On the Endogenous Choice of Bertrand Vs. Cournot Equilibrium in a Duopoly

Flavio Delbono * and Marco Mariotti **

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Abstract

We try to endogenise the choice between Bertrand Equilibrium (BE) and Cournot Equilibrium (CE) in simple duopoly models. The two distinctive features of this paper as compared to the related literature are the following. First, we take the concepts of BE and CE as fundamental and restrict players' choices to these two equilibria. Second, we adopt a forward induction criterion to shrink the Nash equilibrium set of our games. Our findings suggest that the BE seems more vulnerable than the CE whenever forward induction is taken seriously.

Keywords: oligopolistic equilibrium, duopoly, game theory, forward induction.

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* Department of Economics, University of Verona (Italy)

** St. Edmund's College and Department of Applied Economics, University of Cambridge (U.K.)

1. Introduction

"The theory is based on reaction functions expressing profit maximization for given values of the rival's variable. In the initial definition of the concepts, no limitation is introduced on what the variables are. In the detailed analysis of the problem, the technique is handled in such a way that quantity or price may be 'the' variable" (Fellner, 1949, p. 101. emphasis original).

The problem with oligopoly theory that Fellner pointed out more than fourty years ago [1] only recently seems to have attracted the attention that it deserves, as emerges, for instance, from the excellent survey of Shapiro (1989). Only a few recent contributions have dwelled on the question we tackle here, that is, the endogenous determination of the strategic variable chosen in an oligopolistic setting. The contributions that are closest in spirit to this paper are Kreps and Scheinkman (1983), Singh and Vives (1984), Klemperer and Meyer (1986, 1990), Friedman (1988) and Delbono (1989).

Kreps and Scheinkman (1983) model a two-stage game in which firms choose capacity first and then compete in prices. Under a specific hypothesis on the rationing scheme, they show that the Cournot outcome is the final equilibrium in this two-stage game.

Singh and Vives (1984) attempt to treat directly the choice of strategy as a variable. They also consider a two-stage model in which firms first commit themselves to a strategy variable (price or quantity) and then choose quantity or price according to the first stage choice. They show that a dominant strategy for each firm is to select a quantity strategy, leading ultimately to a Cournot equilibrium [2]. Although we are sympathetic with this approach, we adhere to Shapiro's (1989, fn. 34) criticism that "it is fairly unclear what it means for a firm to commit itself to a price or quantity strategy".

Klemperer and Meyer (1986) share with our appraoch the feature that all choices are collapsed in one single stage. However, they focus on the effects of uncertainty on the emergence of a specific equilibrium, an issue beyond our interest here.

Friedman's (1988) contribution is perhaps the most general, in that it examines a very broad class of demand, cost and spillover demand specifications. Moreover, he analyzes three possible strategic structures for the game, with firms either choosing price and quantity simultaneously, or choosing one of the two in a first stage and the other in a second stage. In Friedman's own words, "the verdict is mixed", in the sense that equilibrium may not exist, may be driven by price competition or may be driven by quantity competition depending on the exact specification of the model.

Delbono (1989) considers a duopoly market where firms are differentiated both in cost and move order. He shows that Bertrand or Cournot equilibrium prevails depending on the cost gap between the firms. The more efficient firm choses in the first stage the strategic variable in which firms will compete in the second stage, and its choice obviously depends on the comparison between profits it makes in either equilibrium.

Finally, in their sophisticated contribution, Klemperer and Meyer (1990) introduce the concept of supply function, which relates to each price the quantity produced by a firm. Whether a supply function equilibrium resembles more Cournot or Bertrand depends on the parameters of the model.

In this paper we address the problem well summarised by Fellner by trying to shed some light on the very meaning of "endogenous" determination of the type of game played by the two duopolists. We too shall confine the attention to a one-stage choice between Bertrand and Cournot equilibria and, for the sake of analytical tractability, shall tackle the issue by means of some simple examples. Our suggestion is to rely heavily on an equilibrium approach. This allows us to shrink dramatically the set of candidate strategies. Even so, we face a multiplicity of equilibria. Our way out is to examine how different equilibria survive a refinement based on forward induction.

2. The set-up

The central question we aim at analysing can be summarised as follows: What does it mean to choose between Cournot and Bertrand equilibria? Or, alternatively, can we endogenise such a choice, and if so, how?

