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Abstract

Although death occurs with certainty, the time of death is uncertain. In this paper we build on this conceptualization and show that, although life ends at some point in time, human life can be meaningfully conceptualized as a strive for immortality that is never reached. We consider an intertemporal problem where health investments and consumption choices are made, taking into account that mortality depends on environmental factors, which are not controlled by the agent, and the agent's health condition, which is endogenous to lifestyle and health behavior. Formally, the infinite horizon approach has the advantage that adjustment dynamics to the steady state (i.e. human aging) can be discussed analytically. We explore the determinants of health deficits in this framework and show how individuals choose consumption and health expenditure over their lifetime in order to slow down (biological) aging. We compute analytically the impulse response functions for unexpected parameter changes. Specifically, we investigate how higher prices for medical goods and advancing medical technology affect individual behavior and health deficit accumulation.

Keywords: Endogenous mortality, Life-expectancy, Deficit accumulation, Medical progress

JEL codes: D91, I12, J17

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The Universal Declaration of Human Rights does not say
humans have 'the right to life until the age of ninety'.
It says that every human has a right to life, period.
That right isn't limited by any expiry date.
(Yuval Noah Harari, 2016)

1 Introduction

Human life is finite but the time of death is unknown. In this paper we build on this fact to construct a theoretical model of aging where mortality is stochastic and endogenously affected by individual behavior and health. We show that such scenario can be formalized as an infinite time horizon problem in which human life is conceptualized as a process where the state of constant health, a steady state of "negligible senescence" (Finch, 2009), is a meaningful long-run goal.

Allowing for the possibility of such a steady state is important because historically there was consensus in biology, medicine, and gerontology that human aging, i.e. the progressive loss of bodily function with increasing age, is inescapable (Arking, 2008). Recently, however, scholars observed that the limits to life expectancy are broken (Oeppen and Vaupel, 2002) and that human life span is not immutable but in fact increasing over time (Wilmoth and Robine, 2003; Strulik and Vollmer, 2013). While few scholars agree with de Grey (2013) and Kurzweil and Grossman (2010) who envision human immortality for the near future, many have abandoned the belief that there exists necessarily a "capital T" beyond which human life extension is impossible (e.g. Vaupel, 2010; Kontis et al., 2017).

Our model does neither imply immortality nor the "end of aging". In fact, people accumulate health deficits as in the conventional model (Dalgaard and Strulik, 2014) and their life expectancy is finite and, given a reasonable calibration of the survival function, in line with current observations. The innovation is that human aging does no longer *inevitably* end in death at some finite age. Instead, motivated by the advancements in medical technology, individuals rationally believe that, in principle, aging-related health deficits can be repaired such that the state of negligible senescence becomes a desirable goal, although it is – in practice – never reached. The new view differs from the "perpetual youth" model of conventional macroeconomics (Yaari, 1965) where people do not age and death occurs because of age-unrelated background mortality.

Our approach differs also from the modeling of aging in conventional health economics where people either inevitably die at a finite T or inevitably live forever. In the standard model of health capital accumulation (Grossman, 1972) there exists always a steady state of constant health such that individuals inevitably live forever (Strulik, 2015) . The reason is that for a given rate of health capital depreciation δ, individuals in bad health (with low health capital H) lose relatively little health, i.e. their health depreciation δH is low. This creates an equilibrating force and convergence to a steady state of constant health. To circumvent this problem, the health capital literature usually imposes a finite time horizon T and thus enforces a finite life. In the health deficit model (Dalgaard and Strulik, 2014, 2015), a steady state of constant health exists as well but only for a favorable constellation of parameters. So far, the health deficit literature focused on situations where the steady state does not exist and thus, by design, assumed a finite life.

In the health deficit model, health deficits accumulate approximately at a constant rate if they remain unremedied by health maintenance and repair. This explosive growth of deficits (at the force of aging μ) captures the gerontological notion of aging as the cumulative, progressive, and deleterious loss of bodily function (Arking, 2006). As shown below, conditions for a reachable steady state of constant health are (i) that the force of aging is smaller than the interest rate and (ii) that medical technology is sufficiently powerful in the sense that the returns of health investments are not too strongly decreasing. These conditions are intuitive. The first condition requires that individuals are able to accumulate savings for future health expenditure faster than the pace at which their bodies deteriorate. The second condition requires that health expenditure is effective in reducing health deficits even when individuals spend already significant amounts on their health.

While it can be debated whether these conditions are fulfilled already, it is conceivable that they will be fulfilled at some point in the future. Medical research on aging advanced greatly over the last 20 years. The biological mechanisms of health deficit accumulation are now well understood and for most of the gateways of bodily decay solutions have been suggested and explored in animal studies (Lopez-Otin et al., 2103). The observation that natural scientists started to envision the postponement of aging by health interventions, has motivated us to explore the economic theory of health deficit accumulation in this direction.

A major advantage of the infinite horizon perspective and the existence of a steady state is that it allows for a convenient analytical computation of the comparative dynamics of human aging and longevity. In this respect, we contribute to the literature studying the comparative dynamics properties of intertemporal problems, by proposing a method to analytically assess the impulse response to exogenous shocks at the time when the shock is realized. Our approach can be considered a complement to the comparative dynamics analysis proposed in Caputo (1990,1997) and in Dragone and Vanin (2015), which focus on the long-run response of the steady state, and it can in principle be applied to any intertemporal behavior for which aiming at a stationary state is a meaningful goal.

By showing analytical, sufficient conditions that allow to determine the effect of shocks on behavior, we provide an additional tool to the literature on dynamic optimization models, which typically assesses the impact of policies or shocks either through phase diagram analysis or through numerical simulations. In particular, our results could be useful when the problem involves more than one state variable (in which case phase diagram could be applied only under specific assumptions), or when numerical simulations are too computationally demanding.

As an illustration, we explore the determinants of health and aging in terms of individual characteristics, health-related shocks, which are endogenous to a person's lifestyle and health behavior, and exogenous shocks, which are not under direct control of the agent. The model can be used to derive, with standard methods, analytic conditions to predict the effect of shocks both on impact and in the long run. Our analytical results are obtained for a general survival function and a generic utility function.

The paper is organized as follows. In the next section we set up the infinite horizon model. In Section 3 we explain in general how comparative dynamics are derived analytically. In Section 4 we consider two examples for computing impulse responses to shocks, a rise in the cost of health and an improvement in medical technology. Section 5 concludes.

2 A Life Cycle Model of Health Deficit Accumulation over an Infinite Horizon

Consider an agent whose health condition is represented by the number of health deficits accumulated over her lifetime. Deficit accumulation represents a measure of *biological aging* and it is generally positively correlated with *chronological aging*, although the two notions do not express the same concept (Dalgaard and Strulik, 2014). The accumulation of health deficits depends on the stock of health deficits D and on health investment h at time t,

$$\dot{D} = f\left(D\left(t\right), h\left(t\right)\right). \tag{1}$$

The accumulation of health deficits is faster when health deficits are large, $f_D(D,h) > 0$, and it is slower when the agent engages in health-related behavior, $f_h(D,h) < 0$. As discussed in detail by Dalgaard and Strulik (2014), and based on research in gerontology (Mitnitski et al. 2002, Gavrilov and Gavrilova, 1991), the accumulation of health deficits is well represented by the following quasi-exponential function:

$$f(D(t), h(t)) = \mu (D(t) - a - Ah(t)^{\gamma}), \tag{2}$$

with $\gamma \in (0,1)$ and $D(0) > a$. The parameters A and γ reflect the state of medical technology. The parameter A captures the general efficiency of health expenditure in the repair of health deficits. The parameter γ captures the degree of decreasing returns of health expenditure.

