a_1	0.005	0.01	0.02
	(0.34)	(1.74)	(1.92)
a_5	0.28	0.38	1.08
	(1.09)	(1.24)	(1.75)
a_2	0.33	0.15	0.14
	(1.34)	(1.01)	(2.05)
a_3	0.22	0.30	0.6
	(0.51)	(3.24)	(4.84)
a_4	9.08	10.87	23.5
	(2.54)	(22.3)	(4.9)
a_6	0.22	0.33	0.37
	(6.31)	(6.81)	(5.67)
a_7	0.03	-0.17	0.14
	(0.71)	(-0.5)	(0.36)
a_8	-0.018	-0.001	0.01
	(-0.85)	(-0,21)	(0.55)
R-squared	0.90	0.83	0.81
DW	1.63	1.73	2.36
Serial correlation test	1.33	0.68	1.72
Functional form test	0.84	0.12	0.27
Omoschedasticity test	0.72	2.84	0.70
D-F on fitted residuals	-4.19	-4.4	-6.07
Implied speed of convergence, λ	0.13	0.21	0.27
	(1.8)	(1.2)	(1.5)
Implied $(1-e^{-\lambda})$	0.12	0.19	0.24
	(1.9)	(1.3)	(1.8)

Notes: *t*-statistics in parentheses. Serial correlation test is a Lagrange multiplier test of residual correlation ($\chi^2(1)$); omoskedasticity test statistic ($\chi^2(1)$) is based on the regression of squared residuals on squared fitted values; functional form test is the Ramsey RESET test ($\chi^2(1)$).

The signs of the parameters appearing in Table 3 are consistent with Solow's theory. The magnitudes are quite different from the results of other works: the elasticity of output to physical capital is lower and the labour share in GDP is higher than the values suggested by previous works (see, for instance, Romer (1987, 1989), Barro and Sala I Martin (1992),[10] Durlauf and Jonson (1992) or Mankiw, Romer and Weil (1992)). The value of exogenous technical progress is pretty reasonable, though not highly significant, while the depreciation rate is definitely on the high side.

It is worth stressing that *the speeds of convergence and the error correction factors are not statistically different from zero, thus no significant tendency towards steady state path seems to take place.*

Consider now the linear specification, assuming the constancy across time of parameters[11] and imposing $g + \delta = 0.05$; OLS estimations of equation (4) and tests are reported in Table 4. Also in linear case, none of the diagnostic tests indicate errors in specification. The endogeneity of contemporary regressors is not a serious problem, since only *DSK* results to be endogenous, but Instrumental Variable estimates (using public spending variation as an instrument) are not very different from the OLS ones.

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10 - Both Romer (1989) and Barro and Sala I Martin (1992) point out that the low observed speed of convergence can be reconciled with the (non-stochastic) neoclassical growth model only if the share of capital in GDP is very high. Den Haan (1992) shows that the same speed of convergence is consistent both with low and with high capital share in GDP, if productivity is a persistent stochastic process: in other words, the share of capital in GDP is not important for the speed of convergence, in the standard growth model in the presence of productivity shocks.

11 - Coefficients could vary because they are function of the speed of convergence which is determined by n_t. However, CUSUM and CUSUMQ tests and Chow's second test do not show parameters instability in the regressions of Table 4.

TABLE 4 - Estimates and tests on linear ECM. (Dependent variable is ΔY).