As we mentioned above, we rely heavily on an equilibrium approach. Namely, we assume that players concentrate <u>only</u> on Bertrand or Cournot outcomes and play <u>either</u> (their part in) a Bertrand Equilibrium (BE, henceforth) <u>or</u> (their part in) a Cournot equilibrium (CE, henceforth). In this sense, we make a strong requirement on the rationality of a player, in that we assume that uncertainty concerns the equilibrium according to which his opponent wishes to play rather than his opponent's strategy.

To be more precise, we consider a homogeneous duopoly in which either equilibrium (BE or CE) exists and is unique. Firms decide simultaneously and non-cooperatively the output level and/or the price level. They concentrate on either of the following alternatives:

<u>Bertrand choice</u>: produce the BE output level [3] at the Bertrand price;
<u>Cournot choice</u>: produce the CE output level.

The payoffs when both firms choose BE or both choose CE are automatically determined. The 'disagreement' payoffs are more delicate, as we will see.

In what follows we assume that the BE total output is higher than the CE total output; this requirement seems rather innocuous, as it holds under fairly general conditions (see, for example, Shapiro (1989)). Therefore, when one firm chooses BE, it is possible for the other firm to produce the CE output level at the BE price without exceeding market demand (actually, some customers will be turned away). So, when, say, firm 1 quotes the BE price and sells the BE quantity, while firm 2 chooses CE, what happens is that: (i) firm 1 produces and sells exactly like in the BE; (ii) firm 2 takes the price as given and sells the planned (i.e., CE) output level. All this is possible by the above assumption.

3. The examples

In this section we analyse two examples. In the first one the CE Pareto dominates the BE and therefore firms face a typical coordination problem, whereas in the second example firms have opposite preferences on the equilibria. In both cases we consider a two-fold repetition of the stage game. We will denote the first time and the second time the game is played by t=1 and t=2, respectively.

3.1 (Example 1). In this case we consider identical firms. Let $\bar{p}$ be the BE price and $(\bar{q}_1,\bar{q}_2)$ the CE pair of output levels. Here is the payoff matrix:

	$\bar{p}$	$\bar{q}_2$
$\bar{p}$	0,0	0,-k
$\bar{q}_1$	-k,0	π_{1C},π_{2C}

(Figure 1)

where the BE payoffs have been normalised to zero and π_{iC} is the CE payoff of firm i (i=1,2). It is assumed that:

(a1) $\pi_{iC} > 0$, i =1,2;

(a2) $k > 0$;

(a3) $\pi_{iC} > k$, i =1,2.

(a1) means that CE profits are higher than BE profits; (a2) says that it is worse to play CE quantity than BE quantity at the BE price; (a3) means that the CE profit is greater than the loss in payoff deriving from playing CE quantity instead of BE quantity at the BE price. Assumptions (a1)-(a3) hold in standard cases as the following example confirms.

demand curve: $p = a - (q_1+q_2)$

cost function: $c_i(q_i) = q_i^2/2$

Straightforward calculations show that the matrix of Figure 1 now becomes:

	$\bar{p}$	$\bar{q}_2$
$\bar{p}$	$a^2/18$, $a^2/18$	$a^2/18$, $5a^2/96$
$\bar{q}_1$	$5a^2/96$, $a^2/18$	$3a^2/32$, $3a^2/32$

(Figure 2)

Moreover: $\bar{q}_1 = \bar{q}_2 = a/4$, $p(\bar{q}_1+\bar{q}_2) = a/2$, $\bar{p} = a/3$ is the BE price if $\bar{p} = \delta c_i(q_i*)/\delta q_i*$, from which it follows that the BE output level is $q_i* = a/3$. Therefore it is verified that:

$\bar{p} < p(\bar{q}_1 + \bar{q}_2)$,

$\bar{q}_1 < q_1^*$,

$\pi_{1C}(\bar{q}_1, \bar{p}) < \pi_{1C}(\bar{p}, \bar{p})$,

$\pi_{1C} > \pi_{1C}(\bar{p}, \bar{p})$.

Back to the general case, we notice that there are two Nash equilibria in the game, with the CE dominating the BE. Now suppose that this stage-game is played twice in succession. We want to check whether the Cournot-path and the Bertrand-path survive refinements, where by "-path" we mean the specified equilibrium played in both stages. Clearly, they are both Nash and subgame perfect (since they are composed of strategies which are Nash equilibria of the stage-game at each period).

The refinement concept we adopt is based on forward induction. This means the following. Whenever a player observes a deviation from a proposed equilibrium path at $t=1$ and there is a unique continuation yielding the deviant player a payoff higher than the equilibrium one, he expects the deviant player to play such a continuation (see Kohlberg and Mertens (1986) and van Damme (1989)). Notice that in the example we are considering imposing forward induction amounts to imposing stability in the sense of Kohlberg and Mertens (1986).