The agent faces the following dynamic budget constraint,

$$\dot{k}(t) = rk(t) + Y - c(t) - ph(t), \tag{3}$$

where k is capital, r is the interest rate, Y is income and c is a composite good whose price is normalized to one. The price of health-investment (or health-related behavior) is p, and it includes the cost of medicines, as well as the opportunity cost of health investment.

The agent's problem is to choose consumption and health investment over her lifetime with given discounting rate ρ due to individual impatience. In a deterministic environment, this amounts to consider the following intertemporal utility function:

$$V = \int_0^T e^{-\rho t} U(c(t))\, dt, \tag{4}$$

which depends on instantaneous utility $U(c)$ and on the age at death T.[1] The age at death could be determined ex-ante, as usually in macroeconomic life cycle models of generational accounting (e.g. Erosa and Gervais, 2002), or it could be endogenously determined by individual choices as in most life cycle models in health economics (e.g. Grossman, 1972; Ehrlich and Chuma, 1990; Kuhn et al., 2015). Here we consider a third alternative, where the age at death T is potentially infinite because the exact moment of death is unknown. To account for this uncertainty, let

$$\Omega(D(t), t) = \Pr(T > t) = \int_t^{\infty} g(D(\tau), \tau)\, d\tau \tag{5}$$

[1] As usual, $U(c) > 0$, $U_c(c) > 0$ and $U_{cc}(c) < 0$. Our results qualitatively hold also if the health condition has a utility and a productivity value (Grossman, 1972). Here we neglect these channels and focus on the role of health deficits in affecting the probability of dying.

be the survival function representing the probability that an individual will be alive at t (with $g(D(t), t)$ being the associated density function). The survival function $\Omega(D(t), t) : [0, +\infty) \times [0, +\infty) \rightarrow [0, 1]$ depends on both biological age and chronological age; it is equal to one when $t = 0$ and in absence of health deficits ($D(t) = 0$), and it is strictly decreasing to zero when time and health deficits increase. This formulation is particularly appealing, as it explicitly endogenizes the probability of dying as a function of individual health-related choices. Accordingly, the moment of death cannot be chosen, but only influenced by investing in health. In particular, note that the survival function is defined over an infinite time horizon. This means that, in principle, infinite life is allowed for, although it is likely that such an event will occur with negligible probability. Such an assumption, however, is not restrictive: in the survival literature, which typically focuses on functions such as the exponential, the Weibull, and the Gompertz-Makeham distributions, it it is standard to assume that surviving at very old ages is possible, but unlikely. In addition to these mathematical and rather obvious arguments, the assumption could also be considered from a more philosophical viewpoint: the fact that we have never observed a human being living forever does not mean, *per se*, that human beings cannot reach immortality. In principle, it may be possible that we have not yet observed any human being living forever just because it is an unlikely event. From such a perspective, infinite life, although reached with negligible (but positive) probability, is a meaningful long-run goal.

Under the hypothesis of uncertain time of death, the agent chooses the path of consumption and health investment that solves the following problem

$$\max_{c,h} \mathbb{E}_g \left[\int_0^T e^{-\rho t} U(c(t))\, dt \right] \tag{6}$$

$$\dot{k}(t) = rk(t) + Y - c(t) - ph(t) \tag{7}$$

$$D(0) = D_0 \qquad k(0) = k_0 \tag{8}$$

The above problem differs from the literature considering a deterministic time of death in that the objective function is an expected intertemporal utility function, where the stochastic element is represented by the agent's uncertain time of death. However, as suggested by Yaari, (1965), such a formulation can be conveniently transformed into a more treatable function which weighs the instantaneous utility function by the individual survival probability. Exploiting the definition of expected value and manipulating the corresponding double integral, from (6) one can show (see the Appendix for details)

$$\mathbb{E}_g \left[\int_0^T e^{-\rho t} U(c(t))\, \mathrm{d}t \right] = \int_0^\infty e^{-\rho t} \Omega(D(t), t)\, U(c(t))\, \mathrm{d}t. \tag{9}$$

Expression (9) shows that we can transform an expected intertemporal utility function into a very manageable intertemporal expected utility function which weighs the instantaneous utility function $U(c)$ by the rate of time preference (as represented by $e^{-\rho t}$) and by the probability of death $(\Omega(D(t), t))$. Assume that this probability is multiplicatively separable in biological and chronological aging and consider the following specification

$$\Omega(D(t), t) = S(D(t)) e^{-qt}. \tag{10}$$

The first term $(S(D(t)))$ represents the endogenous component of the survival function, as it depends on individual behavior through the accumulation of health deficits. It is reasonable to assume that it is a decreasing function of health deficits $(S_D(D(t)) < 0)$. The second term (e^{-qt}) represent the exogenous component of the survival function, and it represents the reduction in the survival probability due to the mere passing of time, as it can be appreciated by the fact that, for given health deficits D, $q = -(\partial\Omega(D(t), t)/\partial t)/\Omega(D(t), t)$. The term q sums up the role of environmental factors in affecting chronological aging, and it is assumed to be constant and out of the control of the agent.

3 Solving the Model

In the previous section we have shown that the realistic scenario of a finite, but uncertain life time can be treated as an infinite-time horizon problem where the instantaneous utility function is weighted by time discounting ρ, an exogenous hazard rate q, and the endogenous (health-related) survival probability $S(D(t))$. Formally, this means that we can transform an expected intertemporal utility function into the following intertemporal expected utility function:

$$\mathbb{E}_g\left[\int_0^T e^{-\rho t} U(c(t))\,\mathrm{d}t\right] = \int_0^\infty e^{-(\rho+q)t} S(D(t))\, U(c(t))\,\mathrm{d}t. \tag{11}$$

Based on the formulation presented in (11), we solve the agent's intertemporal problem by constructing the associated current-value Hamiltonian function:

$$H = S(D(t))\, U(c(t)) + \lambda(t)\, \mu\left(D(t) - a - Ah(t)^\gamma\right) + \eta\left(rk(t) + Y - ph(t) - c(t)\right) \tag{12}$$

where $\lambda(t)$ and $\eta(t)$ are the costate variables associated with the dynamics of health deficits and capital, respectively. The corresponding necessary conditions for an internal solution read as

$$H_h = 0 \quad \Leftrightarrow \quad \gamma\mu A\lambda(t)\, h(t)^{\gamma-1} = -p\eta(t) \tag{13}$$

$$H_c = 0 \quad \Leftrightarrow \quad S(D(t))\, U_c(c(t)) = \eta(t). \tag{14}$$

From the first order conditions (13) and (14) we obtain the optimal value of health investment h and consumption c as functions of the state variables, the costate variables and the survival probability. Note that both optimal health investment and consumption do not directly depend on capital, but they depend on its evolution through the shadow price $\eta(t) \geq 0$.[2] The necessary conditions for the costate dynamics are