VARIABLE	USA	FRANCE	ITALY	JAPAN
CONSTANT	0.61	0.84	1.87	5.10
	(0.89)	(0.44)	(1.15)	(2.10)
TREND	0.004	0.0002	0.003	0.004
	(1.25)	(0.06)	(0.87)	(0.76)
SK(-1)	0.11	0.08	0.05	0.12
	(2.6)	(0.95)	(0.63)	(1.11)
SH(-1)	-0.26	0.06	0.11	0.49
	(-1.45)	(0.39)	(1.79)	(1.18)
NGD(-1)	-0.06	-0.12	-0.13	0.008
	(-092)	(-0.42)	(-0.98)	(0.08)
Y(-1)	-0.12	-0.11	-0.19	-0.21
	(-1.9)	(-1.14)	(-1.36)	(-1.37)
DSK	0.28	0.22	0.33	0.38
	(9.4)	(4.93)	(6.59)	(5.36)
DSH	-0.08	0.03	-0.21	0.08
	(-0.33)	(0.72)	(-0.60)	(0.19)
DNGD	-0.01	-0.02	-0.002	0.03
	(-0.93)	(-0.76)	(-0.49)	(0.25)
R-SQUARED	0.90	0.90	0.83	0.81
D-W test	2.10	1.64	1.89	2.39
D-F test	-4.54	-3.72	-3.36	-3.94
Serial Correlation test	0.22	1.23	0.11	2.40
Omoskedasticity test	0.55	0.65	2.32	0.56
Functional form test	0.44	1.12	0.02	0.75
Chow's second test	0.33	0.50	1.00	1.84

Notes: *t*-statistics in parentheses. Tests are the same as in Table 3. Chow's second test statistic (F(3,15)) is based on the three last observations.

The coefficients of $Y(-1)$ --i.e. the error correction coefficients in the ECM interpretation of Solow's model-- are analogous to those obtained in non-linear specification; they do not significantly differ from zero. Once again, this fact indicates the absence of a significant error correction mechanism, able to move the current level of labour productivity towards its equilibrium path. The investigation reveals a weak tendency towards equilibrium (as the negative value of the coefficient of $Y(-1)$ expresses), but it is not statistically significant. This result confirms the absence of cointegration emerging from the first step of Engle-Granger's method.

The investment rate in physical and human capital and (generally) *NGD* have the right sign, but they are not very significant. Also the coefficient of regressor *TREND*, which is connected to exogenous technological progress, is not significant; this fact could indicate that exogenous progress is not a very important factor in the growth process, as endogenous growth models suggest (see Romer (1986), Lucas (1988)).

Phillips and Loretan (1991) point out that the estimators of long-run relation parameters, in small samples and with integrated variables, show better performance in non-linear specification than in linear one. In the present case, however, it is worth noting that the result of main interest is the same in both specifications and, in fact, a LM test on linear restriction --based on the difference between maximum levels of likelihood in linear and non linear specification-- gives $\chi^2(1)=0.06$ for France, $\chi^2(1)=2.98$ for Italy and $\chi^2(1)=0.2$ for Japan, leading to the conclusion that linear restriction can not be rejected in any considered case.

Finally, for the sake of curiosity, the steady state levels of labour productivity, $\ln y^*$, have been computed, basing on NLS estimates for France, Italy and Japan and basing on OLS estimates for USA.[12] Figures 1.a-d show the patterns of $\ln y^*$ (and $\ln y$) in the considered countries. The variability of $\ln y^*$ should not be surprising, at least from an "econometric" point of view:

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12 - For France, Italy and Japan, $\ln y^*$ has been computed as it follows: $\ln y_t^* = \left[a_4 + a_1 T + \frac{a_2}{1-a_2-a_3} \ln s_t^K + \frac{a_3}{1-a_2-a_3} \ln s_t^H - \frac{a_2+a_3}{1-a_2-a_3} \ln(n_t + a_1 + a_5) \right]$, using estimates appearing in Table 3. For USA, referring to the symbols of equation (4), and using the parameter values reported in Table 4, it is: $\ln y_t^* = \frac{1}{-\gamma} \cdot [a_0 + a_1 T + a_2 \ln s_t^H + a_3 \ln s_t^H + a_4 \ln(n_t + .05)]$.

it stems from the variability of its determinants. Note that current productivity always lies below its (fitted) long-run equilibrium level in USA and Japan while it lies above the equilibrium in the case of France; the two series cross each other in Italy.