Hence, for the game in Figure 1, we have the following:

CLAIM 1: the BE path violates the forward induction criterion.

A heuristic proof of the claim is the following. Suppose that the BE path is the proposed equilibrium. Suppose that at $t=1$ firm 1 deviates and plays CE. What sense must firm 2 make of this deviation? It must think: "Let me see, 1 could have got a payoff equal to zero for sure by sticking to the BE path. After the deviation there is only one thing it can do in order to obtain more than a null profit overall after losing k at t=1: this thing is to play CE at t=2. And my best response to CE is CE, so I should play CE at t=2". Given this response by firm 2, firm 1 actually gets more by deviating to CE than by sticking to BE (this is because of (a3)). Here is the tree of the game:

[Insert Figure 3 about here]

By looking at the tree of the game, the forward induction argument is fairly clear. In the path starting with (BE,BE) there is only one sub-path leading to an outcome yielding for player 2 a higher payoff than the subgame perfect equilibrium (SPE) path {(BE,BE),(BE,BE)}. Then, this confirms our claim which can be restated as follows: in no stable path is BE played twice. On the contrary, always playing CE is a stable path, for clearly there is no available gain in deviating from such a path which yields the maximum overall payoff $2\pi_{iC}$.

3.2 (Example 2). When firms are not identical, it can happen that one prefers the CE, while the rival prefers the BE. For instance, this is the case, under constant returns to scale and linear demand, if the cost gap is wide enough, but not so high to create monopoly (see Delbono (1989)). Let the payoff matrix be:

	$\bar{p}$	$\bar{q}_2$
$\bar{p}$	$\pi_B, 0$	$\pi_B - k, 0$
$\bar{q}_1$	$0, \pi_B - h$	π_{1C}, π_{2C}

(Figure 4)

where π_B is the BE profit and $k,h > 0$. We assume that:

(a4) $\pi_{1C} < \pi_B$.

Since firm 2 certainly prefers CE to BE, (a4) guarantees that firms have opposite preferences on the equilibria.

Once again, BE and CE are both Nash equilibria. In this case, though, one of the two equilibria implies playing a weakly dominated strategy and therefore it would not be trambling-hand perfect. However, in our view the temptation of eliminating the BE through such a refinement should be resisted. The reason has to do with the level of the BE price under constant but different marginal costs. There are two different interpretations of such a price. According to the first one, the BE price exactly equals the higher cost; as a consequence, the higher cost firm would be indifferent between any output level, whereas the rival would prefer the inefficient firm to drop out. Under the alternative interpretation, the BE price is set 'just' below (say, μ below, $\mu > 0$) the higher cost and this prevents the inefficient firm to be active in the BE. Our opinion is that the latter interpretation, even though it raises well-known technical difficulties, is to be preferred at the conceptual level.

In order to select between the two equilibria we will then use the same device as before, i.e., we consider a two-fold repetition of the constituent game and use the forward induction criterion. Proceeding thus for the game of Figure 4 we again get the following:

CLAIM 2: the BE path violates the forward induction criterion.

The heuristic proof follows the same lines as for Claim 1, and uses the fact that $\pi_{2C} > 0$. [4]

4. Concluding remarks

In this paper we have tried to endogenise the choice between Bertrand equilibrium and Cournot equilibrium in simple duopoly models. The two distinctive features of this paper as compared to the related literature are the following. First, we have taken the concepts of BE and CE as fundamental and restricted players' choices to these two equilibria. Second, we have adopted a forward induction criterion to shrink the Nash equilibrium set of our simple games [5]. Our findings suggest that the BE seems more vulnerable than the CE whenever forward induction is taken seriously.

Footnotes

(1) As is well known, the earliest reference to this issue is probably Bertrand's 1883 criticism to Cournot.

(2) See Cheng (1985) for a nice geometric treatment of Singh and Vives' findings.

(3) We are aware that at the BE market price the individual output level is not uniquely determined; we bypass this difficulty by attaching to "BE output level" the meaning of "one half of the quantity demanded at the BE price".

(4) This is the case under the first interpretation of the BE price; to be consistent with our own interpretation we should replace $\pi_{2C} < 0$ with $\mu\bar{q}_{2C} < \pi_{2C}$.

(5) Notice that in our second example both the forward induction and the trembling-hand criteria reduce the number of Nash equilibria; this is not the case in some related papers (e.g., Klemperer and Meyer (1986)).