$$\dot{\lambda}(t) = \lambda(t)(q+\rho-\mu) - U(c(t))\, S_D(D(t)) \tag{15}$$

$$\dot{\eta}(t) = \eta(t)(q+\rho-r). \tag{16}$$

To abstract from the dynamics originated by changes in individual wealth and to focus on the role of health deficits accumulation, let $r = q+\rho$. In such a case, the shadow price of capital η does not vary over time. This does not imply that health investment or consumption will be constant over time (as, e.g., in Heckman, 1974), because they depend on health deficits (through the survival probability $S(D)$) and on the shadow price of health deficits λ (see equations 13 and 14). In the proceeding we assume that $\eta(t) = \eta_0$ for all t. This assumption is common in the literature on lifecyle models of intertemporal behavior (see, e.g. Grossman, 1972, Heckman, 1974, 1976, Becker and Murphy, 1988), and it implies focusing on Frisch demand functions where the marginal utility of wealth is constant. Manipulating the above conditions yields (omitting the time notation for brevity):

$$\dot{h} = \frac{h}{1-\gamma}\left(r-\mu+\frac{\gamma\mu A h^{\gamma-1}}{p}\frac{S_D(D)}{S(D)}\frac{U(c)}{U_c(c)}\right) \tag{17}$$

$$\dot{c} = -\frac{S_D(D)}{S(D)}\frac{U_c(c)}{U_{cc}(c)}\dot{D} \tag{18}$$

$$\dot{D} = \mu(D-a-Ah^{\gamma}) \tag{19}$$

$$\dot{k} = rk+Y-ph-c \tag{20}$$

Note that, consistent with the previous observations, the dynamics of optimal consumption follows the dynamics of deficit accumulation, so that consumption decreases as health deficits accumulate. In particular, when health deficits reach a steady state, also consumption stops evolving.

The steady state(s) where consumption, investment, health deficits, and capital are

[2] Note that $\eta(t) \geq 0$ and $\lambda(t) \leq 0$. We require the Arrow sufficient condition for optimality to hold.

constant satisfy the following conditions:

$$h^{ss} = \left[\frac{\mu - r}{\mu}\frac{p}{\gamma A}\frac{S(D^{ss})}{S_D(D^{ss})}\frac{U_c(c^{ss})}{U(c^{ss})}\right]^{\frac{1}{\gamma-1}} \quad (21)$$

$$c^{ss} = U_c^{-1}\left(\frac{\eta_0}{S(D^{ss})}\right) \quad (22)$$

$$D^{ss} = a + A(h^{ss})^{\gamma} \quad (23)$$

$$k^{ss} = \frac{1}{r}(ph^{ss} + c^{ss} - Y). \quad (24)$$

The above conditions allow for multiple steady states. We are particularly interested in positive (interior) steady states. Positive steady states, $h^{ss} > 0$ and $D^{ss} > 0$, exist for $\mu < r$, i.e. when the rate of aging is smaller than the interest rate. As discussed in the Introduction, the condition is intuitive. It requires that wealth can accumulate at a faster rate than the rate at which the body deteriorates (without health investment), a feature that makes it possible to save for the future health expenditures needed to keep health deficits constant. We will focus on saddlepoint-stable steady states, which requires

$$h^{ss} > \left[\frac{1-\gamma}{\gamma A}\frac{S_D(D^{ss})U(c^{ss})}{\Phi(D^{ss})}\right]^{\frac{1}{\gamma}} \quad (25)$$

where $\Phi(D) \equiv U(c(D))S_{DD}(D) - [S_D^2(D)U_c^2(c(D))]/[S(D)U_{cc}(c(D))] < 0$ by Arrow's sufficient condition for optimality (see the Appendix for details). Condition (25) is complex and hard to interpret in its general form. In principle, it requires that medical technology is strong enough. Figure 1 shows the condition in the μ–γ space. Saddlepoint-stability and thus convergence toward the steady state requires that the marginal return of health expenditure in the repair of health deficits (γ) is sufficiently large. Otherwise, the steady state is unstable and an infinite life is not a meaningful goal. The lower the force of aging (μ) the lower the required marginal return of health investment. This intuitive result highlights two paths to life extension, which are both discussed in medical science and gerontology: a slowdown in the force of "natural" aging (μ) achieved through, for example gene therapy, caloric restriction etc., or a sufficiently fast repair of health damages (sufficiently high γ) achieved through elimination of damaged cells, telomerase reactivation etc. See Lopez-Otin et al. (2013) for a detailed discussion.

When condition (25) holds, one eigenvalue associated with the 3×3 Jacobian matrix J constructed on $\dot{h}$, $\dot{k}$ and $\dot{D}$, and denoted with ζ, is negative.[3] Let (ξ_1, ξ_2, ξ_3) be the

[3]Given that there are two state variables but only one negative eigenvalue, the saddle point stability is only "conditional" in the sense that the optimal manifold (and hence the set of initial conditions leading to the steady state) is one dimensional, and not bidimensional.

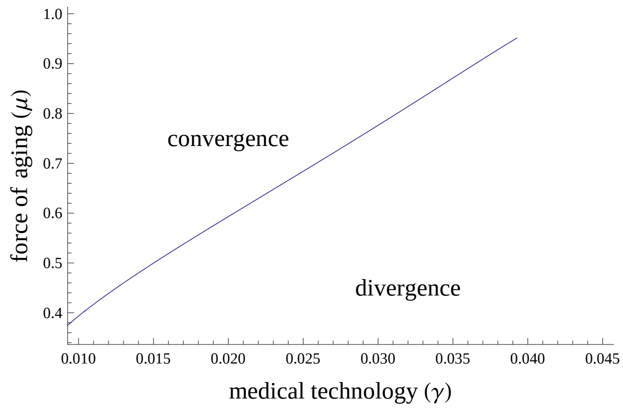

Figure 1: Convergence-divergence threshold. The figure shows condition (25) in the μ-γ space. Above the threshold: saddlepoint stable steady state. Below the threshold: unstable steady state. Parameters as for Figure 2.

associated eigenvector and define

$$\xi \equiv \frac{\xi_1}{\xi_3} = \frac{\mu - \zeta}{\gamma \mu A} \left(h^{ss}\right)^{1-\gamma} > 0. \tag{26}$$

In the neighborhood of the steady state, the term ξ represents the slope of the optimal path of health investment leading to the steady state in the (D, h) space.

Figure 2 shows this path as a function of deficit accumulation using a CES utility function

$$U(c) = \frac{c(t)^{1-\sigma}}{1-\sigma} + b, \tag{27}$$

where σ is the constant elasticity of marginal utility and b a base level utility (see, e.g. Hall, Jones, 2007), and a logistic survival function

$$S(D) = \frac{1+\alpha}{1+\alpha e^{\phi D}}. \tag{28}$$

Under the parametrization used for this figure,[4] two internal steady states exist: an unstable steady state (D^1, h^1) featuring a low level of deficits and health investment, and a saddlepoint-stable steady state (D^2, h^2) associated with a high level of health deficits and health investment. When the agent's health condition is good enough, i.e. $D_0 \in (D^1, D^2)$, she spends most of her income on consumption. As health deficits progressively accumulate, it becomes optimal to spend more income on health investment and to reduce consumption until reaching the steady state level of health deficits $D^{ss} = D^2$. The assumption of a

[4]Parameter values: $p = 1$, $Y = 10$, $A = 0.5$, $a = 0.0001$, $\mu = 0.03$, $q = 0.02$, $\rho = 0.04$, $\sigma = .95$, $\gamma = 0.96$, $\alpha = 0.01$, $\phi = 10$, $b = 0$.