INSERT FIGURES 1.a-d HERE (see last pages)

It has been already noted that $\ln y^*$ can be interpreted as a stochastic process, possibly with a unit root. In fact, ADF(1) statistics for $\ln y^*$ are: -1.66 in USA, -2.00 in France, -1.55 in Italy and -2.07 in Japan. (95% critical value: -3.59), confirming that the (fitted) equilibrium level of labour productivity is a stochastic non-stationary process, in any considered country. This evidence is very important for the analysis based on the interpretation of Solow's model as an error correction model. Both the current and the equilibrium levels of labour productivity are stochastic integrated processes. They should be cointegrated, according to stochastic Solow's model; but this is not the case: the unit root analysis reveals that the disequilibrium $[\ln y^*_t - \ln y_t]$ is integrated of order 1 in any considered country.

5. CONCLUDING REMARKS

We examined Solow's model taking into account that many of its variables are not constant, as the standard version of the model assumes, but they are time series with an important stochastic property: they are integrated of order one. This means that a different *interpretation* of Solow's model is required. Such an interpretation should be consistent with the stochastic nature of the variables. More importantly, this interpretation produces clear implications of Solow's growth model that can be tested.

We showed that the growth rate of employment and the series used as proxies for the investment rates in physical and in human capital, contain a unit root. These variables are the determinants of the steady state level of labour productivity, in Solow's model. As a consequence, also the steady state value of equilibrium productivity can be a stochastic process with a unit root.

The fact that the steady state to which an economy is approaching, is a stochastic --rather than a deterministic-- path, was pointed out by Danny Quah (1993). In this paper, we suggested that the stochastic path can be integrated of order 1. This observation is not in contradiction with the logic of Solow's model. On the contrary, we looked at the stochastic properties of the time series as features which can highlight the consistency of Solow's model with actual data.

Solow's model predicts that the movement of labour productivity depends on the previous difference between the current level of labour productivity and its steady state equilibrium level: if the current level of labour productivity is below (above) its steady state equilibrium level, it will increase (decrease) in next period. In other words, Solow's model predicts that labour productivity movements follow an error correction mechanism: Solow's growth model can be interpreted as an error correction model. Thus, the literature on cointegration and error-correction provide a natural way to test this model.

The stochastic nature of the variables is consistent with Solow's model, only if the labour productivity, the propensities to accumulate physical and human capital and the growth rate of employment --each of these time-series is integrated of order one-- cointegrate. More specifically, the difference between the actual and the equilibrium levels of productivity has to be stationary, so that it determines the subsequent growth rate of labour productivity, which appears, in fact, to be a stationary series. Moreover, the coefficient of error correction --i.e. the coefficient linking the productivity growth rate to the lagged disequilibrium between the steady state and the current level of productivity-- has to be negative.

First, we tested cointegrating relationships; secondly, we run the regressions of an error correction model, looking at the significance of correction coefficient, both in linear and in non linear specifications. The considered data are annual observations (sample 1960-88), coming from USA, France, Italy and Japan.

In most cases, there is evidence of no-cointegration. If we use Johansen's method, cointegrating relationships do not exist in Italy and Japan, while they exist (with rank equal to

1) in USA and France, but --in these cases-- the estimates of the coefficients of cointegrating vector do not support the augmented Solow's model predictions. If we use the Engle-Granger method, cointegration relationships never appear. Moreover, the estimates of the coefficients in the model with error correction mechanism provide a negative --but not statistically significant-- value for the parameter of error-correction: this result appears both in the linear specification of ECM and in non-linear case; it is consistent with the evidence of no-cointegration.

The absence of a significant error correction mechanism, able to move the current level of labour productivity towards its stochastic equilibrium path, seems to be a clear-cut result in our samples. Thus, the most important prediction of Solow's model is not supported by actual data.

Although we are aware that additional investigation is required and improvements in this methodology are possible and necessary, we can make two points: Solow's model can be tested by looking at productivity movements in a country, rather than by considering cross-section predictions of the model; according to this criterion, the augmented and stochastic version of Solow's model does not appear consistent with the pattern of the variables considered.