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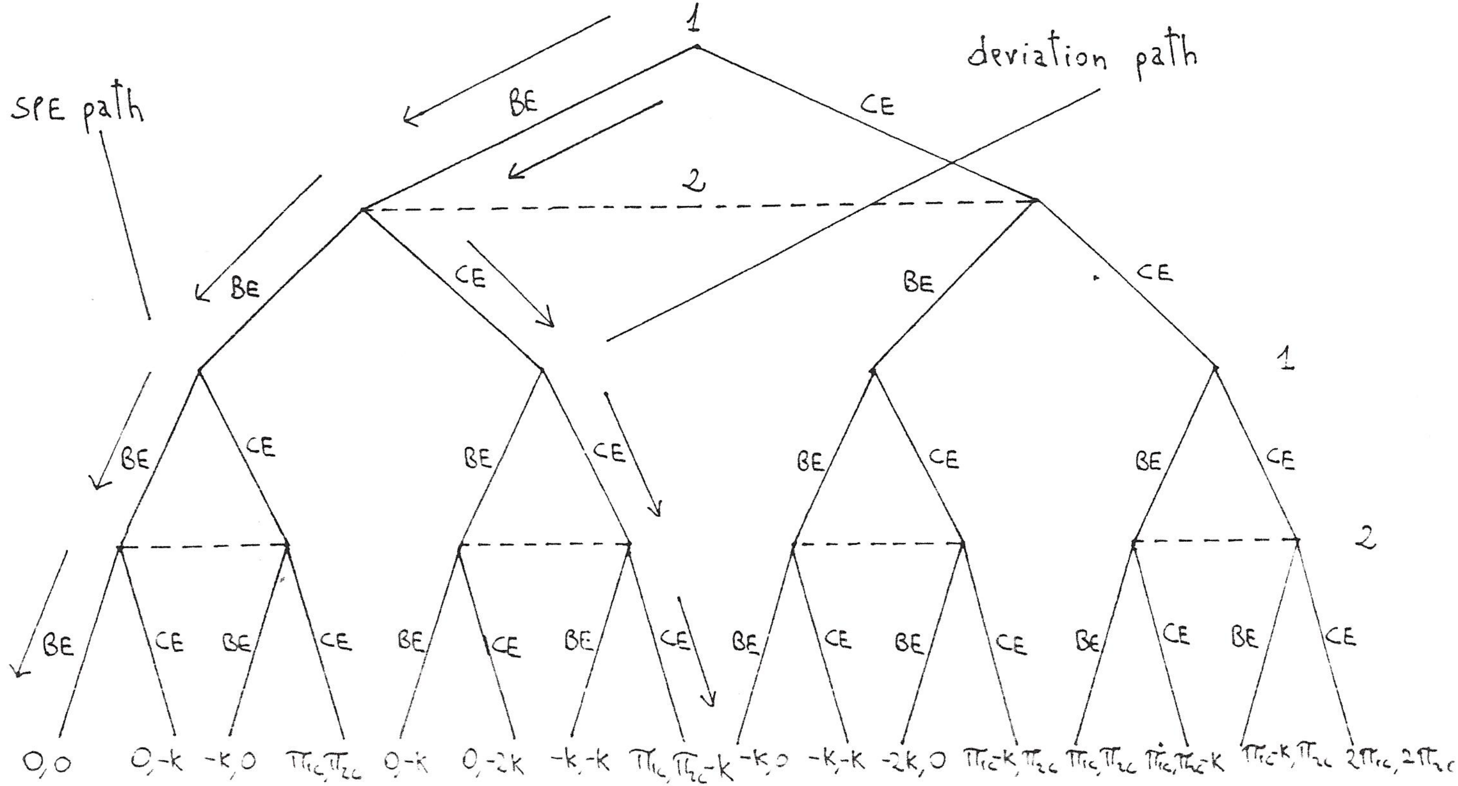
Fig. 3



SPE path
deviation path
1
2
1
2
BE
CE
0,0
0,-k
-k,0
Π_{1c},Π_{2c}
0,-k
0,-2k
-k,-k
$\Pi_{1c},\Pi_{2c}-k$
-k,0
-k,-k
-2k,0
$\Pi_{1c}-k,\Pi_{2c}$
Π_{1c},Π_{2c}
$\Pi_{1c},\Pi_{2c}-k$
$\Pi_{1c}-k,\Pi_{2c}$
$2\Pi_{1c},2\Pi_{2c}$