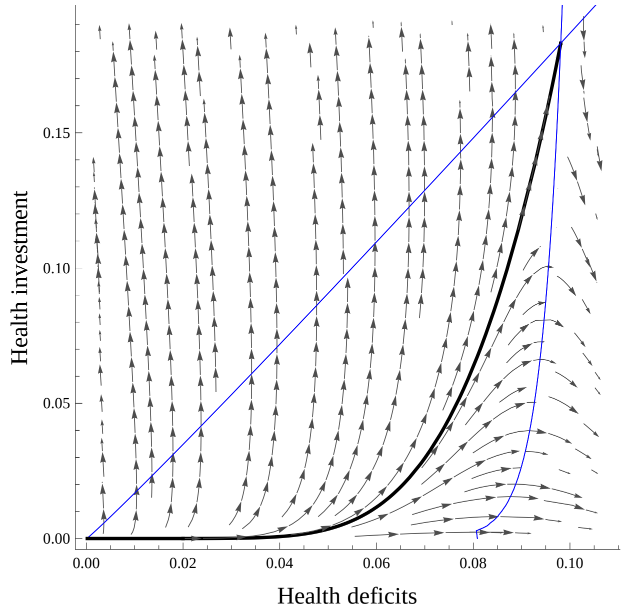

Figure 2: Optimal path starting from the unstable steady state D^1 and leading to the stable steady state D^2 (thick line). The blue lines are the nullclines corresponding to $\dot{h} = \dot{D} = 0$. The black arrows represent the vector field.

reachable steady state does thus not change the prediction from the standard health deficit model that health expenditure increases with age, a prediction in line with observable life time pattern of health expenditure (e.g. Dalgaard and Strulik, 2014; Schuenemann et al., 2017). Define the (expected) value of life as the cumulated utility an agent obtains from time t onwards,

$$V(t) = \int_t^\infty e^{-(\rho+q)\tau} S(D(\tau)) U(c(\tau)) \, \mathrm{d}\tau. \tag{29}$$

As shown in Figure 3, the value of life is larger for younger people (small t) and it decreases at older ages. As health deficits approach D^2, the value of life tends to zero.[5]

The following discussion focuses on the saddle-point stable steady state (D^2, h^2).

[5]In the proceeding figures age 0 should be conceptualized as real age 20 since, by assumption, individuals are "born" as young adults.

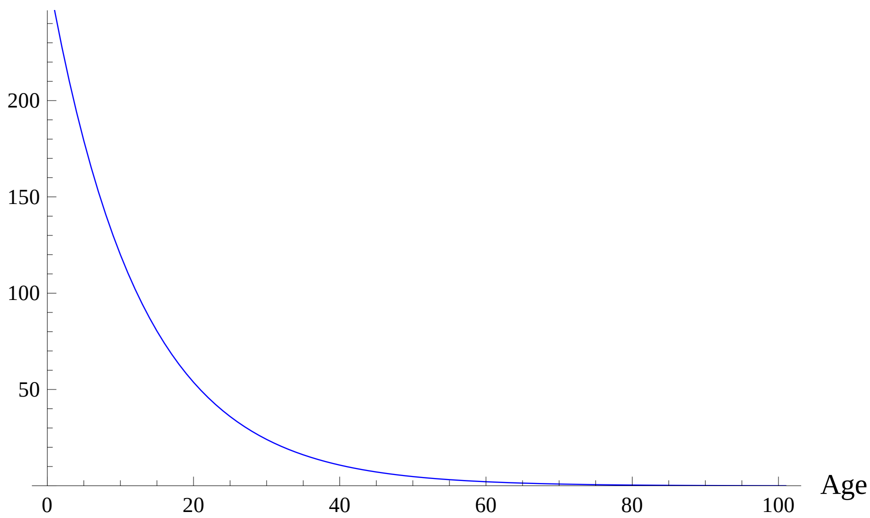

Figure 3: Value of life.

4 Studying the Determinants of Longevity

4.1 Method

One advantage of our model is that it allows to study the determinants of longevity by exploiting how the trajectory leading to the steady state (i.e. the policy function associated with optimal health behavior) is affected by the economic, technological and environmental factors. In this section we perform a comparative dynamics exercise by perturbating some parameters of the model and studying how they affect lifetime behavior, health investment, and health deficits.

Consider an unexpected shock of a parameter ω when $D = D_0$. We are interested in assessing what happens on impact, when the health condition is given and the agent can only adjust her behavior, and what happens in the long run, at the steady state. This will provide valuable information to understand how changes in the determinants of longevity are expected to influence the agent's optimal behavior over time, and how this affects her health, her survival probability and, ultimately, lifetime happiness.

To assess changes in the steady state values we implement the comparative dynamics procedure described in Dragone and Vanin (2015). Essentially, the procedure requires applying the implicit function theorem to the system of equations (17), (19) and (20), computed at the steady state. Denote with h_ω^{ss} and D_ω^{ss} the change in health investment and the level of deficits at the steady state when the parameter ω is perturbed. Hence

$$h_\omega^{ss} = -\frac{|J_{h,\omega}|}{|J|}, \qquad D_\omega^{ss} = -\frac{|J_{D,\omega}|}{|J|}, \tag{30}$$

where

$$J_{h,\omega} \equiv \begin{bmatrix} \frac{\partial \dot{h}}{\partial \omega} & \frac{\partial \dot{h}}{\partial k} & \frac{\partial \dot{h}}{\partial D} \\ \frac{\partial \dot{k}}{\partial \omega} & \frac{\partial \dot{k}}{\partial k} & \frac{\partial \dot{k}}{\partial D} \\ \frac{\partial \dot{D}}{\partial \omega} & \frac{\partial \dot{D}}{\partial k} & \frac{\partial \dot{D}}{\partial D} \end{bmatrix}, \qquad J_{D,\omega} \equiv \begin{bmatrix} \frac{\partial \dot{h}}{\partial h} & \frac{\partial \dot{h}}{\partial k} & \frac{\partial \dot{h}}{\partial \omega} \\ \frac{\partial \dot{k}}{\partial h} & \frac{\partial \dot{k}}{\partial k} & \frac{\partial \dot{k}}{\partial \omega} \\ \frac{\partial \dot{D}}{\partial h} & \frac{\partial \dot{D}}{\partial k} & \frac{\partial \dot{D}}{\partial \omega} \end{bmatrix}. \tag{31}$$

To assess the impulse response, i.e. how the optimal path of health investment $\hat{h}$ leading to the steady state changes when the parameter ω changes, we proceed in three steps.[6] First, recall that the path of both optimal health investment and optimal consumption converging to the steady state is in principle a function of the two state variables D and k. Due to the Frisch compensation, however, $\hat{h}$ depends on the state variable variable D only, as it can be appreciated from inspection of (17) and (18). Using Taylor's rule to approximate the policy function in the neighborhood of the steady state, one can write

$$\hat{h}(D) = h^{ss} + (D - D^{ss})\,\xi \tag{32}$$

where ξ, defined in (26), is the slope (in the (D, h) space) of the eigenvector computed at the steady state and associated with the negative eigenvalue of the Jacobian matrix (Dragone and Vanin, 2015). From (26) and (31) follows that

$$\frac{\partial \hat{h}(D^{ss})}{\partial D} = \xi > 0. \tag{33}$$

Second, using the time-elimination method (Barro, Sala-i-Martin, 1995), the slope of an optimal trajectory in the phase diagram can be computed from the optimal dynamics of h and D,

$$\frac{\dot{h}}{\dot{D}} = \frac{\frac{dh}{dt}}{\frac{dD}{dt}} = \frac{dh}{dD}. \tag{34}$$

Graphically, this method allows studying the slope of the vectors represented in the phase diagram. Hence, studying how this slope changes when perturbing parameter ω, i.e. $\frac{\partial}{\partial \omega}\left(\frac{dh}{dD}\right)$, provides qualitative information on how the slope of the optimal path changes when ω changes. The result will depend on which portion of the phase diagram is considered. Since we are interested in the optimal path leading to the steady state, we will

[6] Throughout the paper we maintain the assumption that the policy function is differentiable with respect to the parameter of interest. This assumption turns out to be satisfied as there are no jumps in the optimal path to the steady state.

restrict our attention to the portion of the phase diagram that contains the policy function $\hat{h} = \hat{h}(D)$.