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FIGURE 1.a

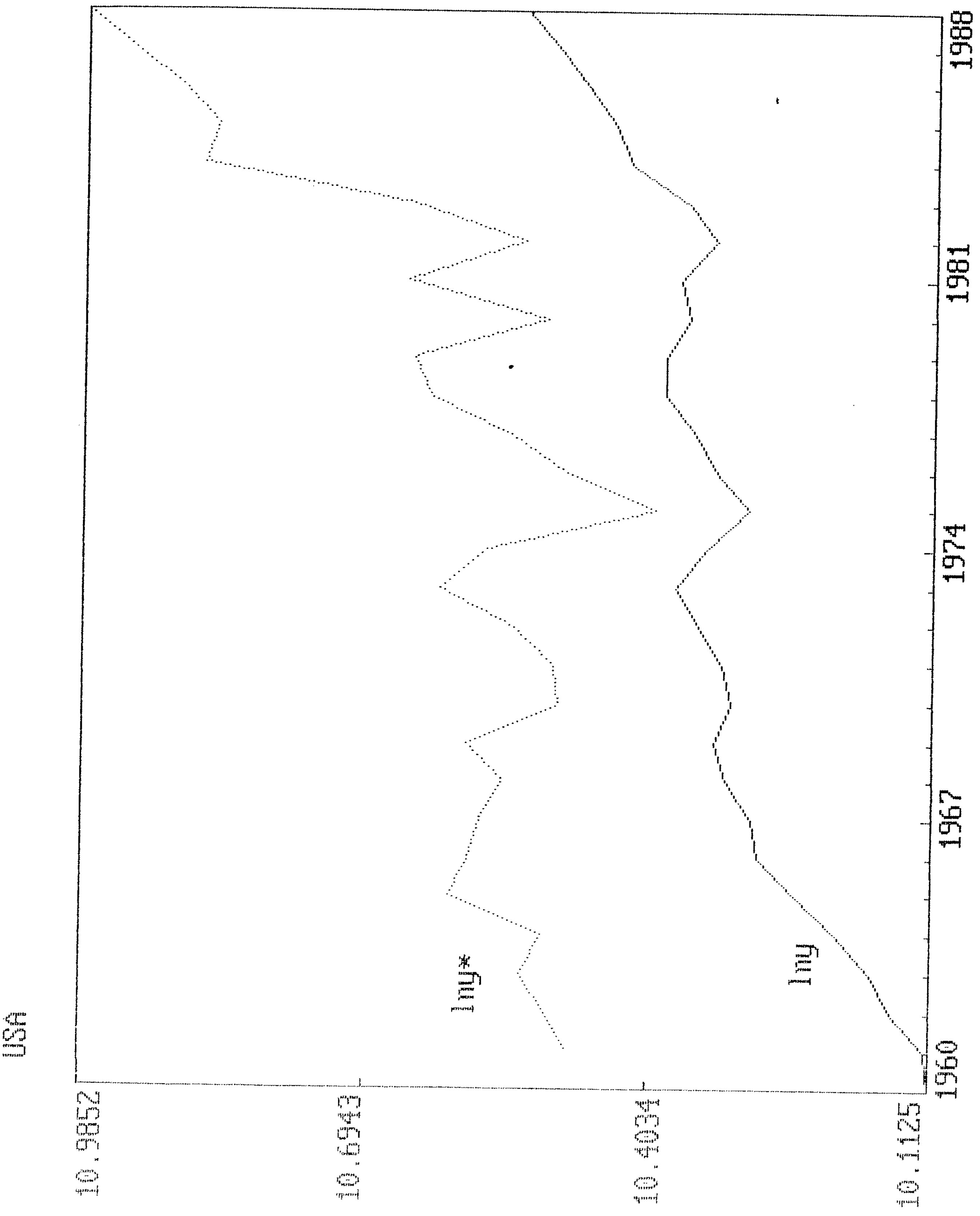
USA
lmy*
lmy
10.9852
10.6943
10.4034
10.1125
1960
1967
1974
1981
1988