Third, the policy function $\hat{h} = \hat{h}(D)$ must satisfy the following expression,

$$h^{ss} = h^0 + \int_{D_0}^{D^{ss}} \frac{\mathrm{d}\hat{h}}{\mathrm{d}D} dD, \tag{35}$$

where h^0 is the optimal health investment when $D = D_0$ and $\mathrm{d}\hat{h}(D)/\mathrm{d}D$ is the slope of the policy function for each D along the optimal path starting at D_0 and ending in D^{ss}. Denote with h^0_ω the response on impact of the optimal health investment when parameter ω unexpectedly and permanently changes, and take the derivative of (35) with respect to the generic parameter ω. Applying Leibniz's rule yields

$$h^{ss}_\omega = h^0_\omega + D^{ss}_\omega \frac{\mathrm{d}\hat{h}}{\mathrm{d}D}_{|D=D^{ss}} + \int_{D_0}^{D^{ss}} \frac{\partial}{\partial \omega}\left(\frac{\mathrm{d}\hat{h}}{\mathrm{d}D}\right) \mathrm{d}D. \tag{36}$$

Replacing $\mathrm{d}\hat{h}/\mathrm{d}D = \xi$ in the second term of (36) and rearranging yields the following Proposition,

Proposition 1 (Impulse response) *The response on impact of health investment after an unexpected permanent change in parameter ω is*

$$h^0_\omega = h^{ss}_\omega - \xi D^{ss}_\omega - \int_{D_0}^{D^{ss}} \frac{\partial}{\partial \omega}\left(\frac{d\hat{h}}{dD}\right) dD. \tag{37}$$

In the proceeding we will use equation (37) to study how changes in the determinants of longevity will affect individual behavior on impact, when health deficits are D_0. Specifically, we will consider changes in the price p of health behavior and in the productivity A of health investment in slowing down the process of deficit accumulation. This comparative dynamics exercise essentially requires knowing how the steady state changes and how the slope of the policy function changes along the optimal path. Suppose the steady state does not change when perturbing ω, i.e. $h^{ss}_\omega = D^{ss}_\omega = 0$, and that $\partial\left(\frac{\mathrm{d}\hat{h}}{\mathrm{d}D}\right)/\partial\omega$ is always positive (resp. negative), then the sign of h^0_ω must be negative (resp. positive). If, instead, the steady state changes, we will assess the sign of the right hand side taking into account both the change in the steady state values, $h^{ss}_\omega - \xi D^{ss}_\omega$, and the change in slope of the policy function over the range (D^0, D^{ss}), i.e. $\int_{D^0}^{D^{ss}} \frac{\partial}{\partial\omega}\left(\frac{\mathrm{d}\hat{h}}{\mathrm{d}D}\right)\mathrm{d}D$.

4.2 Increasing Health Care Costs

In the following we consider the case in which health investment becomes more expensive.[7] All statements will be reversed in sign in case health investment becomes cheaper.

Proposition 2 *Consider an agent who invests in health in order to reach the steady state and let the initial level of deficits be* $D_0 \in (D^1, D^2]$*. Right after an unexpected, permanent increase in the price of health investment (e.g. medicines), the individual will decrease her health investment. Health investment will subsequently increase until, in the long run, the agent will reach a steady state level where health deficits and health investment are higher than they would be in absence of the positive price shock.*

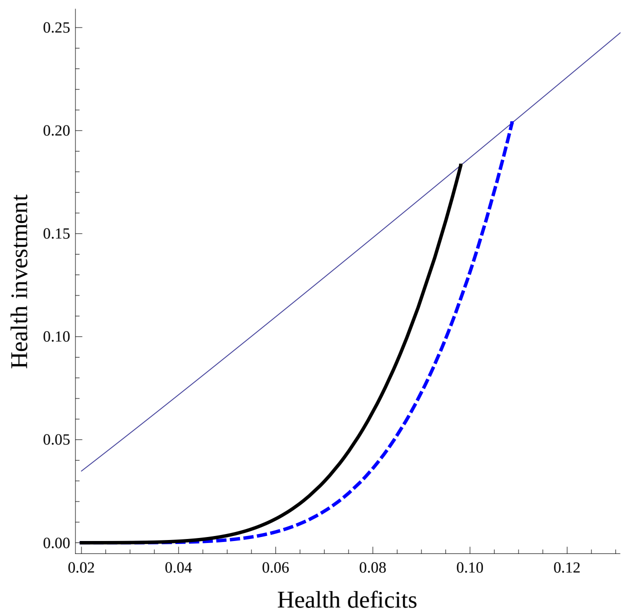

Figure 4: More expensive health investment (from $p = 1$ to $p = 1.1$) occurring when $D_0 = 0.15$. Solid lines: initial paths; Dashed lines: new paths after the price shock. The thin line represents $\dot{D} = 0$.

Proof. In the long run health investment and deficits change as follows

$$\frac{\partial h^{ss}}{\partial p} = \frac{rA\mu^2\gamma c^{ss}\left(h^{ss}\right)^{\gamma}}{p^2\left(1-\sigma\right)\left(1-\gamma\right)|J|}\frac{S_D\left(D^{ss}\right)}{S\left(D^{ss}\right)} > 0 \quad (38)$$

$$\frac{\partial D^{ss}}{\partial p} = \gamma A\left(h^{ss}\right)^{\gamma-1}\frac{\partial h^{ss}}{\partial p} > 0. \quad (39)$$

[7]For general statements on long-run price effects in intertemporal consumer problems, see Dragone and Vanin (2015).

Using the above expressions and considering the first two terms of equation (37) yields

$$h_p^{ss} - \xi D_p^{ss} = \frac{(\mu - r)\, r\zeta}{p\,(1-\gamma)\,|J|} h^{ss} < 0. \tag{40}$$

The integrand of the third term of equation (37) is

$$\frac{\partial}{\partial p}\left(\frac{\mathrm{d}\hat{h}}{\mathrm{d}D}\right) = -\frac{A\gamma c h^{\gamma}}{p^2\,(1-\sigma)\,(1-\gamma)} \frac{S_D\,(D)}{S\,(D)\,\dot{D}} > 0. \tag{41}$$

Since $\dot{D} > 0$, the above expression is positive. Hence, using equation (37), the sign of h_p^0 is negative. ■

As an application of the results from Proposition 2 to the numerically specified model, we consider a 10% increase in price (from $p = 1$ to $p = 1.1$). Results are depicted in Figures 4 and 5. On impact, health investment drops as a response to the higher price of medical care while consumption is not affected (since it depends only on the current level of health deficits). As a consequence of the initial period of reduced health investment, health deficits accumulate at a faster rate. Over time, this will also drive health investment to increase, but the effects of the initial lower investment are persistent and cannot be easily compensated. The agent will still invest in health and aim at a steady state of constant health-deficits and health investment, but such steady state features a higher level of deficits and, correspondingly, higher health investments. Since consumption is negatively related to health deficits, this path is accompanied by a reduction in consumption and utility.

Accordingly, and as it would be expected, the survival probability, consumption, and utility are lower after the increase in health care costs while health investment and health deficits are higher, see Figure 5. Overall, the value of life at time t decreases with losses getting larger with increasing time horizon under consideration.

4.3 Improvement of Medical Technology

We next discuss the comparative dynamics for an improvement of medical technology. Formally, this can be investigated by considering the effect of an increase in A, which measures the impact of health investment on deficit reduction. As one would expect, in the long run the level of deficits and the corresponding health investment to maintain health at a stationary level is lower than before the technological innovation, as stated below:

Proposition 3 *Consider an agent who invests in health in order to reach the steady state. Suppose $\gamma > \bar{\gamma}\,(D)$ for all (h, c) belonging to the optimal path starting at D_0 and leading*

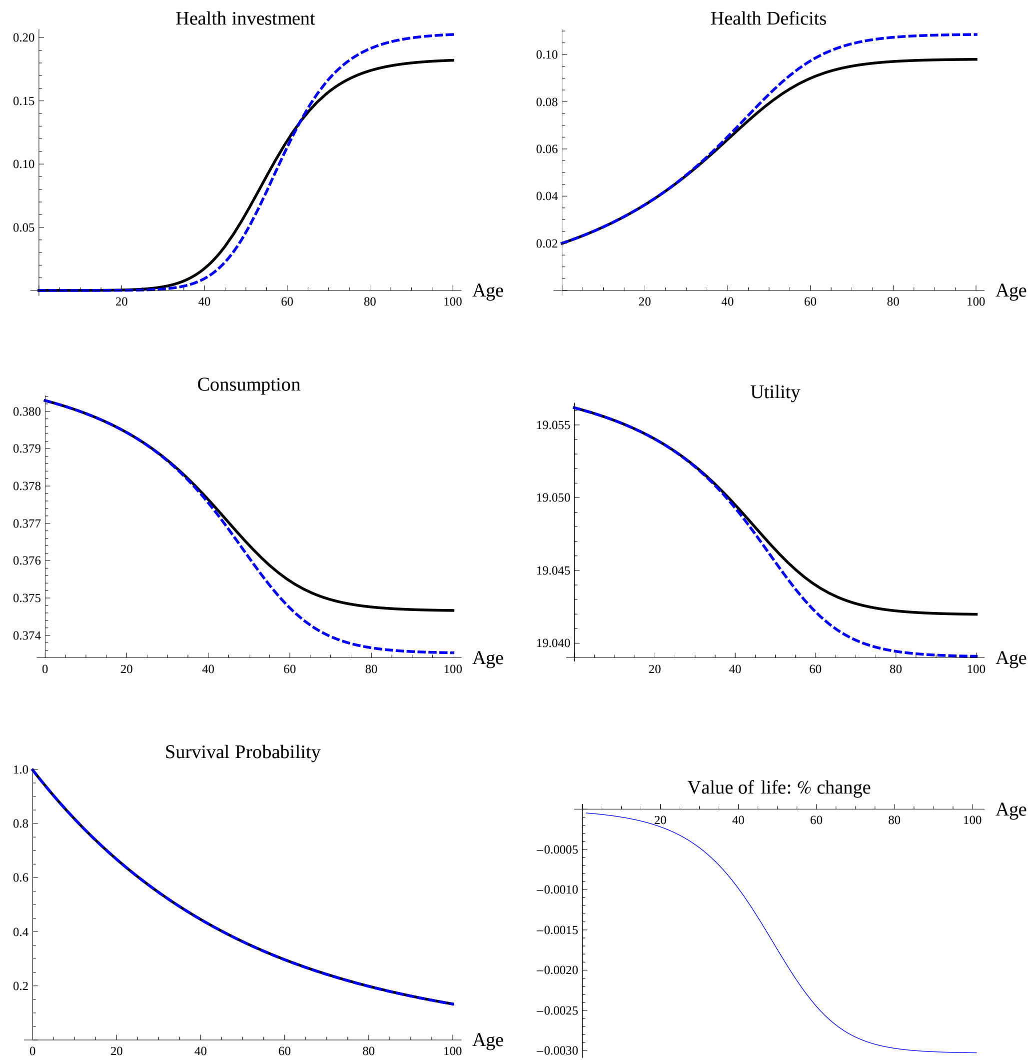

Figure 5: More expensive health investment (from $p = 1$ to $p = 1.1$) occurring when $D_0 = 0.02$. Solid lines: initial paths; Dashed lines: new paths after the price shock. The thin line in the last panel is the percentage loss in the value of life.

to the steady state, with $\bar{\gamma}(D) \equiv 1 - \frac{r-\mu}{\mu}\left(\frac{(D-a)\Phi(D)}{S_D(D)U(c(D))}+1\right)^{-1}$. *Right after an unexpected, permanent improvement in medical technology, the agent will increase her health investment. Health investment will subsequently adjust until, in the long run, the agent will reach a steady state level where health deficits and health investment are lower than they would be in absence of the technological innovation.*

Proof. In the long run, health investment and deficits change as follows

$$D_A^{ss} = \frac{\mu(q+\rho)(q+\rho-\mu)}{(1-\gamma)|J|}(h^{ss})^{\gamma} < 0 \tag{42}$$

$$h_A^{ss} = \frac{h^{ss}}{A\gamma}\left(\frac{D_A^{ss}}{(h^{ss})^{\gamma}}-1\right) < 0. \tag{43}$$

Exploiting the fact that at the steady state

$$|J| = r\zeta(r-\zeta), \tag{44}$$

one can write

$$h_A^{ss} - \xi D_A^{ss} = \frac{h^{ss}}{\gamma A}\left[\frac{(r-\mu)}{(1-\gamma)}\frac{r\zeta}{|J|}-1\right] \tag{45}$$

$$= \frac{h^{ss}}{\gamma A}\left[\frac{(r-\mu)}{(1-\gamma)(r-\zeta)}-1\right]. \tag{46}$$

Let

$$\bar{\gamma}(D) \equiv 1 - \frac{r-\mu}{\mu}\left(\frac{(D-a)\Phi(D)}{S_D(D)U(c(D))}+1\right)^{-1} < 1. \tag{47}$$

Then, in the long run, the following holds:

$$h_A^{ss} - \xi D_A^{ss} > 0 \iff \gamma > \frac{\mu-\zeta}{r-\zeta} \iff \gamma > \bar{\gamma}(D^{ss}). \tag{48}$$

To assess the value of the integrand in equation (37), assume that $\gamma > \bar{\gamma}(D)$ holds not only in steady state, but for all $D \in [D_0, D^{ss}]$, and compute

$$-\frac{\partial}{\partial A}\left(\frac{dh}{dD}\right) = \frac{1}{A\dot{D}}\left[\frac{(r-\mu)h}{1-\gamma} - \mu(D-a)\frac{\dot{h}}{\dot{D}}\right]. \tag{49}$$