FIGURE 1.b

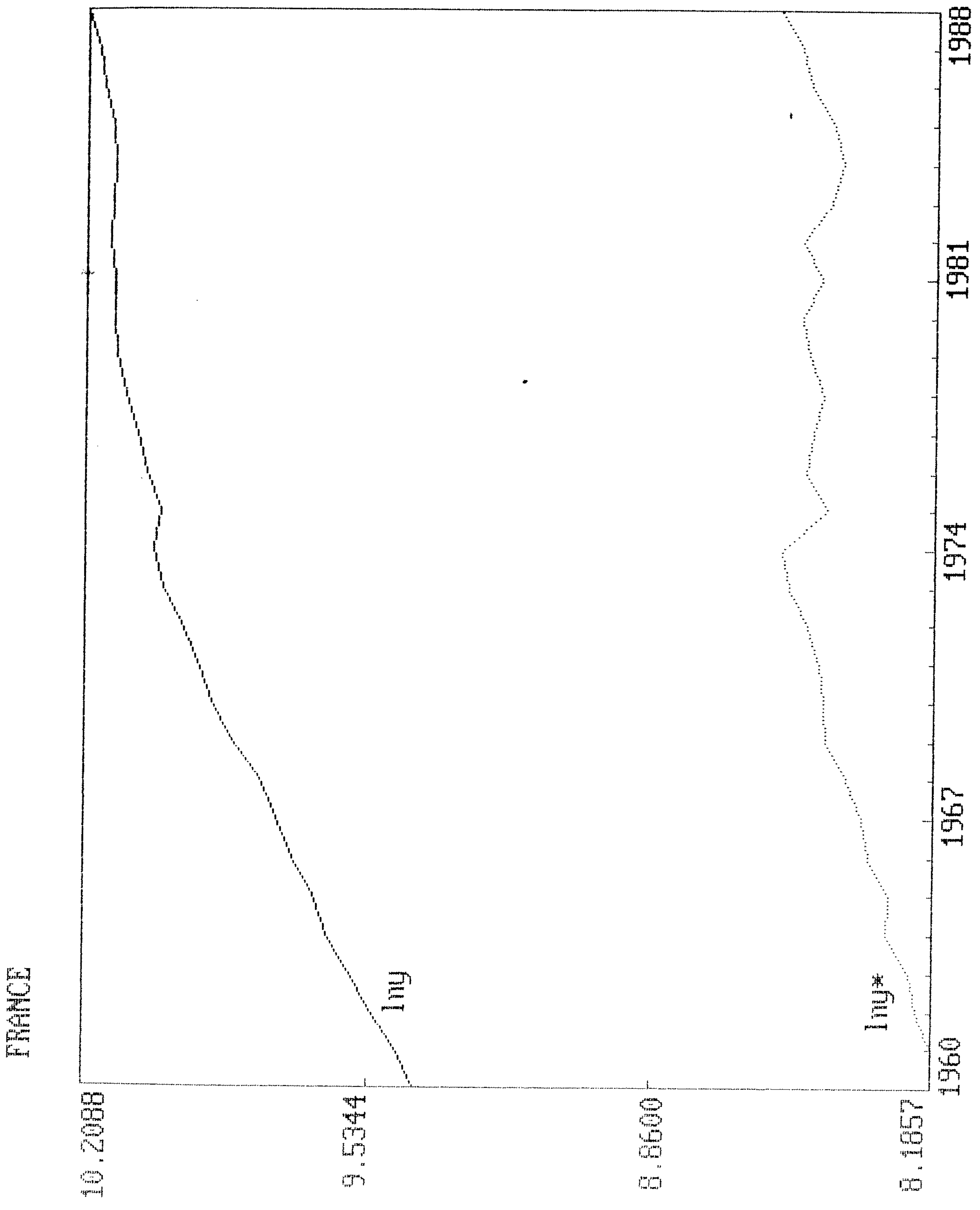
FRANCE
lny
lny*
10.2088
9.5344
8.8600
8.1857
1960
1967
1974
1981
1988

FIGURE 1.c


ITALY
lny*
lny
10.2819
9.9438
9.6058
9.2677
1960
1967
1974
1981
1988

FIGURE 1.d