The first term in square brackets is positive and, when focusing on regions where $\dot{D}/\dot{h} > 0$, the second term is negative. Replacing $\dot{h}$ and $\dot{D}$ from (17) and (19), and considering that optimal consumption is a function of D, such that $c = c(D)$, yields

$$-\frac{\partial}{\partial A}\left(\frac{dh}{dD}\right) > 0 \iff h < \bar{h}(D) \equiv -\frac{\mu}{r-\mu}\frac{\gamma(D-a)}{p}\frac{S_D(D)}{S(D)}\frac{U(c(D))}{U_c(c(D))}. \tag{50}$$

If the optimal health investment $\hat{h}(D)$ is lower than the threshold value $\bar{h}(D)$ for any $D \in [D_0, D^{ss}]$, then we conclude that the impulse response is positive. In the following we will show that this is indeed the case even if we do not explicitly know $\hat{h}(D)$. First, observe that, when (48) holds, (i) $\frac{\partial \bar{h}(D)}{\partial D} > 0$, (ii) $\bar{h}(D^{ss}) = h^{ss}$, and (iii) $\frac{\partial \bar{h}(D^{ss})}{\partial D} < \xi$ when (48) holds (recall that ξ is the slope of the policy function in the neighborhood of the steady state). This means that the locus $h = \bar{h}(D)$ is increasing in D and "hits" the steady state with a flatter slope than the slope of the policy function $\hat{h}(D)$. Hence $\bar{h}(D) > \hat{h}(D)$, at least in some neighborhood on the left of the steady state. If we assume that $\gamma > \bar{\gamma}(D)$ holds for all $D \in [D_0, D^{ss}]$, we can extend this conclusion to the whole policy function by showing that the loci $h = \bar{h}(D)$ and $h = \hat{h}(D)$ do not intersect for $D \in [D_0, D^{ss}]$. Suppose, by contradiction, this is not the case; then there must be a h such that the locus $h = \bar{h}(D)$ (which stays above the policy function when D is close to the steady state) crosses the policy function with a larger slope, i.e. $\frac{\partial \bar{h}(D)}{\partial D} \geq \frac{\partial \hat{h}(D)}{\partial D}$, where equality holds if the two curves were tangent. Recall that, for given D and $\hat{h}(D)$, the slope of the policy function can be described by $\dot{D}/\dot{h} = dh/dD$. Hence crossing would require $\frac{\partial \bar{h}(D)}{\partial D} \geq \frac{dh}{dD}$ if it were true that $\hat{h}(D) = \bar{h}(D)$. Since

$$\frac{\partial \bar{h}(D)}{\partial D} - \frac{dh}{dD} = \gamma \frac{[r + \mu(\gamma - 2)] S_D(D) U(c(D)) - \mu(1-\gamma)(D-a)\Phi(D)}{p(r-\mu)(1-\gamma) S(D) U_c(c(D))}, \tag{51}$$

then

$$\frac{\partial \bar{h}(D)}{\partial D} \geq \frac{dh}{dD} \iff \gamma \leq \bar{\gamma}(D), \tag{52}$$

which contradicts the assumption that $\gamma > \bar{\gamma}(D)$ holds for all $D \in [D_0, D^{ss})$. Hence the two curves never intersect and health investment increases on impact. Using equation (37), we therefore conclude that the sign of h_A^0 is positive. ■

For an intuition of the adjustment dynamics it may be helpful to recall that health deficits are a (slow-moving) state variable. At the point of time when the individual experiences the positive shock of health technology, the state of health is given and the individual responds to the improved efficiency of health spending by increasing health expenditure in the short-run. This slows down the deficit accumulation which, ultimately, will reach a long run level that is lower than it would be without the technological improvement. Figure 6 shows the corresponding adjustment paths for the case in which the technological shock occurs when health deficits are still far from the stationary level.[8] As consumption will be

[8] In case the level of health deficits were close to the former steady state, the agent will still increase her health investment on impact, but will subsequently decrease it over time until the new steady state is reached.

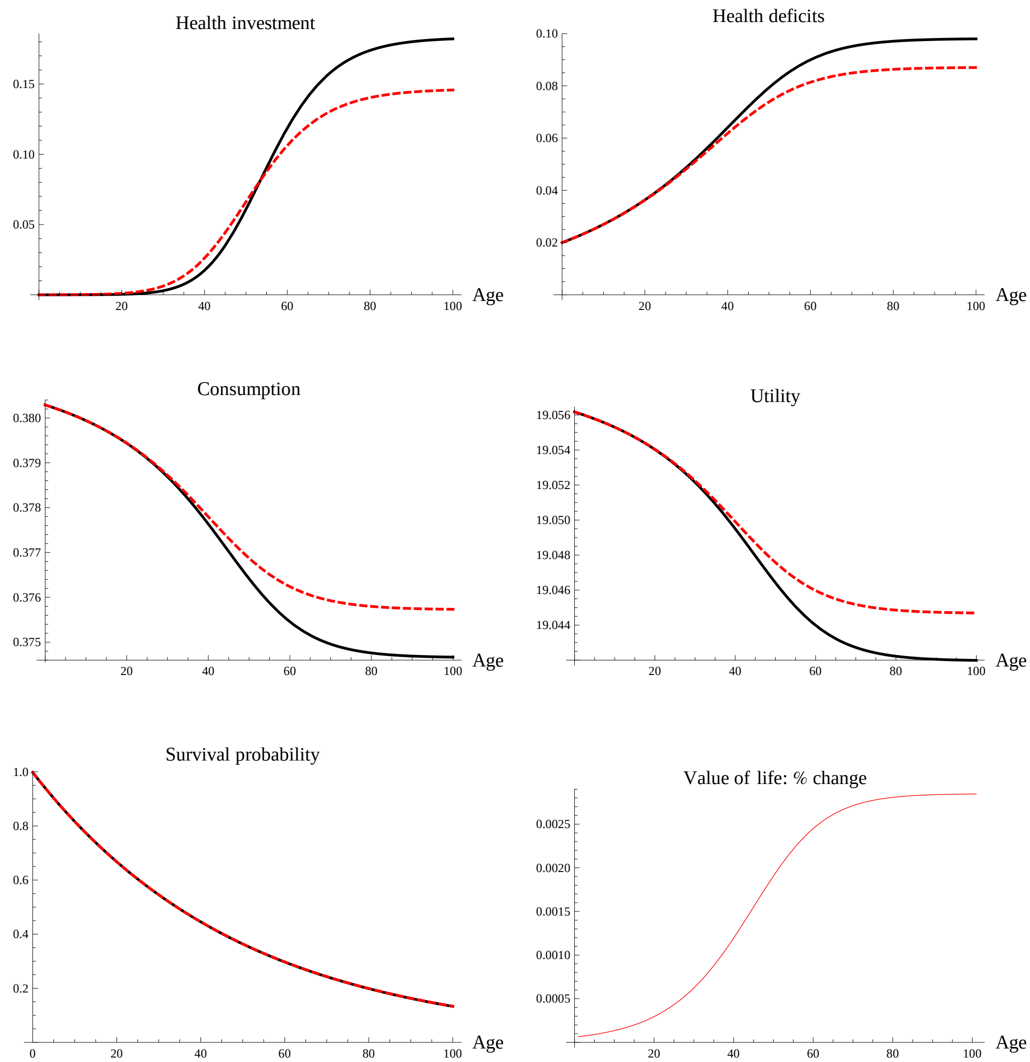

Figure 6: Better health technology (from $A = 0.5$ to $A = 0.55$) occurring when $D_0 = 0.02$. Solid lines: initial paths; dashed lines: new paths after the positive technological shock. The thin line in the last panel is the percentage gain in the value of life.

higher and deficits lower at each point in time after the technology improvement, the value of life increases.

5 Conclusion

In this paper we discussed optimal life cycle health expenditure in a model where individuals conceive an infinite life as a meaningful goal. While humankind had always longed for transcending death, for most time in history these aspirations were confined to religious beliefs and the afterlife. Now, in the 21st century, income and medical progress has advanced far enough that natural scientists as well as philosophers discuss for the first time seriously the possibilities and consequences of an infinite life on earth (Harari, 2016). Naturally, it are wealthy entrepreneurs who have the least problems in imagining and aspiring (infinite) life extension, see Friend (2017). Here we integrated into a simple life cycle model a gerontologically founded law of motion of human aging and showed that a reachable steady state of infinite life requires that the rate of health deficit accumulation falls short of the interest rate and that the marginal return in terms of health deficit repair does not decline too strongly with rising health expenditure. The simple model allows to assess the steady state's characteristics and comparative dynamics analytically. We used this feature to discuss impulse responses to advances in medical technology and increasing health care costs.

Adjustment dynamics towards the steady state are characterized as the continuous repair of health deficits resulting from "natural aging". This view is in contrast to the conventional model of health capital accumulation (Grossman, 1972) but in line with the notion of aging in modern gerontology. In particular optimistic scholars such as de Grey (2013) conceptualize medical gerontology as the endeavor to repair bodily deficits, which, once it succeeds sufficiently well, will end aging. Here we have proposed a simple model that integrates these ideas into an economic life cycle theory for the future.

6 Appendix

To transform the expected intertemporal utility function into an intertemporal expected utility function, exploit the definition of the expectation operator and the resulting double integral:

$$\begin{aligned}\mathbb{E}_g\left[\int_0^T e^{-\rho t}U(c)\,\mathrm{d}t\right] &= \int_0^\infty g(D,T)\left(\int_0^T e^{-\rho t}U(c)\,\mathrm{d}t\right)\mathrm{d}T \\ &= \int_0^\infty e^{-\rho t}U(c)\left(\int_t^\infty g(D,T)\,\mathrm{d}T\right)\mathrm{d}t \\ &= \int_0^\infty e^{-\rho t}\Omega(D,t)\,U(c)\,\mathrm{d}t \end{aligned} \tag{53}$$

Arrow's concavity condition requires

$$\Phi(D) = U(c(D))\,S_{DD}(D) - \frac{S_D^2(D)}{S(D)}\frac{U_c^2(c(D))}{U_{cc}(c(D))} < 0$$

which implies, as a necessary condition, $S_{DD}(D) < 0$.

To assess whether a steady state is reachable, consider the 3×3 Jacobian matrix J constructed on $\dot{h}$, $\dot{k}$ and $\dot{D}$ (note that that the dynamics of c is not linearly independent from the dynamics of D):

$$\begin{aligned} J &= \begin{bmatrix} \frac{\partial\dot{h}}{\partial h} & \frac{\partial\dot{h}}{\partial k} & \frac{\partial\dot{h}}{\partial D} \\ \frac{\partial\dot{k}}{\partial h} & \frac{\partial\dot{k}}{\partial k} & \frac{\partial\dot{k}}{\partial D} \\ \frac{\partial\dot{D}}{\partial h} & \frac{\partial\dot{D}}{\partial k} & \frac{\partial\dot{D}}{\partial D} \end{bmatrix} \\ &= \begin{bmatrix} r-\mu & 0 & -\frac{\gamma\mu A}{p(1-\gamma)}\frac{\Phi(D^{ss})(h^{ss})^\gamma}{S(D^{ss})U_c(c^{ss})} \\ -p & r & \frac{S_D(D^{ss})}{S(D^{ss})}\frac{U_c(c^{ss})}{U_{cc}(c^{ss})} \\ p(r-\mu)\frac{S(D^{ss})}{S_D(D^{ss})}\frac{U_c(c^{ss})}{U(c^{ss})} & 0 & \mu \end{bmatrix} \end{aligned} \tag{54}$$

At the steady state, the associated determinant and trace are:

$$|J| = \mu r(\mu - r)\left(\frac{\gamma A\Phi(D^{ss})}{1-\gamma}\frac{(h^{ss})^\gamma}{S_D(D^{ss})\,U(c^{ss})} - 1\right) \tag{55}$$

$$Tr|J| = 2r \tag{56}$$

The three eigenvalues associated with the Jacobian at the steady state are

$$\frac{1}{2}\left(r \pm \sqrt{(2\mu - r)^2 - 4\frac{\gamma\mu(\mu-r)A\Phi(D^{ss})(h^{ss})^\gamma}{(1-\gamma)S_D(D^{ss})U(c^{ss})}}\right) \text{ and } r. \tag{57}$$

Hence the steady state has saddle point stability (one negative and two positive eigenvalues, hence $|J| < 0$) or it is unstable (three eigenvalues with positive real part, hence $|J| > 0$). Given that $r > \mu$, the former case occurs if $\frac{\gamma A\Phi(D^{ss})}{1-\gamma}\frac{(h^{ss})^\gamma}{S_D(D^{ss})U(c^{ss})} > 1$, and the latter case otherwise.

6.1 Specific functional forms

Using a CES utility function

$$U(c) = \frac{c(t)^{1-\sigma}}{1-\sigma} + b \text{ for } \sigma \neq 1, \tag{58}$$

and a logistic survival function

$$S(D) = \frac{1+\alpha}{1+\alpha e^{\phi D}}, \tag{59}$$

the optimal agent's choices when $\eta(t) = \eta_0$ are

$$h^*(t) = \left(-\frac{\mu\gamma A}{p}\frac{\lambda(t)}{\eta_0}\right)^{\frac{1}{1-\gamma}} \tag{60}$$

$$c^*(t) = \left(\frac{S(D(t))}{\eta_0}\right)^{\frac{1}{\sigma}}. \tag{61}$$

When $c = c^*$ and $h = h^*$ the optimal dynamics are

$$\dot{h} = \frac{h^*}{1-\gamma}\left(r - \mu + \frac{A\mu\gamma c^*(h^*)^{\gamma-1}}{p(1-\sigma)}\frac{S_D(D)}{S(D)}\right) \tag{62}$$

$$\dot{c} = \frac{c^*}{\sigma}\frac{S_D(D)}{S(D)}\dot{D} \tag{63}$$

$$\dot{D} = \mu\left(D - a - A(h^*)^{\gamma}\right) \tag{64}$$

$$\dot{k} = rk + M - ph^* - c^*. \tag{65}$$

The steady state(s) satisfies the following conditions:

$$h^{ss} = \left[\frac{p(1-\sigma)}{A\mu\gamma c^{ss}}\frac{S(D^{ss})}{S_D(D^{ss})}(\mu - r)\right]^{\frac{1}{\gamma-1}} \tag{66}$$

$$c^{ss} = \left(\frac{S(D^{ss})}{\eta_0}\right)^{\frac{1}{\sigma}} \tag{67}$$

$$D^{ss} = a + A(h^{ss})^{\gamma} \tag{68}$$

$$k^{ss} = \frac{1}{q+\rho}\left(ph^{ss} + c^{ss} - Y\right) \tag{69}$$

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